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STABILITY AND REPLICA SYMMETRY IN THE ISING SPIN GLASS : A TOY MODEL

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RESUME

Dans la recherche de solutions symétriques de repliques pour les verres de spin d'Ising (dans l'approximation des arbres) nous étudions un mini modèle dont la distribution des liens est caractérisée par le fait qu'elle possède deux cumulants non nuls (au lieu d'un seul pour la distribution gaussienne).

ABSTRACT

Searching for possible replica symmetric solutions in an Ising spin glass (in the tree approximation) we investigate a toy model whose bond distribution has two non vanishing cumulants (instead of one only as in a gaussian distribution).

In the recent period a conflict has been sharpening up in the approach to spin glasses. On the one hand experiments and computer simulations have given credence to the existence of a well defined transition in dimension three (for references see the review by BINDER and YOUNG^[1]). On the other hand the theory has followed a painful course. Centered mostly on the EDWARDS ANDERSON^[2] model (with a gaussian bond distribution) it delivers, as a starting point the PARISI^[3] ansatz, the symmetric solution for its mean field being unstable due to the "replicon" negative mass. Its loop expansion around the PARISI solution is complicated^[4]. Besides, after a proper regularization tadpole diagrams lead to p^{-6} infrared divergences^[5]. Although it cannot be excluded that via sub-series resummation, one could restore weaker infrared divergencies, one may already wonder how to reconcile the above quoted evidence and that type of theoretical approach. Recently, BOVIER and FROHLICH^[6] have given strong arguments to the

effect that there should be only two states (in zero field) in dimension three thereby excluding a PARISI type of replica symmetry breaking in that dimension. This is also an underlying assumption in the droplet model of FISHER and HUSE^[7] and in the block spin renormalisation group of BRAY and MOORE^[8].

The problem is then the following. How can we build a theory by the standard 6-d loop expansion (if 6 is the upper critical dimension) and reach a result valid in $d = 3$ if the expansion is performed around a "mean field" (tree approximation) that does not allow a replica symmetric solution to start with? Could not one change the starting point by changing the bond distribution (or the dilution) in such a way that the starting point be already symmetric stable?

In this short note we consider a toy model where the bond distribution has two cumulants only (one more than the gaussian distribution). We look at the stability of the replica symmetric solution and show that provided the second cumulant is negative enough (as in a double bumped distribution) we indeed have a range of possible stability.

A ONE PARAMETER TOY MODEL

We are interested in the (unrealistic) one parameter model described by

$$\bar{Z}^n = \text{Tr} \prod_{(ij)} \exp \left\{ \frac{\lambda^2}{2} \left(\sum_{\alpha} \sigma_i^{\alpha} \sigma_j^{\alpha} \right)^2 + \rho \frac{\lambda^4}{2} \left(\sum_{\alpha} \sigma_i^{\alpha} \sigma_j^{\alpha} \right)^4 \right\} \quad (1)$$

where the product (ij) is over nearest neighbour pairs, $\lambda \equiv \beta J = \beta J_0 / \sqrt{z}$ and z is the number of nearest neighbours. In the $\pm J$ model expanded in cumulants $\rho = -1/6$, but we keep here ρ as a free parameter. Rearranging the sum over replicas we can write

$$\begin{aligned} \bar{Z}^n \simeq \text{Tr} \prod_{(ij)} \exp \left\{ \lambda^2 (1 - 8\rho\lambda^2) \sum_{(\alpha\beta)} \left(\sigma_i^{\alpha} \sigma_j^{\beta} \right) \left(\sigma_j^{\alpha} \sigma_i^{\beta} \right) \right. \\ \left. + 12\rho \lambda^4 \sum_{(\alpha\beta\gamma\delta)} \left(\sigma_i^{\alpha} \sigma_j^{\beta} \sigma_i^{\gamma} \sigma_j^{\delta} \right) \left(\sigma_j^{\alpha} \sigma_i^{\beta} \sigma_j^{\gamma} \sigma_i^{\delta} \right) \right\} \quad (2) \end{aligned}$$

where the summation is over distinct pairs and quartets of replicas. As is standard we rewrite (2) now by introducing the fields $y_j^{\alpha\beta}$ and $y_j^{\alpha\beta\gamma\delta}$ as

$$\begin{aligned} \bar{Z}^n \simeq \int \prod_{\substack{j \\ (\alpha\beta) \\ (\alpha\beta\gamma\delta)}} dy_j^{\alpha\beta} dy_j^{\alpha\beta\gamma\delta} \\ \exp \left\{ -\frac{1}{2} \sum_{j1} \left\{ \sum_{(\alpha\beta)} y_j^{\alpha\beta} (K^{-1})_{j1} y_1^{\alpha\beta} + \sum_{(\alpha\beta\gamma\delta)} y_j^{\alpha\beta\gamma\delta} (K^{-1})_{j1} y_1^{\alpha\beta\gamma\delta} \right\} \right. \\ \left. + \sum_j \ln \text{tr}_{\sigma} \exp \left\{ L(y_j; \sigma) \right\} \right\} \quad (3) \end{aligned}$$

$$L(y_j; \sigma) = f \sum_{(\alpha\beta)} y_j^{\alpha\beta} \sigma^{\alpha} \sigma^{\beta} + g \sum_{(\alpha\beta\gamma\delta)} y_j^{\alpha\beta\gamma\delta} \sigma^{\alpha} \sigma^{\beta} \sigma^{\gamma} \sigma^{\delta} \quad (4)$$

Here $K_{ij} = 1$ for (ij) nearest neighbours and zero otherwise, and

$$\begin{aligned} f^2 &= \lambda^2 (1 - 8\rho\lambda^2) \\ g^2 &= 12\rho\lambda^4 \end{aligned} \quad (5)$$

We now have a field theory for the fields $y_j^{\alpha\beta}$, $y_j^{\alpha\beta\gamma\delta}$ the couplings of which can be obtained by properly expanding the $\ln$ trace term of (3) in powers of y_j . This is the model we propose to study now.

THE TREE APPROXIMATION

The saddle point in the field integrals are given by

$$\left(y_j^{\alpha\beta} \right)_c = f \sum_1 K_{j1} \left\langle \sigma^{\alpha} \sigma^{\beta} \right\rangle_c \quad (6)$$

$$\left(y_j^{\alpha\beta\gamma\delta} \right)_c = g \sum_1 K_{j1} \left\langle \sigma^{\alpha} \sigma^{\beta} \sigma^{\gamma} \sigma^{\delta} \right\rangle_c$$

Let us introduce the (site-independent) order parameter $q^{\alpha\beta}$, by

$$\left(y_j^{\alpha\beta} \right)_c = z f q^{\alpha\beta} = z f \left\langle \sigma^{\alpha} \sigma^{\beta} \right\rangle_c \quad (7)$$

$$\left(y_j^{\alpha\beta\gamma\delta} \right)_c = z g q^{\alpha\beta\gamma\delta} = z g \left\langle \sigma^{\alpha} \sigma^{\beta} \sigma^{\gamma} \sigma^{\delta} \right\rangle_c$$

where

$$\langle A \rangle_c = \text{tr}_{\sigma} \exp L(y_c; \sigma) A / \text{tr}_{\sigma} \exp L(y_c; \sigma) \quad (8)$$

If we are looking for symmetric solutions only, then

$$\begin{aligned} q^{\alpha\beta} &= q \\ q^{\alpha\beta\gamma\delta} &= r \end{aligned} \quad (9)$$

and the (stationary) tree Lagrangian becomes

$$\begin{aligned} -\beta f n = + \frac{n}{2} \left\{ \frac{F}{2} q^2 + \frac{G}{4} r^2 \right\} \\ + \ln \text{tr}_{\sigma} \exp \left\{ F q \sum_{(\alpha\beta)} \sigma^{\alpha} \sigma^{\beta} + G r \sum_{(\alpha\beta\gamma\delta)} \sigma^{\alpha} \sigma^{\beta} \sigma^{\gamma} \sigma^{\delta} \right\} \end{aligned} \quad (10)$$

with

$$\begin{aligned} F &\equiv z f^2 \\ G &\equiv z g^2 \end{aligned} \quad (11)$$

The stationary conditions as in (7,8), expanded for small q, r yield the equations of motion

$$(1-F)q = -2(Fq)^2 + 3FGqr - 4(Gr)^2 + \dots \quad (12)$$

$$(1-G)r = +3(Fq)^2 - 16FGqr + 30(Gr)^2 + \dots$$

with the bare masses $1-F$, $1-G$ associated with the fields $y^{\alpha\beta}$, $y^{\alpha\beta\gamma\delta}$. Lowering the temperature down from the paramagnetic region (where $q = r = 0$) the system undergoes a transition at

$$1-F = 0 \quad (\mathcal{C}_2) \quad (13)$$

$$1-G = 0 \quad (\mathcal{C}_4) \quad (14)$$

The curves $\mathcal{C}_2$, $\mathcal{C}_4$ are two parabolas in the plane $x \equiv (\beta J_0)^{-2}$, $y \equiv \rho/z$. They intersect at the origin and at $x_0 = 3/5$, $y_0 = 3/100$. For $y < y_0$ when lowering the temperature $1-F = -\tau$ vanishes first ($\tau < 0$ in the paramagnetic phase) and the equations of motion give

$$q = \frac{\tau}{2} + 0(\tau^2) \quad (15)$$

$$r = \frac{3}{4} \frac{\tau^2}{1-G} + 0(\tau^3)$$

i.e. $r \ll q$ near the transition curve. For $y > y_0$, $1-G$ vanishes first but (12) yields unphysical negative values for q and r hinting perhaps at a first order phase transition. This case is left aside for the time being since the present model is unrealistic and we now turn to the stability of the above solution (15).

FLUCTUATIONS AND STABILITY

We follow the same way as used in the early studies of the gaussian SHERRINGTON KIRKPATRICK^[9] model by PYTTE and RUDNICK^[10], BRAY and MOORE^[11], and de ALMEIDA and THOULESS^[12].

Letting

$$y_j^{\alpha\beta} = z f q + z f \phi_j^{\alpha\beta} \quad (16)$$

$$y_j^{\alpha\beta\gamma\delta} = z g r + z g \phi_j^{\alpha\beta\gamma\delta}$$

We now expand (3,4) in powers of ϕ_j , and keep to the quadratic terms to discuss the eigenmasses and propagators. For each wave vector p , we obtain

$$\delta \mathcal{L}(p) = \delta \mathcal{L}_0(p) + \delta \mathcal{L}_1(p) \quad (17)$$

$$\mathcal{L}(p) = -\frac{1}{2} F \left(1 + \frac{p^2}{z} \right) \sum_{(\alpha\beta)} \left(\phi_p^{\alpha\beta} \right)^2 - \frac{1}{2} G \left(1 + \frac{p^2}{z} \right) \sum_{(\alpha\beta\gamma\delta)} \left(\phi_p^{\alpha\beta\gamma\delta} \right)^2 \quad (18)$$

$$\delta \mathcal{L}_1(p) = + \frac{1}{2} F^2 \sum_{(\alpha\beta)} \phi_p^{\alpha\beta} \left\{ \langle \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \rangle - q^2 \right\} \phi_p^{\gamma\delta} \quad (19)$$

$$+ FG \sum_{(\alpha\beta)} \phi_p^{\alpha\beta} \left\{ \langle \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \sigma^\epsilon \sigma^\mu \rangle - qr \right\} \phi_p^{\gamma\delta\epsilon\mu} \quad (19)$$

$$+ \frac{1}{2} G^2 \sum_{\substack{(\alpha\beta\gamma\delta) \\ (\epsilon\mu\nu\rho)}} \phi_p^{\alpha\beta\gamma\delta} \left\{ \langle \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \sigma^\epsilon \sigma^\mu \sigma^\nu \sigma^\rho \rangle - r^2 \right\} \phi_p^{\epsilon\mu\nu\rho}$$

In the standard approach to critical field theory, the fields $\phi_p^{\alpha\beta\gamma\delta}$ whose mass $1-G$ is non vanishing are usually considered as non fluctuating ($\phi = 0$) their only effect being via the equation of motion. Thus here one would drop the last two lines of (19) and the effect of the $y^{\alpha\beta\gamma\delta}$ field would appear in the changed value of $\langle \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \rangle = 3q^2 + 3Gq^2/(1-G) + 0(q^3)$. However de ALMEIDA and THOULESS have shown that the dangerous fluctuation mode (replicon) is of order τ^2 and the subdominant effect of the non fluctuating fields is in fact precisely of that order thereby affecting e.g. the replicon mass (and the associated stability condition). Working at the order τ^2 ($q \sim \tau$, $r \sim \tau^2$) we are then left with

$$\begin{aligned} \delta \mathcal{L}_1(p) = & + \frac{1}{2} F^2 \sum_{\substack{(\alpha\beta) \\ (\gamma\delta)}} \phi_p^{\alpha\beta} \left\{ \langle \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \rangle - q^2 \right\} \phi_p^{\gamma\delta} \\ & + FGq \sum_{(\alpha\beta\gamma\delta)} \phi_p^{\alpha\beta\gamma\delta} \left[\phi_p^{\alpha\beta} + \phi_p^{\alpha\gamma} + \phi_p^{\alpha\delta} + \phi_p^{\beta\gamma} + \phi_p^{\beta\delta} + \phi_p^{\gamma\delta} \right] \\ & + \frac{1}{2} G^2 \sum_{(\alpha\beta\gamma\delta)} \left(\phi_p^{\alpha\beta\gamma\delta} \right)^2 \end{aligned} \quad (20)$$

Integrating over the $\phi_p^{\alpha\beta\gamma\delta}$ field we obtain

$$\delta \mathcal{L}_1(p) = \frac{1}{2} F^2 \sum_{\substack{(\alpha\beta) \\ (\gamma\delta)}} \phi_p^{\alpha\beta} \left\{ \langle \sigma^\alpha \sigma^\beta \sigma^\gamma \sigma^\delta \rangle - q^2 \right\} \phi_p^{\gamma\delta} \quad (21)$$

$$+ \frac{1}{2} \frac{F^2 G q^2}{1-G+p^2/z} \sum_{(\alpha\beta\gamma\delta)} \left[\phi_p^{\alpha\beta} + \phi_p^{\alpha\gamma} + \phi_p^{\alpha\delta} + \phi_p^{\beta\gamma} + \phi_p^{\beta\delta} + \phi_p^{\gamma\delta} \right]^2$$

The eigenvalues are then easily determined yielding for the replicon

$$\lambda_r = \frac{p^2}{z} \left[1 + \frac{10Gq^2}{(1-G)^2} \right] - \frac{4q^2}{3} \left[\frac{1+2G}{1-G} \right] \quad (22)$$

Note that studying a distinct model (weak dilution, strong bonds) VIANA and BRAY^[13] obtain a effective Lagrangian different from (20) but a replicon eigenvalue equivalent to (22).

According to (22) the symmetric solution is thus a stable one provided $G < -1/2$ i.e. at the transition

$$G < -F/2 \quad (23)$$

and on the $\mathcal{C}_2$ curve

$$y < y_1 = -3/32 \quad (24)$$

$$x > x_1 = +3/2$$

The stability condition (23) is a *general* one valid for any shape of functions F and G (with transition at $F = 1$). Indeed even if we include higher cumulants or fields, in as much as we are looking at the standard transition where $1-F$ vanishes first, these higher cumulants are only contributing terms at least of order τ^3 .

Looking at the $\pm J$ distribution, by stopping at its second cumulant, and using (24), we obtain $z < 16/9$ a not very heartening result in the perspective of 6-d expansion. However the situation should improve when care is taken of all the cumulants, thereby changing the functional dependence of F and G on $\lambda = \beta J_0 / \sqrt{z}$. This will be examined in a separate publication^[11].

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