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Atomic parity violation measurements in the highly forbidden $6S_{1/2} - 7S_{1/2}$ caesium transition.

III. Data acquisition and processing. Results and implications

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Résumé. — Ce papier achève la présentation détaillée de notre expérience de violation de parité sur la transition $6S_{1/2} - 7S_{1/2}$ du Cs. Nous donnons une description détaillée de l'acquisition et du traitement des données. L'accord des résultats de deux mesures indépendantes réalisées sur deux composantes hyperfines $\Delta F = 0$ et $\Delta F = 1$ fournit un recoupement important. Après réanalyse complète des systématiques et de la calibration, la précision est légèrement améliorée et la moyenne pondérée donne $\text{Im } E_1^{\text{PV}}/\beta = -1,52 \pm 0,18$ mV/cm. Les résultats ultérieurs d'un groupe indépendant sont en très bon accord. En utilisant la valeur semi-empirique $\beta = (26,8 \pm 0,8) a_0^3$, on déduit $E_1^{\text{PV}} = (-0,79 \pm 0,10) \times 10^{-11} i |e| a_0$, ce qui à l'aide de calculs atomiques entraîne pour la charge nucléaire faible du césium $Q_w = -68 \pm 9$. Cette valeur est en accord avec le modèle électrofaible standard et conduit pour l'angle d'interaction faible à $\sin^2 \theta_w = 0,21 \pm 0,04$. Nous illustrons la complémentarité entre ces mesures et celles des expériences de hautes énergies.

Abstract. — This paper completes the detailed presentation of our PV experiment on the $6S_{1/2} - 7S_{1/2}$ transition in Cs. A detailed description of the data acquisition and processing is given. The results of two independent measurements made on $\Delta F = 0$ and $\Delta F = 1$ hfs components agree, providing an important cross-check. After a complete reanalysis of systematics and calibration, the precision is slightly improved, leading to the weighted average $\text{Im } E_1^{\text{PV}}/\beta = -1.52 \pm 0.18$ mV/cm. Later results from an independent group agree quite well. With the semi-empirical value $\beta = (26.8 \pm 0.8) a_0^3$, our result yields $E_1^{\text{PV}} = (-0.79 \pm 0.10) \times 10^{-11} i |e| a_0$. Coupled with the atomic calculations, this implies that the weak nuclear charge of Cs is $Q_w = -68 \pm 9$. This value agrees with the standard electroweak theory and leads to a weak interaction angle $\sin^2 \theta_w = 0.21 \pm 0.04$. The complementarity of these measurements with high energy experiments is illustrated.

Introduction.

This paper is the third and last part of a detailed presentation of the measurements of parity violation (PV) in the Cs $6S-7S$ transition performed at ENS in Paris. Part I [1] presented the theoretical analysis and the experimental procedure and apparatus. Part II [2] presented the analysis and control of systematic effects. The present part III describes data acquisition and processing, and analyzes the results and their implications. The numbering of paragraphs, tables and figures continues that of parts I and II.

This part is divided into two independent sections

(§ 4 and 5). In the first section, § 4.1 attempts at a complete description of the data acquisition sequences and of the detection chain. The experimental complexity was unavoidable in order to satisfy two requirements : (1) control in real-time of all systematic effects, whose relevance was demonstrated in part II : (2) collection of maximum information, with both redundancy and cross-checks, as protection against possible omitted systematics. The data processing as performed just after the runs is described in § 4.2. This analysis produced no significant change in the results. In particular no significant correction for systematics had to be applied. Statistical tests and various internal consistency checks are described. The results of our two independent measurements, on the two different hyperfine

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components $\Delta F = 0$ [3] (Exp 1) and $\Delta F = 1$ [4] (Exp 2), are discussed and compared. The detailed reanalysis of the systematics performed in part II, together with a more thorough examination of the calibration procedure after the results were first published [5], allows us to slightly improve the precision initially quoted. Finally, we outline the main experimental tests performed in auxiliary experiments in connection with the PV experiment. These tests provide increased confidence in the results and in the quoted uncertainty. When combined, Exp 1 and Exp 2 yield a final result wherein the statistical uncertainty (11 %) dominates the systematic uncertainty (5 %).

The self-contained second section (§ 5) presents the interpretation of the results. Starting from a phenomenological description of the electron-nucleus weak neutral current interaction and using the atomic theory we extract the weak charge of the caesium nucleus $Q_w = -68 \pm 9$ (the weak charge plays in the weak interaction the same role as the electric charge in the electromagnetic interaction). This value agrees with the prediction -70.0 of the standard electroweak model for the currently accepted value $\sin^2 \theta_w = 0.223$ of its single parameter. In a model-independent interpretation the information about the weak coupling constants contained in Q_w is complementary to that obtained from high energy experiments. Furthermore Q_w is shown to be a sensitive probe of exotic weak interactions suggested by alternative models elaborated with the aim of extending unification to strong and gravitational interactions. Finally, we outline the main conclusions so far extracted from PV experiments in atoms, without any reference to electroweak theories. The incentive for more precise PV experiments in caesium is made clear.

4.1 Description of data acquisition.

We shall review all operations performed first during

data acquisition, then during subsequent data processing.

4.1.1 ELEMENTARY PATTERNS. — The data acquisition sequence consists in repeating in a definite succession four elementary « patterns »: the PV pattern; and three auxiliary patterns in configurations including an additional E or H-field, for calibration or measurements of experimental imperfections. Each pattern consists of $2^2 = 4$ or $2^3 = 8$ integration times (« live times ») of 16 s, separated by « dead times » of 4 s used to reverse 2 or 3 parameters, namely: (i) E_0 -field (η -reversal, with the notations of § 2 in part I); (ii) modulator's $\lambda/4$ voltage (η' -reversal); (iii) additional E or H-field (η'' -reversal) in the auxiliary patterns. Parameter reversals and configuration changes are commanded and checked by the computer program. To avoid accidental correlations with possible linear or periodic drifts, the order (+ - or - +) of each reversal is chosen at random (except the third parameter reversal in the auxiliary patterns). To avoid causal correlations in PV measurements, E_0 is reversed using two relays in series: one is reversed in a fixed + - order and the other at random. In each pattern the hierarchy of the 2 or 3 basic reversals is chosen according to their respective importance: for example in the PV pattern the E_0 -reversal is performed twice more frequently than the $\lambda/4$ -reversal which is in principle redundant with the helicity modulation (cf. § 2.3); in auxiliary patterns with H-field, since the signals of interest are discriminated owing to their odd behaviour under H reversal, the most frequent reversal is assigned to H.

4.1.2 ELECTRONIC DETECTION CHAIN. — The detection chain is shown schematically in figure 16. The contents of the detector's photocurrent and the analog processor have been described (§ 2.9. in Part I). Each lock-in-amplifier (LIA) is set to detect

Fig. 16. — Schematic representation of the detection chain.

a particular modulation induced in the fluorescence signal by a combination of the modulations of the incident polarization and of the fluorescence analyzer. The setting procedure is described below (§ 4.1.4). Each LIA output is integrated in a different channel of the digital integrator. In addition the outputs of the LIA detecting the PV amplitudes ($\mathcal{F}_{e+f}$ or $\mathcal{F}_{e-f}$) are added. The sum constitutes the numerator of an analog ratiometer whose denominator is the output of the LIA detecting the normalization amplitude ($\mathcal{F}_{e+b}$). The output $\mathcal{F}_{ef} = (\mathcal{F}_{e+f} + \mathcal{F}_{e-f}) / \mathcal{F}_{e+b}$ of this analog channel, cross-checks the digital evaluation of this important ratio, from which the PV signal is extracted⁽¹⁾.

Finally two more quantities are integrated : 1) $\mathcal{F}_{e+b}/i_1$, i.e. the unpolarized fluorescence yield. This quantity, independent of the laser intensity i_1 , is in practice essentially sensitive to Cs pressure drifts. 2) The output of the gaussmeter which probes the most harmful stray H-component.

The digitized amplitudes are transmitted to the calculator during the next dead time. After each elementary pattern all interesting quantities Q_i relative to this pattern i are computed and stored on tape. The new mean value m and rms deviation σ are estimated over all the N previous patterns of the same type, according to formulae :

$$m = \left(\sum_{i=1}^N Q_i \right) / N \quad \text{and} \quad \sigma^2 = \left[\sum_i Q_i^2 - \left(\sum_i Q_i \right)^2 / N \right] / N(N-1) .$$

A few correlation coefficients are estimated through :

$$r_{QQ'} = \left[\sum_i Q_i Q'_i - \left(\sum_i Q_i \right) \left(\sum_i Q'_i \right) / N \right] / (N-1) \sigma_Q \sigma_{Q'} .$$

4.1.3 INFORMATION EXTRACTED IN REAL TIME. — Table XIV summarizes the main information extracted at the end of each elementary pattern, in each of the 4 basic configurations. We now review this information.

4.1.3.1 PV pattern : (Table XIV and Fig. 17a). — It consists of 4 « live time-dead time » cycles. Two parameters are reversed : the field E_0 and the modulator $\lambda/4$ voltage (η and η' reversals resp.). The η' -reversal occurs every two live times. Except for small compensating currents in the coils, no magnetic field is applied.

$\alpha)$ PV signal : In table XIV, the quantities $\mathcal{A}$ or $\mathcal{A}'$ measure the PV component of the electronic polarization P^{PV} , using a digital or analog ratio respectively. Since the noise in the electronics is

negligible we expect continuous equality between $\mathcal{A}$ and $\mathcal{A}'$.

$\beta)$ Control signals for consistency checks : Several quantities are expected to remain zero. These are :

- The asymmetry « $\mathcal{A}$ » computed as if the $(+ -)$ or $(- +)$ order of each pair of signs were fixed, while it is actually chosen at random⁽²⁾.
- The difference $\mathcal{D}_1$ between the PV asymmetries measured in the two states of the $\lambda/4$ -voltage.
- The difference $\mathcal{D}_2$ between the PV asymmetries detected at the two frequencies $(\omega_e \pm \omega_f)$.
- The sum and the difference, $\mathcal{A}_+^x$ and $\mathcal{A}_-^x$, of the asymmetries detected at frequencies $(\omega_e \pm \omega_f)$ in phase quadrature with the PV signal. These quantities are monitored to verify that they remain consis-

Fig. 17. — The four basic configurations programmed during data-acquisition.

⁽¹⁾ In addition in Exp 2 two distinct pairs of LIAs were used : EG & G model 5206 and PAR model HR8 for digital and analog evaluations respectively.

⁽²⁾ We have checked that the numbers of $(+ -)$ and $(- +)$ reversals have been nearly equal, as expected.

Table XIV. — Quantities formed in real-time in the four basic configurations, sketched in figure 17. Interpretation in terms of atomic and/or defect parameters. In the definition (column 3) the amplitudes are relative to the same live time (η , η' , η'' fixed) and the summation is performed over all (4 or 8) states of the pattern. The definitions imply the signatures (in column 2) which consist of characteristic modulations through Stokes parameters, and behaviour under reversal. A cross in upperscript implies a modulation in phase quadrature: e.g. $U_2^x = \eta' \sin \omega_e t$. The interpretation (column 4) has been derived in previous appendices (see the quoted equation). The last column indicates how this quantity is used (U) or corrected for (C). Modulations used as phase references to set the lock-in-phases are indicated by the mention « & phase ».

Designation	Signature	Definition	Interpretation	Use (u) or correction (c)
$\uparrow$ A	$U_2 \xi_f \eta$	$\Sigma \eta \eta' (F_{e,f} + F_{e,-f}) / 4 F_{e,b} $	or $\text{Im } E_1^{PV} / \alpha E_0$ (C.7) $\text{Im } E_1^{PV} / \beta E_0$	U PV parameter
A'	$U_2 \xi_f \eta$	$\frac{1}{2} \Sigma \eta \eta' P_{ef}$		
$\leftarrow$ $\mathcal{A}$	*	$\frac{1}{2} [(P_{ef})_1 - (P_{ef})_2 - (P_{ef})_3 + (P_{ef})_4]$	Null quantities	U Consistency Checks
$\mathcal{D}_1$	$U_2 \xi_f \eta \eta'$	$\Sigma \eta' (F_{e,f} + F_{e,-f}) / 4 F_{e,b} $		
$\mathcal{D}_2$	$U_2^x \xi_f^x \eta$	$\Sigma \eta \eta' (F_{e,f} - F_{e,-f}) / 4 F_{e,b} $		
$\mathcal{A}_+$	$U_2 \xi_f^x \eta$	$\Sigma \eta \eta' (F_{e,f}^x + F_{e,-f}^x) / 4 F_{e,b} $		
$\mathcal{A}_-$	$U_2^x \xi_f \eta$	$\Sigma \eta \eta' (F_{e,f}^x - F_{e,-f}^x) / 4 F_{e,b} $		
Correlation between $\mathcal{A}_1$ and $\mathcal{D}_2$:				
$\uparrow$ Pres	$U_2 \xi_f$	$\Sigma \eta' (F_{e,f} + F_{e,-f}) / 8 F_{e,b} $	$\hat{k} \cdot \hat{k}_f$ (G.8)	C Geometrical adjustment + Hanle effect in Exp1.
$\uparrow$ Pres. ⁽¹⁾ ₃	$U_3 \xi_f \eta$	$\Sigma \eta (F_{e,b,f} + F_{e,b,-f}) / 4 F_{e,b} $	$(M_1/\alpha E_0 \text{ or } M_1/\beta E_0) \times 1/\mathcal{P}$ (" ") $\times \tilde{\chi}$ } (E.30)	U $\vec{P}^{(1)}$ suppression factors in the multipass.
$\uparrow$ Pres. ⁽¹⁾ ₁	$U_1 \xi_f \eta$	do. with $F_{e,b,\pm f}^x$		
$\uparrow$ S _e	U_2	$\Sigma \eta' F_e / 8 F_{e,b} $	Error signal w_3 (E.20)	C Laser intensity modulation under
$\uparrow$ S _{μ}	$U_3 U_2$	$\Sigma \eta' F_{ze,b} / 2 F_{e,b} $	Control signal μ (E.19)	U } servo-loop control
$\uparrow$ A _u (η)	$U_3 \eta'$	$\frac{\Sigma \eta' \eta F_{e,b}}{\Sigma \eta' F_{e,b}}$	Pockels retardation misreversal (2.2.23)	C Pockels cell bias
$\uparrow$ A _u (η)	$U_3 \eta$	do. with $\eta \leftrightarrow \eta'$	$\Delta E_y / E_0$ (F.11)	C $\vec{E}_0$ -field voltage bias
$\uparrow$ A _u ^x (η)	$U_1 \eta$	$\Sigma \eta F_{e,b}^x / 4 F_{e,b} $	$\Delta E_x / E_0$ (F.12)	C Auxiliary-field voltage bias
$\uparrow$ F _{e,b}	U_3	$\frac{1}{4} \Sigma F_{e,b}$	Fluo. intensity F_u (3.2)	U Signal normalization & phase $\omega_2 + \omega_3$
$\uparrow$ F _{e,b} /i _l	U_3	$\frac{1}{4} \Sigma F_{e,b} / i_l$	Fluo. efficiency	U Probe of Cs density drifts
$\uparrow$ α_f	$U_3 \xi_f$	$\Sigma (F_{e,b,f} + F_{e,b,-f}) / 4 F_{e,b} $	ξ_f -like modulation in F_u	U Measure of ξ_f -imperfections
$\uparrow$ C	$U_2 \xi_f \eta''$	$\Sigma \eta' \eta'' (F_{e,f} + F_{e,-f}) / 8 F_{e,b} $	$P_{44}^{(2)}$ (C.6)	U Calibration & Phase at $\omega_2 \pm \omega_f$
$\uparrow$ W ₁	$U_1 \xi_f \eta''$	$\frac{1}{2} \Sigma \eta'' (F_{e,b,f}^x + F_{e,b,-f}^x) / [\Sigma \eta'' (F_{e,f} + F_{e,-f})]$	w_1 w_3 } in Exp 1 (E.7)	C Birefringence compensation
$\uparrow$ W ₃	$U_3 \xi_f \eta''$	do. with $F_{e,b,\pm f}$		
$\uparrow$ P _{hf} ^x	$U_1 \xi_f \eta''$	$\Sigma \eta'' (F_{e,b,f}^x + F_{e,b,-f}^x) / F_{e,b} $	P_{hf}^x in Exp 2 (B.5)	U Calibration & Phase at $\omega_2 + \omega_f \pm \omega_l$
$\uparrow$ $\Pi_{\eta,add}(H_k)$	$U_2 \xi_f \eta \eta''$	$\Sigma \eta \eta'' (F_{e,f} + F_{e,-f}) / 8 F_{e,b} $	$\Delta E_y / E_0$ (F.17)	C d.c. voltage of auxiliary field
$\uparrow$ $\Pi_{\eta,even}(H_k)$	$U_2 \xi_f \eta''$	$\Sigma \eta' \eta'' (F_{e,f} + F_{e,-f}) / 8 F_{e,b} $	$\hat{E}_0 \cdot \hat{k}$ (F.16)	C Geometrical adjustment
$\uparrow$ W ₁	$U_1 \eta''$	$(\Sigma \eta'' F_{e,b}^x) / \Sigma \eta' \eta'' F_e$	w_1 w_3 } in Exp 2 (E.10) (E.11)	C Birefringence compensation
$\uparrow$ W ₃	$U_3 \eta''$	do. with $F_{e,b}$		
$\uparrow$ CD	$U_2 \eta''$	$(\Sigma \eta' \eta'' F_e) / 2 \Sigma F_{e,b}$	Magnetic circ. dichroism (E.12)	U Calibration & Phase at ω_e
$\uparrow$ CD _{det}	U_2	$(\Sigma \eta' F_e) / 2 \Sigma F_{e,b}$	w_3' (+ stray $\vec{H} // \hat{k}$) servo-loop unlocked (E.13)	C Birefringence } compensation & (Exp2) stray $\vec{H}_k$ }
$\uparrow$ Π_e	$U_2 \xi_f \eta \eta''$	$\Sigma \eta \eta' \eta'' (F_{e,f} + F_{e,-f}) / 16 F_{e,b} $	$\vec{E}_0 \cdot \hat{k}_f$ (+ stray $\vec{H} // \hat{k}$) (F.13)	C Geometrical adjustment (controlled by Hanle effect in Exp1)

tent with zero. This is a check of the internal consistency of our results.

In addition we check for unexpected cross-correlation between the asymmetries measured at frequencies $\omega_e + \omega_f$ and $\omega_e - \omega_f$. (Such correlation might imply a fluctuating systematic effect present in both channels). Note that the noises at these two frequencies are not uncorrelated : in the detected signal the large modulations at frequencies $\omega_e + \omega_b$ and $\omega_e - \omega_b$ (§ 2.9.2.i) beating with the same noise component at frequency $\omega_b + \omega_f$ (or $\omega_b - \omega_f$), yield correlated noise contributions at frequencies $\omega_e + \omega_f$ and $\omega_e - \omega_f$. Therefore we expect finite cross-correlation between the two frequency channels (-0.10 in the case of white noise, see Appendix H). On the other hand the correlation between their sum ($\mathcal{A}$) and their difference ($\mathcal{D}_2$) involves only the difference between the noise levels in the two channels (eq. H. 7).

γ) Information on experimental imperfections :

- Quantity P_{res} differs from the asymmetry $\mathcal{A}$ or $\mathcal{A}'$ by the η -reversal only (it is η -even) ; it is interpreted as the residual component of $\mathbf{P}^{(0)} + \mathbf{P}^{(2)}$ along $\hat{\mathbf{k}}_r$, associated with a misorthogonality between $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}_r$. During Exp 1 a Hanle effect driven by a digital servo-loop maintained P_{res} below the noise level. In Exp 2 sufficient reduction of P_{res} was achieved by initial geometrical adjustment. For more details see § 3.5 in part II.

- $P_{\text{res},3}^{(1)}$ and $P_{\text{res},1}^{(1)}$ measure the residual $\mathbf{P}^{(1)}$ -components (imperfect compensation in the multipass).

- S_e represents a stray ξ -like modulation amplitude (i.e. a stray circular dichroism). In practice it is induced by residual birefringence in the entrance window of the caesium cell (w_3). It is used as the error signal in the analog servo-loop driving the modulation amplitude in the laser intensity (§ E.3). This amplitude is directly deduced from the modulation amplitude at frequency $2\omega_e + \omega_b$ (signal S_μ).

- $A_u(\eta')$ (asymmetry in the unpolarized fluorescence signal $\mathcal{F}_u$ under η' -reversal) originates in imperfect reversal of the Pockels cell retardation. It is compensated using a digital servo-loop (§ 2.2.3 in part I).

- $A_u(\eta)$ and $A_u^\times(\eta)$ (asymmetries under η -reversal) are interpreted in terms of stray unreversed components of $\mathbf{E}$ (ΔE_y and ΔE_x resp.). Component ΔE_y is compensated using a digital servo-loop.

- The fluorescence intensity ($\mathcal{F}_{e+b}$) and the fluorescence efficiency ($\mathcal{F}_{e+b}/i_1$: i_1 is the laser intensity) are useful to control the stability of the multipass beam intensity, Cs density, etc...

- The last quantity, a_f , measures a stray ξ_f -modulation induced in $\mathcal{F}_u$ by imperfections of the fluorescence polarization analyser.

δ) Control of the noise : For good statistics, it is important to maintain the noise level at its lowest value, i.e. the shot noise limit of the fluorescence photons. The standard deviation σ in the asymmetry estimated over the N previous elementary patterns indicates the intrinsic noise level. To test its stability σ is also estimated over sets of 20 successive PV patterns (total duration 27 min) and compared to the photon noise limit. If the comparison evidences noise excess the measurement is interrupted and some readjustments are performed (§ 4.1.5 below).

4.1.3.2 Pattern with $\mathbf{H}_0 \parallel \mathbf{E}_0$ (Table XIV and Fig. 17b). — This pattern consists of 8 « live time-dead time » cycles with 3 reversed parameters : $\mathbf{H}_0$, Pockels cell voltage, $\mathbf{E}_0$ (η'' , η' , η respectively). The direction of $\mathbf{H}_0$ is along $\mathbf{E}_0$ and its magnitude is taken equal to the Hanle width ΔH . This constitutes the calibration pattern : the standard polarization $P_{44}^{(2)}$ in Exp 1, or $P_{\text{hf}}^\times$ in Exp 2, is derived ⁽³⁾. The quantities W_1 and W_3 (formed in Exp 1) measure the stray birefringences of the entrance window. They are used to adjust the birefringence compensator and to determine the need for manual readjustment.

4.1.3.3 Pattern with $\vec{\mathbf{H}}_k \parallel \hat{\mathbf{k}}$ (Table XIV and Fig. 17c). — This pattern is similar to the previous one with the magnetic field along $\hat{\mathbf{k}}$ ($\mathbf{H}_k$ instead of $\mathbf{H}_0$). Quantities $\Pi(\mathbf{H}_k)$ are interpreted as stray polarization components associated with $\vec{\mathbf{E}}$ -imperfections (ΔE_z or $\vec{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$), which can thus be corrected for. Quantities W_1 and CD_{str} yield the stray birefringences w_1 and w_3 resp. They are used to set the birefringence compensator in Exp 2. (The stray $\mathbf{H}_k$ is compensated prior to the run). W_3 serves as a cross-check quantity of w_3 . CD is the magnetic circular dichroism used to calibrate CD_{str} .

4.1.3.4 Pattern with $\mathbf{e} \parallel \hat{\mathbf{k}}$ (Table XIV and Fig. 17d). — In this pattern the electric field component $\mathbf{e} \parallel \hat{\mathbf{k}}$ replaces the $\mathbf{H}$ -field of the previous patterns. The quantity Π_e is interpreted as a polarization component along $\vec{\mathbf{E}}_0$ spuriously detected because of $\vec{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$ nonorthogonality (or stray Hanle effect). The misalignment can then be corrected for (§ 3.4.3.2.B in part II).

4.1.4 PHASE SETTINGS. — Lock-in detection provides a sensitive means of extracting the signals of interest. The components of the incident polarization (Stokes parameters U_1 , U_2 , U_3) and the detected polarization (ξ_f) are all modulated. The signature of each measured quantity (2nd column of table XIV) is a simple combination of these modulations. The phase of each lock-in amplifier is adjusted using a large, well understood atomic signal involving the considered combination of modulations. The combinations of modulations have been listed in table XII

⁽³⁾ Table XIV gives the correct definition of the quantity C misprinted in part I, equation (2.10).

of part II, with the corresponding frequencies and notations. We now review the signals used as phase standards :

- For U_3 we use the large U_3 -contribution in the unpolarized fluorescence signal $\mathcal{F}_u$ (Eqs. (3.2) in part II).

- U_1 is in phase quadrature with U_3 . To determine the sign, $\mathcal{F}_u$ is given the signature of U_1 by inserting a polarizer of correct orientation.

- For U_2 we use the magnetic circular dichroism (§ 1.3.1.i in part I), particularly large in the $3 \rightarrow 4$ transition. (An alternative method consists in inserting on the incident beam a $\lambda/4$ plate with axes parallel to the bisectrices of x and y . The modulation that usually labels U_2 is now in U_3 and consequently in $\mathcal{F}_u$.) The modulation of the incident intensity (§ 2.4 in part I) is then turned on with a large amplitude to be adjusted for U_2 -phase. In this situation $\mathcal{F}_u$ acquires a large contribution modulated like $U_2 U_3$ (Eq. (E. 16) in part II) allowing for $U_2 U_3$ -phase setting.

- For $U_2 \xi_f$ we use the standard polarization $P_{44}^{(2)}$, detected in a Hanle field $\mathbf{H}_0 // \mathbf{E}_0$ (Eqs. (1.16), (2.4) and (2.7) in part I).

- For $U_1 \xi_f$ we use the polarization P_{hf}^x induced in the $3 \rightarrow 4$ transition by hf mixing in a field $\mathbf{H}_0 // \mathbf{E}_0$ (Eqs. (B.5) and (2.5b) in part I).

- $U_3 \xi_f$ is obtained by turning the phase through $\pi/2$ with the proper sign.

Our signals allow phase adjustment within $< 1^\circ$. Subsequent (thermal) drifts ($< a$ few degrees) are corrected during data acquisition. Actually in our experiment exact phase setting is not crucial inasmuch as we do not expect any systematic from phase error. In particular, phase influence on the calibration factor is eliminated in our calibration procedure (§ 3.7). Yet correct phase is desirable at the PV frequencies for maximum signal. Incorrect phases at other frequencies would slightly affect the estimation of some systematic effects. In particular the phases at $\omega_e + \omega_b \pm \omega_f$ (associated with $U_1 \xi_f$ and $U_3 \xi_f$) need special care since they are used to discriminate between two defect parameters (quantities $P_{res,1}^{(1)}$ and $P_{res,3}^{(1)}$ in the PV pattern, W_1 and W_3 in the $\mathbf{H}_0$ -pattern). In this case the phase uncertainty results in a systematic uncertainty (§ 4.2.3). On the other hand, any erroneous π or $\pi/2$ -phase shift would be automatically detected through aberrant results during the subsequent auxiliary patterns.

4.1.5 DATA ACQUISITION SERIES. — One PV datum is obtained every 80 s, the duration of one PV pattern. This pattern is repeated 20 times, followed by one pattern with H_0 , then one pattern with H_k . The cycle 20 PV – 1 H_0 – 1 H_k ⁽⁴⁾ is repeated auto-

matically then stopped after ≈ 15 repetitions. We thus obtain about 300 PV values in a real time of ≈ 8 h (i.e. 5 h 20 min of effective PV integration time). This constitutes one « series » and provides one PV result with $S/N \sim 1$. Throughout a series, all experimental conditions are held as constant as possible. Data acquisition is stopped automatically in case of laser intensity destabilization or mode hop. Short manual interruptions infrequently take place on any of the following conditions :

- slight (thermal) drift of the laser frequency reference cavity off the centre of the resonance ;
- slight drift of the birefringence compensator ;
- slight (mechanical) drift of the direction of the beam, increasing the noise level.

The cause of interruption is recorded on the data tape. During manual readjustment the « noisemeter » (§ 2.12 in part I) proves very useful as a fast indicator of satisfactory conditions to resume data acquisition. Any interrupted pattern is rejected. Automatic or manual readjustments are performed between two patterns, never within a pattern. At the end of a series further auxiliary measurements are performed. These are :

- i) Atomic controls of misalignments in Exp 1. (In Exp 2 they were included in the patterns) :

- control of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$ in the $3 \rightarrow 4$ transition using $\mathbf{H}_k$ -patterns ;

- control of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ using $\mathbf{e}$ -patterns.

- ii) In Exp 1, measurement of the dc background, by applying a large modulation in the laser intensity and detecting the resulting modulation in the fluorescence with, then without the field $\mathbf{E}_0$.

- iii) In Exp 2, measurement of the signal contamination from $\Delta F = 0$ overlap (§ E.3c in part II).

Before starting a new series, the program's counters are reset. A few changes are required so as to maintain optimum S/N ratio despite Cs density drifts : $|E_0|$ change, dye change, melting the Cs back to the observation region, ... Other changes were performed as a protection against possible « unexpected » systematics : e.g. $\pi/2$ -rotation of the return mirror inside the cell (end of § 2.3.5 in part I) : change of the rotation direction of the two $\lambda/2$ plates of the modulator.

4.2. Off-line data processing : results.

4.2.1 ANALYSIS SERIES BY SERIES.

4.2.1.1 PV data processing. — Once completed, each series of data is submitted to off-line processing and analysis. Off-line processing is restricted to correction for ambient magnetic field fluctuations (in Exp 1), noise peak rejection, and renormalization to standard values of the electric field E_0 and of the calibration polarization $P_{44}^{(2)}$. These three operations, whose influence on the final result is minor, are now described.

- (i) Hanle effect in a fluctuating ambient magnetic

⁽⁴⁾ An additional pattern with the $\mathbf{e}$ -field was inserted every 40 PV patterns during Exp 2.

Table XV. — PV results. Average value and statistical rms uncertainty of $\text{Im } E_1^p/\beta$ (mV/cm) at the successive steps of data-processing.

		Exp 1	Exp 2
Raw data	(^a)	-1.341 ± 0.220	-1.742 ± 0.264
Corrected for earth's field fluctuations	(^a)	-1.331 ± 0.217	
After noise peak rejection	{ (^a) (^b)	-1.305 ± 0.216 -1.331 ± 0.227	-1.785 ± 0.258 -1.684 ± 0.268
After systematic correction	(^a)	-1.326 ± 0.216	-1.749 ± 0.258

(^a) $(1/\sigma_k)^2$ -weighting. (^b) N_k -weighting.

field contributes to the noise, and might cause a systematic error in case of accidental correlation with the E_0 -reversal (§ 3.5.4 in part II). Therefore in Exp 1 the harmful component was recorded and a correction was applied to the data. The correlation between corrected data and field fluctuations is then checked to be consistent with zero (while it was significant before correction). The average correction amounts to only -0.7% of the final asymmetry and yields a noise reduction of 1.3% (Table XV). This indicates that the field fluctuations were a nearly negligible problem.

Since the source polarization of this Hanle effect is one order of magnitude smaller in the $3 \rightarrow 4$ transition, in Exp 2 the effect was simply neglected.

ii) Noise peak rejection (NPR) aims at rejecting accidental aberrant data, without artificially truncating the wings of the noise distribution. It consists in rejecting the most deviant data (in practice ~ 1 in 100), so that none of the remaining data deviates from the mean by more than 3 rms deviations. The rms deviation is estimated over the remaining data of the considered series, but the mean is estimated over all series merged (⁵). The artificial noise reduction associated with the rejected wings of the noise distribution (1.5% for a Gaussian distribution) is then corrected for in the quoted noise values. Since the noise distribution is expected and observed (§ 4.2.1.2.a) to be symmetric, NPR introduces no bias. NPR shifts the result by -2% in Exp 1, $+2.5\%$ in Exp 2 (Table XV). Finally, NPR also appears as a nearly negligible manipulation.

iii) Finally the data are renormalized to a common value of the standard polarization $P_{44}^{(2)}$ and of E_0 . The $P_{44}^{(2)}$ value is smaller than its theoretical value, by typically 10% to 15% , and drifts slowly by a few percent over the duration of a typical series (8 h): it increases while the Cs density decreases (as deduced from the fluorescence rate). This confirms that the depolarization is essentially due to Cs-Cs collisions [5]. During Exp 1 the measurement of the polarization $P_{44}^{(2)}$ is performed every 20 PV patterns (27 min), through each of the three PV channels

(respectively associated with $\mathcal{F}_{e+f}$, $\mathcal{F}_{e-f}$ and $\mathcal{F}_{ef}$). The individual PV data from each channel are renormalized using the « instantaneous » standard value from the corresponding channel, i.e. the average of the last and next values. During Exp 2 the « instantaneous » standard was the polarization P_{44}^x instead of $P_{44}^{(2)}$, and the ratio $P_{44}^x/P_{44}^{(2)}$ was measured every two or three series. Once renormalized the two asymmetries $\mathcal{A}$ and $\mathcal{A}'$, reconstituted from digital and analog ratio resp., are found equal at the percent level or better.

Within a series, E_0 is kept constant at some value between 100 and 140 V/cm for most series of Exp 1 and between 600 and 700 V/cm in Exp 2. The observed E_0 -dependence of the asymmetry is checked to be consistent with the expected $1/E_0$ law.

The results of the series thus processed are plotted in chronological order in figure 19a of § 4.2.1.3.

4.2.1.2 Tests on the processed data. — All test-quantities formed in real-time (expected to be null, Table XIV) are reevaluated using the processed data. The results do not differ significantly. This shows again that the raw data were essentially sound.

Several statistical tests are performed within each series:

a) Noise distribution: Since each datum results from an average over a large number of independent samples, the noise distribution is expected to be Gaussian (central limit theorem). For each series we compare the observed distribution to the expected one by performing the traditional « χ^2 -test of goodness of fit » in the following manner: the quantity

$$Q^2 = \sum_{j=1}^J (n_o - n_e)^2 / n_e \quad (4.1a)$$

is evaluated, where n_o and n_e are the observed and expected numbers of events in intervals of equal probability ($J \sim 31$ is chosen so that $n_e \sim 10$ for any j). Q^2 is expected to be sampled from a (nearly) χ^2 -distribution with $\nu = J - 3 \sim 28$ degrees of freedom (mean value and rms deviation equal to ν and $(2\nu)^{1/2}$ resp). Series with aberrant Q^2 -values should be rejected, yet this never happened. In

(⁵) For the values of our data, replacing the overall mean by 0 leads to rejecting the same data.

Fig. 18. — a) Histograms of quantity Q^2 (Eq. (4.1a)) estimated in all series of nearly equal size (≈ 300 PV data), and expected χ^2_ν distribution (with $\nu = 28$ degrees of freedom). m and σ : mean and rms deviation. b) Histograms of serial correlation parameter t (Eq. 4.1b) for all series, and expected standard normal distribution.

conclusion, in each series, the observed noise is consistent with the assumption of Gaussian noise. The histograms of figure 18a (plotted for the two PV channels separately) summarize the observed Q^2 values. Also plotted are the expected χ^2 -distributions. The consistency of the observed histogram with the curve is again established by a χ^2 -test of goodness of fit (results in Table XVIa). This brings a new confirmation of the Gaussian nature of the observed noise.

b) Serial correlation: Serial correlation (i.e. correlation between two successive data) on either PV channel would imply a slowly drifting systematic effect on this channel. To test the consistency of the observed correlation r with zero, for each series we evaluate the quantity

$$t = r[(n-2)/(1-r^2)]^{1/2} \quad (4.1b)$$

where n is the number of pairs under consideration. In the assumption of zero correlation and of Gaussian noise, t is sampled from a Student's distribution with $n-2$ degrees of freedom. Figure 18b shows the observed histograms and the expected distributions (in practice a standard normal distribution, in view of the large number of degrees of freedom, ~ 300). The consistency is again established by a χ^2 -test. The result of the test is given in table XVIb, as well as the average serial correlation r_{all} estimated over all series merged and the corresponding t -value t_{all} . In conclusion the results are quite consistent with the absence of serial correlation.

4.2.1.3 Magnitudes of the systematics in a

series. — As described in § 4.1.5, 20 % of the integration time is devoted to auxiliary measurements. This allowed precise estimation of the defects. Within a series the statistical rms deviation in the estimation of each systematic was in general $< 1\%$ of the expected PV effect, with the only exception of the systematic associated with the misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ (stat. unc. $\sim 5\%$). For comparison, within the same period the S/N ratio of the PV signal was ~ 1 . The mean of each systematic remained below or at the level of its statistical uncertainty.

The fact that all systematics have remained small throughout data acquisition, is evidenced in figure 19a. The PV results ($\mathcal{A}$) are plotted with error bars equal to ± 1 rms deviation σ^{PV} . Also plotted, at the same scale but from a shifted origin, are the total systematic effects, i.e. the algebraic sums $\sum_i m^{(i)}$

of the observed means of all registered systematics. The error bar is very conservatively defined as :

$$\left\{ \sum_i [m^{(i)}]^2 + [\sigma^{(i)}]^2 + [\Delta^{(i)}]^2 \right\}^{1/2}.$$

Here $\sigma^{(i)}$ is the statistical rms uncertainty and $\Delta^{(i)}$ the systematic uncertainty affecting our real-time estimate of the systematic i , as discussed in part II or below. We thus observe, in each series, that

$$\left| \sum_i m^{(i)} \right| \ll \sigma^{\text{PV}}$$

Table XVI. — Statistical tests of the series results. a) χ^2 -test of goodness of fit of the observed distribution of the quantity Q^2 . (Eq. (4.1a)) by the χ^2 -distribution expected in the assumption of Gaussian noise. The table gives the χ^2 value for the test, the number ν of degrees of freedom, and the probability of reaching or exceeding this χ^2 value when sampling from a χ^2 -distribution. The two PV channels are treated separately. b) χ^2 -test of goodness of fit of the observed distribution of the serial correlation parameter t (Eq. (4.1b)), by the standard normal distribution expected in the double assumption of Gaussian noise and absence of serial correlation (χ^2 , ν and probability as in a)). Also tabulated are the correlation r_{all} estimated over all series merged, the corresponding t -value t_{all} , and the probability of observing an equal or larger correlation as a matter of chance.

	a) Test of distribution of Q^2	b) Tests of serial correlation :						
		distribution of t			average correlation			
	χ^2 ν proba	χ^2 ν proba	r_{all}	t_{all}	proba			
Exp 1 $\left\{ \begin{array}{l} \omega_e + \omega_f \\ \omega_e - \omega_f \end{array} \right.$	10.74 8 0.22	10.42 12 0.58	0.00	0.01	0.99			
	6.83 8 0.56	20.51 12 0.06	– 0.00	– 0.42	0.67			
Exp 2 $\left\{ \begin{array}{l} \omega_e + \omega_f \\ \omega_e - \omega_f \end{array} \right.$	4.44 6 0.62	8.37 7 0.30	– 0.02	– 1.67	0.10			
	2.50 6 0.87	7.59 7 0.37	0.02	1.57	0.12			

and

$$\sum_i [m^{(i)}]^2 + [\sigma^{(i)}]^2 + [\Delta^{(i)}]^2 \ll [\sigma^{\text{pv}}]^2.$$

The 2nd inequality implies that the smallness of the total systematic effect (1st inequality) does not result from compensations between large systematic effects but actually from the smallness of each effect.

4.2.2 COMBINING THE SERIES RESULTS. —

a) Before combining the PV results from all series (plotted in Fig. 19a) we check that the dispersion between the means m_k is consistent with the dispersions σ_k within the series. To this purpose we form the quantity : $Q^2 = \sum_k (m - m_k)^2 / \sigma_k^2$ where $m =$

$$\left(\sum_k m_k / \sigma_k^2 \right) / \left(\sum_k 1 / \sigma_k^2 \right)$$

is the weighted average. Q^2 is expected to be sampled from a χ^2 -distribution with $\nu = K - 1$ degrees of freedom where K is the number of series. We find $\chi^2/\nu = 71/66$ in Exp 1, 34/41 in Exp 2 and 107/107 for Exp 1 and Exp 2 combined (probabilities 0.25, 0.75 and 0.45 respectively, of reaching or exceeding these values as a matter of chance).

Furthermore in the assumption of Gaussian noise the standardized deviation $(m_k - m) / \sigma_k$ in series k is expected to be sampled from a standard normal distribution. The observed distributions are represented by the histograms of figure 19b, together with the expected distributions. A χ^2 -test of goodness of fit establishes the consistency, with $\chi^2/\nu = 7/12$ in Exp 1, 9/7 in Exp 2 and 14/20 when combined (probabilities 0.86, 0.33 and 0.84 resp. of exceeding).

If the weight of series k is taken to be the number N_k of PV data (i.e. all PV data of all series are given

equal weights) the results are found to hardly change (Table XV).

b) The combination of each systematic effect analysed in part II is performed with PV weights $(\sigma_k^{\text{pv}})^{-2}$ ⁽⁶⁾. The results are listed in table XVIIa. The average value of the main imperfections involved are collected in table XVIII. The statistical uncertainties are very small, which confirms that the imperfections did not fluctuate. In addition, in all cases where a systematic arises as the product of two imperfections we have checked the absence of correlation between these two imperfections.

c) The average results of the test-quantities expected to be zero are collected in table XIX. In addition to the « null quantities » defined in table XIV, two quantities are averaged in the final step of data analysis : $\mathcal{D}_3$, the half-difference between the PV's measured with the two $\lambda/2$ plates of the modulator rotating both clockwise and both counterclockwise, in Exp 1 ; $\mathcal{D}_4$, the half-difference between the PV's measured at orientation 0° and 90° of the multipass return mirror, in Exp 2. All the above quantities are found consistent with zero, as evidenced in figure 20. Finally, the small correlation observed between the asymmetry $\mathcal{A}$ and the difference $\mathcal{D}_2$ ($\sim -3\%$ in Exp 1, -8% in Exp 2) is well accounted for by the small noise level difference observed between the two PV channels (§ H.2).

4.2.3 SYSTEMATIC UNCERTAINTIES. — As discussed in part II, we allow for systematic uncertainties affecting our estimates of the systematics. These uncertainties are evaluated using auxiliary measurements performed just before or after the runs. The

⁽⁶⁾ No significant change is observed with weights $[\sigma_k^{(i)}]^{-2}$, where $\sigma_k^{(i)}$ is the rms noise in the estimate of the systematic i over series k .

Fig. 19. — a) Experimental points. $\mathcal{A}$: observed PV asymmetry converted in terms of $\text{Im } E_1^{\text{pv}}/\beta$ (mV/cm). $\mathcal{A}_{\text{syst}}$: estimated systematic asymmetry at the same scale. The successive points correspond to the successive series during which all experimental conditions are kept as constant as possible (typical integration time of 8 h). Signification of the error bars in § 4.2.1.2. b) Distribution of the standardized deviations about the final mean of the PV asymmetry for all points of Fig. 19a, and expected Gaussian curve of same area.

Fig. 20. — Probability of reaching or exceeding the observed values $|m/\sigma|$ as a matter of chance, assuming the true value to be zero (Table XIX). $\bullet$: Exp 1; $+$: Exp 2. For all test-quantities the observed departures from zero remain quite likely. On the contrary, excessively small probabilities make the assumption of zero true value extremely unlikely in the case of the PV asymmetry.

results are collected in table XVIIb. In addition to the uncertainties previously discussed (part II), we allow for instrumental uncertainties arising in the detection electronics. Two of them are relevant, both affect $\mathcal{A}^{(1)}$:

(i) A calibration uncertainty affects $\mathcal{A}^{(1)}(w_3)$ because the signals involved in the estimation of this quantity are detected at frequencies different from the PV frequencies ($\omega_e \pm \omega_f$). (We allow for 5 % uncertainty in each detection channel). Since in Exp2 the relevant signals were mere noise this calibration uncertainty was well below statistics.

(ii) A « phase uncertainty » in the two-phase lock-in amplifiers results in imperfect discrimination between the two residual $P^{(1)}$ -components (quantities $P_{\text{res.1}}^{(1)}$ and $P_{\text{res.3}}^{(1)}$ detected in phase quadrature to one another). In practice $P_{\text{res.3}}^{(1)} \sim (5 - 10) \times P_{\text{res.1}}^{(1)}$, so only $P_{\text{res.1}}^{(1)}$ was affected; in the uncertainty $P_{\text{res.3}}^{(1)} \times \theta$, the phase error θ observed from time to time during the runs was less than 0.1 rad. Combining this uncertainty to the average value of the birefringence parameter w_1 , from equation (3.13) in part II we estimate the systematic uncer-

Table XVII. — (a) *Estimates of the systematic asymmetries (mean $\pm$ 1 rms deviation).* (b) *Systematic uncertainties affecting (a).* (c) *Overall calibration uncertainty.*

		Exp 1 Unit : $10^{-2} \times$ observed PV asymmetry	Exp 2
(a) Systematic asymmetries			
$As^{(1)}(w_3)$	§ 3.2, Eq. (3.13)	1.5 ± 0.02	$\ll 0$
$As^{(1)}(w_1)$		-4.8 ± 0.24	0.2 ± 0.14
$As_{\text{hf}}^{(1)}$	§ 3.3, Eq. (3.14)	≤ 0.5	≤ 0.15
$As^{(2)}(\Delta E_z)$	§ 3.4, Eq. (3.16)	1.0 ± 0.4	1.4 ± 0.7
$As^{(2)}(\Delta E_x)$		$\ll 0$	0.4 ± 0.1
$As^{(0)}$	§ 3.5 { Eq. (3.18) Eq. (3.21)	0.0 ± 0.2 0.7 ± 0.01	$\ll 0$
As_u	§ 3.6	$\ll 0$	$\ll 0$
(b) Systematic uncertainties			
$As^{(1)}(b_1, b_3)$	§ 3.2.2, Eq. (3.10)	0.8	0.3
$As^{(1)}(w_3)$	§ 4.2.3	$\left\{ \begin{array}{l} 0.4 \\ 5 \end{array} \right.$	$\left\{ \begin{array}{l} \ll 0 \\ \ll 0 \end{array} \right.$
$As^{(1)}(w_1)$			
$\Delta^{(1)}$	§ D.2, Eq. (D.22)	1	1
$\Delta^{(2)}$	§ 3.4.4	3	$\ll 0$
$\Delta^{(0)}$	§ 3.5.3, Eq. (3.19)	5	0.7
(c) Calibration uncertainty			
Δ_{cal}	§ 3.7	1	$2(\text{stat}) \pm 2.5(\text{syst})$

$\ll 0$: less than 10^{-3} of observed PV asymmetry.

Table XVIII. — *Average values of the main defect parameters (mean $\pm$ 1 rms deviation). The measured quantities are defined in table XIV.*

Measured quantity		Corresponding defect parameter	
		Exp 1	Exp 2
$P_{\text{res.3}}^{(1)}$	1/8	$\approx 4.5 \times 10^{-3}$	-4×10^{-4}
$P_{\text{res.1}}^{(1)}$	$\tilde{\chi}$	$\approx 4 \times 10^{-4}$	$< 2 \times 10^{-5}$
W_1	w_1 (rad)	$< 2 \times 10^{-4}$	$< 2 \times 10^{-3}$
S_μ or S_e ($\mu = 0$)	w_3 (rad)	-2.2×10^{-3}	$< 10^{-3}$
A_u	ΔE_y (mV/cm)	1 ± 0.6	-14 ± 5
A_u^x	ΔE_x (mV/cm)	-2.6 ± 1.1	41 ± 2
$\Pi_{\eta\text{-odd}}(H_k)$	ΔE_z (mV/cm)	4.4 ± 1.2	6.4 ± 2.3
$\Pi_{\eta\text{-even}}(H_k)$	$\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$ (rad)	$< 2 \times 10^{-3}$	$\lesssim 10^{-3}$
Π_e	$\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ (rad)	$< 2 \times 10^{-3}$	$(1 \pm 0.2) \times 10^{-3}$
P_{res}	P_{res}	$(0.5 \pm 2.0) \times 10^{-7}$	$(-1.8 \pm 0.1) \times 10^{-5}$

tainty :

$$As^{(1)}(w_1) \leq |w_1| \times |2P_{\text{res.3}}^{(1)} \times \theta|$$

$$\left\{ \begin{array}{l} \approx 5 \% \times \mathcal{A}^{\text{PV}} \text{ in Exp 1} \\ \leq 0.1 \% \times \mathcal{A}^{\text{PV}} \text{ in Exp 2.} \end{array} \right.$$

The limit is much smaller for Exp 2 because $P_{\text{res}}^{(1)}$ was much smaller (it hardly emerges from noise) and because the relevant average birefrin-

gence was accurately compensated, while an offset affected the compensation during part of Exp 1 (§ 3.2 in part II).

4.2.4 FINAL RESULTS.

4.2.4.1 *Comparison between results of Exp 1 and Exp 2.* — We first examine the signs of the two results and their ratio, then we discuss their values more in detail. As shown in part I (Appendix C), the PV quantity directly measured in our experiment is :

Table XIX. — Consistency checks. Observed values (mean m and rms deviation σ) of quantities all expected to vanish, and, for comparison, of the observed PV asymmetry. (Quantities $\mathcal{A}'$ to $\mathcal{A}^*$ are defined in table XIV, $\mathcal{D}_3$ and $\mathcal{D}_4$ in § 4.2.1.2.c.) Probabilities p of reaching or exceeding the observed values $|m/\sigma|$ as a matter of chance, assuming the true value to be zero (cf. Fig. 19). While all tests-quantities are consistent with zero, the observed PV asymmetry is obviously not.

Test-quantities	Exp 1			Exp 2		
	$m \pm \sigma$	$ m/\sigma $	p	$m \pm \sigma$	$ m/\sigma $	p
« $\mathcal{A}$ »	0.30 ± 0.22	1.35	0.18	-0.15 ± 0.26	0.58	0.56
$\mathcal{D}_1$	0.01 ± 0.22	0.04	0.97	-0.16 ± 0.29	0.56	0.58
$\mathcal{D}_2$	-0.12 ± 0.22	0.57	0.57	-0.24 ± 0.27	0.91	0.37
$\mathcal{A}_+^*$	-0.17 ± 0.25	0.68	0.50	0.56 ± 0.31	1.84	0.07
$\mathcal{A}_-^*$				-0.43 ± 0.27	1.60	0.11
$\mathcal{D}_3$	-0.25 ± 0.23	1.09	0.28			
$\mathcal{D}_4$				0.29 ± 0.26	1.12	0.26
PV asymmetry $\mathcal{A}$	-1.34 ± 0.22	6.08	1.6×10^{-9}	-1.75 ± 0.26	6.8	1.6×10^{-11}

$$\mathcal{A}_{\text{FF}}^{\text{PV}}/C = 2 P_{\text{FF}}^{\text{PV}}/P_{44}^{(2)} = \begin{cases} -2 \left(\text{Im } E_1^{\text{PV}}/\beta \mid E_0 \mid \right) (1 + \beta/8 \alpha), & \text{in Exp 1 (4} \rightarrow 4) \\ -3 \left(\text{Im } E_1^{\text{PV}}/\beta \mid E_0 \mid \right) (\alpha/\beta) (1 - 5 \beta^2/12 \alpha^2), & \text{in Exp 2 (3} \rightarrow 4) \end{cases}, \quad (4.2)$$

(with $\alpha/\beta = -9.90 \pm 0.10$ from Eq. (1.7)). The differences in sign and size predicted for Exp 1 and Exp 2 by the above expressions result from the fact that P^{PV} originates in a different interference effect in both hfs components ($\alpha E - E_1^{\text{PV}}$ or $\beta E - E_1^{\text{PV}}$ respectively). It is of course of primary importance to check the experimental results against such predictions.

After renormalization of all data to $E_0 = 100$ V/cm in Exp 1 and to $E_0 = 600$ V/cm in Exp 2, the averages of our measurements give :

$$\begin{aligned} \mathcal{A}_{44}^{\text{PV}}/C &= (2.62 \pm 0.43 \text{ (rms stat)}) \times 10^{-5} \text{ in Exp 1,} \\ \mathcal{A}_{34}^{\text{PV}}/C &= (-8.62 \pm 1.28 \text{ (rms stat)}) \times 10^{-5} \text{ in Exp 2.} \end{aligned} \quad (4.3)$$

So the observed signs are opposite, as expected, and the ratio of the magnitudes (3.3 ± 0.7) is consistent with the expected value 2.5.

Considering now the absolute sign is of fundamental importance : it gives the sign of the parity violating interaction, i.e. the sign of the fundamental constants describing the weak electron-quark coupling (§ 5.1). The origin of $P_{44}^{(2)}$ is clearly understood. From the observation $P_{44}^{\text{PV}}/P_{44}^{(2)} > 0$ we deduce that P^{PV} in the excited state points along the direction $E_0 \times \mathbf{J}_{\text{ph}}$ in the case of $4 \rightarrow 4$ excitation ($\mathbf{J}_{\text{ph}}$ denotes the angular momentum of the incident photons). We conclude that $\text{Im } E_1^{\text{PV}}/\beta$ is negative, in the phase convention initially adopted :

$$\langle 7S, m_s' \mid \mathbf{d} \mid 6S, m_s \rangle = \langle m_s' \mid -\alpha \mathbf{E} - i\beta \boldsymbol{\sigma} \times \mathbf{E} - i \text{Im } E_1^{\text{PV}} \boldsymbol{\sigma} \mid m_s \rangle.$$

We now turn to the detailed examination of the results expressed as a value of $\text{Im } E_1^{\text{PV}}/\beta$ (Eqs. (4.4) and (4.5) below).

a) Exp 1 : Our total data sample consists of 67 series totalizing 17 255 individual processed PV data (307 hours of effective integration) out of 17 459 raw data. We apply a net systematic correction of -1.6% (obtained as the algebraic sum of all effects of table XVIIa). We define the typical systematic uncertainty as the quadratic sum of : i) the statistical uncertainty in the systematic correction ; ii) all systematic

uncertainties listed in table XVIIb (dominantly of statistical origin) ; iii) the calibration uncertainty. The result is 8 % of the observed asymmetry. Our final result is thus

$$\text{Im } E_1^{\text{PV}}/\beta \text{ (Exp 1)} = -1.33 \pm 0.22 \text{ (stat)} \pm 0.11 \text{ (syst)} \text{ mV/cm} . \quad (4.4)$$

b) Exp 2 : Our total data sample consists here of 41 series providing 12 016 processed PV data (214 hours) out of 12 424 raw data. The net systematic correction amounts to +2 %, with a systematic uncertainty of 3.5 % defined as in Exp 1. The final result is

$$\text{Im } E_1^{\text{PV}}/\beta \text{ (Exp 2)} = -1.75 \pm 0.26 \text{ (stat)} \pm 0.06 \text{ (syst)} \text{ mV/cm}^{(7)} . \quad (4.5)$$

An important test of reliability of the whole experiment relies on the comparison between the two results (4.4) and (4.5). They involve interference effects of different nature. Control methods for most systematics, as well as important experimental parameters (E_0 magnitude, Cs and He pressures, length of the interaction region) were different. For these reasons they can be considered to a large extent as independent measurements. The difference between results (4.4) and (4.5) is 1.2 times the corresponding rms statistical deviation, consequently it is quite consistent with zero even disregarding the systematic uncertainty (probability ~ 0.23 of reaching or exceeding the observed difference as a matter of chance). So the two results cross-check one another satisfactorily.

Table XX summarizes the numerous tests and cross-checks that establish the reliability of the results. The tests of the data have just been presented. The tests of the control methods of the systematics can be found in part II. This unusual atomic transition has now been studied in detail by several groups, with a concern for an exhaustive understanding of the underlying physics and for cross-checking experimental methods (part I, especially tables III and IV and references [19-21, 30-36], and reference [26] in part III). We have found no indication of any omitted systematic effect and all registered systematic effects contribute less to the uncertainty than the statistical noise. On the basis of these arguments, we feel that combining the results (4.4) and (4.5) is fully justified.

4.2.4.2 Combining the results of Exp 1 and Exp 2.

— A possible approach to combine (4.4) and (4.5) is to assign equal weights to each experiment, as we did initially [4] : the statistical uncertainty is reduced by $\sim \sqrt{2}$ while the systematic uncertainty is taken as the arithmetic average of the systematic error bars in

(4.4) and (4.5). This leads to :

$$\text{Im } E_1^{\text{PV}}/\beta = -1.54 \pm 0.17 \text{ (stat)} \pm 0.08 \text{ (syst)} \text{ mV/cm}^{(8)} . \quad (4.6a)$$

The parity violation clearly stands out above both statistics (9.2σ) and systematic uncertainty (5 %).

Another philosophy (closer to that adopted in related experiments) consists in affecting each result with a net uncertainty (quadratic sum of the statistical and systematic uncertainties) and weighting each result by the squared reciprocal of that uncertainty. This yields :

$$\text{Im } E_1^{\text{PV}}/\beta = -1.52 \pm 0.18 \text{ mV/cm} , \quad (4.6b)$$

very close to (4.6a) with a less conservative uncertainty. Using the semi-empirical value of β given in part I (Eq. (1.8)) : $\beta = (26.8 \pm 0.8) a_0^3$, which agrees well with the theoretical prediction $27.3 a_0^3$ [19], we can deduce :

$$E_1^{\text{PV}} = (-0.79 \pm 0.10) i | e | a_0 \times 10^{-11} . \quad (4.7)$$

In a recent experiment the Boulder group [6] has observed another PV effect on the 6S-7S transition, namely a parity violating circular dichroism in static E and H -fields crossed with the laser beam, as suggested earlier [7]. Their result $\text{Im } E_1^{\text{PV}}/\beta = -1.65 \pm 0.13 \text{ mV/cm}$ is in good agreement with (4.6). Statistics constitute most of their quoted 8 % uncertainty. It should be noted, however, that their result includes a net systematic correction of 14 %. It is interesting to point out that PV measurements in caesium presently constitute the only case of untested agreement between measurements performed by different groups using different methods.

5. Interpretation of the results.

5.1 THE ELECTRON-NUCLEUS WEAK NEUTRAL CURRENT INTERACTION AND ITS MANIFESTATION.

5.1.1 PV Hamiltonian. Weak charge and weak axial moment of the nucleus. — In view of the extremely short range of the weak interaction on an atomic scale, the electron-nucleon neutral-current interaction in atoms is a contact interaction. Mathematically

⁽⁷⁾ This result slightly differs from our initially published value [4]. The mean value (-1.78) was not corrected for the systematics which were included in the systematic uncertainty. In addition the latter was larger for two other reasons : i) one systematic uncertainty, $\Delta^{(0)}$, was overestimated (cf. footnote in § 3.5.3.2 of part II) ; ii) the calibration uncertainty of systematic origin had been added linearly instead of quadratically. The complete examination of the calibration procedure after Exp 2 [5] justifies the less conservative treatment adopted here.

⁽⁸⁾ See footnote to equation (4.5).

Table XX. — *Tests of the experimental method.*

<ul style="list-style-type: none"> — Detailed understanding of all atomic signals. <ul style="list-style-type: none"> • Hyperfine profile of the unpolarized fluorescence intensity : absence of background and molecular structure. • Dependence of $P^{(1)}$ and $P^{(2)}$: <ul style="list-style-type: none"> — on E- and H-fields (direction and magnitude); — on the hf quantum numbers F, F'. — Tests of the excitation and detection techniques by cross-checking independent measurements of $M_1/\beta, M_1^{hf}/M_1, \alpha/\beta$. — Tests of the procedure used to calibrate P^{pv} : <ul style="list-style-type: none"> • checks of the isotropy of the relaxation; • recourse to an absolute polarization standard. — Tests of the methods used <ul style="list-style-type: none"> • in the control of experimental imperfections; • in the reconstitution of systematic effects. — Statistical tests on the data. — Internal consistency checks in asymmetry measurements. — Cross-checking the first measurement ($\alpha E - E_1^{pv}$ interference, Exp1) against an independent one ($\beta E - E_1^{pv}$ interference, Exp2). 	<p>Part I, §§ 1.2, 1.3</p> <p>Part I, Tables III, IV, and refs. therein [5]</p> <p>Part II, § 3.2.3. §§ 3.2.4, 3.4.3.5. III, § 4.2.1.2 & Table XVI. III, § 4.1.3.1. α) & Table XIX. III, § 4.2.4.1.</p>
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electrons and nucleons are represented as currents. The weak neutral-current Hamiltonian appears as the product of an electron current and a nucleus current evaluated at the same point. Each current is the sum of a polar vector and of an axial vector. As a consequence the Hamiltonian contains two parts

which violate parity, $\mathcal{K}_1$ and $\mathcal{K}_2$, associated with the axial-electronic vector-nucleonic ($A_e V_n$) and vector-electronic axial-nucleonic ($V_e A_n$) currents respectively :

$$\mathcal{K}_1 = \frac{G_F}{\sqrt{2}} \left[\sum_{p=1}^Z C_p^{(1)} \int d^3r (\bar{\psi}_e \gamma^\mu \gamma_5 \psi_e) (\bar{\psi}_p \gamma_\mu \psi_p) + \sum_{n=1}^N id. (p \rightarrow n) \right]$$

$$\mathcal{K}_2 = \frac{G_F}{\sqrt{2}} \left[\sum_{p=1}^Z C_p^{(2)} \int d^3r (\bar{\psi}_e \gamma^\mu \psi_e) (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_p) + \sum_{n=1}^N id. (p \rightarrow n) \right]$$

γ^μ, γ_ν are the Dirac matrices with the convention $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$; ψ_e, ψ_p and ψ_n are the field operators of the electron, proton and neutron respectively. The Fermi constant $G_F = 4 \times 10^{-14}$ Rydberg $\times a_0^3$ gives the overall strength of the interaction. The coupling constants $C_{p,n}^{(1)}$ and $C_{p,n}^{(2)}$ are related to the fundamental coupling constants $C_{u,d}^{(1)}$ and $C_{u,d}^{(2)}$ of the u and d quarks. Since the vector currents are conserved, the coupling constants $C^{(1)}$ simply add :

$$C_p^{(1)} = 2 C_u^{(1)} + C_d^{(1)}, \quad C_n^{(1)} = C_u^{(1)} + 2 C_d^{(1)}. \quad (5.1)$$

In contrast the axial currents are not conserved and a model of the nucleon is needed to express $C_p^{(2)}$ or $C_n^{(2)}$ in terms of $C_u^{(2)}$ and $C_d^{(2)}$.

In the standard model of the electroweak theory the predictions for the electron-quark coupling constants depend only on the weak interaction angle θ_w , related to the masses of the intermediate $W^\pm$ and Z^0 bosons through

$$\sin^2 \theta_w = 1 - (M_W/M_{Z^0})^2, \quad (5.2)$$

whose presently accepted value is close to 0.22 (see § 5.1.2 below). In the lowest approximation the predictions for the coupling constants $C^{(1)}$ are :

$$C_p^{(1)} = -0.5 (1 - 4 \sin^2 \theta_w), \quad C_n^{(1)} = 0.5. \quad (5.3)$$

In the standard model the dependence of $C_p^{(2)}$ and $C_n^{(2)}$ on the nucleon structure can be accounted for using the empirical ratio ($\lambda = 1.25$) of vector to axial coupling in neutron β -decay. The results at the lowest order are :

$$C_p^{(2)} = -C_n^{(2)} = -\frac{\lambda}{2} (1 - 4 \sin^2 \theta_w). \quad (5.4)$$

For more physical insight it is worth considering $\mathcal{K}_1$ and $\mathcal{K}_2$ in the non-relativistic limit⁽⁹⁾ where they become effective parity-violating potentials, respectively :

$$V_{pv}^{(1)} = \frac{G_F}{4\sqrt{2}} Q_w(Z, N) \left[\boldsymbol{\sigma} \cdot \frac{\mathbf{p}}{m_e c} \delta(\mathbf{r}) + \text{h.c.} \right] \quad (5.5a)$$

$$V_{pv}^{(2)} = \frac{G_F}{4\sqrt{2}} A_w(Z, N) \left[\left(\frac{\mathbf{I}}{I} \cdot \frac{\mathbf{p}}{m_e c} + i \frac{\mathbf{I}}{I} \cdot \boldsymbol{\sigma} \times \frac{\mathbf{p}}{m_e c} \right) \delta(\mathbf{r}) + \text{h.c.} \right] \quad (5.5b)$$

Here $\hbar\boldsymbol{\sigma}/2$ and $\mathbf{p}$ are the electron spin and momentum ; $\mathbf{I}$ is the nuclear spin. Both $V_{pv}^{(1)}$ and $V_{pv}^{(2)}$ describe contact interactions and involve the electron velocity $\mathbf{p}/m_e$ at the nucleus.

In potential $V_{pv}^{(1)}$ the boson Z^0 couples to the nucleons like a heavy photon. The parameter Q_w , called the nuclear weak charge, plays the same role as the nuclear electric charge in the electromagnetic interaction. In particular the weak charges of the nucleus constituents add like charges so that

$$Q_w(Z, N) = -2 \left[(2Z + N) C_u^{(1)} + (Z + 2N) C_d^{(1)} \right] \quad (5.6)$$

(from the additivity of the $C^{(1)}$'s; Eq. (5.1)).

In contrast, in potential $V_{pv}^{(2)}$ the contributions of the various constituents add like spins. The parameter involved in the sum is the weak axial moment :

$$A_w(Z, N) I/I = 2 \left[\sum_p C_p^{(2)} \boldsymbol{\sigma}_p + \sum_n C_n^{(2)} \boldsymbol{\sigma}_n \right], \quad (5.7)$$

where $\hbar\boldsymbol{\sigma}_{p,n}/2$ represent the spins of protons and neutrons. From (5.6 and 5.7) it is clear that in heavy atoms Q_w increases roughly as the atomic number while $A_w(Z, N)$ remains approximately of the size of $C_{p,n}^{(2)}$, which both nearly cancel in the standard model (Eq. (5.4) with $\sin^2 \theta_w \simeq 0.22$). Therefore in heavy atoms the $A_e V_n$ interaction ($\propto Q_w$) is expected to be by far dominant. For the Cs nucleus ($Z = 55, N = 78$) the standard model at the lowest order predicts :

$$Q_w^{\text{lo}}(\text{Cs}) = -23 - 220 \sin^2 \theta_w. \quad (5.8)$$

It is also possible to predict A_w/Q_w in terms of $C_p^{(2)}$ provided a specific nuclear model is assumed. For instance the single-particle shell model leads to :

$$A_w/Q_w(\text{Cs}) = +2.2 C_p^{(2)} \quad (5.9)$$

However in view of the discrepancy between the empirical value and this model's prediction of the nuclear magnetic moment of Cs, we must expect the uncertainty in this prediction to be $\sim 50\%$.

5.1.2 Radiative corrections. — Radiative corrections in the zero momentum transfer limit relevant for atomic physics experiments have been computed using two different approaches. With the results of Marciano and Sirlin [9] one can express the effective coupling constants, corrected for radiative corrections, in terms of $\sin^2 \theta_w$ as defined by equation (5.2) ⁽¹⁰⁾

$$\begin{aligned} C_p^{(1)} &= 2 C_u^{(1)} + C_d^{(1)} \\ &= -0.487 + 1.942 \sin^2 \theta_w \\ C_n^{(1)} &= C_u^{(1)} + 2 C_d^{(1)} = 0.487. \end{aligned}$$

Using equation (5.6) this leads to the corrected weak charge :

$$Q_w(\text{Cs}) = -22.40 - 213.62 \sin^2 \theta_w. \quad (5.10)$$

⁽⁹⁾ i.e. neglecting terms quadratic in v_e/c and terms linear in v_N/c (v_e, v_N : electronic and nuclear velocities).

⁽¹⁰⁾ The parameter used in reference [9] is : $\sin^2 \hat{\theta}_w(M_w^2) = \sin^2 \theta_w / 1.006$; see [10].

The weak neutral current interaction has been observed in a variety of reactions, each one providing a value of $\sin^2 \theta_w$ when radiative corrections are properly included. A recent fit to all existing data yields the result [10]:

$$\sin^2 \theta_w = 0.223 \pm 0.004, \quad (5.11)$$

leading to the prediction:

$$Q_w (\text{Cs}) = -70.0 \pm 0.9. \quad (5.12)$$

Another approach followed by B. Lynn uses as empirical inputs the Z^0 mass instead of $\sin^2 \theta_w$ [11]. With the average result obtained at CERN in $p\bar{p}$ scattering ($M_{Z^0} = 92.7 \text{ GeV}/c^2$), the value deduced for Q_w practically coincides with (5.12).

Concerning $C_p^{(2)}$ and $C_n^{(2)}$ radiative corrections are especially important in view of the smallness of the quantity $1 - 4 \sin^2 \theta_w$. The result [9] is

$$\left. \begin{aligned} C_p^{(2)} &= -0.6567 (1 - 4.056 \sin^2 \theta_w) \simeq -0.063 \\ C_n^{(2)} &= +0.6047 (1 - 4.098 \sin^2 \theta_w) \simeq +0.052. \end{aligned} \right\} \quad (5.13)$$

5.1.3 The PV electric dipole of the 6S-7S Cs transition. — To first order of the perturbation theory, the PV potential $V_{pv} = V_{pv}^{(1)} + V_{pv}^{(2)}$ produces no energy shift in an atomic state of well-defined parity but mixes states of opposite parity. Because we have to deal with a contact interaction, only S and P states are involved in the case of a single valence electron atom like Cs. The PV electric dipole amplitude is found to be of the form

$$\langle 7S, m'_s | \mathbf{d}^{pv} | 6S, m_s \rangle = -i \text{Im } E_1^{pv} \langle m'_s | \boldsymbol{\sigma} + \eta \mathbf{I} + i \eta' \boldsymbol{\sigma} \times \mathbf{I} | m_s \rangle \quad (5.14)$$

where η and η' are real coefficients. The term independent of the nuclear spin originates in the $A_c V_n$ interaction (potential $V_{pv}^{(1)}$), while the two other terms mainly originate in the $V_e A_n$ interaction (potential $V_{pv}^{(2)}$). With the assumption that any other contribution to η and η' is negligible, the ratio η'/η is a purely atomic quantity which has been computed for the 6S-7S Cs transition [12]: $\eta'/\eta \simeq 2$. The parameter η has a simple interpretation in the electroweak theory: $\eta = A_w/Q_w$, and is expected to be small in the standard model: $\eta = (1 \text{ to } 2) \times 10^{-5}$ from (5.9 and 5.13). In the following first-order interpretation of the experimental results this fully justifies the neglect of the nuclear spin-dependent terms in (5.14) (as already done in part I, Eq. (1.5)). Yet, without reference to the electroweak theory, expression (5.14) represents the most general description of $\mathbf{d}^{pv}$, provided the PV interaction preserves time-reversal invariance. For this reason it is worthwhile to also consider a purely phenomenological discussion of the experimental results starting from (5.14) without any assumptions concerning the value of η (§ 5.6).

5.1.4 Magnitude of E_1^{pv} in the standard model. —

The PV amplitude E_1^{pv} defined above is the product of the weak charge Q_w and a purely atomic quantity. In the chosen case, several favourable circumstances converge towards making the theoretical evaluation of this quantity reliable: first, the single-valence-electron structure of Cs favours reliable calculations; second, Cs is spectroscopically well-known, so that numerous well established empirical data have provided tests of the corresponding predictions of the model; and third, the detailed experimental study of the 6S-7S transition has resulted in the

determination of new parameters that test the model directly on this transition.

The parameter E_1^{pv} is sensitive to the wavefunctions outside the core (through the interaction with the radiation field) as well as inside the core (due to the practically zero range of the weak interaction). Accordingly it has been established (cf. Table III in part I) that the atomic model of reference [13] correctly predicts not only oscillator strengths, lifetimes and polarizabilities, but also the hyperfine interaction, a contact interaction like V_{pv} . In addition we have obtained experimental evidence that contributions to the hf interaction from many-body effects between electrons are very small. This results from the agreement between the measured value of the 6S-7S hf mixing amplitude M_1^{hf} , with the value accurately predicted under the assumption that many-body effects can be neglected [14]. (A serious discrepancy would have indicated that many-body effects could lead to important modifications of the V_{pv} matrix elements. In such a case the single-particle approximation used in the part of the calculation involving the parity mixing amplitude [13] would have been considered inadequate).

Among all available predictions of E_1^{pv} in Cs, we retain here those which result from the most complete calculations (Table XXI). They incorporate many-body effects either semi-empirically [13] or from first principles [15, 16] ⁽¹⁾. Taking their average we obtain the prediction:

$$(\text{Im } E_1^{pv})^{\text{th}} = (0.91 \pm 0.04) \times Q_w / N \times 10^{-11} |e| a_0, \quad (5.15)$$

⁽¹⁾ We do not include the unpublished result of reference [17] with which other authors have subsequently disagreed [16, 18].

Table XXI. — *Parity violating amplitude in units of $10^{-11} |e| a_0$: theory and experiment. The quantity $\text{Im } E_1^{\text{pv}}/(Q_w/N)$ (where $N = 78$ for Cs is the number of neutrons) is purely atomic. The weak nuclear charge Q_w is computed in the framework of the standard electroweak model.*

Theory		Experiment	
Atomic theory (*)	& stand. model ($\rightarrow Q_w = -70.0$)	Experiment (assuming $\beta = 26.8 a_0^3$)	& stand. model ($\rightarrow Q_w = -70.0$)
$\text{Im } E_1^{\text{pv}}/(Q_w/N)$	$\text{Im } E_1^{\text{pv}}$	$\text{Im } E_1^{\text{pv}}$	$\text{Im } E_1^{\text{pv}}/(Q_w/N)$
0.88 ± 0.03 [15]	-0.79	-0.79 ± 0.10 (a)	0.88 ± 0.11
$0.888 \pm \text{a few } \%$ [16]	-0.797	-0.86 ± 0.07 (b)	0.96 ± 0.08
$0.97 \pm \sim 0.10$ [13]	-0.87		

(*) Only calculations including many-body effects are considered.

(a) This work.

(b) Reference [6].

where the error bar represents the fairly small rms dispersion⁽¹²⁾. In the standard model (using Eq. (5.12) and $N = 78$) the prediction becomes :

$$(\text{Im } E_1^{\text{pv}})^{\text{th}} = - (0.82 \pm 0.04) \times 10^{-11} |e| a_0 \quad (5.16)$$

5.2 DETERMINATION OF $Q_w(\text{Cs})$. — Since the purely atomic quantity $\text{Im } E_1^{\text{pv}}/Q_w$ is known from atomic calculations (Eq. (5.15)), from the measured value of $\text{Im } E_1^{\text{pv}}$ (Eq. (4.7)) we extract an empirical determination of the weak charge of the Cs nucleus :

$$Q_w^{\text{exp}}(\text{Cs}) = -68 \pm 9 \quad (5.17)$$

(Theoretical and experimental uncertainties have been summed quadratically). It should be emphasized that this result does not resort to any electroweak model.

5.3 INTERPRETATION OF $Q_w^{\text{exp}}(\text{Cs})$ IN THE FRAMEWORK OF THE STANDARD ELECTROWEAK MODEL. — With the accepted value $\sin^2 \theta_w \simeq 0.223$, the prediction of the standard electroweak model ($Q_w(\text{Cs}) = -70.0$) is close to the empirical determination (5.17).

Conversely, starting from the empirical result (5.17), we obtain through (5.10) a determination of the weak interaction angle

$$(\sin^2 \theta_w)_{\text{Cs}} = 0.21 \pm 0.04, \quad (5.18)$$

in agreement with the values extracted from high-energy experiments of very different types [10]

⁽¹²⁾ We are aware that taking such an average value is somewhat arbitrary. However, within the quoted uncertainties (Table XXI) the results are consistent. Taking instead one or the other prediction does not significantly change the following analysis in view of the experimental uncertainty.

(Fig. 21). The interest of this agreement lies in the considerable difference in momentum transfer between high energy experiments ($\geq 1 \text{ GeV}/c$) and atomic physics (typically $2.4 \text{ MeV}/c$ in Cs [19]). Therefore PV measurements in Cs offer a crucial test of the standard model at low momentum transfers not accessible so far to experiments performed with large accelerators. This illustrates one aspect of the complementarity between the two types of experiments. The model-independent interpretation in the next section reveals another aspect of this complementarity.

Fig. 21. — Values of $\sin^2 \theta_w$ from our results, and from totally different types of experiments at much higher momentum transfer values q_- [10] in the scattering of various partners (N : nuclei ; pp : proton-antiproton experiment at CERN ; e : electron ; D : deuterium ; ν : neutrino). The dashed line represents the average of all data : $\sin^2 \theta_w = 0.223 \pm 0.004$ (including radiative corrections).

5.4 MODEL-INDEPENDENT INTERPRETATION OF $Q_w^{\text{exp}}(\text{Cs})$. — Since the atomic wave functions have nearly the same phase over the entire nucleus, the individual effects of all nucleons add coherently. From this results the model-independent form of Q_w (Eq. (5.6).

At high energy the situation is totally different. So far only one experiment provides information about the weak charges of the quarks (SLAC exp. [20]). It involves inelastic scattering of longitudinally polarized high energy electrons on deuterons. The nucleons are broken and the quarks act incoherently. Then the electromagnetic amplitude of a quark ($\propto$ electric charge $2/3$ or $-1/3$ for u or d) can only interfere with the weak amplitude of the same quark ($\propto C_u^{(1)}$ or $C_d^{(1)}$). The experiment involving u and d quarks in equal proportions is thus sensitive to the linear combination $\frac{2}{3} C_u^{(1)} - \frac{1}{3} C_d^{(1)}$, which is nearly orthogonal to the combination that determines Q_w (Eq. (5.6)). This is represented in figure 22. Either experimental result restricts the possible values of $C_u^{(1)}$ and $C_d^{(1)}$ to a band in the $C_u^{(1)}$, $C_d^{(1)}$ plane. Combining the results of both experiments allows both $C_u^{(1)}$ and $C_d^{(1)}$ to be determined, while neither experiment can achieve this alone.

The agreement of the above model-independent interpretation with the standard model prediction is noteworthy. Each point of the graduated segment of figure 22 represents this prediction for a value of the parameter $\sin^2 \theta_w$. The intersection of the two experimentally allowed bands is consistent with the prediction for $\sin^2 \theta_w \approx 0.2$, precisely the value obtained from various other high energy experiments.

Fig. 22. — Regions of $C_u^{(1)}$ and $C_d^{(1)}$ allowed by our result (Eq. (5.17)) and the SLAC result $C_u^{(1)} - 0.5 C_d^{(1)} = -0.45 \pm 0.12$ (1 rms stat.), when error bars are multiplied by 1.64 to obtain 90 % confidence level. The graduated segment is the prediction of the standard model.

5.5 IMPLICATIONS CONCERNING ALTERNATIVE MODELS. — A huge theoretical effort is currently devoted to extending electroweak unification to strong and gravitational interactions. This gives rise to numerous alternatives to the standard electroweak theory. Because atomic physics experiments test the weak interaction at long distances ($\sim a_0/Z$), they turn out to be sensitive to exotic weak interactions. Most of the alternative models imply the existence of a second neutral boson U , whose mass and coupling constant g_U are generally constrained by the relation

$$g_U^2/M_U^2 < G_F \approx g^2/M_Z^2.$$

In high energy physics the effect of a very light boson ($(M_U c)^2 \ll q^2 \ll (M_Z c)^2$) is suppressed by a factor $(M_U c/q)^2$. In an atom, the U boson behaves like the Z^0 and contributes to the weak charge, by as much as $\Delta Q_w \approx Q_w$, if (i) it couples to quarks as a vector and (ii) the range of its interaction $\hbar/M_U c \leq a_0/Z$. Consequently from the good agreement between the empirical determination of Q_w and the standard model prediction, one can deduce stringent constraints on the mass of an additional boson and on the (axial or vector) nature of its coupling to the quarks.

For instance, in a specific model such as the one proposed by P. Fayet [21] where the U boson is assumed to couple identically to quarks u and d , the additional contribution to the weak charge is

$$\Delta Q_w = 3(Z + N) \lambda_q R(M_U) \text{ with } |\lambda_q| \leq 1.$$

The function $R(M_U)$ that accounts for the finite mass of the boson with $R(M_U \geq M_w) = 1$, has been explicitly computed by C. Bouchiat and C. A. Piketty [19]. In a cartesian plane where the coupling constant λ_q is plotted versus M_U (Fig. 23), the reinterpretation of our result allows one to exclude a huge region : for values $M_U > 10 \text{ MeV}/c^2$ only a U -quark coupling of axial nature is in practice allowed. It is noted that even at high energy (above $1 \text{ GeV}/c^2$) the constraints from the Cs experiment are more severe than those deduced from the SLAC experiment [19, 21].

5.6 PHENOMENOLOGICAL INTERPRETATION OF THE RESULTS.

5.6.1 Information about the PV nuclear spin-dependent interaction. — We now turn to a phenomenological interpretation allowing for a PV nuclear spin-dependent interaction, as for instance the $V_e A_n$ interaction associated with fundamental constants $C_u^{(2)}$, $C_d^{(2)}$. We expect the matrix elements of the PV electric dipole (Eq. (5.14)) to depend on the

Fig. 23. — Constraints on the electronic-axial quark-vector coupling constant λ_q versus the U boson mass, as implied by our determination of Q_w (Eq. (5.17)) when the error bar is multiplied by 1.64 to obtain 90 % confidence interval.

initial and final hf states of the transition. Each of our measurements yields the value of the ratio

$$d_{FF'} = \frac{\langle 7S, F' m_{F'} | \mathbf{d}^{PV} | 6S, F m_F \rangle}{\langle 7S, F' m_{F'} | \boldsymbol{\sigma} | 6S, F m_F \rangle} \quad (5.19)$$

for $F = F' = 4$ in Exp 1 and $F = 3, F' = 4$ in Exp 2. The dependence of $d_{FF'}$ on F and F' is displayed in table XXII. From the table and for $\eta'/\eta \approx 2$ the ratio of our two results (Eqs. (4.4) and (4.5) is interpreted as :

$$R = d_{34}/d_{44} = \left(1 + \frac{15}{7} \eta\right) / (1 + \eta). \quad (5.20)$$

In principle, this determines η . In practice in view of the experimental uncertainties a categorical answer is premature. Yet figure 24 shows that the results rule out values of η close to -1 or $-7/15$ (the total predicted effect would vanish in one or the other transition), and that values close to zero (pure $A_e V_n$ coupling) are somewhat favoured against large (positive or negative) values (nearly pure $V_e A_n$ coupling). In fact the (4-4 ; 3-4) pair of hfs components, chosen with the view of measuring different electroweak interferences, is not the best choice to determine η .

The recent measurements in Boulder, mentioned in § 4.2.4.2, involve the 3-4 and 4-3 components whose ratio is more sensitive to the nuclear spin-dependent interaction through η' (rather than η). Their present result constitutes an improved test supporting the hypothesis $\eta \leq 1$ (i.e. the $A_e V_n$ nature of the interaction). Table XXII suggests that a comparison of the two $\Delta F = 0$ hf components

Table XXII. — Quantity $d_{FF'}$ (Eq. (5.19)) for the various hyperfine components of a $nS \rightarrow nS'$ alkali transition (I : nuclear spin; η and η' defined by Eq. (5.14)).

$F \rightarrow F'$	$d_{FF'}/i \text{Im } E_1^{PV}(n', n)$
$I + 1/2 \rightarrow I + 1/2$	$1 + \eta$
$I - 1/2 \rightarrow I - 1/2$	$1 - \eta(I + 1)/I$
$I - 1/2 \rightarrow I + 1/2$	$1 - \eta/2 I + \eta'(2I + 1)/2 I$
$I + 1/2 \rightarrow I - 1/2$	$1 - \eta/2 I - \eta'(2I + 1)/2 I$

Fig. 24. — Constraints on η as implied by our results (4.4) and (4.5). The part of the curve of ordinate $> p$ gives the confidence region at the confidence level $1 - p$. (Example : at the confidence level 90 %, the confidence region is the interval $-8.8 \times 10^{-2} < \eta < 7.6$). Explicit definition of p in note [22].

would provide as sensitive a test with the additional advantage of being independent of η' and thereby independent of the atomic calculations. This is going to be taken into account in our experimental work.

5.6.2 Information about the range of the PV interaction. — It is interesting to compare the Cs results to that obtained in a heavier atom such as Tl. The current value of E_1^{PV} in Tl, $(0.69 \pm 0.13) \times 10^{-10} ea_0$ [23], turns out to be ~ 9 times larger than in Cs, whereas allowed E_1 amplitudes are of nearly equal magnitudes in both atoms. This proves that the parity mixing amplitudes (such as e.g. $\langle n' P_{1/2} | V_{pv} | n S_{1/2} \rangle$) are larger in Tl than in Cs by the same factor. Independent of any electroweak model, with the assumption that V_{pv} is a contact interaction, then the matrix elements of $V_{pv}^{(1)}$ depend on Z like $Q_w Z^2 K_r$ and those of $V_{pv}^{(2)}$ like $Z^2 K_r$ [24], where K_r is a relativistic factor (2.8 for

Cs, 9 for Tl). The whole factor $Z^2 K_r$ reflects the large increase of the s wave and of the gradient of the p wave at the nucleus. It grows by as much as a factor 7 from Cs ($Z = 55$) to Tl ($Z = 81$), while the additional factor Q_w varies only by 1.5. The fact that the experimental results in Cs and Tl turn out to be consistent with the $Q_w Z^2 K_r$ law, essentially associated with short range effects, reveals the very short range of the PV interaction in a model-independent way.

6. Conclusion and prospects.

6.1 CONCLUSION. — The essential result obtained in this work is the measurement of a parity violating effect in the Cs atom, observed through an electric dipole transition amplitude of only $8 \times 10^{-12} ea_0$ connecting the 6S and 7S states. At present Cs is the simplest and lightest atom wherein a parity violating effect has been observed. From two independent measurements we obtain

$$\text{Im } E_1^{\text{pv}}/\beta = (-1.52 \pm 0.18) \text{ mV/cm.}$$

The off-diagonal polarizability

$$\beta = (26.8 \pm 0.8) a_0^3,$$

determined semi-empirically agrees well with a theoretical prediction.

In order to make this result significant, numerous precautions have been taken. The systematics have been reduced and monitored using rigorous procedures (part II), so as to make corrections superfluous. Numerous tests have been performed in auxiliary experiments to increase confidence in the experimental method and in the atomic theory required to interpret E_1^{pv} . A recent determination by an independent group agrees well with ours.

Our results complement the high energy results and thus further specify the nature of the weak electron-nucleon interaction which violates parity. They imply that the weak charge of the Cs nucleus is $Q_w^{\text{exp}} = -68 \pm 9$, in good agreement with the prediction of the standard electroweak theory $Q_w(\text{Cs}) = -70.0$ (for $\sin^2 \theta_w = 0.223$). Without reference to any theoretical electroweak model, a few conclusions are now established: (1) the electron-nucleus interaction whose manifestation is observed, violates parity (since it shows up as a right-left asymmetry); (2) it preserves time-reversal invariance (since there is a $\pi/2$ -phase shift between the PV electric and the spontaneous magnetic dipole amplitudes); and (3) its range is very short (as deduced from the comparison between our result in Cs and the Berkeley result in Tl).

6.2 PROSPECTS. — There is considerable incentive to improve the accuracy of the Cs result. One motivation is to arrive at a precise determination of the nuclear weak charge. Radiative corrections in

Q_w amount to about 6 % ⁽¹³⁾. Therefore at the percent level the interest of a comparison between experiment and theory will acquire a new dimension. This currently motivates new experimental projects as well as new ideas to attack atomic structure calculations. In addition such advances would prove valuable in making more stringent the constraints imposed by atomic results on alternative electroweak theories.

In addition, a question fully open from an experimental point of view is that of a possible weak neutral current interaction involving the nucleon's spin. Such an interaction, while theoretically predicted, is still unobserved even at high energy. Although its enhancement in heavy atoms loses one factor Z with respect to the nucleon spin-independent contribution, checking its existence does not seem hopeless. For instance measuring the ratio of the PV amplitudes on two hf components of the 6S-7S transition in Cs with an accuracy of about 1 % would already yield important information about the axial moment of the Cs nucleus. However, for the same reason as the Cs nuclear magnetic moment is not a simple function of the proton's and neutron's magnetic moments, this axial moment cannot provide $C_p^{(2)}$ unambiguously. By no means can such measurements replace experiments in hydrogen which would yield $C_p^{(2)}$ directly.

Concerning our measurements in Cs, the most obvious improvement required is that of the statistics, since systematic corrections were small and since most systematic uncertainties were of statistical origin. The most promising gain expected is in detection efficiency. To insure high spectral selection and good polarization analysis we had to accept that only one 7S excitation in ≈ 2000 is detected. In a new project we hope to monitor the stimulated emission in the 7S-6P transition induced by a probe beam rather than the spontaneous fluorescence emission. The right-left asymmetry created in the 7S state will then show up in the dependence of the probe gain on the polarizations [25]. Because all photons are emitted in the direction and at the frequency of the probe, high efficiency is expected. Different experimental approaches are currently being investigated by the Zurich [26] and Boulder [6] groups. Results from independent groups will again provide valuable cross-checks.

One can thus expect that parity violation measurements in Cs will continue to provide novel information of fundamental importance.

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⁽¹³⁾ This represents the difference between Q_w (Eq. 5.12), and Q_w^b (Eq. 5.8) calculated for the value of $\sin^2 \theta_w$ extracted from deep inelastic $\nu_\mu - N$ scattering analysed at lowest order.

interpretation of the results. We benefited also from fruitful discussions with Michel Davier and Claude-Annette Piketty. One of us (LH) gratefully acknowledges financial support from the Collège de France, a Joliot-Curie fellowship and the Direction des Recherches et Etudes Techniques.

Appendix H.

H.1. The aim is here to compute the correlation between the instantaneous outputs of the two lock-in amplifiers at frequencies $\omega_e + \omega_f$ and $\omega_e - \omega_f$ ($\mathcal{F}_{e+f}$ and $\mathcal{F}_{e-f}$ in Fig. 16) hereafter called $S_+(t)$ and $S_-(t)$. The instantaneous fluorescence intensity received by the detector has the form :

$$\mathcal{J}(t) = I(t) \left\{ 1 \mp \left[\cos(\omega_e - \omega_b)t - \cos(\omega_e + \omega_b)t \right] / 2 \right\}, \quad (\text{H.1})$$

(- for the $\Delta F = 0$ transition, + for the $\Delta F = 1$ transition). The modulations at frequencies $\omega_e \pm \omega_b$ originate in Stokes parameter U_3 of the laser polarization. $I(t)$, the intensity of the fluorescence beam, is to some extent noisy (mainly due to photon noise). Therefore $I(t)$ has to be regarded as a random function of time, i.e. :

$$\langle I(t) \rangle = I = \text{constant}, \quad \langle I(t) I(t') \rangle = \text{function of } (t-t').$$

Introducing the Fourier transform $\hat{I}(\nu)$ of $I(t)$ we thus have

$$\langle \hat{I}(\nu) \rangle = I \delta(\nu) \quad (\text{H.2})$$

$$\langle \hat{I}(\nu) \hat{I}^*(\nu') \rangle = \gamma(\nu) \delta(\nu - \nu'), \quad \gamma(\nu) \geq 0.$$

The reality of $I(t)$ implies $\hat{I}(\nu) = \hat{I}^*(-\nu)$. (H.3)

In the lock-in amps the signal $\mathcal{J}(t)$ is first multiplied by a reference $\pm \sin(\omega_e \pm \omega_f)t$ ⁽¹⁴⁾, then integrated.

The Fourier transforms $\hat{S}_{\pm}(\nu)$ of the output signals $S_{\pm}(t)$ are easily computed. For a $\Delta F = 0$ transition, we obtain

$$\begin{aligned} \hat{S}_{\pm}(\nu) = \pm \frac{1}{2} G(\nu) \times & \left\{ \left[\hat{I}(\nu - f \mp e) - \frac{1}{4} \hat{I}(\nu - e + b - f \mp e) - \frac{1}{4} \hat{I}(\nu + e - b - f \mp e) + \right. \right. \\ & + \frac{1}{4} \hat{I}(\nu - e - b - f \mp e) + \frac{1}{4} \hat{I}(\nu + e + b - f \mp e) \left. \right] - i \left[\hat{I}(\nu + f \pm e) - \frac{1}{4} \hat{I}(\nu - e + b + f \pm e) - \right. \\ & \left. \left. - \frac{1}{4} \hat{I}(\nu + e - b + f \pm e) + \frac{1}{4} \hat{I}(\nu + e + b + f \pm e) + \frac{1}{4} \hat{I}(\nu - e - b + f \pm e) \right] \right\} \end{aligned} \quad (\text{H.4})$$

Here e, b, f denote the frequencies $\omega_e, \omega_b, \omega_f$ respectively, i is the complex number ($i^2 = -1$) and $G(\nu)$ is the gain of the integrator.

To compute the noise at each frequency $\sigma_{\pm}^2 = \langle S_{\pm}^2(t) \rangle$, and the correlation product $\langle S_+(t) S_-(t) \rangle$, we use equations (H.2-4) and the fact that $G(\nu)$ is zero apart from a narrow band centred at frequency $\nu = 0$, i.e. in particular

$$\int_{-\infty}^{+\infty} G(\nu) h(\nu) d\nu \simeq h(0) \int_{-\infty}^{+\infty} G(\nu) d\nu = Ch(0).$$

We obtain

$$\left. \begin{aligned} \sigma_{\pm}^2 = \frac{C}{2} & \left[\gamma(e \pm f) + \frac{1}{16} \gamma(b + f) + \frac{1}{16} \gamma(b - f) \right. \\ & \left. + \frac{1}{16} \gamma(b + f \pm 2e) + \frac{1}{16} \gamma(b - f \mp 2e) \right] \end{aligned} \right\} \quad (\text{H.5}) \quad (15)$$

$$\langle S_+(t) S_-(t) \rangle = -\frac{C}{32} [\gamma(b + f) + \gamma(b - f)]$$

⁽¹⁴⁾ For the definition of the phases, see Eq. (2.8) in Part I.

⁽¹⁵⁾ For the $\Delta F = 1$ transition the signs before the coefficients in (H.4) are opposite. Results given by (H.5) are not altered.

Since $C > 0$ and $\gamma > 0$, the correlation coefficient

$$r = \langle S_+(t) S_-(t) \rangle / \sigma_+ \sigma_- < 0$$

is non-zero and negative. In the simple case of white noise ($\gamma(\nu) = \text{constant}$) it takes on the value -0.1 .

H.2. Independently of the above calculation, we consider the half-sum $\mathcal{A}$ and the half-difference $\mathcal{D}$ of the two outputs :

$$\mathcal{A}(t) = [S_+(t) + S_-(t)]/2$$

$$\mathcal{D}(t) = [S_+(t) - S_-(t)]/2.$$

From the cross-product

$$\langle \mathcal{A}(t) \mathcal{D}(t) \rangle = (\sigma_+^2 - \sigma_-^2)/4,$$

we obtain the correlation coefficient of $\mathcal{A}$ and $\mathcal{D}$

$$r(\mathcal{A}, \mathcal{D}) = (\sigma_+^2 - \sigma_-^2) / 4 \sigma_{\mathcal{A}} \sigma_{\mathcal{D}}. \quad (\text{H.7})$$

This quantity clearly vanishes if the noise power is exactly the same at the two frequencies. If not, it can be predicted simply from the observed noise levels σ_+ , σ_- , $\sigma_{\mathcal{A}}$ and $\sigma_{\mathcal{D}}$.

Note added in proof. — In a very recent work (C. Bouchiat and C. A. Piketty, *Europhysics Lett.* **2** (1986) 511), a new method is used to compute E_1^{PV} . The result $E_1^{\text{PV}} = 0.93 \times 10^{-11} \text{ i } |e| a_0 Q_w / N$ agrees with equation (5.15). It is affected by an uncertainty of empirical origin (2 %) and by a theoretical uncertainty (3 %, which ultimately could be pushed down to the 1 % level). Simultaneously V. A. Dzuba *et al.* have improved the accuracy of their calculation and announced 0.89 with only 2 % uncertainty (Adriatico Research Conference on Relativistic Many-Body Problems (Trieste, July 1986) to be published in *Physica Scripta*.)

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$$D(\eta) = (y_1 - y_2) (d_1^2 + d_2^2)^{-1/2}$$
 is considered to be sampled from a standard normal distribution. Plotted in figure 24 for each value of η is the probability p for a normal random variable that its modulus equals or exceeds the observed value $|D(\eta)|$.
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