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HAL Id: jpa-00210347
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Submitted on 1 Jan 1986

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Influence of Dzyaloshinsky-Moriya interactions on the critical behaviour of metallic spin-glasses

N. de Courtenay, H. Bouchiat, H. Hurdequint and A. Fert
Laboratoire de Physique des Solides, 91405 Orsay Cedex, France

(Reçu le 14 mars 1986, accepté le 13 mai 1986)

Résumé. — Nous présentons des mesures d’aimantation non linéaire en fonction du champ à différentes températures $T \sim T_g$ effectuées sur des alliages CuMn 0,2 % et AgMn 0,5 % dopés avec différentes concentrations d’or de manière à faire varier leur anisotropie. Nous montrons que l’anisotropie doit être considérée comme un paramètre relevant si l’on veut obtenir une description complète et cohérente du comportement critique de ces systèmes. Nous avons déterminé un ensemble d’exposants critiques Ising valables dans les bas champs et près de $T_g$, et un ensemble d’exposants critiques Heisenberg pour les hauts champs et loin de $T_g$. Les valeurs du champ et de la température caractéristiques du passage d’un régime à l’autre augmentent toutes deux avec l’anisotropie du système ; ces variations conduisent à des lois d’échelles dont nous tironns les exposants critiques.

Abstract. — In this paper, we present non-linear magnetization measurements as a function of field at different temperatures $T \sim T_g$ for CuMn 0.2 % and AgMn 0.5 % alloys doped with various amounts of gold impurities in order to change the anisotropy of the systems. We show that one has to consider anisotropy as a relevant scaling parameter in order to obtain a complete coherent description of the critical behaviour of these systems. In particular we can define a set of Ising critical exponents in the range of small fields and temperatures near $T_g$ and a set of Heisenberg exponents in the range of high fields and temperatures far from $T_g$. The values of the field and temperature characteristic of the occurrence of the crossover between the two regimes both increase as a function of anisotropy; this increase involves power laws from which we determine the exponents.

1. Introduction.

For several years experimentalists have been striving to answer the question of whether there is a spin-glass phase transition. One approach widely adopted is to study the non-linear magnetization which mean field theories predict to diverge at the transition temperature $T_g$ in the same way as the order parameter $\langle M \rangle$. All experiments agreed in showing such a divergence. Unfortunately, when it came to derive, from scaling analysis the critical exponents governing the divergence, the results differed from one system to another, thus impeding any clearcut conclusion regarding the point at issue [2]. It has been recently suggested [3], however, that it could be that differences in the ranges of field and temperatures in which the critical exponents are determined are responsible for such a dispersion among the results for at least two reasons :

1.1 The regular terms present in the description of the magnetization may yield higher apparent values

\[ \tau = \frac{T - T_g}{T_g} > 0.1 \]

1.2 The Dzyaloshinsky-Moriya (DM) anisotropy present in metallic spin glasses may induce a crossover from one type of critical behaviour at low fields, to another at higher fields with different values of the critical exponents.

Torque measurements on CuMnAu alloys [4] have already shown that DM anisotropy makes the critical line in the $(H, T)$ plane Ising-like at low fields; a Heisenberg-like critical line reappears above a threshold field, which increases with the concentration of gold, i.e. with the strength of the DM anisotropy. More recently Yeshurun and Sompolinsky [5] have shown that the non-linear susceptibility is also affected by the presence of anisotropy : the scaling law observed in CuMn undergoes deviations at low field when gold impurities are added to enhance the anisotropy. They have ascribed these deviations to an...
anisotropy-induced crossover from an Ising to a Heisenberg critical behaviour. However, their limited accuracy in the low-field regime prevented them from characterizing the Ising critical behaviour and from determining the exponent describing the anisotropy-induced crossover between the two regimes.

The purpose of this paper is to investigate in more detail the phenomenon observed by the previous authors [5]. To this end, we have measured the non-linear magnetization as a function of field at different temperatures \( T \geq T_g \) of CuMn 0.2 \% and AgMn 0.5 \% alloys which were doped with various amounts of gold impurity so as to change the anisotropy of the systems. We are able to perform accurate non-linear measurements between 10 G and 10 kG. Our results show that one must distinguish between two different regimes, each with its own set of critical exponents: a low-field regime characteristic of an Ising type of critical behaviour and a high-field regime characteristic of a Heisenberg type of critical behaviour. We find that the Ising exponents are smaller and closer to the mean-field values than the Heisenberg exponents and that the crossover field between the two regimes increases with the anisotropy. Renormalizing the magnetic field by the anisotropy-dependent crossover field of each sample and the non-linear susceptibility by another anisotropy-dependent parameter, we were able to describe with one unique curve the behaviour of the non-linear susceptibility versus field at \( T_g \) for all our samples. Our work reconciles the scattered data on CuMn [6] and AgMn [3] systems obtained in different ranges of field and temperature. We hope it will also clarify the situation concerning the critical exponents observed in other spin glasses.

2. Experimental method.

We have studied CuMn 0.2 \% and AgMn 0.5 \% alloys which were doped with various concentrations of gold impurity: \( C_{Au} = 0, 0.06, 0.3 \) at \% in CuMn and \( C_{Au} = 0, 0.1, 0.25, 3 \) at \% in AgMn. The samples were prepared by twice melting the starting materials (usually a master alloy of CuMn or AgMn and pure Cu, Ag or Au metals) in an induction furnace in a quartz crucible under argon. The CuMn 0.2 \% Au \% samples were annealed 24 h at 900 °C in vacuum [7]. Cylinders 6 mm in diameter and about 10 mm long where machined out of the ingots.

The transition temperature of each alloy was derived from both the position of the ac-susceptibility cusp (\( f = 10 \) Hz) and from the maximum in the low-field dc-magnetization. The two criteria gave the same value of \( T_g \) within ± 1 \%. We present a list of our samples with their transition temperatures in table I.

The magnetization measurements were made in a Foner vibrating-sample magnetometer having a sensitivity of \( 10^{-6} \) emu. The field range required for our analysis (zero to 10 kG) led us to use two different magnetic field set ups. In our low-field device (10 G to 1.5 kG), the magnetic field is produced by a pair of Helmholtz coils. The earth’s magnetic field is balanced out along the coil axis. By measuring the current in the Helmholtz coils using a high-precision shunt we determine the magnetic field to better than \( 10^{-4} \) above 10 G. The measurement of the non-linear contributions down to \( 10^{-3} \) of the linear magnetization is made possible by compensating out the linear part of the magnetization (the principle of this technique has already been described in reference [3]). In the high-field set up (500 G to 10 kG), the magnetic field is created by an electromagnet and measured with a Hall-effect probe to an accuracy of 1 G. As in this range of fields the non-linear magnetization \( (M_{NL}) \) can reach more than 50 \% of the total magnetization, there was no need to compensate the linear magnetization. The low-and high-field measurements on each sample were joined together by adjusting the high-field points to coincide with the low-field ones in the overlapping field range \((H = 500 \text{ G to } 1.5 \text{ KG})\). The low-field experiment was taken as the reference since it gives a more accurate estimation of \( M_{NL} \).

The samples were in a helium bath, and the temperature was stabilized in the range 2-4.2 K to better than 0.1 \%. The alloy concentrations in this work were chosen so that \( T_g \) fell in the right range for this type of precise temperature control.

Data acquisition and magnetic field runs were made via a micro-computer.

3. Experimental results at \( T_g \) and analysis.

For each sample the temperature was first stabilized at some value \( T \geq T_g \); the field was then gradually increased from zero to 1.5 kG, or 10 kG, depending on the setup used. Typical sets of curves yielding the variations of \( \Delta M/H \) as a function of \( H \) at different temperatures are shown in figure 1 (\( \Delta M \) stands for the compensated magnetization). It can be readily observed from the raw data that at fixed field the absolute value of the non-linear magnetization is systematically smaller in AgMn 0.5 \% Au 1 \% than in AgMn 0.5 \%.

At \( T \geq T_g \), spin glasses can be described by one unique order parameter \( q = \langle S_i \rangle^2 \) (where \( \langle \rangle \) and \( \langle \rangle \) are respectively the thermal and the disorder averages). It has been shown that the field and temperature dependence of the non-linear magnetization contains the singular behaviour of \( q(H^2, T) \) in the neighbourhood of the transition temperature \( T_g \). We have thus analysed our measurements of the non-linear magnetization as a function of field and temperature in the framework of the scaling hypothesis:

\[
M_{NL}/H \propto h^{2/\beta} f(h^2/\tau^4)
\]

where \( \tau = (T - T_g)/T_g \), \( h = H/T_g \), \( \phi = \beta + \gamma \) and \( \delta = \phi/\beta \) (in the mean field case \( \beta = \gamma = 1 \)).
Table I. — In table I are given for each sample studied:
1. The value of $T_g$. Small variations in the Mn concentrations can be partly responsible for the differences in the values of $T_g$ among the CuMn 0.2 % Au and the AgMn 0.5 % Au samples. However the slight increase of $T_g$ with the gold concentration is in agreement with the results of Vier and Schultz [23].
2. The values $\Delta \log H$ and $\Delta \log K_{NL}$ of the translations, respectively along the $\log H$ and the $\log K_{NL}$ axis, necessary to make the curve of the considered sample coincide with that of the AgMn 0.5 % sample.
3. The values of the crossover field $H^*(d)$ and of $a^*(d)$ defined by $H^*(d) = H^*(\text{AgMn}) 10^{\Delta \log H}$ and $a^*(d) = a^*(\text{AgMn}) 10^{\Delta \log K_{NL}}$.
4. The values of the anisotropy parameter $d = \langle D_0^2 \rangle / \langle J_0^2 \rangle$, evaluated as depicted in the text.

<table>
<thead>
<tr>
<th></th>
<th>$T_g$ K</th>
<th>$\Delta \log H^*$</th>
<th>$\Delta \log K_{NL}$</th>
<th>$H^*(d)$ G</th>
<th>$a^*(d)$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CuMn 0.2 %</td>
<td>2.70 ± 0.03</td>
<td>-0.8</td>
<td>-0.45</td>
<td>60 ± 10</td>
<td>0.35 ± 0.05</td>
<td>0.036</td>
</tr>
<tr>
<td>CuMn 0.22 % Au 0.059 %</td>
<td>2.69</td>
<td>-0.215</td>
<td>-0.18</td>
<td>230 ± 40</td>
<td>0.74 ± 0.1</td>
<td>0.086</td>
</tr>
<tr>
<td>CuMn 0.2 % Au 0.37 %</td>
<td>2.81</td>
<td>+0.48</td>
<td>+0.08</td>
<td>1 150 ± 200</td>
<td>1.2 ± 0.2</td>
<td>0.209</td>
</tr>
<tr>
<td>AgMn 0.5 % Cu 0.51 %</td>
<td>2.70</td>
<td>0</td>
<td>0</td>
<td>380 ± 80</td>
<td>1</td>
<td>0.095</td>
</tr>
<tr>
<td>AgMn 0.5 % Cu 0.1 %</td>
<td>2.58</td>
<td>0</td>
<td>0</td>
<td>380 ± 80</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>AgMn 0.54 % Au 0.24 %</td>
<td>3.09</td>
<td>+0.76</td>
<td>+0.24</td>
<td>2 200 ± 440</td>
<td>1.75 ± 0.3</td>
<td>0.239</td>
</tr>
<tr>
<td>AgMn 0.52 % Au 3.1 %</td>
<td>3.10</td>
<td>+0.85</td>
<td>+0.28</td>
<td>2 700 ± 540</td>
<td>1.90 ± 0.3</td>
<td>0.81</td>
</tr>
</tbody>
</table>

$f$ is a function of the variable $x = h^2/\tau^g$ which verifies:

$$\lim_{x \to 0} f(x) = x^{7/6} \quad \text{lim} \quad \frac{M_{NL}}{H}(h, \tau) \sim h^{2/\tau^g} \text{ when } h^2 \ll \tau^g$$

and

$$\lim_{x \to \infty} f(x) = 1 \quad \text{lim} \quad \frac{M_{NL}}{H}(h, \tau) \sim h^{2/\tau^g} \text{ when } H = T_g$$

In figures 2a and 2b, the quantity $K_{NL} = M_{NL}/\chi_0 H$ (where $\chi_0$ is the linear susceptibility, and where $M_{NL}$ is obtained via the extrapolation of $\Delta M/H$ to $H$ equal zero : $M_{NL} = \Delta M/H - \lim_{H \to 0} \Delta M/H$) is plotted versus the magnetic field on a double logarithmic scale for each sample, between 10 G and 10 kG. In figure 2a, for display purposes, we have arbitrarily shifted the curves along the $\log K_{NL}$ axis while maintaining their relative positions; the original figure shows the same features as figure 2b.

1. At high fields all the curves coincide. They gradually move apart at lower fields; the higher the concentration of gold, the lower the absolute value of $K_{NL}$.
2. All curves show a continuous curvature with two asymptotic behaviours. The remarkable feature displayed by figure 2a and 2b is that the low field asymptotic behaviour on the one hand and the high field asymptotic behaviour on the other are described for all samples by two sample independent slopes ($s_L$ and $s_H$). We find:

- in low fields: $s_L = 0.64 \pm 0.02$
- in high fields: $s_H = 0.44 \pm 0.02$.

However, the field extension of both asymptotic behaviours depends on the sample; it is clear in figures 2a and 2b that the « low field » asymptotic behaviour extends to higher fields when the gold concentration is increased.
Fig. 1. — Set of curves yielding $\Delta M/H$ ($\Delta M$ denotes the magnetization after compensation of the linear part) versus $H$, measured for identical values of $\tau = (T - T_g)/T_g$ on the AgMn 0.5 % and the AgMn 0.5 % Au 0.1 % samples. Note that the non-linear magnetization is systematically smaller in AgMnAu than in AgMn. The scales have been arbitrary shifted along the $Y$ axis for each value of $\tau$.

It obviously follows from the foregoing that a single power law of $H$ cannot fit $K_{NL}$ over the entire range of fields (10 G to 10 kG) : two different scaling laws in two different field ranges are required :

- From 10 G to a certain field $H_1$ characteristic of the sample :
  \[ K_{NL} \propto H^{\delta_L}. \]

In the spirit of the scaling hypothesis we identify $s_L$ with $2/\delta_L$ ($\delta = \delta_L$ in (3)). We find :
  \[ \delta_L = 2/s_L = 3.1 \pm 0.2. \]

This regime will be referred to as the « low-field regime ».

- At $H_1$, $K_{NL}$ deviates from the power law (4) and can no longer be described by a power law up to a higher field $H_2$ (also characteristic of the sample), above which a straight line is recovered.

- From $H_2$ to at least 10 kG (our highest field)

$K_{NL}/H$ again follows a power law :
  \[ K_{NL} \propto H^{\delta_H} \equiv H^{2/\delta_H} \]

with $\delta_H = 2/s_H = 4.5 \pm 0.2$.

This regime will be called the « high-field regime ».

The crossover between the two regimes is quite smooth and stretches over approximately a third of a decade in field for all our samples.

The value of $\delta_L$ is in agreement with the experimental determinations of $\delta$ by H. Bouchiat on AgMn [3] and Simpson on CuMnAl [8] in a range of fields where $K_{NL} < 0.1$.

On the other hand, the value of $\delta_H$ is in agreement with the determination of $\delta$ by Barbara et al. on CuMn 4.6 % [9] and by Omari et al. on CuMn 1 % [6] in a range of field where $K_{NL} > 0.1$.

As already suggested above, the only difference between the curves presented in figures 2a and 2b is...
that the crossover from the low-field to the high-field regime is shifted towards higher fields as the concentration of Au impurities increases. When the curves of figures 2a and 2b are shifted with respect to one of them, which is taken as a reference, both along the field and the non-linear susceptibility logarithmic axes, by values $\Delta \log H$ and $\Delta \log K_{NL}$ depending on each sample, all curves are brought onto one unique universal curve. In figure 3a we show the universal curve obtained when AgMn 0.5% is taken as the reference sample. (It should be noticed (Fig. 3a) that this procedure establishes a definite order among our samples, which will be explained in the following.) The values of $\Delta \log H$ and $\Delta \log K_{NL}$ corresponding to each sample are given in table I. These values enabled us:

1. To define a crossover field $H^*$ between the two regimes for each sample in the following way:

$$H^* = H^*(\text{AgMn 0.5%}) \times 10^{\Delta \log H}$$

where $H^*(\text{AgMn 0.5%})$ has been arbitrarily defined as the value of the field at which the two straight lines of slope $s_L$ and $s_H$ intersect for this particular sample.

2. To define for each sample the value $\alpha^*$ renormalizing the curves along the $K_{NL}$ axis:

$$\alpha^* = 10^{\Delta \log K_{NL}}$$

where $\alpha^*(\text{AgMn 0.5%})$ has been taken to 1.

The values of $H^*$ and $\alpha^*$ for each sample are given in table I. The results in figure 3a can thus be summarized by:

$$K_{NL} = \alpha^* f(H/H^*)$$

where the function $f$ is sample independent with $\lim_{x \to 0} f(x) = x^{1/(\delta_L)}$ and $\lim_{x \to +\infty} f(x) = x^{1/(\delta_H)}$.

Finally, it is important to note that adding 0.5% of copper to AgMn does not produce any shift in the crossover field of $K_{NL}(H) = f(H)$. This strongly suggests that the effect we have seen is closely related to the nature of the impurity added and is not simply a mean-free-path effect.

4. Interpretation of the results.

Adding gold (or any other impurity giving rise to strong spin orbit scattering of the conduction electrons) enhances the strength of the anisotropic Dzyaloshinsky-Moriya interactions. These DM interactions contribute to the exchange Hamiltonian in the following way [10]:

$$\mathcal{H}_{DM} \propto \sum_{i,j,k} V_{ijkl} \mathbf{S}_i \times \mathbf{S}_j$$

where $V_{ijkl}$ depends on both the matrix and the spin-orbit scatterer. The DM interactions couple the spins $\mathbf{S}_i$ and $\mathbf{S}_j$ to the lattice when the quantity $\mathbf{S}_i \times \mathbf{S}_j$ is non-zero. The system is thus isotropic in the paramagnetic regime far from $T_S$ and the anisotropy is induced by the correlations among the spins whose characteristic range is related to the divergence of the non-linear susceptibility in the neighborhood of $T_S$.

In this sense the DM anisotropy is very different from an anisotropy of magneto-crystalline origin; furthermore, the DM anisotropy is unidirectional [11] in contrast to the crystalline anisotropy which is uniaxial.

Following Kotliar and Sompolinsky in [12] we characterize the anisotropy to exchange ratio by the parameter:

$$d = D/J$$

where

$$J = k_B T_S = \sqrt{N} \langle J_0^2 \rangle^{1/2}$$

$$D = \sqrt{N} \langle |D_{ij}|^2 \rangle^{3/2}.$$  

According to Bray and Moore [13] and Levy et al. [14] $J$ and $D$ can be expressed as:

$$J = V_o/a^2 \sqrt{C_{MN}}$$

$$D \approx \frac{1}{d^2} \left[ \ln \frac{R_{ij}}{R_0} + \frac{1}{2} \right]^{1/2} \left[ (V_{1}^{MN} C_{MN})^2 + (V_{1}^{Au} C_{MN} C_{Au}) \right]^{1/2}$$
where $C_{\text{Mn}}$ and $C_{\text{Au}}$ are the concentrations of Mn and Au, $R_c \approx 42 \text{Å}, r_0 = 2.55 \text{Å}$ in CuMn, $R_c \approx 39 \text{Å}$, and $r_0 = 2.89 \text{Å}$ in AgMn. From equations (8) and (9):

$$
d = \left[ \ln \left( \frac{R_c}{r_0} \right) + \frac{1}{2} \right] \left[ \left( \frac{V_{1\text{Me}}}{V_0} \right)^2 + \left( \frac{V_{1\text{Au}}}{V_0} \right)^2 \times \right.
$$

$$
\times \frac{C_{\text{Au}}}{C_{\text{Mn}}} \left. \right]^{1/2}. \tag{11}
$$

For the CuMnAu alloys we have derived $d$ from equation (11) by using the values of $V_{1\text{Me}}/V_0$ and $V_{1\text{Au}}/V_0$ calculated by Levy et al. [14] and Goldberg et al. [15], i.e. $(V_{1\text{Me}}/V_0)_{\text{CuMn}} = 2 \times 10^{-2}$ and $(V_{1\text{Au}}/V_0)_{\text{CuMn}} = 8.4 \times 10^{-2}$. For the AgMn alloys there do not exist previous determinations of $V_{1\text{Me}}(\text{AgMn})$ and $V_{1\text{Au}}(\text{AgMn})$. To obtain these parameters, we have performed ESR experiments. It is well known that, below $T_g$, the shift of the resonance is related to the macroscopic anisotropy energy $K$ and the magnetization $M$ by [16]:

$$
\Delta H \sim K/M
$$

while, in alloys containing Mn and Au impurities, $K$ is related to the DM constants $V_{1\text{Me}}$ and $V_{1\text{Au}}$:

$$
K \sim \left( V_{1\text{Me}}^2 C_{\text{Mn}}^2 + V_{1\text{Au}}^2 C_{\text{Mn}}C_{\text{Au}} \right) k_B T_g.
$$

By comparing the shift of the resonance at 4.2 K in the alloys CuMn 1%, AgMn 2.6%, and AgMn 2.6% Au 0.5%, which have approximately the same transition temperature 10.5 K, we have obtained:

$$(V_{1\text{Me}}/V_0)_{\text{AgMn}} \approx 2.64 (V_{1\text{Me}}/V_0)_{\text{CuMn}} \approx 5.28 \times 10^{-2}
$$

$$(V_{1\text{Au}}/V_0)_{\text{AgMn}} \approx 3.46 (V_{1\text{Au}}/V_0)_{\text{AgMn}} \approx 18.3 \times 10^{-2}.
$$

We have calculated the values of $d$ listed in Table I by introducing these values of $(V_{1\text{Me}}/V_0)_{\text{AgMn}}$, $(V_{1\text{Me}}/V_0)_{\text{CuMn}}$ and the above mentioned values of $V_{1\text{Me}}/V_0$ in CuMn and $(V_{1\text{Au}}/V_0)_{\text{CuMn}}$ in equation (11).

We show in figure 3b the variation of $H^*$ and $\alpha^*$ defined in (6) and (7) as a function of the estimated value of $d$ for all the alloys studied. One can see that, except for the AgMn 0.5% Au 0.5% sample, the enhancements of $H^*$ and $\alpha^*$ with $d$ can both be described by power laws:

$$
H^*(d) \propto d^{x_h} \quad \text{with} \quad x_h = 1.7 \pm 0.1 \tag{12}
$$

$$
\alpha^*(d) \propto d^{x_a} \quad \text{with} \quad x_a = 0.9 \pm 0.1 \tag{13}
$$

This yields a description at $T_g$ of the universal function displayed in figure 3a in terms of a scaling function of two parameters:

$$
T = T_g : K_{\text{NL}}(h, d) = d^{x_a} f(x) \tag{14}
$$

with $x = (h/d^{2x_h})^2$, $\lim f(x) = x^{1/\delta_h}$, which is the limit where the effect of the anisotropy dominates the effect of the magnetic field and $\lim f(x) = x^{1/\delta_h}$, which is the limit where the effect of the anisotropy dominates the effect of the magnetic field and $\lim f(x) = x^{1/\delta_h}$, which is the limit where the effect of the anisotropy dominates the effect of the magnetic field and $\lim f(x) = x^{1/\delta_h}$, which is the limit where the effect of the anisotropy dominates the effect of the magnetic field and $\lim f(x) = x^{1/\delta_h}$, which is the limit where the effect of the anisotropy dominates the effect of the magnetic field and $\lim f(x) = x^{1/\delta_h}$, which is the limit where the effect of the anisotropy dominates the effect of the magnetic field and $\lim f(x) = x^{1/\delta_h}$, which is the limit where the effect of the anisotropy dominates the effect of the magnetic field and $\lim f(x) = x^{1/\delta_h}$, which is the limit where the effect of the anisotropy dominates the effect of the magnetic field and $\lim f(x) = x^{1/\delta_h}$, which is the limit where the effect of the anisotropy dominates the effect of the magnetic field.

By analogy with the anisotropy-induced crossovers observed in classical phase transitions [17], we can interpret $\delta_L$ and $\delta_H$ as being respectively the Ising (1) and Heisenberg values of the critical exponent $\delta$ for the spin-glass phase transition in 3-dimensions. We note that the Ising value $\delta_L = 3.1 \pm 0.2$ is closer to the mean-field value $\delta = 2$ than is the Heisenberg value $\delta_H = 4.5 \pm 0.2$.

We have already noticed that in the limit of high fields $K_{\text{NL}}(h, d)$ is independent of $d$. This implies the following relation between the exponents:

$$
x_h = 2 x_a / \delta_H. \tag{15}
$$

From (12) and (5) we deduce $2 x_a / \delta_H = 0.8 \pm 0.1$ which ensures the validity of (15) and is a check of}

---

(1) The term « Ising » is not quite appropriate since the DM interactions have a unidirectional character. We have here conformed to the terminology of Kotliar and Sompolinsky in [12] where it refers to the strong-anisotropy regime in which the spin components are mixed through the random-anisotropic interactions.
the self-consistency of our description. We have, however, to explain why the scaling does not seem to hold for the most anisotropic of our samples: AgMn 0.5% Au 3%. The extrapolation of the results for Au 0.25% 0.1 and 0% samples would yield a value for $H^*$ equal to 12 kG. Deviations from the Ising behaviour are already observed above 3 kG. We attribute these deviations to higher-order terms than $H^{2/d_L}$ in the expansion of $K_{NL}(H)$. We have shown in [3] in the exactly soluble mean-field equations that the presence of such terms are at the origin of an apparent crossover in the field dependence of $K_{NL}(H)$ at $T_*$ which is very similar to the crossover we observe here. The existence of such corrections to the scaling behaviour (which may be important as soon as $K_{NL}$ is not negligible compared to 1) may also spoil the determination of the Heisenberg exponents, which are always measured in a range of field and temperature where $K_{NL} > 0.1$ and $(T - T_*)/T_* > 0.1$.

5. Generalization at $T > T_*$

The results at $T_*$ suggest that the whole scaling of the non-linear magnetization involves three parameters: $h$, $\tau$ and $d$ as has already been suggested in [5] :

$$K_{NL}(d, \tau, h) = h^{2/d_L} g(h^2/\tau^{\phi_H}, h^2/d^{\phi_H}) \cdot (16)$$

The behaviour at $T_*$ is indeed :

$$K_{NL}(d, 0, h) = h^{2/d_L} f(h^2/\phi_H) \cdot (17)$$

(17) is equivalent to (14) when $\phi_A \phi_H = 2 x_h$.

The behaviour far from $T_*$ (i.e. $h^2/\phi_H \ll 1$) is given by :

$$K_{NL} \approx h^{2/d_L} d^{\phi_H} \cdot (18)$$

$$K_{NL} \approx h^{2/d_L} d^{\phi_H} \cdot (19)$$

The temperature dependence of the quadratic term in the field dependence of $M_{NL}(H) = f(H)$ at the inflexion point. These quantities are displayed in figure 4 as a function of $\tau = (T - T_*)/T_*$ on AgMn 0.5% and AgMn 0.5% Au 0.1% on a double logarithmic scale between $\tau = 2 \times 10^{-2}$ and $\tau = 0.4$. For the AgMn 0.5% system, as it has already been quoted in (3), $a(\tau)$ and $p(\tau)$ are well described between $T_*$ and 1.1 $T_*$ by :

$$a(\tau) \propto \tau^{-\gamma_A} \quad \text{with} \quad \gamma_A = 2.2 \pm 0.1 \quad (20)$$

$$p(\tau) \propto \tau^{-\psi_{HL}} \quad \text{with} \quad \psi_{HL} = 0.6 \pm 0.05 \cdot (21)$$

We have shown in [3] that $\psi_{HL}$ is related to the standard exponents by $\psi_{HL} = (\gamma_L - \beta)/2$.

Above 1.1 $T_*$, $a(\tau)$ and $p(\tau)$ are not described any more by (19) and (20) but can be fitted between 1.2 $T_*$ and 1.5 $T_*$ by power laws involving higher exponents :

$$a(\tau) \propto \tau^{-\gamma_H} \quad \text{with} \quad \gamma_H = 3.3 \pm 0.3 \quad (22)$$

$$p(\tau) \propto \tau^{-\psi_H} \quad \text{with} \quad \psi_H = 1 \pm 0.1 \cdot (23)$$

For the AgMn 0.5% Au 0.1% system the expressions (20) and (21) with the same values of $\gamma_A$ and $\psi_{HL}$ properly describe the temperature dependences of $a(\tau)$ and $p(\tau)$ in the whole temperature range between $(T - T_*)/T_* = 2 \times 10^{-2}$ and $(T - T_*)/T_* = 0.4$.

We interpret our results by the existence of anisotropy-induced crossovers in the temperature dependences of $a(\tau)$ and $p(\tau)$. The position of these crossovers is shifted to higher temperatures when the anisotropy is increased from the AgMn 0.5% sample to the AgMn 0.5% Au 1% sample. Our limited reduced-temperature range of exploration (from $2 \times 10^{-2}$ to 0.5) does not enable us to determine directly $\phi_A$, but
we can estimate both the Ising exponents:
\[
\gamma_L = 2.2 \pm 0.1 \\
\beta_L = \gamma_L - 2 \psi_L = 1 \pm 0.1 \\
\phi_L = \beta_L + \gamma_L = 3.2 \pm 0.2 ,
\]
determined in the limit \( \tau \ll d^{\phi_L} \).

and the Heisenberg exponents:
\[
\gamma_H = 3.3 \pm 0.3 \\
\beta_H = 1 \pm 0.2 \\
\phi_H = \beta_H + \gamma_H = 4.3 \pm 0.5 ,
\]
determined in the limit \( \tau \gg d^{\phi_H} \).

The exponents we derive in the Ising regime are in agreement with those obtained on AgMn [3] and CuMnAl [8] at low temperatures \( \tau < 0.1 \). It is interesting to note that they are also in agreement with those obtained in the same conditions on random-anisotropy ferromagnets [19] (2).

Our Heisenberg exponents, on the other hand, are in agreement with the exponents \( \gamma \) and \( \beta \) determined on CuMn by Omari et al. [6] in the range of temperature where \( \tau > 0.1 \).

From the values of \( \phi_H \) and \( x_b \) determined above we can deduce from (18):
\[
\phi_L = 2 x_b / \phi_H = 0.8 \pm 0.1 . \quad (24)
\]

Similar experiments have recently been performed by Yeshurun and Sompolinsky [5] on the CuMn 1.2 at \% system doped with gold impurities \( (C_{Au} = 0, 1, 3 \%) \). They have measured the non-linear magnetization in the range of magnetic field between 500 G and 50 kG.

In the pure CuMn the non-linear magnetization at \( T_g \) is well described by:
\[
M_{NL} / H \propto H^{2/\delta_H}
\]
where \( \delta_H = 5.0 \pm 0.3 \) which is in agreement with our own high-field determination \( \delta_H = 4.5 \pm 0.3 \).

Systematic deviations of this behaviour towards lower effective values of \( \delta \) are observed when increasing the gold concentration. These observations are in qualitative agreement with our results. However, it is not possible to extract from these data the Ising behaviour of the non-linear magnetization. This may be attributed to, first, a lack of data in the low-field range and, second, a systematic error in the subtraction of the linear part of the magnetization, which is deduced from a measurement at \( H_{\text{min}} = 50 \) G.

6. Comparison with torque experiments.

The influence of the DM anisotropy on the transition line in the \((H, T)\) plane, \( H_g(T) \), has also been shown by the torque measurements of de Courtenay et al. [4] on CuMn alloys. These authors find that the equation of the transition line, \( H_g(T) \) changes above a crossover field \( H_e^{\ast} \) which depends on the concentration of gold, i.e. on the strength of the DM anisotropy. Below \( H_e^{\ast} \) the equation of \( H_g(T) \) is given by:
\[
H_e = \tau^\theta \quad \text{with} \quad \theta \sim 3/2 \\
\tau = (T_g - T) / T_g ,
\]
For \( H > H_e^{\ast} \) the critical line is vertical and verifies:
\[
\tau = \tau_e^{\ast} .
\]

The position of the crossover is shifted towards higher fields and lower temperatures by adding gold impurities. It has thus been suggested that this crossover is indeed a crossover from an Ising type of behaviour (at low fields near \( T_g \)) towards a Heisenberg type of behaviour in higher magnetic fields and at lower temperature.

These results are in agreement with the mean-field study of the Heisenberg spin glass with Dzyaloshinsky-Moriya interactions performed by Kotliar and Sompolinsky [20]. According to their prediction the low-field behaviour of the critical line has the form deduced by de Almeida and Thouless [21] for the Ising mean-field spin-glass. The high field behaviour of the critical line is the one calculated by Gabay and Toulouse [22] for the transverse freezing in the Heisenberg mean-field spin-glass. The crossover field \( H_e^{\ast} \) between the low- and high-field forms of the critical line is expected to vary as \( d^{3/2} \). Such a variation has indeed been observed by torque experiments [4]. The torque results are presented in figure 3b.

The agreement between the normalized values of \( H_e^{\ast}(d) \) and \( H_e^{\ast}(d) \) is rather striking; furthermore, the equation of the irreversibility line determined in the Ising regime is:
\[
H_g(t) \propto t^{\theta/2}
\]
where \( \theta = 3 \pm 0.3 \).

The prediction of Sompolinsky and Fischer [20] below \( d = 6 \) is indeed:
\[
\theta = \beta_L + \gamma_L ,
\]
in agreement with our determinations of \( \gamma_L \approx 2 \) and \( \beta_L \approx 1 \). However, the physical connection between the equation of the critical line \( H_g(T) \) and the critical exponents of the transition in zero field seems to us still very speculative and the agreement between our results and the predictions of [20] may be fortuitous and has to be confirmed. More puzzling is the interpretation of the high-field, the so called Heisenberg, regime. In our present analysis what we suggest to be the Heisenberg exponents are indeed determined in
a range of reduced temperatures and fields where the equation of the critical line determined in torque experiments is vertical: \( \tau_c = d \). Such a behaviour for the critical line is interpreted in [4] as characteristic of a crossover region rather than being the Heisenberg behaviour expected to take place in a higher range of field.

6. Conclusion.

In this work we have thus reached a coherent description of the scaling in the CuMnAu and AgMnAu alloys when the anisotropy is considered as a scaling parameter just like the temperature and the magnetic field. We were able to define a set of Ising exponents and a set of Heisenberg exponents corresponding to the respective limits; \( h \ll d^{\Delta_H} \), \( \tau \ll d^{\Delta_H} \) and \( h \gg d^{\Delta_H} \) or \( \tau \gg d^{\Delta_H} \). The crossover exponents \( \eta_H \) and \( \phi_A \) could be measured.

More work is needed, however, to understand what is suggested to be the Heisenberg regime, and its relation with the vertical critical line observed in Torque experiments.

Acknowledgments.

We are very indebted to I. A. Campbell, F. Hippert and P. Monod for stimulative discussions.

References

[7] It has been shown that AgMn alloys do not need any annealing to reach a good homogeneity of concentration through the sample. Chamberlin, R. V., Hardiman, M. and Orbach, R., J. Appl. Phys. 52 (1981) 1771.
[18] We have assumed in this analysis that according to the predictions of (12) the transition temperature corresponding to the Heisenberg behaviour is the same as the one determined in the Ising regime. There is however another prediction coming from Fisher, K. H., Z. Phys. B (1985) 151 which suggest on the contrary that the critical temperatures for the Ising and Heisenberg regimes do not coincide.

A shift of \( T_* \) as far as it verifies \( \Delta T_\ast \ll d^{\Delta_H} \) would however only affect the experimental determinations of \( \gamma_H \) and \( \delta_H \) by a quantity smaller than our accuracy on these exponents which are determined in high fields and far from \( T_* \). The fact that \( \gamma_H \) and \( \delta_H \) are found to be sample independant confirm this assumption.