Atomic parity violation measurements in the highly forbidden $6S_{1/2}$ - $7S_{1/2}$ caesium transition. II. Analysis and control of systematic effects

M.A. Bouchiat, J. Guena, L. Pottier

► To cite this version:

M.A. Bouchiat, J. Guena, L. Pottier. Atomic parity violation measurements in the highly forbidden $6S_{1/2}$ - $7S_{1/2}$ caesium transition. II. Analysis and control of systematic effects. Journal de Physique, 1986, 47 (7), pp.1175-1202. 10.1051/jphys:019860047070117500 . jpa-00210307

HAL Id: jpa-00210307

<https://hal.science/jpa-00210307>

Submitted on 1 Jan 1986

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Classification

Physics Abstracts

32.80 — 32.90 — 11.30E

Atomic parity violation measurements in the highly forbidden $6S_{1/2}$ - $7S_{1/2}$ caesium transition. II. Analysis and control of systematic effects

M. A. Bouchiat, J. Guena and L. Pottier

Laboratoire de Spectroscopie Hertzienne de l'ENS (*), 24, rue Lhomond, 75231 Paris Cedex 05, France

(Reçu le 5 décembre 1985, accepté le 14 mars 1986)

Résumé. — Le signal de violation de parité est discriminé par son comportement spécifique dans une inversion de la chiralité de l'expérience. Néanmoins, la conspiration d'imperfections expérimentales peut engendrer de faux signaux de parité. Nous établissons un modèle d'effets systématiques, dont les paramètres sont les imperfections dans les renversements de chiralité réalisés expérimentalement. Nous décrivons comment leurs valeurs réelles sont mesurées et minimisées, pour la plupart au moyen de contrôles continus de signaux atomiques pendant l'acquisition des données. Cela permet l'estimation et la réduction des effets systématiques possibles. Ceux-ci demeurent finalement au niveau ou en dessous de quelques pour cent de l'effet VP. Nous estimons des limitations possibles dans les contrôles en temps réel, ou dans le modèle lui-même (par exemple à cause de corrélations spatiales dans les imperfections). Ces limitations, ainsi que les incertitudes statistiques dans les contrôles, seront incluses dans l'incertitude systématique finale (Partie III).

Abstract. — The parity-violating signal is discriminated by its specific behaviour when the handedness of the experiment is reversed. Yet conspiracy of experimental imperfections can generate false parity signals. We establish a model of systematic effects, whose parameters are the imperfections in the handedness reversals realized experimentally. We describe how their actual values are measured and minimized, mostly by continuous control of atomic signals during data acquisition. This allows estimation and reduction of possible systematic effects. These remain finally at or below a few percent of the PV effect. We estimate possible limitations in the real-time controls, or in the model itself (e.g. due to spatial correlations in the imperfections). These limitations, together with statistical uncertainties in the controls, will be included in the final systematic uncertainty (Part III).

Introduction.

This paper is the second part of a detailed presentation of the measurements of parity violation (PV) in the Cs $6S$ - $7S$ transition performed at ENS in Paris [1]. Part I [2] presented the theoretical analysis and the experimental procedure and apparatus. The present part II is devoted to the problem of systematic effects. Part III will describe data acquisition and processing, and analyse the results and their implications.

We think that correcting the data for the systematic effects is not really satisfactory, since it requires confidence in the details of a model. Minimizing the imperfections of the set-up seems much safer. Here we present the reduction and control of the imperfections, and the estimation of systematic effects. This long and delicate work is in our opinion crucial : it was the only way to reduce each potential systematic effect

below a few percent of the observed PV effect, so as to avoid the need of corrections ⁽¹⁾.

As shown in part I, in the ideal situation where both geometry and apparatus are perfect, the PV signal differs from each of the present parity-conserving (PC) signals by at least *two* features (e.g. orthogonal directions, or opposite even/odd behaviour under reversal of a certain parameter). Consequently, in the real situation systematic effects appear only as the product $\delta_1 \delta_2$ of at least *two* small imperfections. This makes them second-order small. In addition, for given uncertainties $d\delta_1$ and $d\delta_2$ of the defects, the uncertainty $\delta_1 \cdot d\delta_2 + \delta_2 \cdot d\delta_1$ in the product is minimized with δ_1 and δ_2 . Thus reducing the defects rather than correcting for them in the final result acquires additional interest.

(*) Associé au CNRS.

⁽¹⁾ Recalling that the statistical rms uncertainty is $\approx 15\%$ in each measurement.

One leading idea of this work is the following : in order to achieve reliability in estimating the imperfections, we think that the most convincing way consists in using the *atoms themselves as a probe during the experimental run*. To this purpose we measure, continuously or periodically, atomic quantities, other than the PV signal, sensitive to these imperfections :

i) In many cases, deliberately increasing one imperfection δ_1 up to a large known value makes the systematic $\delta_1\delta_2$ measurable, so that δ_2 can be determined ; and conversely. (Such measurements are performed periodically, in auxiliary configurations.) Since we illustrated the above principles in the control of spurious electric fields [3], we have made constant intensive use of them. They have also been of use in PV experiments performed in thallium [4] and under progress in hydrogen [5].

ii) In addition, numerous atomic controls are continuously available owing to our polarization modulation, which gives continuous access to the response of the atoms to all possible states of the incident polarization [2].

A noteworthy feature of our experiment is to be free of background difficulty. The polarization analysis of the fluorescence and/or the selection of the incident polarization vector with respect to the favoured direction of the applied E-field, prove to be extremely efficient in eliminating the background (which originates in collisions or Cs_2 dimers). The minute residual background effects, considered here for completeness, are shown to play a negligible role.

An essential problem central to all atomic PV experiments performed so far is *the need of cleanly controlled light polarizations*. We believe that our experimental work corresponds to some progress : by providing a fast and clean exploration of all possible states of the incident polarization, our modulator ensures precise control of polarization defects in the optics. For instance we succeeded in measuring birefringences as small as 10^{-6} rad in the mirrors of our multipass cavity. More generally our analysis of light polarization modifications in the multipass cell can also concern quite different high precision experiments performed in similar optical cavities : e.g. gravity-wave detection in Garching [6] and detection of the vacuum polarization induced by a magnetic field at CERN [7].

To mitigate the rather technical aspect of the analysis presented here, the details of the procedures are relegated in appendices, only the physical gist is given in the text. The numbering of paragraphs, tables and figures continues that of part I.

3.1 GENERALITIES.

3.1.1 Building up the model. Contents. — In an attempt to list all potentially harmful imperfections, we start from the theoretical expressions of all PC

fluorescence signals analysed in part I in a configuration of complete generality : arbitrary directions of $\mathbf{E}$ (electric field), $\mathbf{k}$ (laser propagation direction), $\mathbf{k}_f$ (detection direction), arbitrary polarization vector $\mathbf{\epsilon}$, without and with H-fields (App. A and B in Part I). Then each experimental parameter is replaced by the sum of the ideal parameter plus a small defect parameter, and the general expressions are developed up to a sufficient order (App. D to G in present Part II). From this general analysis we derive sensitive procedures to control (and reduce) the relevant defects, either in the particular configuration of the PV measurement, or in auxiliary configurations using additional electric and/or magnetic fields.

Table VIIIa summarizes the origins and characteristics of the fluorescence signals which may generate systematic effects, i.e. all signals present in zero magnetic field and the most harmful signal induced by a stray H-field. For each PC signal (column 1) the table displays two (or more) criteria to discriminate it against the PV signal. Experimental defects affecting these criteria (Table VIIIb, column 3) conspire to make the PC signal generate a systematic effect (b, column 1). To each PC signal thus corresponds a systematic effect, of second (or higher) order in the imperfections. Table VIIIb actually contains all systematics considered in this work.

The main systematics originate in a PC component of the 7S electronic polarization. The first main class [8] is associated with the component ($\mathbf{P}^{(1)}$) independent of the helicity ξ of the laser polarization $\mathbf{\epsilon}$. It involves the conspiracy between spurious modulations in $\mathbf{\epsilon}$ and geometric imperfections in the beam multipass (§ 3.2, App. D-E). A closely related effect may arise from the component spuriously induced by hyperfine mixing in a stray H-field ($\mathbf{P}_{\text{hf}}^{(1)}$) and imperfectly cancelled in the multipass (§ 3.3). The second main class of systematics [3] is associated with the helicity-dependent components ($\mathbf{P}^{(2)}$ and $\mathbf{P}^{(0)}$). It involves the conspiracy of stray electric fields and misalignments or stray Hanle effects (§§ 3.4, 3.5; App. F-G).

Third-order effects appear when one allows for possible (instrumental or atomic) backgrounds (§ 3.5), or for perturbation of the unpolarized fluorescence signal by imperfections in the polarization analyser (§ 3.6). Finally we analyse possible systematics in the calibration procedure (§ 3.7).

In practice the imperfections δ may vary over the extension of the observed interaction region. Basically, our method amounts to replacing the true systematic effect $\langle \delta_1\delta_2 \rangle$ by the estimate $\langle \delta_1 \rangle \langle \delta_2 \rangle$. For this to be legitimate, the difference, i.e. the *correlation* of δ_1 and δ_2 , should be negligible. Correlation deserves particular attention when $\langle \delta_1 \rangle$ and/or $\langle \delta_2 \rangle$ *vanish artificially*, as the result of adjustments or compensation designed precisely to cancel them. On the contrary, if the average value $\langle \delta_1 \rangle$ of an imperfection spontaneously remains always below the noise level,

Table VIII. — a) Summary of origins and characteristics of the PV signal, and of the PC signals considered as potential sources of systematics. The criteria used to discriminate the PV signal against each PC signal are displayed. b) Designations and origins of systematic asymmetries analysed in the text. The combination of defects (column 3), source of each systematic, is deduced from the combination of criteria characterizing each PC signal of related name in table a)

(a) Fluo. signals	Interference	Associated polarization component & Criteria used for discrimination	Incident Stokes param. involved.
$\mathcal{F}^{PV}$	$\text{Im } E_1^{PV} \propto E$ $\text{Im } E_1^{PV} \propto \beta E$	$\mathbf{P}^{PV} \propto \xi \hat{\mathbf{E}} \times \hat{\mathbf{k}}$	U_2
$\mathcal{F}^{(1)}$ $\mathcal{F}_{hf}^{(1)}$	$M'_1 \propto E$ $M'_1 \propto \beta E$	$\mathbf{P}^{(1)} \propto \hat{\mathbf{E}} \times \hat{\mathbf{k}}$ $\mathbf{P}^{(1)} \not\parallel \hat{\mathbf{E}}_\perp (*)$ $\mathbf{P}_{hf}^{(1)} \propto (\mathbf{H} \cdot \hat{\mathbf{k}}) \hat{\mathbf{E}} \times \xi \hat{\mathbf{k}}$	U_0, U_3 U_0, U_1 U_2
$\mathcal{F}^{(2)}$ $\mathcal{F}^{(0)}$	$\alpha \beta E^2$ $\beta^2 E^2$ $M_1'^2$	$\mathbf{P}^{(2)} \propto \xi \hat{\mathbf{k}}$ $\propto (\hat{\mathbf{E}} \cdot \xi \hat{\mathbf{k}}) \hat{\mathbf{E}}$ $\mathbf{P}^{(0)} \propto \xi \hat{\mathbf{k}}$	U_2
$\mathcal{F}_u$	$\alpha^2 E^2$ $\beta^2 E^2$	None unpolarized signal	U_0, U_3

(b) Syst. asymmetry	PC source		Combination of defects
Signal	Polarization component		
$\mathcal{A}_S^{(1)}$	$\mathcal{F}^{(1)}$	$\mathbf{P}^{(1)}$	birefringences stray H-field
$\mathcal{A}_{S_{hf}}^{(1)}$	$\mathcal{F}_{hf}^{(1)}$	$\mathbf{P}_{hf}^{(1)}$	
$\mathcal{A}_S^{(2)}$ $\mathcal{A}_S^{(0)}$	$\mathcal{F}^{(2)}$ $\mathcal{F}^{(0)}$	$\mathbf{P}^{(2)} \not\parallel \mathbf{E}$ $\mathbf{P}^{(0)}, \mathbf{P}^{(2)} \not\parallel \hat{\mathbf{k}}$	misalignments & misreversal of $\mathbf{E}$ — in direction — in magnitude (& offset)
$\mathcal{A}_{S_u}$	$\mathcal{F}_u$	None	imperfect analysis of fluo polarization & birefringence & misreversal of $\mathbf{E}$ in magnitude

(*) $\hat{\mathbf{E}}_\perp$ represents a unit vector along the $\mathbf{E}$ -component normal to $\hat{\mathbf{k}}$.

the possibility that correlations with other imperfections δ_2 might be important seems rather unlikely. This will be discussed in each particular case. The correlation effects are accounted for as systematic *uncertainties*.

Hereafter we express each possible systematic effect in terms of measurable defect parameters. Then we

discuss the origin and magnitude of the imperfections, and present the methods used to control and reduce them during data acquisition.

3.1.2 Choice of coordinate axes. — Since the directions of physical significance (electric field $\mathbf{E}$, laser beam $\hat{\mathbf{k}}$, polarization $\boldsymbol{\varepsilon} \perp \hat{\mathbf{k}}$) depart from the ideal

configuration (§ 1.2.3 in Part I), and are slightly inhomogeneous, we introduce a *global* reference frame defined as follows (Fig. 13) : z is the multipass axis (line joining the centres of the ellipses drawn by the beam impacts on the mirrors); it is oriented like the incoming laser beam. y is the projection on a plane $\perp z$, of the normal to the plane of the main electrodes⁽²⁾. y is oriented so that the third direction x of the direct trihedral (x, y, z) points towards the detector. We see that each beam passage deviates little from $\pm z$. The direction of E at each point of the observation region is nearly along y , and the average detection direction deviates little from x .

In our analysis of systematic effects, the imperfections described in this global frame are inserted in the expressions of all fluorescence signals. Since the correct treatment is somewhat cumbersome, we adopt below a slightly simplified presentation. We express the true Stokes parameters of the laser beam (referred to axes linked to the *local* directions of the beam and field) in terms of approximate Stokes

Fig. 13. — Definition of the global reference frame xyz (not to scale).

parameters defined by the usual equations written in the *global* reference frame :

$$\left. \begin{aligned} u_0 &= |\varepsilon_x|^2 + |\varepsilon_y|^2 = \text{unpolarized intensity} . \\ u_1 &= 2 \operatorname{Re}(\varepsilon_x^* \varepsilon_y) = \text{linearly polarized intensity, axes at } 45^\circ \text{ to } x \text{ and } y . \\ u_2 &= 2 \operatorname{Im}(\varepsilon_x^* \varepsilon_y) = \text{circularly polarized intensity} . \\ u_3 &= |\varepsilon_x|^2 - |\varepsilon_y|^2 = \text{linearly polarized intensity, axes } \parallel x \text{ and } y . \end{aligned} \right\} \quad (3.1)$$

To first order in the small angles $|\hat{k} \times \hat{z}|$ and $\hat{E}(\mathbf{r}) \cdot \hat{x}$, the transformation from the true Stokes parameters to approximate ones is equivalent to a rotation of angle $\hat{E} \cdot \hat{x}$ around the z axis :

$$\left. \begin{aligned} u_0 &\leadsto u_0 \\ u_1 &\leadsto u_1 + 2(\hat{E} \cdot \hat{x}) u_3 \\ u_2 &\leadsto u_2 \\ u_3 &\leadsto u_3 - 2(\hat{E} \cdot \hat{x}) u_1 \end{aligned} \right\} \begin{array}{l} \text{parameters} \\ \text{in local frame} \end{array} \quad \begin{array}{l} \text{parameters} \\ \text{in global frame} \end{array}$$

For example, in the frequently used expression of the unpolarized fluorescence signal

$$\mathcal{F}_u = n_F [\delta_{FF} \alpha^2 |\mathbf{E} \cdot \boldsymbol{\varepsilon}|^2 + h_{FF} \beta^2 |\mathbf{E} \times \boldsymbol{\varepsilon}|^2 + h_{FF} M_1'^2 |\boldsymbol{\varepsilon}|^2], \quad (3.2a)$$

one must insert

$$\begin{aligned} 2|\hat{E} \cdot \boldsymbol{\varepsilon}|^2 &= u_0 - u_3 + 2(\hat{E} \cdot \hat{x}) u_1 \\ 2|\hat{E} \times \boldsymbol{\varepsilon}|^2 &= u_0 + u_3 - 2(\hat{E} \cdot \hat{x}) u_1 . \end{aligned} \quad (3.2b)$$

Comparison with the complete formulation shows that the error introduced by omitting higher orders does not alter the analysis of systematic effects. The physical reason is that the *handed* Stokes parameter u_2 which labels the PV signal, and the *unhanded* parameters u_0 , u_1 and u_3 , are not mixed at any order.

⁽²⁾ Or more rigorously to the bisector plane of the electrodes imperfectly parallel within 5×10^{-3} rad.

Note that the sign of the circular Stokes parameter u_2 (referred to the global frame) is the same for all passages, while the sign of the helicity

$$\xi = \operatorname{Im} \hat{\mathbf{E}}^* \times \hat{\mathbf{E}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{z}} u_2 / u_0 \quad (3.3)$$

(which refers to the forward or backward direction $\hat{\mathbf{k}}$ of the beam) is opposite in forward and backward passages.

3.2 EFFECT OF IMPERFECT POLARIZATION MODULATION. — In part I we have analysed all contributions to the polarized fluorescence signal. The parity-violating contribution $\mathcal{F}^{pv}$ (Eq. (1.15)) is proportional to ξ , i.e. to the circular Stokes parameter u_2 (Eq. (3.3)); it is written :

$$\mathcal{F}^{pv} \propto (E_1^{\text{ind}} \operatorname{Im} E_1^{pv}) u_2 \quad (3.4)$$

(where $E_1^{\text{ind}} \approx \alpha E$ or $= \beta E$ for a $\Delta F = 0$ or ± 1 transition respectively), while the PC contribution $\mathcal{F}^{(1)}$, independent of ξ (Eq. (1.17)), is a linear combination of the three unhanded Stokes parameters :

$$\begin{aligned} \mathcal{F}^{(1)} &\propto E_1^{\text{ind}} M_1(\hat{\mathbf{k}} \cdot \hat{\mathbf{z}}) \times \\ &\times \{ u_0 - (-1)^{\Delta F} [u_3 + (\hat{E} \cdot \hat{x} - \hat{\mathbf{k}}_f \cdot \hat{y}) u_1] \} \end{aligned} \quad (3.5)$$

(with the same normalization). In principle a specific modulation labels each Stokes parameter (Table XII in App. E), and discriminates $\mathcal{F}^{pv}$ against $\mathcal{F}^{(1)}$. In

addition $\mathcal{F}^{(1)}$, odd under beam direction reversal, is in principle cancelled in the multipass.

In practice this cancellation is imperfect. Moreover, optical defects of the multipass cell transfer into u_0 , u_1 and u_3 , a small amount of the modulation assigned to u_2 . As a result, a spurious contribution simulating $\mathcal{F}^{pv}$ is present in $\mathcal{F}^{(1)}$.

The modification of the beam polarization in the multipass cell has been studied thoroughly, both theoretically and experimentally [9]. The relation between the Stokes parameters U_i of the incident beam and the parameters u_j after a certain number of reflections, can be written :

$$\begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = M \begin{bmatrix} U_0 \\ U_1 \\ U_2 \\ U_3 \end{bmatrix} \quad (3.6a)$$

Matrix elements $M_{2,i}$ ($i = 0, 1$ and 3) are responsible for transfer of labelling from U_2 into u_i . Under assumptions carefully checked in our practical case [9], the real 4×4 transfer matrix M (known as the Mueller matrix) takes on a simple symmetric form with only 7 real degrees of freedom :

$$M = \mathbb{1} + \begin{bmatrix} \cdot & \cdot & w_0 & \cdot \\ \cdot & \cdot & w_1 & \cdot \\ w_0 & -w_1 & \cdot & -w_3 \\ \cdot & \cdot & w_3 & \cdot \end{bmatrix} \quad (3.6b)$$

($w_0, w_1, w_3 \ll 1$). Only the elements relevant to our problem are written here explicitly. Symmetric elements M_{ij} ($i \neq j$) are either equal ($ij = 0$) or opposite

($ij \neq 0$). This provides the useful possibility of controlling M_{ij} either directly or by measuring M_{ji} (§ 3.2.2). In the simple symmetric case each matrix element has a simple physical interpretation : w_0 stands for circular dichroism ; w_1 represents a birefringence of axes x and y , and of retardation w_1 ; similarly w_3 is the retardation of a birefringence whose axes are at 45° to x and y [9].

3.2.1 Expression of the systematic effect. — We now outline the derivation of the systematic asymmetry. The detailed analysis is given in appendix D. The definition of the parameters successively introduced hereafter are summarized in table IX.

In a first step we consider only the systematic effect occurring through u_3 in equation (3.5) (this turns out to be the main contribution), and we allow for *only one passage* of the beam, along either $+z$ or $-z$. According to (3.6), the Stokes parameter u_3 seen by the atoms is connected to the ideal Stokes parameters U_i at the output of the modulator ⁽³⁾ by :

$$u_3 = U_3 + w_3 U_2 (+ \text{ terms irrelevant here}). \quad (3.7)$$

$w_3 \ll 1$ describes stray birefringence of axes at 45° to x and y in the entrance window of the caesium cell.

⁽³⁾ The analysis presented here does not explicitly include imperfections of the modulator. These are small ($\approx 10^{-4}$; § 2.2). A more detailed analysis shows that their dominant systematic effect is cancelled by the automatic compensation of § 3.2.3.2, and that the systematic asymmetry really overlooked here does not finally exceed 0.1 % of the PV signal.

Table IX. — *Definition of the parameters involved in $\mathcal{A}_s^{(1)}$.*

U_i ($i = 0$ to 3)	Stokes parameters perfectly labelled.
w_0, b_0, f_0	circular dichroism of <u>w</u> indow, <u>b</u> ack & <u>f</u> ront mirrors.
w_1, b_1, f_1	birefringence of xy axes
w_3, b_3, f_3	birefringence at 45° } of <u>w</u> indow, <u>b</u> ack & <u>f</u> ront mirrors.
χ	geometrical misalignment : $(\hat{\mathbf{k}}_f \cdot \hat{\mathbf{y}} - \hat{\mathbf{E}} \cdot \hat{\mathbf{x}})$.
$\bar{\chi}$	average of χ taken over all passages.
$\tilde{\chi}$	suppression factors of geometrical origin.
$1/8$	
$\Delta^{(1)}$	contribution to $\mathcal{A}_s^{(1)}$ arising from correlation effects in the multipass.
$P_{\text{res},1}^{(1)}$	$\mathbf{E}_0$ -odd polarization ratio of $\begin{Bmatrix} U_1 \\ U_3 \end{Bmatrix}$ signature (from residual $\mathbf{P}^{(1)}$ subsisting in the multipass).
$P_{\text{res},3}^{(1)}$	
W_1	effective birefringences of window + Babinet compensator (with laser modulation adjusted by servo-loop).
W_3^{eff}	
w_{hf}	offset in measurement of W_1 (hf-mixing effect).

Inserting (3.7) in (3.5) we find in $\mathcal{F}^{(1)}$ a term

$$\propto (-1)^{\Delta F+1} (E_1^{\text{ind}} M_1) (\hat{\mathbf{k}} \cdot \hat{\mathbf{z}}) w_3 U_2,$$

which mimics $\mathcal{F}^{\text{pv}}$ (Eq. (3.4)). The ratio of the resulting asymmetry to the genuine PV asymmetry is thus :

$$\mathcal{A}S^{(1)}/\mathcal{A}^{\text{pv}} \approx (-1)^{\Delta F+1} (M_1/\text{Im } E_1^{\text{pv}}) (\hat{\mathbf{k}} \cdot \hat{\mathbf{z}}) w_3. \quad (3.8a)$$

Since $\text{Im } E_1^{\text{pv}}/M_1 \approx 0.5 \times 10^{-4}$, a birefringence w_3 of 10^{-4} rad would be enough to mimic the expected PV effect (in the single-passage case).

Now we allow for *one double-passage* of the laser beam. The factor $\hat{\mathbf{k}} \cdot \hat{\mathbf{z}}$ in (3.8a) predicts exact compensation between the forward passage and the backward one. This compensation is in fact limited, mainly by two causes :

i) Because of reflection losses, and also because the forward passage and the backward one slightly differ and are therefore viewed slightly differently by the detector, compensation is not strict, but amounts only to reduction by some large factor S (typically 200-2 000). So far we thus have :

$$\mathcal{A}S^{(1)}/\mathcal{A}^{\text{pv}} \approx (-1)^{\Delta F+1} (M_1/\text{Im } E_1^{\text{pv}}) w_3/S.$$

ii) Reflection on a mirror causes a slight polarization change [9]. In the corresponding Mueller matrix we shall distinguish birefringence b_3 , similar to the w_3 of equation (3.6b). (b stands for back mirror.) The reflection adds in the return passage an additional systematic contribution, given by (3.8a) with b_3 in place of w_3 . On the whole, the systematic asymmetry for a double-passage is :

$$\mathcal{A}S^{(1)}/\mathcal{A}^{\text{pv}} \approx (-1)^{\Delta F+1} (M_1/\text{Im } E_1^{\text{pv}}) (w_3/S - \frac{1}{2} b_3). \quad (3.8b)$$

A birefringence b_3 of 2×10^{-4} rad in the back mirror would be large enough to simulate the PV signal. On the contrary, because of the compensation between the forward and backward passage with $S > 200$, birefringence in the window (w_3) would have now to reach the level of 10^{-2} rad to mimic the expected signal.

In the real configuration with N (≈ 60 to 70) *double-passages*, equation (3.8b) has to be written for each double-passage. But now we must account for the effect of birefringences accumulated in previous double-passages. One must then average over all double-passages.

A similar analysis applies to the systematic effects occurring through the u_1 and u_0 terms present in $\mathcal{F}^{(1)}$ (Eq. (3.5)). The contribution of u_0 terms (explicitated in Eqs. (D.21)) involves circular dichroisms shown to be negligible ($w_0, b_0 \lesssim 10^{-6}$) [9]. So it is omitted here for simplicity. The u_1 term has two origins :

i) an electronic polarization along y , i.e. orthogonal to the laser beam in the plane containing the beam and electric field $\mathbf{E}$. In the ideal case this component, normal to the detection direction $\hat{\mathbf{k}}_r$, is not detected ;

ii) an electronic polarization along x , i.e. orthogonal to the plane containing the beam and electric field $\mathbf{E}$. In the ideal case this component has u_3 -signature, but if the direction of $\mathbf{E}$ is inhomogeneous it acquires u_1 -signature (referring to the global frame, § 3.1.2). Thus in practice the u_1 -term is detected proportionally to a misorthogonality angle

$$\chi = \hat{\mathbf{k}}_r \cdot \hat{\mathbf{y}} - \hat{\mathbf{E}} \cdot \hat{\mathbf{x}}, \quad (3.9)$$

accounting for the inhomogeneities of both $\hat{\mathbf{E}}$ and $\hat{\mathbf{k}}_r$.

Including all contributions the final result is (Eqs. (D.21)) :

$$\mathcal{A}S^{(1)}/\mathcal{A}^{\text{pv}} \approx (-1)^{\Delta F+1} (M_1/\text{Im } E_1^{\text{pv}}) [w_3/S + w_1 \tilde{\chi} - \frac{1}{2} b_3 - \frac{1}{2} \bar{\chi} b_1] + \Delta^{(1)}/\mathcal{A}^{\text{pv}}. \quad (3.10)$$

The first two terms involve the two components of the window's birefringence. Both are attenuated by a suppression factor ($1/S$ or $\tilde{\chi}$) of geometric origin. $\tilde{\chi}$ accounts for correlations between the sign of the misalignment χ and the passage direction. As expected since large correlations are unlikely, $\tilde{\chi}$ is measured to be $\ll 1/S$. Thus birefringence w_3 is more harmful than w_1 . The 3rd and 4th terms in (3.10) describe the effect of the average birefringence, b_3 or b_1 , in reflections on the back mirror. The effect of b_3 , not reduced in the multipass, is potentially harmful, while that of b_1 is much attenuated by the small geometric factor $\bar{\chi}$ ($\lesssim 2 \times 10^{-2}$ rad). Here $\bar{\chi}$ describes the spatial average value of the misalignment χ . The last term $\Delta^{(1)}$ involves only correlation effects associated with polarization imperfections accumulated in the successive reflections : they appear in case of a systematic

difference in efficiencies of forward and backward passages (or in case of a systematic difference in misalignments, for instance due to $\partial \hat{\mathbf{E}} \cdot \hat{\mathbf{x}} / \partial x$).

Owing to the attention paid to avoid birefringences in the reflections, the b_i terms are small (see below). Using a realistic model including correlations between imperfections in the multipass, $\Delta^{(1)}$ is estimated in terms of the b_i 's and shown to be smaller than 1 % of $\mathcal{A}^{\text{pv}}$ (§ D.2). On the contrary since the window birefringences are by far dominant ($w_i \gg b_i$) both terms w_3/S and $w_1 \tilde{\chi}$ have to be kept under continuous real-time control. This is described in § 3.2.3.

3.2.2 Estimation of the effect of mirror birefringences. — Two types of birefringence act in each reflection on either mirror :

(i) the birefringence due to non-normal incidence

is typically 1.3×10^{-5} rad/reflection⁽⁴⁾. Its axes are on the average those of the multipass ellipses. These are nearly degenerated into their long axis which is oriented $\parallel x$ within 10^{-2} rad (§ 2.3.5 in Part I). This brings the corresponding contribution in b_3 (or in f_3 , the front mirror birefringence) well below 10^{-6} rad;

(ii) the intrinsic birefringence is especially small ($\lesssim 10^{-5}$ rad) after sorting the mirrors. Adjustment of the axes along x and y in principle cancels b_3 (and f_3) and in practice reduces it by at least a factor of 5.

We estimate the average angle $\bar{\chi}$ to be less than 2×10^{-2} rad (§ D.2). Finally taking the values of b_1 and b_3 from table X, equation (3.10) yields

$$\mathcal{A}^{(1)}/\mathcal{A}^{PV} \text{ (from back mirror)} \begin{cases} < 1 \% , \text{ in Exp 1 }^{(5)} \\ < (+2 - 2) \% \approx 0.3 \% , \text{ in Exp 2 }^{(5)} . \end{cases}$$

Table X. — *Birefringence components of the multipass back and front mirrors. Unit 10^{-6} rad.*

		Exp 1	Exp 2
Axes $\parallel xy$	intrinsic $\begin{cases} b_1 \\ f_1 \end{cases}$	3	+ 10, - 10 (*)
	incidence b_1, f_1	9	12
		13	13
Axes at 45° to xy	intrinsic $\begin{cases} b_3 \\ f_3 \end{cases}$	0.6	+ 2, - 2 (*)
	incidence b_3, f_3	1.8	2.4
		0.1	0.1

(*) Each $\pm$ value corresponds to one of the two orientations of the back mirror periodically rotated during Exp 2.

(The $\pm 2\%$ for Exp 2 correspond to the two orientations of the mirror which was periodically rotated in its plane by $\pi/2$ so as to reverse the intrinsic birefringence. The incidence birefringence is not reversed but oriented in the less harmful direction. Finally $\mathcal{A}^{(1)}/\mathcal{A}^{PV}$ is less than 0.3 %.)

3.2.3 Control, correction and estimation of the effect of the window birefringences in real time. — Here we present the real-time reduction and estimation of the contribution of the window birefringences to the systematic effect (terms involving w_1 and w_3 in Eq. (3.10)).

Our methods to control w_1 and w_3 make use of signals independent of the laser propagation direction. These signals reflect the net birefringences of the multipass cell, which include both an imperfection in the entrance window, and an average imperfection accumulated in the reflections :

$$\text{or} \quad \left. \begin{aligned} \frac{1}{2}(b_1 + f_1) \times N &\approx 10^{-3} \text{ rad} \\ \frac{1}{2}(b_3 + f_3) \times N &\lesssim 7 \times 10^{-5} \text{ rad} \end{aligned} \right\} \quad (3.11)$$

⁽⁴⁾ This value ≈ 3 times less than quoted in [9] results from improvement in coating realization (Spectra Physics).

⁽⁵⁾ As in part I, Exp 1 and Exp 2 designate our two measurements performed in the $4 \rightarrow 4$ and $3 \rightarrow 4$ hyperfine components respectively.

This estimation, done for $N \approx 60$ reflections on each mirror, uses the values quoted in table X. (Detailed analysis in § E.1.) These last contributions are negligible compared with the uncompensated value of the window birefringence, w_1 or w_3 , measured to be $\approx 5 \times 10^{-3}$ rad. In real time we reduce w_1 and w_3 by a Bravais compensator located before the cell window, down to $\pm 2 \times 10^{-3}$ and $\pm 10^{-3}$ respectively. The effect of w_3 is further reduced to that of a birefringence $\lesssim 10^{-4}$ rad (§ 3.2.3.1). So the mirror contributions given by (3.11) do not exceed the size of the effective window + compensator birefringences. Therefore we will neglect them hereafter. The error thus introduced in $\mathcal{A}^{(1)}$ is at most 0.5 % of the PV effect.

3.2.3.1 Measurement of the birefringence w_3 and compensation of its systematic effect.

— In zero magnetic field, both the unpolarized fluorescence signal $\mathcal{F}_u$ and the polarized signal $\mathcal{F}^{(1)}$ are linear combinations of u_0 and u_3 (Eqs. (3.2) and (3.5) where u_1 -terms associated with small misalignments are here irrelevant). In other terms $\mathcal{F}_u$ and $\mathcal{F}^{(1)}$ respond to the laser polarization much like two linear polarizers, but of slightly different extinction ratios⁽⁶⁾. Birefringence w_3 (or circular dichroism w_0) causes u_3 (or u_0), and consequently $\mathcal{F}_u$ and $\mathcal{F}^{(1)}$, to acquire spurious U_2 -like modulations. The systematic effect, i.e. the spurious modulation in $\mathcal{F}^{(1)}$, can be compensated by means of a slight modulation of opposite phase in the laser intensity U_0 . (This modulation acts like a circular dichroism.)

To adjust the amplitude of this modulation we cannot use the spurious modulation in $\mathcal{F}^{(1)}$, indistinguishable from the genuine PV modulation in $\mathcal{A}^{PV}$. (That is why it is a systematic effect.) Instead we control the stray modulation in $\mathcal{F}_u$. The required modulation amplitude corresponds to reducing the latter not quite to zero but to a known small fraction of its spontaneous value (as a result of the different

⁽⁶⁾ First because α and β do not contribute identically to $\mathcal{F}_u$ and $\mathcal{F}^{(1)}$, second because of the small E-independent contribution in $\mathcal{F}_u$ only.

response of $\mathcal{F}_u$ and $\mathcal{F}^{(1)}$ to u_0 and u_3 ; detailed theory in § E.3, realization in § 2.4.2).

Actually in Exp 1 the stray modulation in $\mathcal{F}_u$ was exactly compensated to zero thus leaving a small one in $\mathcal{F}^{(1)}$. The corresponding systematic effect ($\approx 1.5\%$) was corrected for in the PV result. In Exp 2 exact automatic compensation of the systematic effect was performed. The amplitude of the compensating modulation continuously yields a measure of birefringence w_3 . The result cross-checks the value obtained from time to time by measuring the stray modulation with the compensation turned off (§ E.3).

3.2.3.2 Other birefringence measurements yielding both w_1 and w_3 . — Here we describe the methods used to measure the less harmful birefringence parameter w_1 . They also provide an independent measurement of w_3 (yet less accurate than the one just described). These methods are based on the symmetry of the Mueller matrix $M_{ij} = \pm M_{ji}$ (§ 3.2.1). Instead of looking for spurious U_2 labellings in u_1 or u_3 , we look for spurious U_1 or U_3 labelling in u_2 . This is convenient because the excited atomic vapour is easily made equivalent to a circular analyser, i.e. to a probe of the circular parameter u_2 . It simply requires the application of an H-field component in a judicious

direction. (The relevant new effects have been analysed in Part I, § 1.3.1 and App. B.)

i) In Exp 1 ($\Delta F = 0$ transition), when a Hanle magnetic field is applied along $\mathbf{E}$, the large spin polarization $\mathbf{P}^{(2)}$ proportional to u_2 yields a large H-odd signal proportional to $u_2 \xi_f$ (Eq. (B.14)). So H-odd modulations at the (common) frequency and the (orthogonal) phases of $U_1 \xi_f$ and $U_3 \xi_f$ yield a measure of the birefringences w_1 and w_3 respectively. (Actually the quantity W_1 thus measured (Eq. (E.7)) differs from w_1 by a small well-understood systematic bias, $w_{hf} \approx 4 \times 10^{-3}$ rad in practical conditions, induced by hf mixing in the Hanle field.)

ii) In Exp 2 ($\Delta F = 1$ transition) hyperfine mixing in a magnetic field applied along the laser beam induces a large well-understood circular dichroism proportional to u_2 (Eq. (B.8)). In this case w_1 and w_3 are measured by extracting the H-odd contributions of signature U_1 and U_3 from $\mathcal{F}_u$.

The detailed procedures are given in § E.2. Such measurements are periodically performed during data acquisition. The results are used to adjust the Bravais compensator. The reliability of the various controls of w_1 and w_3 is attested by their agreement at the level of 10^{-3} rad.

3.2.3.3 Measurement of the geometrical reduction factors $1/S$ and $\tilde{\chi}$. Reconstitution of $\mathcal{A}s^{(1)}$ during Exp 1 and Exp 2. — The reduction factor $1/S$ and the effective misalignment $\tilde{\chi}$ are measured by looking for residual (i.e. uncompensated in the multipass) $\mathbf{P}^{(1)}$ -components. This is done by extracting E-odd contributions of signature $U_3 \xi_f$ or $U_1 \xi_f$ from the polarization ratio. We denote them $P_{res,3}^{(1)}$ or $P_{res,1}^{(1)}$ respectively (Eqs. (E.30-31)). Typical results are

$$P_{res,3}^{(1)} \approx \begin{cases} 12.5 \times 10^{-5} \\ \pm 4 \times 10^{-5} \end{cases}; \quad P_{res,1}^{(1)} \lesssim \begin{cases} 2.5 \times 10^{-5}, & \text{in Exp 1,} \\ 1.5 \times 10^{-5}, & \text{in Exp 2,} \end{cases} \quad (3.12)$$

corresponding to

$$1/S \approx \begin{cases} 5 \times 10^{-3} \\ \pm 5 \times 10^{-4} \end{cases}; \quad \tilde{\chi} \approx \begin{cases} 10^{-3}, & \text{in Exp 1,} \\ 2 \times 10^{-4}, & \text{in Exp 2.} \end{cases}$$

In real time the systematic effect due to the window + Bravais compensator birefringence is directly given by the product of the quantities $P_{res,3}^{(1)}$ (or $P_{res,1}^{(1)}$) and W_3^{eff} (or W_1) measuring the effective residual birefringence

$$\mathcal{A}s^{(1)}(w_1, w_3) \simeq \begin{cases} -2 P_{res,3}^{(1)} W_3^{\text{eff}} - 2 P_{res,1}^{(1)} (W_1 + w_{hf}), & \text{in Exp 1,} \\ -2 P_{res,3}^{(1)} W_3^{\text{eff}} - 2 P_{res,1}^{(1)} W_1, & \text{in Exp 2.} \end{cases} \quad (3.13)$$

Here $w_{hf} \approx 4 \times 10^{-3}$ is the offset of atomic origin in the birefringence measurement performed in the $\Delta F = 0$ component (§ 3.2.3.2). In practice, typically $W_1 \approx \pm 2 \times 10^{-3}$, and W_3^{eff} is the inaccuracy in the automatic compensation of the birefringence effect by the modulation in the laser intensity, estimated in real time from empirical parameters (§ E.3b-c) :

$$W_3^{\text{eff}} \begin{cases} \approx w_3/10 < 10^{-4}, & \text{in Exp 1;} \\ < 7 \times 10^{-5} \text{ (1 rms stat.)} \pm 8 \times 10^{-5} \text{ (syst.)}, & \text{in Exp 2.} \end{cases}$$

These results together with (3.12) yield finally from (3.13) :

$$\mathcal{A}S^{(1)}/\mathcal{A}^{PV} \approx \begin{cases} 1 \% (\text{from } w_3) & \text{and } 4 \% (\text{from } w_1), \text{ in Exp 1;} \\ 0.1 \% (\text{from } w_3) & 0.2 \% (\text{from } w_1), \text{ in Exp 2.} \end{cases}$$

(Without the automatic compensation the effect from w_3 would have been 10 times larger.)

3.2.4 A test of the procedure of reconstitution of $\mathcal{A}S^{(1)}$. — We have checked the reconstitution of the systematic effect $\mathcal{A}S^{(1)}$ using (3.13) and the methods described above, in an auxiliary experiment where large effects were induced artificially. We operated with a single passage of the laser beam (thus $S = 1$: no suppression effect) and a large birefringence ($w_3 \approx \text{a few} \times 10^{-2}$ rad) given by the Bravais « compensator ».

At first no modulation was applied in the laser intensity. Using the controls of w_3 performed in the $\Delta F=0$ component, the systematic effect $\mathcal{A}S^{(1)}$ was estimated to be $(-15.2 \pm 1.5) \times 10^{-5}$ (rms stat.). The simultaneous measurement of the false PV signal (after 7 min integration) was consistent : $(-18.8 \pm 1.5) \times 10^{-5}$ (~ 50 times the true PV signal). In a second test we turned on the automatic compensation of birefringence w_3 (i.e. modulation in the laser intensity). We thus expected to reduce the effective birefringence by a large known factor ($-\beta/4 \alpha \simeq 1/40$; cf. Eq. (E.23) with $b_{dc} \approx 0$ here), i.e. to bring the false PV signal down to $\sim -0.5 \times 10^{-5}$. The observed value had the expected order of magnitude $(-1.1 \pm 0.6) \times 10^{-5}$ (40 min integration). The value simultaneously estimated using (3.13) and direct measurements of $P_{res,3}^{(1)}$ and W_3^{eff} is consistent : $(-0.4 \pm 0.1) \times 10^{-5}$.

3.3 EFFECT OF HYPERFINE MIXING IN A STRAY H-FIELD.

3.3.1 Expression of the systematic effect. — In § 3.2 we assumed that the PC signal $\mathcal{F}^{(1)}$, associated with $P^{(1)}$, remains ξ -independent in the absence of light polarization imperfections. However a new effect arises if a magnetic field component is spuriously present along the beam. As seen in part I (§ 1.3) such a component generates, by hf mixing, the ξ -odd polarization $P_{hf}^{(1)} \propto (\mathbf{k} \cdot \mathbf{H}) \xi \mathbf{k} \times \hat{\mathbf{E}}$ (a M_1 -Stark interference effect like $P^{(1)}$). This spurious contribution to $P^{(1)}$ is along the detection direction and is already provided with the U_2 -signature of the PV component. Fortunately $P_{hf}^{(1)}$, just like $P^{(1)}$, reverses in a beam reversal (since $\mathbf{k} \cdot \mathbf{H}$ then is reversed). Therefore the beam multipass reduces its magnitude by the geometric factor S already introduced (§ 3.2.3.3). The magnitude of the systematic effect is finally given by :

$$\mathcal{A}S_{hf}^{(1)}/\mathcal{A}^{PV} \approx \begin{cases} 9 h_k/S, & \text{if } \Delta F = 0 (\text{Exp 1}) \\ 100 h_k/S, & \text{if } \Delta F = 1 (\text{Exp 2}) \end{cases} \quad (3.14)$$

where h_k is the magnitude (in gauss) of the spurious H-component along z . (We have used Eqs. (B.1-2),

(B.9-10), (A.5), (A.16), $M_1/\text{Im } E_1^{PV} \approx 2 \times 10^4$ and values quoted in table IV.)

3.3.2 Control of $\mathcal{A}S_{hf}^{(1)}$. — The factor S is measured in real-time (§ 3.2.3.3). In our set-up the earth's field is of the order of 0.07 G along the beam. With an additional component of magnitude smaller than ± 0.1 G deliberately applied in Exp 1 (§ 3.4.3.2b), the net h_k was always less than 0.2 G. Combining this value with $S \approx 200$ leads to a spurious effect $\mathcal{A}S_{hf}^{(1)}/\mathcal{A}^{PV} < \pm 1\%$ (in Exp 1).

The systematic effect would have been larger in Exp 2 (Eq. (3.14)). Therefore we decided to compensate the earth's component h_k . Prior to data acquisition h_k was measured using the magnetic circular dichroism (CD) simultaneously induced by hf mixing too (§ 1.3 in Part I), and monitored *via* the unpolarized fluorescence signal $\mathcal{F}_u$. To eliminate a false CD due to residual birefringence in the optics, the CD signal was measured in the two $\Delta F = \pm 1$ transitions : the magnetic CD's are opposite, while the birefringence effect is the same. (Detailed procedure in § E.2b.) With a constant current in the coils, h_k remained compensated within ≈ 30 mG. (Its small fluctuations were monitored both with a fluxgate and through the CD signals which were periodically measured during Exp 2.) In addition, the reduction factor of the multipass was quite large in Exp 2 : $S > 2000$. Thus in that case equation (3.14) leads to $\mathcal{A}S_{hf}^{(1)}/\mathcal{A}^{PV} < 0.15\%$.

3.4 EFFECTS OF IMPROPER E-REVERSAL. — We now turn to the second main class of systematics, associated with the ξ -dependent PC components of the electronic polarization ($P^{(2)} + P^{(0)}$). Then the effects which mimic PV involve improper E-reversal. We first concentrate on the systematic effect originating in the component of $P^{(2)}$ along $\mathbf{E}$. The discussion concerning the component of $P^{(2)}$ along $\mathbf{k}$ (and $P^{(0)}$) is postponed to section 3.5.

3.4.1 Expression of the systematic asymmetry. — The $P^{(2)}$ -component $\mathcal{P}E$ is $\propto (\hat{\mathbf{E}} \cdot \xi \hat{\mathbf{k}}) \hat{\mathbf{E}}$. It appears only in case of imperfect orthogonality between the beam and E-field direction ($\hat{\mathbf{E}} \cdot \hat{\mathbf{k}} \neq 0$). Since it is created along $\hat{\mathbf{E}}$, it is in principle undetected, unless the detection direction $\hat{\mathbf{k}}_f$ is not perfectly orthogonal to $\hat{\mathbf{E}}$. However, thus far the detected component $\propto (\hat{\mathbf{E}} \cdot \hat{\mathbf{k}})(\hat{\mathbf{E}} \cdot \hat{\mathbf{k}}_f)$ is even under E-reversal so it does not simulate P^{PV} . This is no longer true if during E-reversal the direction $\hat{\mathbf{E}}$ undergoes a small tilt caused by a

stray field $\Delta\mathbf{E}$. Then the detected variation of $\mathbf{P}^{(2)}$, which is simply

$$\delta P^{(2)} \propto \xi[(\Delta\mathbf{E} \cdot \hat{\mathbf{k}})(\hat{\mathbf{E}} \cdot \hat{\mathbf{k}}_f) + (\hat{\mathbf{E}} \cdot \hat{\mathbf{k}})(\Delta\mathbf{E} \cdot \hat{\mathbf{k}}_f)],$$

mimics PV. A more rigorous calculation is performed

$$\mathcal{A}S^{(2)}/\mathcal{A}^{PV} = (\beta/\text{Im } E_1^{PV}) [\Delta E_z(\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f) + (\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}) \Delta E_x - 2 \Delta E_y(\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}})(\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f)]. \quad (3.16)$$

We see that the unreversed field components in the plane $xz \perp \hat{\mathbf{E}}_0$ act to first order in the misalignment angles $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$ and $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$; the unreversed component along y combines to their product only. Therefore a change of the modulus of $\mathbf{E}$ under reversal is much less dangerous than a change of its direction. Since $|E_1^{PV}/\beta| \approx 1.7$ mV/cm the stray component $|\Delta\mathbf{E}|$ able to simulate $\mathcal{A}^{PV}$ is of the order of $1.7/\varphi$ mV/cm (φ is the misalignment), i.e. 170 mV/cm if $\varphi \approx 10^{-2}$ rad. As seen below the stray field and the misalignment can be controlled in fact at the level of 15 mV/cm and 10^{-3} rad respectively. This ensures the control of $\mathcal{A}S^{(2)}$ at the percent level of the observed PV effect.

3.4.2 Origins and magnitudes of the imperfections. — Here we discuss the origin of the defects involved in (3.16). (Yet our control methods measuring them *in situ*, from atomic signals, make no assumption concerning this origin.)

3.4.2.1 Misalignments; stray magnetic fields. —

(i) $(\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}})$ describes the non-orthogonality of the field and beam directions, averaged over the observed atoms. Because of edge effects in the finite capacitor plates, $\hat{\mathbf{E}}_0$ shows a dispersion around the normal to the plates (§ 2.3.4.3 in Part I). The directions of the beam passages also show a dispersion ($\approx 10^{-2}$) around the multipass axis (centre line of the two ellipses drawn on the mirrors). Nevertheless, at the price of some precautions in the setting (adjustment of the multipass between the plates, focusing of the detection optics onto the centre of the interaction region), then by symmetry these inhomogeneities tend to average to zero. In practice the dominant defect appears to be the non-orthogonality of the ellipses centre line with respect to the normal to the plates, which can be adjusted to 10^{-3} rad (§ 2.9). The real-time control of the defect as probed by the atoms themselves (§ 3.4.3) turns out to agree with the geometrical adjustment.

(ii) $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ describes the average non-orthogonality of the field and detection directions. The mechanical axis of the detection assembly is initially oriented parallel to the capacitor plates (§ 2.7). Yet the coincidence of the average detection direction with the mechanical axis is not guaranteed. In addition, edge effects in $\hat{\mathbf{E}}_0$ are larger along $\hat{\mathbf{k}}_f$ than along $\hat{\mathbf{k}}$ given the shape of the observed interaction region. Therefore we cannot rely on a pure geometric control of the misalignment.

in appendix F: the static field $\mathbf{E}$ is replaced by $\eta\mathbf{E}_0 + \Delta\mathbf{E}$ ($\eta = \pm 1$ is the sign of the voltage, $\Delta\mathbf{E}$ is the non-reversing stray field). Up to first order in $|\Delta\mathbf{E}|/E_0 \ll 1$, and to second order in the (small) misalignments $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$, $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$, and $\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}_f$, the spurious asymmetry $\mathcal{A}S^{(2)}$ is computed to be (Eq. (F.10)):

So far we have assumed the absence of magnetic field. A stray $\mathbf{H}$ -field would rotate the electronic polarizations (through Hanle effect). Such an effect simulates a misalignment but cannot be controlled geometrically. The polarization responsible for the systematic effect involving $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ is created along $\hat{\mathbf{E}}_0 \parallel \hat{\mathbf{y}}$, the vertical direction. So only horizontal $\mathbf{H}$ -components are harmful. These components, measured first with a fluxgate at the place of the centre of the cell, then using atomic signals, are ≈ 0.07 G (in Exp 1) or $\lesssim 0.03$ G (in Exp 2) along $\hat{\mathbf{k}}$, and $\lesssim 0.03$ G along $\hat{\mathbf{k}}_f$. Given the Hanle width $\Delta H \approx 10$ G, they can simulate misalignments $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \approx 7 \times 10^{-3}$ or $\lesssim 3 \times 10^{-3}$ rad, and $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}} \lesssim 3 \times 10^{-3}$ rad respectively. Conversely, Hanle effect in a controlled magnetic field can be used to compensate geometric misalignments. During Exp 1 this method was actually chosen to compensate $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ (cf. § 3.4.3).

3.4.2.2 Stray electric fields. — The atoms between the plates are relatively well shielded against external static fields by the thick grounded metallic oven, by a partially conducting Cs film on the cell walls and by the plates themselves. A more worrying cause of defect is associated with surface impurities on the (yet carefully cleaned and outgassed) stainless steel capacitor plates. The resulting field inhomogeneities, differently screened by the space charge when the voltage has one or the other sign, have to be averaged over the observation volume and may change with the status of the cell. In view of the difficulty of estimating a reliable value for $\Delta\mathbf{E}$, we detect atomic signals able to reveal it directly in real-time (§ 3.4.3).

3.4.3 Real-time controls. — The procedures described below follow the basic principle mentioned in introduction. They consist in magnifying one imperfection δ_1 so as to make the corresponding systematic effect $\delta_1\delta_2$, and thereby the small conjugated imperfection δ_2 , easily detectable (irrespective of the origin of δ_2).

3.4.3.1 Control of ΔE_z and $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$.

a) Principle. — The controls of the stray field component ΔE_z and of the misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$ make use of the spurious $\mathbf{P}^{(2)}$ -component $\propto \xi(\hat{\mathbf{E}} \cdot \hat{\mathbf{k}}) \hat{\mathbf{E}}$ created along the direction $\hat{\mathbf{E}}$ of the total field $\mathbf{E} = \eta\mathbf{E}_0 + \Delta\mathbf{E}$ ($\eta = \pm 1$). By applying a Hanle field $\mathbf{H}_k$ along $\hat{\mathbf{k}}$, this component is rotated around $\hat{\mathbf{k}}$ towards the third direction, i.e. essentially the detection direction $\hat{\mathbf{k}}_f$. The stationary value of the polarization along $\hat{\mathbf{k}}_f$ is $\frac{1}{2}$ of the initial source term when H_k is equal

to the Hanle width (§ B.2 in Part I). (The method is equivalent to magnifying $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ up to $\frac{1}{2}$.) The magnified spurious effect is identified owing to its oddness under $\mathbf{H}_k$ -reversal. It is proportional to the imperfection $\hat{\mathbf{E}} \cdot \hat{\mathbf{k}}$. Then the η -odd and η -even parts of the effect yield directly the spurious field components $\Delta \mathbf{E} \cdot \hat{\mathbf{k}} \simeq \Delta E_z$ and $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$ respectively. Exact reconstitution is given in § F.2c.

b) Practical procedures and corrections. — In practice the identification of ΔE_z making use of *five* reversals⁽⁷⁾, is very good. A small effect ($\lesssim 40$ mV/cm) emerged above the noise level during Exp 1 only. It was on the average compensated by aid of a small voltage v_1 across the auxiliary electrodes (§ 2.3.4 and Fig. 9 in Part I).

The identification of the misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$ which does not benefit of the η -reversal is valid for a $\Delta F = \pm 1$ transition only. In $\Delta F = 0$ transitions the measurement can be falsified due to the presence of the large polarization component $\nabla \mathbf{k}$. This latter might be detected if $\mathbf{H}_k$ is not strictly normal to $\hat{\mathbf{E}}_0$, thereby giving rise to a signal of same signature as the signal able to reveal the misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}$. For this reason during both Exp 1 and Exp 2 the control was performed in the $\Delta F = 1$ transition. The method is very sensitive : a misalignment of a few 10^{-4} rad is detected in 1 min. Fine readjustment of the beam direction then allowed one to achieve $\hat{\mathbf{E}}_0 \perp \hat{\mathbf{k}}$ with an accuracy better than 10^{-3} rad. The residual imperfection remained stable at this level.

3.4.3.2 Control of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$.

a) Principle. — Interchanging the rôle of the imperfections $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ and ΔE_z conspiring in (3.16), we now magnify ΔE_z in order to measure a stray $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$. For this we simply apply a large known voltage across the auxiliary electrodes (§ 2.3.4), which induces an electric field $\mathbf{e} \nabla \mathbf{k}$. The potential configuration is represented in figure 14. Typical values (averaged over the observed volume) are $e/E_0 \approx 4.5 \times 10^{-2}$ in Exp 1 and 1.5×10^{-2} in Exp 2. In this modified configuration the polarization $\xi(\hat{\mathbf{E}} \cdot \hat{\mathbf{k}}) \hat{\mathbf{E}}$ created along $\hat{\mathbf{E}}$ acquires a fairly large contribution $\propto \xi(e/E_0) \hat{\mathbf{E}}$, whose component detected along $\hat{\mathbf{k}}_f$ is proportional to the sought-for misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$. This component, Π_e , is unambiguously discriminated owing to its oddness under reversal of either $\mathbf{e}$ or $\mathbf{E}_0$. The proportionality constant involving the magnified angle $\hat{\mathbf{E}} \cdot \hat{\mathbf{k}} \simeq e/E_0$ is measured by momentarily applying the Hanle field $\mathbf{H}$ along $\hat{\mathbf{k}}$ (just as in the previous control of $\hat{\mathbf{E}} \cdot \hat{\mathbf{k}}$) and measuring the large $\mathbf{H}_k$ -odd part of Π_e ⁽⁸⁾.

⁽⁷⁾ $\mathbf{H}_k$, η , plus the three reversals (two in ξ , § 2.2; one in ξ_f) which characterize any $\mathbf{P}^{(2)}$ -component.

⁽⁸⁾ Since in this measurement both conspiring defects are simultaneously magnified, the measured polarization is large. Its expected variations as a function of $\mathbf{E}_0$ or $\mathbf{e}$ or $\mathbf{H}_k$ have been checked experimentally [10], providing a test of our understanding of the systematic effect discussed here.

Fig. 14. — Typical voltages applied on the main and auxiliary electrodes for the control of the misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ during (a) Exp 1 and (b) Exp 2.

The misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ is finally deduced from

$$\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f = \Pi_e(H=0)/\text{sgn } g_F [\Pi_e(\Delta H_k) - \Pi_e(-\Delta H_k)].$$

(For exact definition of the measured quantities, see § F.2b.)

b) Correction of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$. — The analysis of the Hanle effect shows that, once measured, $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ can be corrected using a field $\mathbf{h}_k$ applied along $\hat{\mathbf{k}}$, of magnitude $h_k/\Delta H = \text{sgn } g_F \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$. ($\Delta H \simeq 10$ G is the « Hanle width ».) However as seen in § 3.3 such a component might be troublesome through hyperfine mixing effect, quite especially in the $\Delta F = 1$ component used in Exp 2. For this reason the « Hanle » correcting procedure was practised in Exp 1 ($\Delta F = 0$ case) only and the applied field remained sufficiently small ($\lesssim 0.1$ G) for the hf mixing effect to remain negligible. The procedure allowed one to correct geometric misalignment up to 10^{-2} rad and to keep the effective value of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ at the level of a few mrad. Yet a small offset ($\approx 2 \times 10^{-3}$ rad) affected the measurement. It originates in the combination of : i) a dissymmetry in the design of the auxiliary electrodes such that a common potential generates a field along x ; ii) a modification in the choice of their common potential, between the PV configuration and the control configuration, whose consequences occurred to us only after Exp 1.

Since in Exp 2 the field component $\mathbf{h}_k$ was cautiously corrected and controlled (§ 3.3.2), the misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ had essentially a *geometric* origin. For Exp 2 we improved the mechanical adjustment of the detection optics, in particular of the spherical mirror used to bring back the fluorescence emitted along $-\hat{\mathbf{k}}_f$ (Fig. 11 in Part I). In addition, the atomic control of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ had better statistics in Exp 2 than in Exp 1, because the error signal Π_e was larger by a factor of ~ 5 (Eqs. (F.13c)). This allowed finer control as well as better reproducibility of the residual geometric imperfection. Having more sensitivity to probe the edge effects in the $\mathbf{E}$ -field, their average value could be controlled by fine centring of the multipass ellipses (i.e. of the interaction region). Finally by purely geometric means, the average misalignment was reduced to the level of a few mrad during Exp 2 too.

3.4.3.3 Control of ΔE_x and ΔE_y .

— Our procedure

for controlling ΔE_x and ΔE_y , is based on the fact that the 7S population, a quadratic function of $\mathbf{E}$, can acquire a small odd contribution under unrigorous field reversal. In both hf transitions, the unpolarized fluorescence signal $\mathcal{F}_u$ is a linear combination of $|\mathbf{E} \cdot \boldsymbol{\varepsilon}|^2$ and $|\mathbf{E} \times \boldsymbol{\varepsilon}|^2$ (Eq. (3.2a)). A development up to first order in $|\Delta \mathbf{E}|/E_0$ yields the η -odd terms

$$|\mathbf{E} \cdot \boldsymbol{\varepsilon}|_{\text{odd}}^2 = 2 \eta E_0 \Delta E_x \text{Re}(\varepsilon_x \varepsilon_y^*) + 2 \eta E_0 \Delta E_y |\varepsilon_y|^2, \quad (3.17)$$

$$|\mathbf{E} \times \boldsymbol{\varepsilon}|_{\text{odd}}^2 = -2 \eta E_0 \Delta E_x \text{Re}(\varepsilon_x \varepsilon_y^*) + 2 \eta E_0 \Delta E_y |\varepsilon_x|^2.$$

ΔE_x and ΔE_y are obtained from the stray η -odd signals having the signature of $U_1 = 2 \text{Re}(\varepsilon_x \varepsilon_y^*)$ and $U_3 = |\varepsilon_x|^2 - |\varepsilon_y|^2$ respectively. (Exact reconstitution in § F.2a.) ΔE_y is compensated by adding a small unreversed voltage to the reversed main voltage. The procedure is controlled by a digital servo-loop using the atomic error signal. Modifications of the correction voltage are postponed until completion of the current elementary sequence of PV data acquisition. ΔE_x can be compensated by a small common constant voltage applied to the auxiliary electrodes. This was used during part of Exp 1.

3.4.3.4 Final order of magnitude of $\mathcal{A}S^{(2)}$. — Inserting in (3.16) a typical value of ≈ 15 mV/cm for the unreversed field components and a typical misalignment value of 10^{-3} rad, we obtain, for $|E_1^{\text{PV}}/\beta| \approx 1.7$ mV/cm, the typical estimate : $\mathcal{A}S^{(2)}/\mathcal{A}^{\text{PV}} \lesssim 2\%$, in both Exp 1 and Exp 2.

3.4.3.5 A test of the procedure of $\mathcal{A}S^{(2)}$ -reconstitution. — In an auxiliary experiment, we artificially induced large defects (of known sign and known magnitude), in view of checking the actual value of the resulting systematic effect against the value reconstituted as just described. We applied (i) a Hanle field h_k (≈ 0.5 G), magnifying the effective value of the misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$, and (ii) an unreversed voltage across the auxiliary electrodes, magnifying ΔE_z . First, the measured values of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ and ΔE_z (control of §§ 3.4.3.1 and 3.4.3.2) were in agreement with the values expected from the applied current and voltages, yielding the estimate $\mathcal{A}S^{(2)} \approx 1.5 \times 10^{-4}$. Second, the false PV signal easily measured was found to be consistent : $(1.35 \pm 0.15 \text{ (stat)}) \times 10^{-4}$.

3.4.4 Conspiracy between electric field gradients : uncertainty in $\mathcal{A}S^{(2)}$. — We have been concerned with a possible uncertainty in our estimation of the systematic effect $\mathcal{A}S^{(2)}$ (Eq. (3.16)), due to inhomogeneities in the misalignments $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \simeq \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{x}}$ and $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}} \simeq \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{z}}$. These inhomogeneities might be harmful if they were correlated with corresponding inhomogeneities of the conjugated imperfections ΔE_z and ΔE_x .

Inhomogeneities in the misalignments are well described by the two gradient components $\partial \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{x}}/\partial x \simeq 2 \times 10^{-2} \text{ mm}^{-1}$ and $\partial \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{z}}/\partial z \simeq 3 \times 10^{-3} \text{ mm}^{-1}$

(§ 2.3.4.3). The conjugated imperfections are thus the gradient components $\partial \Delta E_z/\partial x$ and $\partial \Delta E_x/\partial z$ respectively. First, we remark that such quantities break the symmetry of the observed region, which makes them unlikely. Second, the stray field $\Delta \mathbf{E}$ remained below the noise level for part of Exp 1 and all Exp 2. When it emerged above the noise, the signs of ΔE_x and ΔE_z fluctuated. We have experimental indication that $\Delta \mathbf{E}$ depends on the caesium film distribution on the plates and walls, in which no particular symmetry is expected. Concerning the compensating non-reversing field applied during part of Exp 1, its (non-random) distribution essentially preserves the symmetry. In conclusion we had no reason to suspect the presence of the gradients under consideration. However, we have included, in the final result, a systematic uncertainty $\Delta^{(2)}$ of $3\% \times \mathcal{A}^{\text{PV}}$: this corresponds to the value of $\mathcal{A}S^{(2)}$ computed for 30 mV/cm (average of the stray field modulus $|\Delta E_z|$) conservatively combined with a misalignment of 2×10^{-3} rad (uncertainty in the control of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{x}}$). A more precise control of this correlation effect could be achieved. The principle of the method is described in § F.3.

3.5 EFFECT OF THE ξ -DEPENDENT PC POLARIZATION $\nparallel \hat{\mathbf{k}}$.

3.5.1 Origin and expression of the false asymmetry. — We now analyse possible systematic effects associated with the PC electronic polarization created along the beam, including :

(i) the component of $\mathbf{P}^{(2)} \propto \xi \hat{\mathbf{k}}$, originating in $\alpha\beta$ interference. Created with a large magnitude in the $\Delta F = 0$ transitions it gives, by line overlap, a small contribution in the much less intense $\Delta F = 1$ line ;

(ii) $\mathbf{P}^{(0)} \propto \xi \hat{\mathbf{k}}$, originating in a (small) square term M_1^2 and affected by a (small) background term induced by Cs-He collisions. The magnitude of each component is given in table XIIIa in appendix G.

A first common characteristic of these components is their direction ($\nparallel \hat{\mathbf{k}}$) which makes them in principle undetected ($\hat{\mathbf{k}}_f \perp \hat{\mathbf{k}}$). A second common characteristic providing a second discriminant against $\mathbf{P}^{\text{PV}}$, is that they do not reverse with $\mathbf{E}$. At first sight, therefore, spurious effects involving any of these components appear as second order effects : combination of mis-orthogonality of $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}_f$ and of imperfect $\mathbf{E}$ -reversal.

For the (small) component $\mathbf{P}^{(0)}$ this second order analysis goes as follows. $\mathbf{P}^{(0)}$ provides, in the polarized fluorescence signal $\mathcal{F}_p$, an E -independent contribution proportional to the average value of the misalignment $\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}_f$. Subsequent normalization of $\mathcal{F}_p$ by the unpolarized signal $\mathcal{F}_u$ ($\propto E_y^2$) brings an E_y -dependent term in the polarization ratio $\mathcal{F}_p/\mathcal{F}_u$. If the reversal of E_y is imperfect, this ratio thus acquires an E_y -odd contribution which simulates the PV signal.

Fortunately this analysis does not apply to the large Stark-induced component of $\mathbf{P}^{(2)}$. Its contribution to $\mathcal{F}_p$ is proportional to $E_y^2 \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}_f$. So in the polarization ratio $\mathcal{F}_p/\mathcal{F}_u$, the E_y -dependence disappears (irres-

pective of E_y -misreversal), unless an offset b_0 affects differently $\mathcal{F}_u$ and $\mathcal{F}_p$. The corresponding systematic effect finally involves the third order term $b_0(\mathbf{k} \cdot \mathbf{k}_f)(\Delta E_y/E_0)$. We stressed in § 1.2 that detection of $\mathcal{F}_u$ (through modulation amplitude $\mathcal{F}_{e+b}$) is essentially free from any background. An instrumental background (e.g. a slight offset at the output of the lock-in amplifier), however, would not be eliminated since no reversal is used to extract $\mathcal{F}_{e+b}$.

From the detailed analysis of appendix G a correct estimation of the net systematic effect discussed here, $\mathcal{A}S^{(0)}$, is given in each $F \rightarrow F'$ component in terms of measurable quantities by :

$$\mathcal{A}S^{(0)} \simeq -2[\delta_{FF'} P_{\text{res}}^{(2)} b_0 + P_{\text{res}}^{(0)}] \times A_u + \Delta^{(0)}. \quad (3.18)$$

Here $P_{\text{res}}^{(0)} + P_{\text{res}}^{(2)} = P_{\text{res}}$ is the « residual polarization », a signal extracted exactly like the PV signal *except it is E-even* (Eq. (G.8a)). In the ideal experiment P_{res} is zero. In the real case it is associated with the E-even, ξ -dependent PC component $\mathbf{P}^{(0)} + \mathbf{P}^{(2)}$ detected through some effective misalignment (including geometrical misalignments and Hanle effect in stray H-fields; Eqs. (G.8) and detailed account in Table XIIIb). The two individual contributions $P_{\text{res}}^{(0)}$ and $P_{\text{res}}^{(2)}$ are known by combining measurements with and without E. The offset b_0 combining to $P_{\text{res}}^{(2)}$ is defined as the fractional E-independent contribution ($\lesssim 10^{-3}$) in $\mathcal{F}_{e+b}$. The quantity A_u represents the asymmetry in $\mathcal{F}_{e+b}$ when E is reversed, reflecting imperfect reversal of E_y (§ 3.4.3.3). The last term $\Delta^{(0)}$ allows for possible spatial correlations between quantities $P_{\text{res}}^{(2)}$ and A_u , associated with inhomogeneities in $\mathbf{k} \cdot \mathbf{k}_f$ and ΔE_y (Eq. (G.7)). The estimation of $\Delta^{(0)}$ is given in § 3.5.3.

3.5.2 Real-time reduction of the residual polarization and of the stray field ΔE_y . Estimate of $\mathcal{A}S^{(0)}$. — In the conditions of Exp 1 ($\Delta F = 0$, $E = 100$ V/cm), $P_{\text{res}}^{(2)} \approx 25 \times P_{\text{res}}^{(0)}$. In view of the factor $b_0 \approx 10^{-3}$, it appears much less harmful than $P_{\text{res}}^{(0)}$ as regards the systematic effect (3.18). Yet $P_{\text{res}}^{(2)}$ may be large ($\sim 100 \times P^{\text{PV}}$ assuming a misalignment $\mathbf{k} \cdot \mathbf{k}_f$ of 1 mrad). So $P_{\text{res}} \approx P_{\text{res}}^{(2)}$ might have been a source of noise. For this reason we decided to cancel it. In practice it was kept below the noise level ($< 10^{-6}$) using a digital servo-loop controlling a Hanle field along y ($h_0 < 0.1$ G; § G.2c). $P_{\text{res}}^{(0)}$ measured from time to time was $\sim 10^{-5}$.

During Exp 2, the earth's field compensation was kept constant and the orthogonality $\mathbf{k} \perp \mathbf{k}_f$ was adjusted within ≈ 3 mrad. P_{res} , and thereby also $P_{\text{res}}^{(0)}$, was thus maintained below $\sim 2 \times 10^{-5}$. We combine these figures with the (statistical) limit of 2×10^{-4} for the asymmetry $A_u = 2 \Delta E_y/E_0$ which was compensated by digital servo-loop (§ 3.4.3.3). From equation (3.18) this yields a systematic effect $\mathcal{A}S^{(0)} \approx (4 \text{ or } 8) \times 10^{-9}$, which is 0.1 % of the PV asymmetry in Exp 1 or Exp 2. (Without servo-loop $\mathcal{A}S^{(0)}$ would have been 10 times larger.)

3.5.3 Spatial correlation between inhomogeneities of the conjugated defects $\mathbf{k} \cdot \mathbf{k}_f$ and ΔE_y .

3.5.3.1 Analysis. — The correlation effect $\Delta^{(0)}$ in (3.18) arises as follows : at each point of the observed region, the average observation direction is given by the line joining this point to the centre of the light-collecting lens. Consequently $\mathbf{k} \cdot \mathbf{k}_f$ varies linearly with z , and the residual polarization associated with $\mathbf{P}^{(2)} \cdot \mathbf{k}_f$ is inhomogeneous : $\partial P_{\text{res}}^{(2)}/\partial z \neq 0$. On the other hand since the fluorescence is proportional to E_y^2 , a spurious unreversed gradient $\partial \Delta E_y/\partial z$ would shift the effective centre of the observed region when E_y is reversed. Conspiracy of both effects yields an E_y -odd contribution to $\mathbf{P}^{(2)} \cdot \mathbf{k}_f$, which simulates P^{PV} . Yet, some compensation is expected between the effects associated with « direct » fluorescence, emitted along $+x$, and with « indirect » fluorescence, emitted along $-x$ then collected *via* the spherical mirror (Fig. 15 in App. G). The reason is that reflection at small angles in this mirror is physically equivalent to (imperfectly) reflecting the observed region with respect to the $z = 0$ plane, so the shifts seen « directly » and « indirectly » are opposite. If the $z \rightarrow -z$ symmetry were perfectly realized, the net correlation effect would cancel out.

Each gradient can be measured by looking for a forward-backward (i.e. $z > 0 - z < 0$) asymmetry either in $P_{\text{res}}^{(2)}$ or in A_u (unpolarized fluorescence asymmetry $\propto \Delta E_y$). This is done in auxiliary measurements performed while masking the detection mirror and one half of the detector, so as to observe separately the $z > 0$ or $z < 0$ half of the observation region (Fig. 15a in App. G). (The mirror would reduce the sensitivity to $\partial \Delta E_y/\partial z$). The correlation effect is finally obtained from :

$$\Delta^{(0)} = \frac{2}{3} [P_{\text{res}}^{(2)}(z > 0) - P_{\text{res}}^{(2)}(z < 0)] \times [A_u(z > 0) - A_u(z < 0)]/S', \quad (3.19)$$

where S' accounts for the suppression effect of the mirror as deduced by combining gradients measurements with and without mirror (§ G.3).

3.5.3.2 Results. — Separate measurements of $P_{\text{res}}^{(2)}$ for $z > 0$ and $z < 0$ yield results of opposite signs, as expected, and of magnitude consistent with the geometry of the detection optics ($(\partial \mathbf{k} \cdot \mathbf{k}_f/\partial z)^{-1} \approx D$, distance of the entrance lens; Fig. 15). The forward-backward asymmetry in A_u did not emerge above the noise level. (This corresponds to $\partial(\Delta E_y/E_0)/\partial z \leq 2 \times 10^{-5} \text{ mm}^{-1}$ in both Exp 1 and Exp 2.) Using the conservative limit $1/S' < 0.2$ (empiric determination $1/S' = -0.08 \pm 0.1$), equation (3.19) finally yields

$$\Delta^{(0)}/\mathcal{A}^{\text{PV}} < 5 \% \text{ or } 0.5 \% (\text{stat}) \text{ in Exp 1 or Exp 2.} \quad (3.20)$$

As will appear in conclusion (Table XI), $\Delta^{(0)}$ is a dominant systematic uncertainty in Exp 1. The limit is smaller in Exp 2 essentially because the Stark-

Table XI. — (a) *Estimates of the systematic asymmetries (mean ± 1 rms deviation)* (b) *Systematic uncertainties affecting (a).* (c) *Overall calibration uncertainty.*

		Exp 1 Unit : $10^{-2} \times$ observed PV asymmetry	Exp 2
(a) Systematic asymmetries			
$\mathcal{A}s^{(1)}(w_3) \}$	§ 3.2, Eq. (3.13)	1.5 ± 0.02	« 0 »
$\mathcal{A}s^{(1)}(w_1) \}$		-4.8 ± 0.24	0.2 ± 0.14
$\mathcal{A}s_{hf}^{(1)} \}$	§ 3.3, Eq. (3.14)	≤ 0.5	≤ 0.15
$\mathcal{A}s^{(2)}(\Delta E_z) \}$	§ 3.4, Eq. (3.16)	1.0 ± 0.4	1.4 ± 0.7
$\mathcal{A}s^{(2)}(\Delta E_x) \}$		« 0 »	0.4 ± 0.1
$\mathcal{A}s^{(0)}$	§ 3.5 $\left\{ \begin{array}{l} \text{Eq. (3.18)} \\ \text{Eq. (3.21)} \end{array} \right.$	0.0 ± 0.2	« 0 »
$\mathcal{A}s_u$	§ 3.6	« 0 »	« 0 »
(b) Systematic uncertainties			
$\mathcal{A}s^{(1)}(b_1, b_3)$	§ 3.2.2, Eq. (3.10)	0.8	0.3
$\mathcal{A}s^{(1)}(w_3) \}$	*	0.4	« 0 »
$\mathcal{A}s^{(1)}(w_1) \}$		5	« 0 »
$\Delta^{(1)}$	§ D.2, Eq. (D.22)	1	1
$\Delta^{(2)}$	§ 3.4.4	3	« 0 »
$\Delta^{(0)}$	§ 3.5.3, Eq. (3.19)	5	0.5
(c) Calibration uncertainty			
Δ_{cal}	§ 3.7	1	$2 \text{ (stat)} \pm 2.5 \text{ (syst)}$

* : Instrumental uncertainties in the real-time control of $\mathcal{A}s^{(1)}$ (to be discussed in Part III).

« 0 » : less than 10^{-3} of observed PV asymmetry.

induced $\mathbf{P}^{(2)}$ -component along $\hat{\mathbf{k}}$ is much smaller in the $\Delta F = 1$ line⁽⁹⁾.

3.5.4 Correction associated with possible correlation between E-reversal and fluctuations of the ambient magnetic field.

3.5.4.1 Origin and expression. — Hanle effect in stray $\mathbf{H}$ -fields is included in the analysis of §§ 3.4 and 3.5.1 provided it remains *time-independent*, since it then plays the same rôle as a misalignment (§ G.2c). Here we allow for fluctuations of the vertical component of the ambient field h_0 , which cause a fraction of the largest PC polarization component created along the beam ($\mathbf{P}^{(0)} + \mathbf{P}^{(2)}$) to be detected. Consequently : (i) « random » fluctuations contribute to the noise in the measured polarization; (ii) fluctuations *correlated with the reversal of E* make the residual polarization look E-odd, thereby simulating P^{pv} . The false asymmetry thus generated is simply

$$\mathcal{A}s(h_0) = \Pi \times (h_0^+ - h_0^-)/\Delta H, \quad (3.21a)$$

⁽⁹⁾ The above analysis shows that this effect was overestimated in our first report of Exp 2 [1]. The present estimate leads to a substantial reduction in the overall systematic uncertainty to be discussed in part III.

where $h_0^\pm$ denotes the time-average value of the ambient field component for either sign of the E-field voltage. Π is an electronic polarization ratio, both predicted theoretically and measured by deliberately inducing a large h_0 -asymmetry ($\Pi \approx 8 \times 10^{-2}$ in Exp 1 and 10^{-2} in Exp 2; § F.2d and last line of Table XIIIb).

3.5.4.2 Control. — Before the measurements, the absence of *causal* correlation between h_0 and E-reversal (e.g. generated by relay coils) was verified with a flux-gate probe placed near the cell centre. The result, $(h_0^+ - h_0^-) < 1.5 \times 10^{-6}$ G, yields the limits :

$$\mathcal{A}s(h_0)/\mathcal{A}^{pv} \begin{cases} < 5 \times 10^{-3}, & \text{if } \Delta F = 0, \\ < 6 \times 10^{-4}, & \text{if } \Delta F = 1. \end{cases} \quad (3.21b)$$

Although no troublesome effect was expected, during Exp 1 the fluxgate was used to record the earth's field h_0 and extract its *accidental* correlations with E-reversal. The false asymmetry thus accurately known was corrected from each individual PV data. The average correction amounted to 1 % only and led to 1.4 % noise reduction.

3.6 EFFECT OF IMPERFECTIONS ACTING ON THE UNPOLARIZED FLUORESCENCE SIGNAL. — The unpolarized fluorescence signal $\mathcal{F}_u$ is the largest of all detected PC signals ($\mathcal{F}_u/\mathcal{F}^{PV} \approx 10^6$). This signal, however, differs from the PV signal by three characteristics (Eqs. (3.2) and (3.4)) : it is even under reversal of $\mathbf{E}$ or ξ , and it is free from the ξ_r modulation which characterizes the polarized signal $\mathcal{F}_p$. Therefore $\mathcal{F}_u$ cannot simulate $\mathcal{F}^{PV}$ without conspiracy of imperfect reversal of $\mathbf{E}$, imperfect labelling of ξ and imperfection in ξ_r . For $|E_1^{PV}/\beta| = 1.7$ mV/cm and with the imperfection values quoted in part I (Table VI) the systematic effect is estimated to be totally negligible. ($\mathcal{A}_{s_u}/\mathcal{A}^{PV} \approx 6 \times 10^{-4}$ in Exp 1, at $E = 100$ V/cm, and $\approx 3 \times 10^{-5}$ in Exp 2, at $E = 600$ V/cm.)

3.7 CALIBRATION UNCERTAINTIES. — The calibration procedure has been presented thoroughly in part I : principle and validity (§ 1.3.2); practice (§ 2.9) and interpretation of the measured quantity in the *ideal* case (App. C). Here we allow for experimental imperfections.

Basically the procedure consists in forming the ratio $P^{PV}/P_{44}^{(2)}$ where $P_{44}^{(2)}$ (polarization $P^{(2)}$ in the $4 \rightarrow 4$ transition) is observed by Hanle effect. Since P^{PV} and $P_{44}^{(2)}$ have the same $\xi\xi_r$ -dependence, they are measured using the same excitation elements as well as the same detection optics and electronics. Thus the experimental ratio eliminates most instrumental calibration factors. One remaining factor (actually the only one if $F = F' = 4$ as in Exp 1) results from the different dependence of each signal on the electric (and magnetic) field.

The dependence on H_0 is the Hanle factor, $(H_0/\Delta H)/[1 + (H_0/\Delta H)^2]$, whose maximum can be measured with 10^{-4} accuracy. No absolute calibration of H_0 is necessary.

The E-dependent factor in the ideal case is $1/E_y$. Taking into account actual E-inhomogeneities it turns out to be $\langle E_y \rangle / \langle E_y^2 + E_x^2 \rangle$, i.e. $0.99/\langle E_y \rangle$ in the model mentioned in part I (§ 2.3.4.3). On the whole, we estimate the uncertainty in the E-dependent factor to be 1 %.

Misalignments of $\hat{\mathbf{k}}$, $\hat{\mathbf{k}}_r$ and $\hat{\mathbf{H}}_0$, including their dispersion over the observed volume, yield a second-order correction, in practice less than 10^{-3} .

In addition, in the $3 \rightarrow 4$ component the measured polarization, $P^{PV} \propto \mathcal{F}_{e\pm f}/\mathcal{F}_{e+b}$, must be corrected for the $\Delta F = 0$ line overlap reducing the normalization amplitude $\mathcal{F}_{e+b}$. The correction factor, periodically measured during data acquisition, is $1 - p = 0.92 \pm 0.01$ (cf. § E.3c and Eq. (E.24)). Moreover, during Exp 2 we used an intermediate (instantaneously available) standard, provided by the large polarization $P_{\text{hf}}^x(3 \rightarrow 4)$ discussed in part I (§ B.1a). The ratio $P_{\text{hf}}^x(3 \rightarrow 4)/P_{44}^{(2)}$ was periodically measured. This indirect procedure avoided changing frequently the laser frequency but introduces an additional 2 % calibration uncertainty of statistical origin.

As seen in part I (App. C), from the quantities measured in Exp 1 and Exp 2 we extract the PV parameter $\text{Im } E_1^{PV}/\beta$ and $(\text{Im } E_1^{PV}/\beta) \times (\alpha/\beta)$ respectively (Eqs. (C.7)). The net calibration uncertainty in $\text{Im } E_1^{PV}/\beta$, parameter finally determined, is :

$$\Delta_{\text{cal}} \approx \begin{cases} 1\%, & \text{in Exp 1;} \\ 2\%(\text{stat}) \pm 2.5\%(\text{syst})^{(10)}, & \text{in Exp 2.} \end{cases}$$

4. Conclusion.

In this work we have aimed at a both complete and detailed understanding of all atomic or optical phenomena underlying the parity violation experiment. This appeared to us essential in order to master the systematics, since an overlooked signal might signify an overlooked systematic. The price paid is that the analysis may seem here or there somewhat involved. Actually, whereas the above presentation puts all systematics together, they have been studied one by one at great length throughout the five years work summarized here. As reported above or in the quoted references, numerous measurements have been performed to ascertain the analysis of each effect. In many cases the conclusions have been directly checked by experimental simulation of the imperfections, and different control methods have been cross-checked. So we have achieved great confidence in the analysis.

This methodical critical analysis of our experiment leads to a model where each possible systematic effect (Table VIII) is expressed in terms of measurable imperfections of the apparatus. Recording these imperfections during PV data acquisition allows quantitative estimation of each systematic effect. As can be seen from the results summarized in table XIa (practical organization and discussion will be given in Part III), owing to active reduction and continuous or repeated control, only two systematic effects ($\mathcal{A}^{s(1)}$ and $\mathcal{A}^{s(2)}$) attain a few percent of the observed PV signal. Consequently no significant correction had to be operated on the observed result. (In provision against possible *overlooked* systematic effects, consistency checks were performed on the data; Part III.)

We have been concerned with several limitations or uncertainties in the experimental procedure (Table XI) : (i) calibration uncertainty ; (ii) statistical and sometimes also systematic uncertainties affecting the imperfection controls ; (iii) uncertainties associated with possible inhomogeneities and spatial correlations of various parameters (electric field; detection efficiency of the beam passages;...). In these last uncertainties, estimated using auxiliary inhomogeneity measurements, the statistical part dominates. In the final PV result we have chosen to include all above uncertainties in the *reported sys-*

⁽¹⁰⁾ Combining uncertainties in $\langle E_y \rangle$, in p (overlap parameter) and in α/β .

tematic uncertainty. (Detailed evaluation in Part III.) This conservative attitude is in our opinion a factor of confidence in the reported uncertainty.

Acknowledgments.

We are grateful to Larry Hunter, who stimulated some aspects of this work by penetrating questions and contributed to the elaboration of methods used in the atomic control of experimental imperfections. We thank him also for a detailed critical examination of the whole analysis.

Appendix D.

D.1 SYSTEMATIC ASYMMETRY $\mathcal{A}_S^{(1)}$. — In this appendix our aim is to compute in terms of measurable quantities the false asymmetry arising from $\mathcal{F}^{(1)}$ because of light polarization modifications in the multipass cavity.

Here stray unreversed E-fields are irrelevant (since $\mathcal{F}^{(1)}$ is *E*-odd like $\mathcal{F}^{PV}$). Misalignments are treated up to first order. Coordinates and Stokes parameters are expressed in the *global* system *xyz* defined in § 3.1.2, in particular

$$\hat{\mathbf{k}}_n \cdot \hat{\mathbf{z}} = (-1)^{n+1}$$

for the *n*th passage ($n = 1$ to $2N$); even (resp. odd) *n* labels backward (resp. forward) passages. Since each passage may be detected with a different efficiency we introduce weights r_n , with the normalization :

$$\sum_{n=1}^{2N} r_n = 1. \quad (\text{D.1})$$

Starting from equation (A.18), we deduce the contribution of the *n*th passage to $\mathcal{F}^{(1)}$:

$$\mathcal{F}_n^{(1)} = n_{F'} p_{F'} M'_1 \eta E_0 \xi_f (-1)^{n+1} r_n \times [(\delta_{FF'} \alpha + g_{FF'} \beta) u_{0,n} - (\delta_{FF'} \alpha - g_{FF'} \beta) (v_{3,n} + \chi_n u_{1,n})]. \quad (\text{D.2})$$

We have set $\mathbf{E}_n = \eta E_0 \hat{\mathbf{E}}_{0,n}$, where $\eta = \pm 1$ is the sign of the cell voltage and $\hat{\mathbf{E}}_{0,n}$ is the field direction probed by the *n*th passage (¹¹). The $u_{i,n}$'s denote the Stokes parameters of the *n*th passage, and the χ_n 's involve small misalignments

$$\chi_n = (\hat{\mathbf{k}}_f \cdot \hat{\mathbf{y}} - \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{x}})_n. \quad (\text{D.3})$$

The total $\mathcal{F}^{(1)} = \sum_n \mathcal{F}_n^{(1)}$ is thus written

$$\mathcal{F}^{(1)} = n_{F'} p_{F'} M'_1 \eta E_0 \xi_f \times [(\delta_{FF'} \alpha + g_{FF'} \beta) \tilde{U}_0 - (\delta_{FF'} \alpha - g_{FF'} \beta) (\tilde{U}_3 + \tilde{U}_1)]. \quad (\text{D.4})$$

Here we have introduced « effective Stokes parameters »

$$\tilde{U}_0 = \sum_n (-1)^{n+1} r_n u_{0,n}, \quad (\text{D.5a})$$

$$\tilde{U}_3 = \sum_n (-1)^{n+1} r_n u_{3,n}, \quad (\text{D.5b})$$

$$\tilde{U}_1 = \sum_n (-1)^{n+1} r_n \chi_n u_{1,n}. \quad (\text{D.5c})$$

The r_n 's are nearly equal within small fluctuations σ_n . We thus let :

$$r_n = (1 + \sigma_n)/2N, \quad |\sigma_n| \ll 1, \quad \sum_n \sigma_n = 0, \quad (\text{D.6})$$

in accordance with (D.1). The misalignment χ_n (Eq. (D.3)) varies essentially because of inhomogeneities of $\mathbf{E}$ (§ 2.3.4). Its average value

$$\bar{\chi} = \sum_n (\hat{\mathbf{k}}_f \cdot \hat{\mathbf{y}} - \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{x}})_n / 2N \quad (\text{D.7})$$

results in practice from imperfect centring of the multipass between the capacitor plates. No other assumption is made concerning the σ_n 's and the χ_n 's.

Since the effects of the optics on the light polarization remain small, a simple formalism (Eqs. (3.6)) can be used to express the $u_{i,n}$'s in terms of the ideal U_i 's delivered by the modulator (¹²). The relevant dangerous imperfections (present in the entrance window and in the mirrors) are those polluting u_0 , u_1 or u_3 with u_2 , thus contaminating the effective Stokes parameters $\tilde{U}_i$ with the U_2 -signature (and thereby $\mathcal{F}^{(1)}$ with the PV signature). They are described by equations of the type :

$$u_{3,n} = U_3 + (w_3 + m_{3,n}) \times U_2. \quad (\text{D.8})$$

Here w_3 and $m_{3,n}$ are small real numbers. They respectively describe birefringence of axes at 45° to *x* and *y* in the entrance window, and birefringence of same axes accumulated in the mirrors, in the reflections previous to the *n*th passage. The difference of birefringences between one backward passage and the previous forward one :

$$m_{3,2p} - m_{3,2p-1} = b_{3,p}, \quad p = 1 \text{ to } N, \quad (\text{D.9})$$

appears below; $b_{3,p}$ is nothing but the birefringence

(¹¹) Variations of $|E_0|$ are accounted for by the r_n 's.

(¹²) See footnote (3) in § 3.2.1.

in the reflection on the back mirror in the middle of the p th double-passage.

Relations similar to (D.8-9) with indices 1 (or 0) instead of 3, can be written for $u_{1,n}$ (or $u_{0,n}$). They involve parameters w_1 , $m_{1,n}$, $b_{1,p}$ (or w_0 , $m_{0,n}$, $b_{0,p}$) describing birefringences of axes xy (or circular dichroisms).

Using (D.6), (D.8-9) and their analogues, the $\tilde{U}_i$'s (Eq. (D.5)), and then $\mathcal{F}^{(1)}$ (Eq. (D.4)), can be expressed in terms of the U_i 's. We discuss explicitly $\tilde{U}_3$ whose effect is dominant. Extension to $\tilde{U}_0$ (less harmful since circular dichroism is negligible compared to birefringences) and to $\tilde{U}_1$ (less harmful too, since it involves birefringence combined to small misalignments) is straightforward. We obtain :

$$\tilde{U}_3 = \left[\sum_{p=1}^N (\sigma_{2p-1} - \sigma_{2p})/2N \right] \times (U_3 + w_3 U_2) - \left[\sum_{p=1}^N b_{3,p}/2N \right] \times U_2 + \left[\sum_{n=1}^{2N} (-1)^{n+1} \sigma_n m_{3,n}/2N \right] \times U_2 \quad (\text{D.10})$$

(the two first sums now run over N double-passages instead of $2N$ passages). In this equation :

i) the 1st term would exactly vanish if each forward passage were followed by a backward passage of same efficiency ($\sigma_{2p-1} = \sigma_{2p}$). In fact it yields a harmful ($\propto U_2$) contribution from the window birefringence w_3 , and a harmless ($\propto U_3$) contribution present even in absence of birefringences. We define a « suppression factor » $\mathcal{S}$ by :

$$\sum_{p=1}^N (\sigma_{2p-1} - \sigma_{2p})/2N = 1/\mathcal{S}. \quad (\text{D.11})$$

It describes the mean efficiency of the successive double-passages in compensating the signal $\mathcal{F}^{(1)}$ in the absence of birefringences other than the window's. The experimental determination of $\mathcal{S}$ is given in appendix E;

ii) the 2nd term involves the quantity :

$$b_3 = \sum_{p=1}^N b_{3,p}/N, \quad (\text{D.12})$$

i.e. the average birefringence in one reflection on the back mirror. This term is present even when all passages have equal weights. A detailed study of b_3 is performed before the PV measurements;

iii) the 3rd term involves the « effective average birefringence » of the multipass :

$$\tilde{m}_3 = \sum_{n=1}^{2N} (-1)^{n+1} \sigma_n m_{3,n}/2N. \quad (\text{D.13})$$

It is associated with a correlation between the passage direction, the fluctuation (σ_n) in the passage efficiency and the polarization defect accumulated in previous reflections (through $m_{3,n}$). We expect $\tilde{m}_3 \ll b_3$ (§ D.2). Inserting (D.11-13) in (D.10) we obtain :

$$\tilde{U}_3 = U_3/\mathcal{S} + [w_3/\mathcal{S} - \frac{1}{2} b_3 + \tilde{m}_3] U_2, \quad (\text{D.14})$$

and similarly

$$\tilde{U}_0 = U_0/\mathcal{S} + [w_0/\mathcal{S} - \frac{1}{2} b_0 + \tilde{m}_0] U_2, \quad (\text{D.15})$$

where $\tilde{m}_0$ describes an « effective average circular dichroism » in the multipass, similar to $\tilde{m}_3$ (Eq. (D.13)). The result for $\tilde{U}_1$ (Eq. (D.5)) which involves misalignments is written :

$$\tilde{U}_1 = \chi U_1 + [\tilde{\chi} w_1 - \frac{1}{2} \tilde{\chi} b_1 + \tilde{m}_1] U_2, \quad (\text{D.16})$$

where :

$$b_1 = \left(\sum_{p=1}^N b_{1,p} \right) / N, \quad (\text{analogue to (D.12)}); \quad (\text{D.17})$$

$$\tilde{\chi} = \sum_{n=1}^{2N} (-1)^{n+1} r_n \chi_n; \quad (\text{D.18})$$

$$\tilde{m}_1 = \sum_{n=1}^{2N} (-1)^{n+1} (r_n \chi_n - \tilde{\chi}/2N) m_{1,n}. \quad (\text{D.19})$$

$\tilde{\chi}$ is an effective misalignment involving correlations between the passage direction and the misalignment. $\tilde{m}_1$ is an effective birefringence involving correlations between the passage direction, the misalignment (χ_n) and the polarization defect (through $m_{1,n}$). We expect $\tilde{m}_1 < b_1$ (§ D.2).

Inserting (D.14-16) in (D.4), $\mathcal{F}^{(1)}$ is finally written :

$$\begin{aligned} \mathcal{F}^{(1)} = n_F p_F M'_1 \eta E_0 \xi_f \{ & (\delta_{FF'} \alpha + g_{FF'} \beta) U_0/\mathcal{S} - (\delta_{FF'} \alpha - g_{FF'} \beta) (U_3/\mathcal{S} + \tilde{\chi} U_1) + \\ & + [(\delta_{FF'} \alpha + g_{FF'} \beta) w_0/\mathcal{S} - (\delta_{FF'} \alpha - g_{FF'} \beta) (w_3/\mathcal{S} + \tilde{\chi} w_1)] U_2 \\ & - \frac{1}{2} [(\delta_{FF'} \alpha + g_{FF'} \beta) b_0 - (\delta_{FF'} \alpha - g_{FF'} \beta) (b_3 + \tilde{\chi} b_1)] U_2 \\ & + [(\delta_{FF'} \alpha + g_{FF'} \beta) \tilde{m}_0 - (\delta_{FF'} \alpha - g_{FF'} \beta) (\tilde{m}_3 + \tilde{m}_1)] U_2 \}. \end{aligned} \quad (\text{D.20})$$

Several comments are relevant concerning this equation :

— the $U_{i \neq 2}$ terms are present even in the absence of polarization defects. They are reduced with respect

to the single-passage case by either the suppression factor $1/S$ or the effective misalignment $\tilde{\chi}$. They provide *in situ* measurement of these parameters (App. E);

— the three U_2 -terms, which simulate $\mathcal{F}^{PV}$, originate in polarization degradation by the window (1st U_2 -term) or by the mirrors (2nd and 3rd U_2 -terms). Comparing these terms with $\mathcal{F}^{PV}$ (Eq. (A.16)) we express the false asymmetry in terms of the genuine PV asymmetry :

$$\mathcal{A}^{(1)}/\mathcal{A}^{PV} = (M'_1/\text{Im } E_1^{PV}) [w_0/S - \frac{1}{2} b_0 + \tilde{m}_0 - [(\delta_{FF'} \alpha - g_{FF'} \beta)/(\delta_{FF'} \alpha + g_{FF'} \beta)] \times \\ \times (w_3/S - \frac{1}{2} b_3 - \tilde{m}_3 + \tilde{\chi} w_1 - \frac{1}{2} \tilde{\chi} b_1 - \tilde{m}_1)] \quad (\text{D.21a})$$

or, using equation (C.5a),

$$\mathcal{A}^{(1)} = 2 \Pi_{FF'}^{(1)} [(w_0/S - \frac{1}{2} b_0 + \tilde{m}_0)(\delta_{FF'} \alpha + g_{FF'} \beta)/(\delta_{FF'} \alpha - \delta_{FF'} \beta) - \\ - (w_3/S - \frac{1}{2} b_3 - \tilde{m}_3 + \tilde{\chi} w_1 - \frac{1}{2} \tilde{\chi} b_1 - \tilde{m}_1)]. \quad (\text{D.21b})$$

Here we have introduced the polarization ratio associated with the electronic component $\mathbf{P}^{(1)}$:

$$\Pi_{FF'}^{(1)} = 2 p_{F'} (\delta_{FF'} \alpha - g_{FF'} \beta) M'_1 / |g_{FF'} \alpha^2 - h_{FF'} \beta^2| E_0. \quad (\text{D.22})$$

Except the $\tilde{m}_i$'s, all parameters in (D.21) are measured either in real-time (case of the w_i terms; App. E) or just before the measurements (case of the b_i 's). The $\tilde{m}_i$ terms involve correlation effects in the multipass. Their contribution given by

$$\Delta^{(1)}/\mathcal{A}^{PV} = (M'_1/\text{Im } E_1^{PV}) [\tilde{m}_0 - (\tilde{m}_3 + \tilde{m}_1)(\delta_{FF'} \alpha - g_{FF'} \beta)/(\delta_{FF'} \alpha + g_{FF'} \beta)], \quad (\text{D.23})$$

is estimated by refining the model as outlined now.

D.2 ESTIMATION OF $\Delta^{(1)}$.

a) *Model*. — To compute the effective birefringences $\tilde{m}_3$ and $\tilde{m}_1$ (Eqs. (D.13) and (D.19)) we need a model of the fluctuations σ_n in the weights, of the misalignments χ_n , and of the birefringences $m_{1,n}$ and $m_{3,n}$ accumulated in the reflections.

The $m_{i,n}$'s are given by :

$$m_{i,n} = \sum_{k=1}^n m_k^{(i)} \quad (\text{D.24})$$

where $m_k^{(i)}$ is the birefringence in reflection k (preceding the k th passage). From the results of reference [9], $m_k^{(i)}$ is the sum of two contributions :

— incidence birefringence. The axes are in, and normal to, the incidence plane. The retardation varies like the incidence squared ($\delta = Ki^2$, $K \approx 3.4 \times 10^{-2} \text{ rad}^{-1}$);

— intrinsic birefringence independent of the incidence (actual values in Table X). Using the analytical description of the multipass given in appendix A of reference [9], the accumulated birefringences (D.24) are then exactly computed.

Concerning the weight fluctuations, we account i) for the small intensity attenuation at each reflection (1 to 3×10^{-3}); ii) for variations linear or quadratic along x (long axis of the multipass ellipses) ⁽¹³⁾, of maximum amplitude $\sigma < 10^{-2}$. (These variations

may exist in the detection efficiency or originate in inhomogeneities of $|E_0|$.)

According to equations (3.2), the unpolarized fluorescence signal $\mathcal{F}_u$ depends on u_3 and acquires u_1 -dependence under the tilt $\hat{\mathbf{E}} \cdot \hat{\mathbf{x}}$ responsible for the misalignment χ . Since u_1 and u_3 are modulated at the same frequency with orthogonal phases, variations of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{x}}$ appear through a phase rotation in the modulation of $\mathcal{F}_u$, when the multipass axis is systematically translated along x in the plane $y = 0$ (Fig. 13) so as to probe x -dependence. The observed rotation leads to adopt for χ_n a linear variation along x ($2 \times 10^{-2} \text{ rad/mm}$ with maximum amplitude $\chi_M < 0.16 \text{ rad}$). On the other hand, we estimate the imperfection in centring the multipass to be less than 1 mm , yielding $\tilde{\chi} < 2 \times 10^{-2} \text{ rad}$ ⁽¹⁴⁾.

b) *Results*. — In this model the effective birefringences $\tilde{m}_1$ and $\tilde{m}_3$ (Eqs. (D.13) and (D.19)) are exactly computed (by analytical summations over the $2N$ passages). The *linear* variation of the weights combining with the field-gradient along x yields the dominant contribution in $\tilde{m}_1$, while their *quadratic* variation is the most harmful in the case of $\tilde{m}_3$. The orders of

⁽¹³⁾ Variations along y are irrelevant since the multipass is nearly degenerated into the xz plane.

⁽¹⁴⁾ As described in § 3.4.3 the average value of the misalignment $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ ($\simeq \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{x}} + \hat{\mathbf{k}}_f \cdot \hat{\mathbf{y}}$) is measured *in situ* and compensated within a few mrad. This is not inconsistent with $\tilde{\chi} < 2 \times 10^{-2} \text{ rad}$ because of the difference $2 \hat{\mathbf{k}}_f \cdot \hat{\mathbf{y}}$ between $\langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle$ and $\tilde{\chi}$ (Eq. (D.7)), estimated to be $< 10^{-2} \text{ rad}$ (accuracy of our geometrical adjustment; § 2.7 in Part I).

magnitude are given by

$$\tilde{m}_3 \approx \frac{1}{4}(b_3 + f_3) \sigma / \cos \theta$$

and

$$\tilde{m}_1 \approx \frac{1}{8}(b_1 + f_1) \chi_M \sigma / \cos \theta. \quad (\text{D.25})$$

Here f_1 and f_3 are the birefringence components of the front mirror (analogues of b_1 and b_3 , Eqs. (D.12) and (D.17) resp.). $\cos \theta = 1 - d/R$ depends on the spacing d and on the curvature radius R of the mirrors. In our nearly confocal multipass cavity $\cos \theta$ can be as small as 2.5×10^{-2} . Equations (D.25) show today that a different configuration would have efficiently reduced $\tilde{m}_1$ and $\tilde{m}_3$.

Using $|b_i + f_i| < |b_i| + |f_i|$ and values quoted in table X, we finally obtain an upper limit for the systematic effect associated with (D.23) : $|\Delta^{(1)}/\mathcal{A}^{\text{pv}}| < 1\%$, in both Exp 1 and Exp 2.

Appendix E.

Here we describe the methods used to estimate *in situ* the effect of the window birefringence to the systematic effect $\mathcal{A}^{(1)}$ (w_i terms of Eqs. (D.21)).

E.1 POLARIZATION PARAMETERS UNDER CONTROL. — The atomic signals relevant to birefringence controls are independent of the passage direction. So they probe the average Stokes parameters :

$$\bar{u}_i = \sum_{n=1}^{2N} r_n u_{i,n}, \quad i = 1 \text{ to } 4.$$

Using the model and notations of appendix D, we obtain :

$$\bar{u}_i = U_i + w'_i U_2 \quad \text{with} \quad w'_i = w_i + \bar{m}_i, \\ \text{if } i = 0, 1 \text{ or } 3. \quad (\text{E.2})$$

Here $\bar{m}_1$ and $\bar{m}_3$ denote the average birefringence components (and $\bar{m}_0$ the average circular dichroism), accumulated in the $2N$ multipass reflections :

$$\bar{m}_i = \sum_{n=1}^{2N} m_{i,n} r_n, \quad i = 0, 1 \text{ or } 3. \quad (\text{E.3a})$$

In the analytical model of § D.2, and since $N \gg 1$, they are computed to be :

$$\bar{m}_i \simeq N[b_i + f_i]/2, \quad i = 0, 1 \text{ or } 3 \quad (\text{E.3b})$$

Moreover, using the symmetry of the Mueller matrix (Eq. (3.6b))

$$\bar{u}_2 = U_2 + w'_0 U_0 - w'_1 U_1 - w'_3 U_3. \quad (\text{E.4})$$

⁽¹⁵⁾ With our definitions (Eqs. (3.3), (3.6)), b_0 is the circular dichroism of the back mirror, but f_0 is the *opposite* of the c.d. of the front mirror. So equations (E.3b) involve the *difference* of the c.d., as expected for two mirrors facing each other.

Relations (E.2) and (E.4) imply that our control signals (explicited hereafter) are sensitive to the *global* imperfection degrading the polarization in the multipass cell (w'_i), which differs from the window's imperfection (w_i) by the mirror's one ($\bar{m}_i$). The orders of magnitude discussed in the text (§ 3.2.3) justify the approximation $w'_i \simeq w_i$ made hereafter.

E.2 BIREFRINGENCE MEASUREMENTS USING A H-FIELD. CONTROL OF A STRAY $\mathbf{H} \parallel \hat{\mathbf{k}}$. — In this section we use the results of appendix B in part I and we assume the ideal configuration ($\hat{\mathbf{k}} = \hat{\mathbf{z}}$, $\hat{\mathbf{k}}_r = \hat{\mathbf{x}}$, $\mathbf{E}_0 = \eta E_0 \hat{\mathbf{y}}$ $\eta = \pm 1$).

a) $\Delta F = 0$ components. $\mathbf{H}_0 \parallel \mathbf{E}$. — The H_0 -odd polarized fluorescence signal receives two contributions of different physical origin :

i) $\mathcal{F}_p(H_0\text{-odd}, \xi\text{-odd})$ (Eq. (B.13)) from Hanle effect; it involves $\bar{u}_2$ through ξ . In practice only the $\alpha\beta$ term contributes (up to terms $< 1\%$ neglected hereafter).

ii) $\mathcal{F}_{\text{hf}}^x$ (Eq. (B.5)) from hf mixing; it involves $\bar{u}_1$ through the quantity

$$2 \operatorname{Re} [\hat{\mathbf{E}} \cdot \boldsymbol{\varepsilon}(\hat{\mathbf{E}} \times \boldsymbol{\varepsilon}^*)] = -u_1 \hat{\mathbf{z}},$$

and is 100 times smaller than signal i).

Using equations (B.5, 6, 11, 13) and the sign conventions given in appendix C (Eq. (C.1)) we obtain at the centre of one $\Delta F = 0$ component :

$$\mathcal{F}_p(H_0\text{-odd}) \simeq \eta'' \mathcal{J}_{F'} \xi_f (\bar{u}_2 - w_{\text{hf}} \bar{u}_1), \\ \eta'' = \pm 1 \text{ (sign of } H_0),$$

with :

$$\mathcal{J}_{F'} = -n_{F'} p_{F'} \operatorname{sgn} g_{F'} \alpha \beta E_0^2 \mathcal{D}(H_0) \quad (\text{E.5})$$

and

$$w_{\text{hf}} = (1 - g_{F'}) |\mu_B \Delta H / \hbar| (1/\omega_7 - 1/\omega_6) \\ (\approx 4.2 \times 10^{-3}). \quad (\text{E.6})$$

(from Eqs. (B.1) and (B.3)). Then using equations (E.2) and (E.4)

$$\mathcal{F}_p(H_0\text{-odd}) \simeq \eta'' \mathcal{J}_{F'} \xi_f [U_2 + w_0 U_0 - w_3 U_3 - \\ - (w_1 + w_{\text{hf}}) U_1].$$

The relevant modulation amplitudes, as defined in table XII, are written :

$$\mathcal{F}_{e \pm f}(H_0\text{-odd}) \simeq \frac{1}{2} \eta' \eta'' \mathcal{J}_{F'} \\ \mathcal{F}_{e+b \pm f}^x(H_0\text{-odd}) \simeq -\frac{1}{4} \eta'' (w_1 + w_{\text{hf}}) \mathcal{J}_{F'} \\ \mathcal{F}_{e+b \pm f}(H_0\text{-odd}) \simeq \frac{1}{4} \eta'' w_3 \mathcal{J}_{F'}.$$

The quantities :

$$W_1 = \left[2 \sum_{\eta, \eta', \eta''} \eta'' (\mathcal{F}_{e+b+f}^x + \mathcal{F}_{e+b-f}^x) \right] / \\ \left[\sum_{\eta, \eta', \eta''} \eta' \eta'' (\mathcal{F}_{e+f} + \mathcal{F}_{e-f}) \right] \quad (\text{E.7a}) \\ W_3 = \{ \text{same with } \mathcal{F}_{e+b \pm f} \}$$

Table XII. — *Ideal modulations of the polarization parameters involved in the detected fluorescence signals.*

	Stokes notation	Polarization parameters			Designation of fluo signal amplitude
		Complete modulation	Relevant modulation frequency & phase	amplitude	
$ \varepsilon ^2$	U_0	1	—	—	—
$2 \operatorname{Re}(\varepsilon_x^* \varepsilon_y)$	U_1	$-\sin \omega_e t \cdot \cos \omega_b t$	$-\sin(\omega_e + \omega_b) t$	1/2	$\mathcal{F}_{e+b}^x$
$ \varepsilon ^2 \xi$	U_2	$\eta' \cos \omega_e t$	$\cos \omega_e t$	η'	$\mathcal{F}_e$
$ \varepsilon_x ^2 - \varepsilon_y ^2$	U_3	$\sin \omega_e t \cdot \sin \omega_b t$	$\cos(\omega_e + \omega_b) t$	-1/2	$\mathcal{F}_{e+b}$
$ \varepsilon ^2 \cdot \xi_f$	$U_0 \xi_f$	$\sin \omega_f t$	$\sin \omega_f t$	1	$\mathcal{F}_f$
$2 \operatorname{Re}(\varepsilon_x^* \varepsilon_y) \xi_f$	$U_1 \xi_f$	$-\sin \omega_e t \cdot \cos \omega_b t \cdot \sin \omega_f t$	$-\cos(\omega_e + \omega_b + \omega_f) t$ $\cos(\omega_e + \omega_b - \omega_f) t$	-1/4 -1/4	$\mathcal{F}_{e+b+f}^x$ $\mathcal{F}_{e+b-f}^x$
$ \varepsilon ^2 \xi \xi_f$	$U_2 \xi_f$	$\eta' \cos \omega_e t \cdot \sin \omega_f t$	$\sin(\omega_e + \omega_f) t$ $-\sin(\omega_e - \omega_f) t$	$\eta'/2$ $\eta'/2$	$\mathcal{F}_{e+f}$ $\mathcal{F}_{e-f}$
$(\varepsilon_x ^2 - \varepsilon_y ^2) \xi_f$	$U_3 \xi_f$	$\sin \omega_e t \cdot \sin \omega_b t \cdot \sin \omega_f t$	$\sin(\omega_e + \omega_b + \omega_f) t$ $-\sin(\omega_e + \omega_b - \omega_f) t$	-1/4 -1/4	$\mathcal{F}_{e+b+f}$ $\mathcal{F}_{e+b-f}$
$\xi(\varepsilon_x ^2 - \varepsilon_y ^2)$	$U_2 U_3$	$\eta' \cos \omega_e t \cdot \sin \omega_e t \cdot \sin \omega_b t$	$\cos(2\omega_e + \omega_b) t$	$-\eta'/4$	$\mathcal{F}_{2e+b}$

are thus interpreted as :

$$\left. \begin{aligned} W_1 &\simeq -w_1 - w_{\text{hf}} \\ W_3 &\simeq w_3 \end{aligned} \right\} \text{(valid for } \Delta F = 0). \quad (\text{E.7b})$$

b) $\Delta F = \pm 1$ components, $\mathbf{H}_k \nparallel \hat{\mathbf{k}}$. — Accounting for the magnetic circular dichroism (CD; Eq. (B.8)) the complete expression of the unpolarized fluorescence signal (Eqs. (A.20) and (A.8)) in $\Delta F = \pm 1$ components is :

$$\mathcal{F}_u = \mathcal{F}_{F'} [\bar{u}_0 + \bar{u}_3 + \operatorname{sgn} g_{F'} (\mathbf{H} \cdot \hat{\mathbf{k}}/\mathcal{H}) \bar{u}_2] \quad (\text{E.8})$$

with

$$\mathcal{F}_{F'} = \frac{1}{3} n_{F'} (1 - g_{F'}) \beta^2 E_0^2$$

and

$$1/\mathcal{H} = -2 |\mu_B/\hbar| (1/\omega_7 - 1/\omega_6) \alpha/\beta \simeq 9.7 \times 10^{-3} \text{ G}^{-1}. \quad (\text{E.9})$$

Inserting (E.2) and (E.4) in (E.8) and using table XII we obtain :

$$\begin{aligned} \mathcal{F}_{e+b} &= \mathcal{F}_{F'} \left[-\frac{1}{2} + \operatorname{sgn} g_{F'} (\mathbf{H} \cdot \hat{\mathbf{k}}/\mathcal{H}) w_3 \right] \\ \mathcal{F}_{e+b}^x &= \mathcal{F}_{F'} [-\operatorname{sgn} g_{F'} (\mathbf{H} \cdot \hat{\mathbf{k}}/\mathcal{H}) w_1] \\ \mathcal{F}_e &= \mathcal{F}_{F'} \eta' [\operatorname{sgn} g_{F'} (\mathbf{H} \cdot \hat{\mathbf{k}}/\mathcal{H}) + w_0 + w_3]. \end{aligned}$$

We now write $\mathbf{H} \cdot \hat{\mathbf{k}} = \eta'' H_k + h_k$ where $\eta'' = \pm 1$ is the sign of the applied field, H_k is its magnitude and $|h_k| \ll H_k$ is a stray unreversed component. We thus

obtain the quantities :

$$W_1 = \left(\sum_{\eta' \eta''} \eta'' \mathcal{F}_{e+b}^x \right) / \left(\sum \eta' \eta'' \mathcal{F}_e \right) = -w_1 \quad (\text{E.10})$$

$$W_3 = (\sum \eta'' \mathcal{F}_{e+b}) / (\sum \eta' \eta'' \mathcal{F}_e) = +w_3 \quad (\text{E.11})$$

$$\begin{aligned} \text{CD} &= \frac{1}{2} (\sum \eta' \eta'' \mathcal{F}_e) / |\sum \mathcal{F}_{e+b}| = \\ &= \operatorname{sgn} g_{F'} H_k / \mathcal{H} \quad (\text{E.12}) \end{aligned}$$

$$\begin{aligned} \text{CD}_{\text{stray}} &= \frac{1}{2} (\sum \eta' \mathcal{F}_e) / |\sum \mathcal{F}_{e+b}| = w_0 + w_3 + \\ &+ \operatorname{sgn} g_{F'} h_k / \mathcal{H}, \quad (\text{E.13}) \end{aligned}$$

where all sums run over η , η' , and η'' . Equation (E.13) shows that in a given transition the stray magnetic CD (h_k term) cannot be distinguished from circular dichroism (w_0) or birefringence (w_3) in the optics. Yet comparison of the two $\Delta F = \pm 1$ components having opposite $g_{F'}$ provides measurement of h_k through the difference

$$\begin{aligned} \text{CD}_{\text{str}}(3 \rightarrow 4) - \text{CD}_{\text{str}}(4 \rightarrow 3) &\simeq h_k / \mathcal{H} \simeq \\ &\simeq 2 \times 10^{-2} h_k (\text{G}). \quad (\text{E.14}) \end{aligned}$$

Methods a) and b) used to determine the same birefringence in $\Delta F = 0$ and $\Delta F \neq 0$ lines give consistent results.

E.3 COMPENSATION OF THE w_3 EFFECT BY A MODULATION IN THE LASER INTENSITY. — Here no **H**-field is present so that the magnetic CD can be omitted. We suppose stray **H** to be sufficiently compensated. Now we assume that a small U_2 -like modulation is artificially added in the laser intensity. Thus all u_i 's, and thereby all fluorescence signals, are multiplied by $(1 + \mu U_2)$, with $\mu \ll 1$.

a) Effective birefringence $\tilde{w}_3$. Error signal. — Examination of (D.20) shows that (to first order) μ will act in $\mathcal{F}^{(1)}$ just as w_0 (circular dichroism in the optics). Therefore the systematic effect $\mathcal{A}_S^{(1)}$ is still given by (D.21) with $w_0 + \mu$ instead of w_0 . We define an effective window birefringence $\tilde{w}_3$ equal to the net imperfection combining to $1/8$ in (D.21b)

$$\tilde{w}_3 = w_3 - (w_0 + \mu) (\delta_{FF'} \alpha + g_{FF'} \beta) / (\delta_{FF'} \alpha - g_{FF'} \beta). \quad (\text{E.15})$$

$$\left. \begin{aligned} \mathcal{F}_{e+b} &= \frac{1}{4} n_{F'} E_0^2 (\delta_{FF'} \alpha^2 - h_{FF'} \beta^2) \\ \mathcal{F}_{2e+b} &= \frac{1}{8} n_{F'} E_0^2 (\delta_{FF'} \alpha^2 - h_{FF'} \beta^2) \mu \eta' \\ \mathcal{F}_e &= \frac{1}{2} n_{F'} E_0^2 [(\delta_{FF'} \alpha^2 + h_{FF'} \beta^2) (w_0 + \mu + \mu b_{dc}) - (\delta_{FF'} \alpha^2 - h_{FF'} \beta^2) w_3] \eta'. \end{aligned} \right\} \quad (\text{E.17})$$

In $\mathcal{F}_e$ we have introduced the small, readily measured parameter :

$$b_{dc} = B_u^2 \left/ \left[\frac{1}{2} n_{F'} E_0^2 (\delta_{FF'} \alpha^2 + h_{FF'} \beta^2) \right] \right. \simeq \mathcal{F}_{dc}(E_0 = 0) / \mathcal{F}_{dc}(E_0) \ll 1. \quad (\text{E.18})$$

The η' -odd amplitudes $\mathcal{F}_{2e+b}$ and $\mathcal{F}_e$ provide two new control signals :

$$\text{a « } \mu\text{-control signal » : } S_\mu = \sum_{\eta\eta'} \eta' (\mathcal{F}_{2e+b} / 2 \mathcal{F}_{e+b}) \quad (\text{E.19a})$$

$$\text{an « error signal » : } S_e = \sum_{\eta\eta'} \eta' (\mathcal{F}_e / 8 \mathcal{F}_{e+b}), \quad (\text{E.20a})$$

interpreted from (E.17) as

$$S_\mu = \mu \quad (\text{E.19b})$$

$$S_e = [w_0 + \mu(1 + b_{dc})] (\delta_{FF'} \alpha^2 + h_{FF'} \beta^2) / (\delta_{FF'} \alpha^2 - h_{FF'} \beta^2) - w_3. \quad (\text{E.20b})$$

As shown below S_e is sufficiently close to $-\tilde{w}_3$ (Eq. (E.15)) to constitute an appropriate error signal.

b) Procedure during Exp 1 ($\Delta F = 0$). — Here $h_{FF'} \beta^2 / \alpha^2 = 5 \beta^2 / 12 \alpha^2 \ll 1$, $|g_{FF'} \beta / \alpha| = |\beta / 8 \alpha| \ll 1$ since $\alpha / \beta \simeq -9.9$; in practice $w_0 \ll w_3$ and $b_{dc} \simeq 8.5\%$ at $E_0 = 100$ V/cm.

This is exactly the quantity we would like to cancel by adjusting μ . An error signal as close to $\tilde{w}_3$ as possible is thus required.

To compute the U_2 -type modulation present in $\mathcal{F}_u$ we start from the expression :

$$\mathcal{F}_u = \left\{ \frac{1}{2} n_{F'} E_0^2 [(\delta_{FF'} \alpha^2 + h_{FF'} \beta^2) \bar{u}_0 - (\delta_{FF'} \alpha^2 - h_{FF'} \beta^2) \bar{u}_3] + B_u^2 \bar{u}_0 \right\} \times (1 + \mu U_2). \quad (\text{E.16})$$

The $\hat{\mathbf{E}} \cdot \hat{\mathbf{x}}$ terms of equations (3.2) which had to second-order effects, are omitted here. The B_u^2 term accounts for the existence of a contribution independent of both E_0 and the laser polarization (contribution omitted so far because it was not modulated)⁽¹⁶⁾. Inserting (E.2) in (E.16) and using table XII we extract the relevant modulation amplitudes :

When $\mu = 0$ (servo-loop unlocked), the effective birefringence (E.15) and the error signal (E.20b) are respectively given by :

$$\tilde{w}_3(\mu = 0) \simeq w_3 \quad \text{and} \quad S_e(\mu = 0) \simeq -w_3. \quad (\text{E.21})$$

During Exp 1, S_e (Eq. (E.20b)) was servo-locked to zero by applying the adequate amplitude

$$\mu \simeq w_3 / (1 + b_{dc}). \quad (\text{E.22})$$

This reduces the effective birefringence (E.15) to :

$$\tilde{w}_3 \simeq (b_{dc} - \beta/4 \alpha) w_3 / (1 + b_{dc}) (\simeq w_3/10 \text{ in practice})$$

or from (E.22), to :

$$\tilde{w}_3 \simeq (b_{dc} - \beta/4 \alpha) \mu = W_3^{\text{eff}}. \quad (\text{E.23})$$

W_3^{eff} is used as a real-time estimate of $\tilde{w}_3$. It is obtained by measuring μ (signal S_μ ; Eqs. (E.19a)) and b_{dc} (Eq. (E.18)). Moreover we periodically check the consistency between μ , the error signal $S_e(\mu = 0)$, and the quantity W_3 . We expect

$$\mu = -S_e(\mu = 0) / (1 + b_{dc}), \quad \text{from (E.21-22)},$$

⁽¹⁶⁾ In **H** = **0** the absence of **E**-independent $u_{i \neq 0}$ terms, expected from § 1.2.1 of part I, has been checked experimentally.

and

$$S_e(\mu = 0) = -W_3, \quad \text{from (E.21) and (E.7).}$$

c) *Procedure during Exp 2* ($\Delta F = 1$).

α) *Principle of the compensation of $\tilde{w}_3$.* — For a complete treatment in the $\Delta F = 1$ component we must account for signal contamination by $\Delta F = 0$ line overlap : typically 8 % and 0.3 % in $\mathcal{F}_u$ and $\mathcal{F}^{(1)}$ respectively, resulting in an additional offset between S_e and $\tilde{w}_3$. Since $\Delta F = 0$ lines contribute essentially through α -terms, the contamination in $\mathcal{F}_u$ (Eq. (E.16)) appears as a non-zero effective value $\delta_{FF'}^{\text{eff}} (\simeq 3 \times 10^{-4})$. Introducing the relevant parameter

$$p = \delta_{FF'}^{\text{eff}} \alpha^2 / h_{FF'} \beta^2 (\approx 8 \times 10^{-2} \ll 1, \text{ for } \Delta F = 1), \quad (\text{E.24})$$

the error signal (E.19) is now written :

$$S_e \simeq -(1+p)(1+b_{dc})\mu/(1-p) - w_3 \quad (\text{E.25})$$

(using $w_0 \ll w_3$). Since the contamination in $\mathcal{F}^{(1)}$ is negligible the effective birefringence is still given by (E.15), i.e. :

$$\tilde{w}_3 \simeq w_3 + \mu, \quad (\text{E.26})$$

or, in terms of the error signal measured with or without μ (servo-loop locked or unlocked respectively), by :

$$\tilde{w}_3 \simeq -[(1-p)S_e^\mu + (b_{dc} + 2p)S_e(\mu = 0)] / (1+p)(1+b_{dc}) = W_3^{\text{eff}}. \quad (\text{E.27})$$

It cancels out provided the error signal is locked to

$$S_e^\mu = \kappa S_e(\mu = 0) \quad \text{where} \quad \kappa = -(b_{dc} + 2p)/(1-p), \quad (\text{E.28})$$

a non-zero quantity because of the dc background and of line overlap contamination. From (E.26) the required modulation amplitude is in principle just opposite to the true birefringence ($\mu = -w_3$).

β) *Practical uncertainty in the compensation.* — $S_e(\mu = 0)$ is periodically measured (servo-loop unlocked); b_{dc} and p , and thereby the offset parameter κ in (E.28), are known from auxiliary measurements involving the measurement of $\mathcal{F}_u$ when the exciting polarization is made linear $\nearrow x$ then $\nearrow y$, with E_0 then without E_0 . Typical values for $E_0 = 650$ V/cm

are :

$$b_{dc} \simeq (12 \pm 1) \times 10^{-2}, \quad p \simeq (8 \pm 1) \times 10^{-2}, \\ \text{yielding } \kappa = 0.28 \pm 0.02. \quad (\text{E.29})$$

During PV measurements the loop is locked and μ adjusted to satisfy (E.28), i.e. $\tilde{w}_3 = 0$ (the gain in the loop is sufficient not to limit the compensation efficiency). The μ -control signal is used as consistency check of the procedure : from equations (E.25-26) we expect $\mu = S_e(\mu = 0)$, and both must agree with quantity $-W_3$ defined by (E.11). The dominant uncertainty in the procedure comes from uncertainties in b_{dc} and in p , quoted in (E.29). Using (E.25-27), we estimate the uncertainty in the compensation of the effective birefringence $\tilde{w}_3$ to be $\sim 4\%$ of its free value w_3 .

E.4 MEASUREMENTS OF $1/S$ AND $\tilde{\chi}$. — Using the modulations of $\xi_f U_3$ and $\xi_f U_1$ (Table XII) we extract the modulation amplitudes in signal $\mathcal{F}^{(1)}$ (Eq. (D.20)) :

$$\mathcal{F}_{e+b\pm f} = \frac{1}{4} n_{F'} p_{F'} M_1' (\delta_{FF'} \alpha - g_{FF'} \beta) \eta E_0 / S, \\ \mathcal{F}_{e+b\pm f}^x = \frac{1}{4} n_{F'} p_{F'} M_1' (\delta_{FF'} \alpha - g_{FF'} \beta) \eta E_0 \tilde{\chi}.$$

Both are η -odd. We define two polarization ratios :

$$P_{\text{res.3}}^{(1)} = \sum_{\eta'} \sum_{\eta} \eta (\mathcal{F}_{e+b+f} + \mathcal{F}_{e+b-f}) / 4 | \mathcal{F}_{e+b} |, \\ P_{\text{res.1}}^{(1)} = \sum_{\eta'} \sum_{\eta} \eta (\mathcal{F}_{e+b+f}^x + \mathcal{F}_{e+b-f}^x) / 4 | \mathcal{F}_{e+b} |, \quad (\text{E.30a})$$

interpreted, using (E.17) for $\mathcal{F}_{e+b}$ and the definition (D.22), as

$$P_{\text{res.3}}^{(1)} = \Pi_{FF'}^{(1)} / S, \\ P_{\text{res.1}}^{(1)} = \Pi_{FF'}^{(1)} \tilde{\chi}. \quad (\text{E.30b})$$

They represent the polarization ratios associated with the residual $P^{(1)}$ -components of U_3 and U_1 signature respectively, and they are just the factors combining to the window's imperfections w_3 and w_1 in $\mathcal{A}^{(1)}$ (Eq. (D.21b)).

E.5 RECONSTITUTION OF THE SYSTEMATIC EFFECT OF THE WINDOW. — Using equations (E.30), (E.15), (E.7b or E.10), we reconstitute the window contribution to (D.21) by :

$$\mathcal{A}^{(1)}(w_1, w_3) \simeq \begin{cases} -2 P_{\text{res.3}}^{(1)} W_3^{\text{eff}} - 2 P_{\text{res.1}}^{(1)} (W_1 + w_{\text{hf}}), & \text{in Exp 1,} \\ -2 P_{\text{res.3}}^{(1)} W_3^{\text{eff}} - 2 P_{\text{res.1}}^{(1)} W_1, & \text{in Exp 2.} \end{cases} \quad (\text{E.31})$$

The quantity W_3^{eff} , which measures the effective birefringence $\tilde{w}_3$, is either equal to the small offset (deduced from Eq. (E.23)) in Exp 1, or, to the uncertainty affecting the exact compensation of $\tilde{w}_3$ in

Exp 2 (Eq. (E.27) and § E.3c β). All quantities involved are measured *in situ*, except the atomic factors β/α (in (E.23)) and w_{hf} (Eq. (E.6)), determined from previous experiments.

Appendix F.

In this appendix we derive the complete expression of the systematic effect $\mathcal{A}^{(2)}$ arising from the ξ -dependent PC component of the spin polarization along $\mathbf{E}$ (cf. Table VIII), and we define the quantities used to measure the relevant imperfections.

F.1 THE SYSTEMATIC ASYMMETRY $\mathcal{A}^{(2)}$. — We start from the expression of $\mathcal{F}^{(2)}$ obtained in appendix A in the most general configuration (Eqs. (A.17)). Here we consider the $(\mathbf{E} \cdot \hat{\mathbf{k}}) \cdot \mathbf{E}$ term, associated with a polarization component along $\mathbf{E}$:

$$\mathcal{F}^{(2)} = n_{F'} p_{F'} (\delta_{FF'} \alpha + g_{FF'} \beta) \beta (\mathbf{E} \cdot \hat{\mathbf{k}}) \mathbf{E} \cdot \xi |\varepsilon|^2 \xi_f \hat{\mathbf{k}}_f + \dots, \quad (\text{F.1})$$

We replace $\mathbf{E}$ by $\eta \mathbf{E}_0 + \Delta \mathbf{E}$ where $\eta = \pm 1$ is the sign of the Stark voltage : $\eta \mathbf{E}_0$ is the field that reverses exactly with the voltage while $\Delta \mathbf{E}$ is the stray field ($|\Delta \mathbf{E}| \ll E_0$). Within small misalignments the trihedral $(\hat{\mathbf{k}}_f, \hat{\mathbf{E}}_0, \hat{\mathbf{k}})$ coincides with $(x, y, (-1)^{n-1} z)$. (Here $\hat{\mathbf{k}}$ denotes the laser beam direction in passage $\# n$; the reference system (x, y, z) is defined in § 3.1.2 and Fig. 13.) We develop (F.1) up to first order in $|\Delta \mathbf{E}|/E_0$, and express each coefficient to the lowest non-vanishing order in the misalignments $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}, \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f, \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}_f$. We obtain :

$$\begin{aligned} \mathcal{F}^{(2)} = n_{F'} p_{F'} (\delta_{FF'} \alpha + g_{FF'} \beta) E_0^2 |\varepsilon|^2 \xi_f \{ (\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}) (\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f) + \\ + \eta [(\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f) (\Delta \mathbf{E} \cdot \hat{\mathbf{k}}/E_0) + (\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}) (\Delta \mathbf{E} \cdot \hat{\mathbf{k}}_f/E_0)] \}. \end{aligned} \quad (\text{F.2})$$

A similar development of $\mathcal{F}_u$ (Eq. (A.20)) yields

$$\begin{aligned} \mathcal{F}_u = n_{F'} [(\delta_{FF'} \alpha^2 |\varepsilon_y|^2 + h_{FF'} \beta^2 |\varepsilon_x|^2) E_0^2 (1 + 2 \eta \Delta E_y/E_0) + h_{FF'} M_1'^2 |\varepsilon|^2 + \\ + 2 \eta (\delta_{FF'} \alpha^2 - g_{FF'} \beta^2) \text{Re}(\varepsilon_x \varepsilon_y^*) E_0 \Delta E_x]. \end{aligned} \quad (\text{F.3})$$

The η -odd term in $\mathcal{F}^{(2)}$ has the complete signature of $\mathcal{F}^{\text{pv}}$ (Eq. (A.16)). Using table XII we extract the relevant modulation amplitudes :

$$\mathcal{F}_{e \pm f} = \frac{1}{2} n_{F'} p_{F'} (\delta_{FF'} \alpha + g_{FF'} \beta) \beta E_0^2 \eta' \{ \langle (\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}}) \cdot (\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f) \rangle + \eta [\langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \Delta E_z/E_0 \rangle + \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \Delta E_x/E_0 \rangle] \} \quad (\text{F.4})$$

$$\mathcal{F}_{e+b} = \frac{1}{4} n_{F'} (\delta_{FF'} \alpha^2 - h_{FF'} \beta^2) E_0^2 (1 + 2 \eta \langle \Delta E_y/E_0 \rangle) \quad (\text{F.5})$$

$$\mathcal{F}_{e+b}^x = \frac{1}{2} n_{F'} (\delta_{FF'} \alpha^2 - h_{FF'} \beta^2) E_0^2 \eta \langle \Delta E_x/E_0 \rangle \quad (\text{F.6})$$

where the notation $\langle \rangle$ implies an average over the observed atoms. In (F.4) the unit vector $\hat{\mathbf{j}}$ is defined by $\hat{\mathbf{j}}_n = (-1)^{n-1} \hat{\mathbf{k}}_n$ (i.e. $\hat{\mathbf{j}}$ remains close to $+z$ both for forward and backward beam passages). Up to first order in $|\Delta \mathbf{E}/E_0|$ and second order in misalignments, the polarization ratio is written

$$\begin{aligned} \mathcal{F}_{e \pm f} / |\mathcal{F}_{e+b}| = \Pi_{FF'}^{(2)} \eta' \{ \langle (\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}}) (\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f) \rangle (1 - 2 \eta \langle \Delta E_y/E_0 \rangle) + \\ + \eta [\langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \Delta E_z/E_0 \rangle + \langle E_0 \cdot \hat{\mathbf{j}} \Delta E_x/E_0 \rangle] \} \end{aligned} \quad (\text{F.7})$$

where :

$$\left. \begin{aligned} \Pi_{FF'}^{(2)} &= 2 p_{F'} (\delta_{FF'} \alpha + g_{FF'} \beta) \beta / |\delta_{FF'} \alpha^2 - h_{FF'} \beta^2| \\ &\simeq \frac{2}{3} (1 + 2 g_{F'}) \beta / \alpha, & \text{if } \Delta F = 0 \text{ (Exp 1),} \\ &= 1 + 2 g_{F'}, & \text{if } \Delta F = 1 \text{ (Exp 2),} \end{aligned} \right\} \quad (\text{F.8})$$

(using (A.8) and the approximation $|g_{F'} \beta / \alpha| \simeq 1/80 \ll 1$ in the case $\Delta F = 0$). Inserting (F.7) in the definition of the asymmetry

$$\mathcal{A} = \sum_{\eta=\pm 1} \sum_{\eta'=\pm 1} \eta \eta' (\mathcal{F}_{e+f} + \mathcal{F}_{e-f}) / 4 |\mathcal{F}_{e+b}|, \quad (\text{F.9})$$

we obtain the false asymmetry in terms of the true one (Eq. (C.5a)) :

$$\begin{aligned} \mathcal{A}^{(2)} / \mathcal{A}^{\text{pv}} \simeq (\beta / \text{Im } E_1^{\text{pv}}) [\langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle \langle \Delta E_z \rangle + \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \rangle \langle \Delta E_x \rangle - \\ - 2 \langle \Delta E_y \rangle \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \rangle] + \Delta^{(2)} / \mathcal{A}^{\text{pv}}. \end{aligned} \quad (\text{F.10})$$

Terms between brackets are written as if $\langle \delta_1 \delta_2 \rangle = \langle \delta_1 \rangle \langle \delta_2 \rangle$ for all pairs of imperfections conspiring in (F.7). Yet $\Delta^{(2)}$ is a correction allowing for the difference $\langle \delta_1 \delta_2 \rangle - \langle \delta_1 \rangle \langle \delta_2 \rangle$, i.e. for spatial correlation between the relevant imperfections. We describe below i) how each imperfection parameter involved in $\mathcal{A}^{(2)}$ is measured *in situ* (§ F.2); ii) how $\Delta^{(2)}$ can be controlled *in situ* too (§ F.3).

F.2 *In situ* CONTROL OF $\mathcal{A}^{(2)}$.

a) *Measurements of $\langle \Delta E_x \rangle$ and $\langle \Delta E_y \rangle$.* — The η -odd parts of $\mathcal{F}_{e+b}$ and $\mathcal{F}_{e+b}^x$ yield the asymmetries A_u and A_u^x of the unpolarized fluorescence signals under E_0 -reversal :

$$A_u = \frac{1}{2} \sum_{\eta'=\pm 1} \left[\left(\sum_{\eta} \eta \mathcal{F}_{e+b} \right) / \left(\sum_{\eta} \mathcal{F}_{e+b} \right) \right] \quad (\text{F.11a})$$

and

$$A_u^x = \frac{1}{4} \sum_{\eta'=\pm 1} \sum_{\eta=\pm 1} (\eta \mathcal{F}_{e+b}^x / \mathcal{F}_{e+b}). \quad (\text{F.12a})$$

Inserting (F.5 and F.6) in (F.11a and F.12a) we obtain :

$$A_u = 2 \langle \Delta E_y / E_0 \rangle \quad (\text{F.11b})$$

$$A_u^x = 2 \langle \Delta E_x / E_0 \rangle. \quad (\text{F.12b})$$

b) *Measurements of $\langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle$.* — Here we consider the case where an additional electric field is applied along the beam direction $\hat{\mathbf{z}}$. Replacing $\mathbf{E}$ by $\eta_1 E_0 \hat{\mathbf{E}}_0 + \eta_2 e \hat{\mathbf{z}}$ ($\eta_1 = \pm 1$, $\eta_2 = \pm 1$) in equations (A.15-19), the ξ -dependent term in $\mathcal{F}_p$ is found to be :

$$\mathcal{F}_p = n_{F'} p_{F'} \{ (\delta_{FF'} \alpha + g_{FF'} \beta) \beta E_0 [\eta_1 \eta_2 e (\hat{\mathbf{z}} \cdot \hat{\mathbf{k}} \hat{\mathbf{E}}_0 + \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}} \hat{\mathbf{z}}) + \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}} \hat{\mathbf{E}}_0] + (-\delta_{FF'} \alpha \beta E_0^2 + g_{FF'} \beta^2 e^2 + g_{FF'} M_1'^2) \hat{\mathbf{k}} \} \cdot |\boldsymbol{\varepsilon}|^2 \xi \xi_f \hat{\mathbf{k}}_f.$$

In

$$\mathcal{F}_u = n_{F'} [(\delta_{FF'} \alpha^2 - h_{FF'} \beta^2) E_0^2 |\varepsilon_y|^2 + h_{FF'} (\beta^2 E^2 + M_1'^2) |\boldsymbol{\varepsilon}|^2]$$

only the first term contributes to the modulation amplitude $\mathcal{F}_{e+b}$. The η_1 -odd, η_2 -odd part in the polarization ratio, defined by :

$$\Pi_e = \sum_{\eta'} \sum_{\eta_1} \sum_{\eta_2} \eta' \eta_1 \eta_2 (\mathcal{F}_{e+f} + \mathcal{F}_{e-f}) / 16 |\mathcal{F}_{e+b}|, \quad (\text{F.13a})$$

is finally interpreted as :

$$\Pi_e = \Pi_{FF'}^{(2)} (e/E_0) \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle \quad (\text{F.13b})$$

(since $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}} \ll \hat{\mathbf{z}} \cdot \hat{\mathbf{k}} \sim 1$). Quantitatively, using equations (F.8), $g(F' = 4) = 1/8$, $\alpha/\beta \simeq -9.9$,

$$\Pi_e \simeq \begin{cases} -4 \times 10^{-3} \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle, & \text{in Exp 1 } (\Delta F = 0; e/E_0 = 4.5 \times 10^{-2}) \\ 2 \times 10^{-2} \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle, & \text{in Exp 2 } (\Delta F = 1; e/E_0 = 1.5 \times 10^{-2}). \end{cases} \quad (\text{F.13c})$$

When in addition a Hanle field is applied along z ($\mathbf{H} = \eta'' \Delta H \hat{\mathbf{z}}$, $\eta'' = \pm 1$), from the results of appendix B (Eqs. (B.13), (B.11)), Π_e acquires an $\mathbf{H}$ -odd part simply given by (F.13b) with $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ replaced by $\text{sgn } g_{F'} \mathcal{D}(\Delta H) \hat{\mathbf{H}} \times \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ ($= -\frac{1}{2} \eta'' \text{sgn } g_{F'}$). Thus finally the ratio :

$$\Pi_e(\mathbf{H} = 0) \left/ \sum_{\eta''=\pm 1} \eta'' \Pi_e(\Delta H) \right. = -\text{sgn } g_{F'} \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle \quad (\text{F.14})$$

provides an *absolute* measurement of $\langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle$.

c) *Measurements of $\langle \Delta E_z \rangle$ and of $\langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \rangle$.* — Here we assume that a Hanle field is applied along the beam ($\mathbf{H} \parallel \hat{\mathbf{z}}$), and we allow for small misalignments of $\mathbf{H}$. Inserting $\mathbf{E} = \eta E_0 \hat{\mathbf{E}}_0 + \Delta \mathbf{E}$ in equations (B.13) describing Hanle effect, up to first order in misalignments and in $|\Delta \mathbf{E}/E_0|$, we obtain the $\mathbf{H}$ -odd polarization ratio :

$$\mathcal{F}_{e\pm f} / |\mathcal{F}_{e+b}| = -\Pi_{FF'}^{(2)} \text{sgn } g_{F'} \mathcal{D}(H) \eta' \eta'' \{ \eta \Delta \mathbf{E} \cdot \hat{\mathbf{j}} / E_0 + \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} + \hat{\mathbf{H}} \cdot (\hat{\mathbf{j}} \times \hat{\mathbf{k}}_f) [\delta_{FF'} / (1 + g_{F'} \beta / \alpha) - (\lesssim 10^{-2})] \} \quad (\text{F.15})$$

($\eta'' = \pm 1$ is the sign of $\mathbf{H}$ along z). The last term ($\lesssim 10^{-2}$) comes from the $M_1'^2$ and B_p^2 contributions in (B.13).

We build two quantities, respectively η -odd and η -even :

$$\Pi_{\eta\text{-even}}(H_k) = \sum_{\eta\eta'\eta''} \eta' \eta'' (\mathcal{F}_{e+f} + \mathcal{F}_{e-f})/8 | \mathcal{F}_{e+b} |, \quad (\text{F.16a})$$

$$\Pi_{\eta\text{-odd}}(H_k) = \sum_{\eta\eta'\eta''} \eta \eta' \eta'' (\mathcal{F}_{e+f} + \mathcal{F}_{e-f})/8 | \mathcal{F}_{e+b} |, \quad (\text{F.17a})$$

whose interpretation, easily deduced from (F.15), is

$$\Pi_{\eta\text{-even}}(H_k) = - \Pi_{FF'}^{(2)} \text{sgn } g_{F'} \{ \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \rangle + \hat{\mathbf{H}} \cdot (\hat{\mathbf{j}} \times \hat{\mathbf{k}}_f) \cdot [\delta_{FF'}/(1 + g_{F'} \beta/\alpha) - (< 10^{-2})] \} \quad (\text{F.16b})$$

$$\Pi_{\eta\text{-odd}}(H_k) = - \Pi_{FF'}^{(2)} \text{sgn } g_{F'} \langle \Delta E_z/E_0 \rangle \quad (\text{F.17b})$$

(for $H = \Delta H$ i.e. $\mathcal{D}(H) = \frac{1}{2}$). Equation (F.16b) shows that in a $\Delta F = 0$ transition ($\delta_{FF'} = 1$) the misorthogonality of $\hat{\mathbf{H}}$ and $\hat{\mathbf{y}}$ adds to the misorthogonality of $\hat{\mathbf{E}}_0$ and $\hat{\mathbf{j}}$, while it is practically irrelevant in a $\Delta F = \pm 1$ transition. Therefore measurement of $\Pi_{\eta\text{-even}}(H_k)$ yields the defect $\langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \rangle$ unambiguously in the latter case only. The η -odd quantity (F.17b) provides a measure of $\langle \Delta E_z/E_0 \rangle$ in either hf component.

F.3 EFFECT OF SPATIAL CORRELATION IN $\mathcal{A}^{(2)}$.

a) Expression. — We explicit the correlation term $\Delta^{(2)}$ in equation (F.10). We take into account the spatial variations of the misalignments involved in the two first terms of $\mathcal{A}^{(2)}$:

$$\begin{aligned} \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f &\simeq a_x x, \quad \text{with } a_x \simeq \partial \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{x}} / \partial x (\approx 2 \times 10^{-2} \text{ mm}^{-1}), \\ \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} &\simeq a_z z, \quad \text{with } a_z \simeq \partial \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{z}} / \partial z (\approx 3 \times 10^{-3} \text{ mm}^{-1}). \end{aligned}$$

To first order in the space variables

$$\begin{aligned} \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \Delta E_z \rangle - \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle \langle \Delta E_z \rangle &= a_x g_{zx} \langle x^2 \rangle, \\ \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \Delta E_x \rangle - \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \rangle \langle \Delta E_x \rangle &= a_z g_{zx} \langle z^2 \rangle, \end{aligned} \quad (\text{F.18})$$

where $g_{zx} = \partial \Delta E_z / \partial x$ and $g_{zx} = \partial \Delta E_x / \partial z$ satisfy $g_{zx} = g_{xz}$ since $\text{rot } \Delta \mathbf{E} = \mathbf{0}$. We thus obtain :

$$\Delta^{(2)}/\mathcal{A}^{\text{pv}} = (\beta/\text{Im } E_1^{\text{pv}}) (a_x \langle x^2 \rangle + a_z \langle z^2 \rangle) g_{zx}. \quad (\text{F.19})$$

b) In situ control. — We now reexpress $\Delta^{(2)}$ in terms of observable quantities. Assuming $\Delta E_x = g_{zx} z$ in (F.12b) and averaging separately over $z > 0$ or $z < 0$ (by masking one half of the sensitive area), we deduce the existence of a forward-backward asymmetry in A_u^x (Eq. (F.12a)) :

$$A_u^x(z > 0) - A_u^x(z < 0) = 2 Z g_{zx} / E_0 \quad (\text{F.20})^{(17)}$$

($2 Z$ is the length of the observation region along z). (F.19) is finally rewritten :

$$\Delta^{(2)}/\mathcal{A}^{\text{pv}} = (\beta E_0 / \text{Im } E_1^{\text{pv}}) K [A_u^x(z > 0) - A_u^x(z < 0)] \quad (\text{F.21})$$

where $K = (a_x X^2 + a_z Z^2)/6$ Z is a factor of purely geometrical origin (it involves the observed lengths along x and z , and the gradient of $\hat{\mathbf{E}}_0$, induced by edge effects).

During its measurement, quantity (F.20) remained below the noise level, yet statistics was poor because the relevance of an accurate control appeared to us only after Exp 2.

Appendix G.

This appendix, very similar to appendix F, is devoted to the systematic effect $\mathcal{A}_s^{(0)}$ associated with the ξ -dependent polarization component along $\hat{\mathbf{k}}$ (Table VIII).

G.1 SYSTEMATIC ASYMMETRY $\mathcal{A}_s^{(0)}$. — The relevant fluorescence terms are now all terms proportional to $\xi \hat{\mathbf{k}} \cdot \xi_f \hat{\mathbf{k}}_f$ in $\mathcal{F}_p$ explicit in equations (A.17), (A.19), (A.21). Up to first order in $|\Delta \mathbf{E}|/E_0$, their net contribution is :

$$\mathcal{F}_p = n_{F'} p_{F'} | \varepsilon |^2 \langle \xi \hat{\mathbf{k}} \cdot \xi_f \hat{\mathbf{k}}_f [- \delta_{FF'} \alpha \beta E_0^2 (1 + 2 \eta \Delta E_y / E_0) + g_{FF'} M_1'^2 + k_{F'} B_p^2] \rangle \quad (\text{G.1})$$

⁽¹⁷⁾ Equation valid provided the spherical mirror detecting along $-x$ is removed. As discussed in § G.3b this mirror gives rise to a suppression effect in the gradient measurements.

where $k_{F'} = (\sum n_{F'} p_{F'})/n_{F'} p_{F'}$. In addition we allow for a possible offset $\mathcal{F}_{e+b}(E_0 = 0) \neq 0$ in the normalization amplitude (F.5). We define the fraction :

$$b_0 = \mathcal{F}_{e+b}(E_0 = 0)/\mathcal{F}_{e+b} \simeq 10^{-3}, \quad (G.2)$$

in the conditions of both Exp 1 and Exp 2. Up to first order in $\Delta E_y/E_0$, the polarization ratio arising from (G.1) is :

$$\mathcal{F}_{e\pm f}/|\mathcal{F}_{e+b}| = \Pi_{FF'}^{(2)} \eta' [\langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle (1 - 2 \eta \langle \Delta E_y/E_0 \rangle b_0) + 2 \eta (\langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \Delta E_y/E_0 \rangle - \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle \langle \Delta E_y/E_0 \rangle)] + \Pi_{FF'}^{(0)} \eta' \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle (1 - 2 \eta \langle \Delta E_y/E_0 \rangle) \quad (G.3)$$

where :

$$\Pi_{FF'}^{(2)} = -2 p_{F'} \delta_{FF'} \alpha \beta / |\delta_{FF'} \alpha^2 - h_{FF'} \beta^2| \quad (G.4)$$

$$\Pi_{FF'}^{(0)} = 2 p_{F'} (g_{FF'} M_1'^2 + k_{F'} B_p^2) / |\delta_{FF'} \alpha^2 - h_{FF'} \beta^2| E_0^2 \quad (G.5)$$

(polarization ratios associated with $\mathbf{P}^{(2)}$ and $\mathbf{P}^{(0)}$ along $\hat{\mathbf{k}}$). The η -odd, η' -odd part of (G.3) yields, in the experimental asymmetry (F.9), a spurious contribution :

$$\mathcal{A}_s^{(0)} = -2[\Pi_{FF'}^{(2)} b_0 + \Pi_{FF'}^{(0)}] \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_j \rangle \langle 2 \Delta E_y/E_0 \rangle + \Delta^{(0)}, \quad (G.6)$$

where the last term

$$\Delta^{(0)} = 4 \Pi_{FF'}^{(2)} [\langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \Delta E_y/E_0 \rangle - \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle \langle \Delta E_y/E_0 \rangle] \quad (G.7)$$

is a pure correlation effect.

We now reexpress each contribution to (G.6) in terms of measurable quantities.

G.2 RESIDUAL POLARIZATION. *In situ* CONTROL OF $\mathcal{A}_s^{(0)}$.

a) *Residual polarization.* — We form the η -even, η' -odd part of the polarization ratio :

$$P_{\text{res}} = \sum_{\eta'} \sum_{\eta} \eta' (\mathcal{F}_{e+f} + \mathcal{F}_{e-f}) / 8 |\mathcal{F}_{e+b}|, \quad (G.8a)$$

which represents a residual polarization along $\hat{\mathbf{k}}_f$. Summing the contributions (F.7) and (G.3) we obtain up to second order in misalignments :

$$P_{\text{res}} = P_{\text{res}}^{(2)} + P_{\text{res}}^{(0)}$$

with :

$$P_{\text{res}}^{(2)} = \Pi_{FF'}^{(2)} \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle + \Pi_{FF'}^{(2)} \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \rangle \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle, \quad (G.8b)$$

$$P_{\text{res}}^{(0)} = \Pi_{FF'}^{(0)} \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle. \quad (G.8c)$$

The 2nd quantity, $P_{\text{res}}^{(0)}$, is known by measuring the amplitudes $\mathcal{F}_{e\pm f}$, involved in (G.8a), when $E_0 = 0$. Then the difference $(P_{\text{res}} - P_{\text{res}}^{(0)})$ yields the « Stark » contribution $P_{\text{res}}^{(2)}$.

b) $\mathcal{A}_s^{(0)}$ -reconstitution. — Using now (F.11b) and (G.8b), (G.6) is rewritten :

$$\mathcal{A}_s^{(0)} \simeq -2[\delta_{FF'} P_{\text{res}}^{(2)} b_0 + P_{\text{res}}^{(0)}] \times A_u + \Delta^{(0)} \quad (G.9)$$

(with the approximation $P_{\text{res}}^{(2)} = \Pi_{FF'}^{(2)} \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle$ in the case $\delta_{FF'} = 1$).

c) *Remark about Hanle effect.* — Equations (G.8-9) account explicitly for misalignments of *geometric* origin. But they also account for Hanle effect in stray **H**-fields acting on the direction of the spin

Fig. 15. — (a) Scheme showing the regions OA and OA' imaged directly and *via* the spherical mirror M respectively, onto one half of the sensitive area. (b) Enlarged scheme. O : centre of the observation region. O' : centre of the entrance lens L of the detection optics. ξ and ξ_f : helicities of the incident and detected photons respectively.

polarization. In fact this effect is equivalent to a misalignment $\hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f = \text{sgn } g_{F'} (h_0/\Delta H)$ where h_0 is the **H**-component along $\hat{\mathbf{j}} \times \hat{\mathbf{k}}_f$, ΔH the Hanle width, and $\text{sgn } g_{F'}$ determines the sign of the Larmor precession (§ B.2 in Part I). Yet as explained in appendix B, the background contribution to Hanle effect ($\propto B_p^2$) is reduced by the factor $\sum n_{F'} p_{F'} \text{sgn } g_{F'} / \sum n_{F'} p_{F'} = 4/11$, with respect to its contribution through the geometric misalignment).

Table XIII. — *Origin and absolute magnitude of the E_0 -even, ξ -odd polarization components : (a) along the laser beam (Eqs. (G.4), (G.5)); E_0 in V/cm; (b) detected as « residual polarization » (Eqs. (G.8) and § G.2c).*

Using $\alpha/\beta \simeq -10$; $k_F = 22/15$ and $p_{F'} = 5/12$ for $F' = 4$; and

$$F = \begin{cases} 4 \\ 3 \end{cases} \quad \delta_{FF'} = \begin{cases} 1 \\ 3 \times 10^{-4} (+) \end{cases} \quad g_{FF'} = \begin{cases} 1/8 \\ 7/8 \end{cases} \quad h_{FF'} = \begin{cases} 5/12 \\ 7/12 \end{cases} \quad M'_1/\alpha = \begin{cases} 3 \text{ V/cm} \\ 3.5 \text{ V/cm} \end{cases} \quad B_p/\alpha = \begin{cases} 5 \text{ V/cm, in Exp 1} \\ 3.3 \text{ V/cm, in Exp 2.} \end{cases}$$

(a)	Exp 1	Exp 2
$\Pi'^{(2)}$	8.4×10^{-2}	$4.2 \times 10^{-3} (*)$
$\Pi^{(0)} \begin{cases} M_1'^2 \\ B_p^2 \end{cases}$	$0.93 \times 10^{-4} (100/E_0)^2$ $3 \times 10^{-3} (100/E_0)^2$	$4.3 \times 10^{-3} (600/E_0)^2$ $6.2 \times 10^{-3} (600/E_0)^2$
(b)	at $E_0 = 100$ V/cm	at $E_0 = 600$ V/cm
$P_{\text{res}} \begin{cases} P_{\text{res}}^{(2)} \\ P_{\text{res}}^{(0)} \end{cases} \begin{matrix} \text{via} \\ \text{misalignment} \end{matrix}$	$8.4 \times 10^{-2} (\langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle - \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \rangle \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle)$ $3 \times 10^{-3} \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle$	$4.2 \times 10^{-3} \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle - \frac{5}{4} \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}} \rangle \langle \hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f \rangle$ $10^{-2} \langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle$
$P_{\text{res}} \text{ via Hanle effect}$	$8.5 \times 10^{-2} h_0/\Delta H$	$1.1 \times 10^{-2} h_0/\Delta H$

(+) Effective value in (G.4) to account for $\Delta F = 0$ line overlap.

(*) From $\Delta F = 0$ line overlap.

d) *Quantitative interpretation of P_{res} .* — Table XIIIb summarizes the various quantitative contributions to the residual polarization, expected under the conditions of Exp 1 and Exp 2. Experimentally each of them can be deduced from various measurements performed with and without $\mathbf{E}$ -field, at and out of each hf resonance, with and without Hanle $\mathbf{H}_0$ -field, with and without misalignment magnification⁽¹⁸⁾. Expected and observed values fully agree.

Examination of table XIIIb shows that P_{res} can be cancelled using a Hanle field h_0 of adequate magnitude ($h_0/\Delta H \simeq -\langle \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f \rangle$, if $\Delta F = 0$). The actual values of P_{res} during Exp 1 and Exp 2 are discussed in the text (§ 3.5.2).

G.3 CORRELATION EFFECT $\Delta^{(0)}$. — $\Delta^{(0)}$ (Eq. (G.7)) arises from inhomogeneities in conjugated defects $\hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f$ and $\Delta E_y/E_0$. The misalignment $\hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f$ varies along the z -direction of the observed region. So we have to allow for variations of $\Delta E_y/E_0$ along z . For simplicity here we consider the case where only ΔE_y depends on z . However the final result is easily shown to include also a possible z -dependence in E_0 .

⁽¹⁸⁾ In practice $\hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f = \pm \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}_f$ is magnified, up to $\sim 5 \times 10^{-2}$ rad, and sign-reversed, by masking one or the other half of the sensitive area (Fig. 15a). $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{j}}$ is magnified up to $\sim \pm 10^{-2}$ rad by applying the main reversed voltage to either auxiliary electrode and grounding the other one. The effective value of $\hat{\mathbf{E}}_0 \cdot \hat{\mathbf{k}}_f$ is magnified using a Hanle field along $\hat{\mathbf{k}}$.

a) *Analysis without detection mirror.* — At the lowest order

$$\hat{\mathbf{j}} \cdot \hat{\mathbf{k}}_f = a_z z \quad \text{and} \quad \Delta E_y(z) = g_{yz} z, \quad (\text{G.10})$$

where a_z and g_{yz} denote the gradients. This yields the correlation effect

$$\Delta^{(0)} = \frac{4}{3} \Pi_{FF'}'^{(2)} Z^2 g_{yz} a_z / E_0 \quad (\text{G.11})$$

(2 Z is the length of the observed region). On the other hand introducing (G.10) in (G.8b) and (F.5), then averaging separately over $z > 0$ or $z < 0$ we deduce :

$$P_{\text{res}}^{(2)}(z > 0) = -P_{\text{res}}^{(2)}(z < 0) = \frac{1}{2} \Pi_{FF'}'^{(2)} a_z Z \quad (\text{G.12})$$

and

$$A_u(z > 0) - A_u(z < 0) = 2 Z g_{yz} / E_0, \quad (\text{G.13})$$

(from the definition (F.11a) of the fluorescence asymmetry A_u). These quantities thus reveal the gradients of interest. Combining them, (G.11) is simply rewritten :

$$\Delta^{(0)} = \frac{2}{3} [P_{\text{res}}^{(2)}(z > 0) - P_{\text{res}}^{(2)}(z < 0)] \times [A_u(z > 0) - A_u(z < 0)]. \quad (\text{G.14})$$

Separate measurements for $z > 0$ and $z < 0$ are achieved by masking the corresponding half of the detector's sensitive area.

b) Suppression effect of the detection mirror. — Figure 15 shows that the real situation is somewhat more complex : in addition to fluorescence emitted directly towards the detector by atoms at location A, using the spherical mirror we collect fluorescence emitted in a nearly opposite direction by atoms A' ⁽¹⁹⁾. From (G.10) the fields at A(z) and A'(-z) are opposite. Therefore the contributions from « direct » and « indirect » fluorescence in (G.11) have opposite signs. Their relative magnitudes depend on the mirror's efficiency and geometry. As seen in figure 15, detection *via* the mirror involves more

oblique rays. This tends to increase the « indirect » contribution to $\Delta^{(0)}$ (through gradient a_z). Yet this effect is counter-balanced by less efficient collection of the indirect fluorescence. For a quantitative estimation of the net effect, we perform gradient measurements with, then without, mirror : i) measurements of a_z (Eq. (G.12)) in the two configurations provide the geometrical factor ; ii) measurements of a well-known magnified field gradient ⁽²⁰⁾ provide the efficiency factor. On the whole we estimate the net correlation effect $\Delta^{(0)}$ to be reduced by a factor $1/S' = -0.08 \pm 0.1$, with respect to its value in the absence of detection mirror.

⁽¹⁹⁾ The signs of both ξ_f and $\hat{k}_f$ are opposite for rays detected directly and *via* the mirror. Thus the quantity $\xi_f \hat{k}_f$ (angular momentum of the emitted photons) involved in $\mathcal{F}_p$ (Eq. (G.1)) is the same, and the two contributions add.

⁽²⁰⁾ The chosen gradient, $\partial \hat{E}_0 \cdot \mathbf{z} / \partial z$, is induced by edge effects magnified, for the present purpose, by grounding the auxiliary electrodes.

References

- [1] BOUCHIAT, M. A., GUÉNA, J., HUNTER, L. R. and POTTIER, L., *Phys. Lett.* **117B** (1982) 358 and **134B** (1984), 463.
- [2] BOUCHIAT, M. A., GUÉNA, J. and POTTIER, L., *J. Physique* **46** (1985) 1897.
- [3] BOUCHIAT, M. A., GUÉNA, J. and POTTIER, L., *J. Physique Lett.* **41** (1980) L-299.
- [4] BUCKSBAUM, P., COMMINS, E. and HUNTER, L., *Phys. Rev. Lett.* **46** (1981) 640 ; and *Phys. Rev. D* **24** (1981) 1141.
- DRELL, P. and COMMINS, E., *Phys. Rev. A* **32** (1985) 2196.
- [5] ADELBERGER, E. G. *et al.*, *Nucl. Instrum. Meth.* **179** (1981) 181 ;
- LEVY, L. P., D. Phil. Thesis. Univ. of Michigan (1982) ;
- HINDS, E. A. and HUGHES, V. W., Private commun.
- [6] LEUCHS, G., Private commun. and to be published.
- [7] IACOPINI, E., SMITH, B., STÉFANINI, G. and ZAVATTINI, E., *Nuovo Cim.* **61B** (1981) 21.
- [8] BOUCHIAT, M. A., COBLENTZ, A., GUÉNA, J. and POTTIER, L., *J. Physique* **42** (1981) 985.
- [9] BOUCHIAT, M. A. and POTTIER, L., *Appl. Phys.* **B 29** (1982) 43.
- BOUCHIAT, M. A. and POTTIER, L., *Opt. Commun.* **37** (1981) 229.
- [10] BOUCHIAT, M. A., GUÉNA, J. and POTTIER, L., *Opt. Commun.* **37** (1981) 265.