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Statistical properties of photons in a three-level plus two-mode model

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Resume. — On étudie les caractéristiques des propriétés statistiques des photons dans un modèle analytique à trois niveaux et deux modes du champ. Le modèle consiste en un émetteur unique, à trois niveaux en lambda, initialement dans le niveau le moins excité, interagissant avec deux modes du champ quantifié. On calcule les variances, en ordre normal, du nombre de photons et les fonctions de corrélation croisées. Tous les résultats sont obtenus sans utiliser, ni théorie de perturbation, ni d'approximations de décorréléation. On montre que les états du mode « signal » du champ, obtenus à partir du vide initial, sont sub-poissoniens à tout instant. On décrit l'évolution des propriétés statistiques des photons quand le mode « signal » et le mode « pompe » sont initialement dans un état cohérent de Glauber. Dans des conditions convenables, on pourrait obtenir dans ce système, un dégroupement de photons et une anti-corrélation entre les modes.

Abstract. — The characteristics of the photon statistical properties are studied in a soluble three-level plus two-mode model. The model consists of a single lambda-configuration three-level emitter interacting with two modes of the quantized radiation field, the emitter being initially in the lowest level. The normally ordered variances of the photon numbers and the cross-correlation function are calculated. All the results are obtained without using perturbation theory and decorrelation approximations. It is shown that the states of the signal-mode field produced from its initial vacuum are sub-Poissonian for all times. For the case when both the signal-mode and pump-mode fields are initially in the Glauber coherent states the temporal behaviour of the photon statistical characteristics are described. Under suitable conditions photon antibunching and anticorrelation between the modes are potentially possible in the system.

1. Introduction.

During the last ten years the photon antibunching effect has attracted considerable interest, since it has no counterpart in classical electrodynamics and, hence, provides new evidence of the wave-particle dualism. Photon antibunching is characterized by a nonclassical state of the field in which the variance of the number of photons is less than the mean number of photons, i.e. the photons exhibit sub-Poissonian statistics. While experiments thus far are restricted to the study of resonance fluorescence from single atoms [1-3], various physical mechanism that generate such a state have been shown to be theoretically possible (see Refs. [4-20] and the reviews [21, 22]). Among them are second harmonic generation [4-7] and three-wave parametric interaction [8] in a nonlinear medium, resonance fluorescence from two-level atoms [9-15], degenerate four-wave mixing [16], two-photon absorption [17, 18], free-electron-laser generation [19, 20]. Recently, the generation of squeezed states, another class of nonclassical states [28], has also become the object of active researches (see [28-30] and refs. therein).

In this paper the production of sub-Poissonian non-classical states by using two-mode processes in a three-level system will be discussed. On the basis of the exactly soluble model, studied in [23] we shall examine the time behaviour of the normally ordered variances of the photon numbers and of the cross correlation between the modes. It will be shown that under suitable conditions photon antibunching and anticorrelation between the modes are possible in the system.

The remainder of the paper is organized as follows. In section 2 the model Hamiltonian and the necessary previous results are given. In section 3 we study the time behaviour of the normally ordered variances of the photon numbers. In section 4 the cross correlation between the modes is calculated. Finally, the conclusions are briefly summarized in section 5.
2. Model Hamiltonian and basic equations. — The lambda-configuration three-level emitter considered here is shown in figure 1. The upper level 3 is connected with the levels 1 and 2 by dipole transitions whereas the transition 1-2 is forbidden. The Hamiltonian describing the interaction of the emitter with the two-mode resonant radiation field in the dipole and rotating wave approximations is [23]

\[ H = \sum_{j=1}^{3} \hbar \Omega_j \hat{R}_{jj} + \sum_{a=1}^{2} \hbar \omega_a \hat{a}_a^+ \hat{a}_a + \sum_{a=1}^{2} \hbar \omega_a (\hat{a}_a \hat{R}_{3a} + \hat{a}_a^+ \hat{R}_{2a}) \]  

(1)

Here, the operator \( \hat{R}_{jj} = |j\rangle \langle j| \) describes the population of level \( j \) with the corresponding energy \( \hbar \Omega_j \) and state vector \( |j\rangle \). \( \hat{R}_{ij} = |i\rangle \langle j| \) is the operator of transition from level \( j \) to level \( i \). The operators \( \hat{R}_{ij}, i, j = 1, 2, 3 \), obey the following relations:

\[ \hat{R}_{ij} \hat{R}_{kl} = \delta_{ik} \hat{R}_{jl}, \quad \sum_{i=1}^{3} \hat{R}_{ii} = 1. \]

(2)

The photon operators \( \hat{a}_a, \hat{a}_a^\dagger \) describe two modes of the radiation field with the resonance frequencies \( \omega_a = \Omega_a - \Omega_e \) and \( g_a \)'s are the corresponding parameters of emitter-mode coupling.

The three-level two-mode model (1) has extensively been investigated [31, 23] (see also [32] and Refs. therein). In the framework of this model one can study not only various purely quantum effects in population dynamics and photon statistics [31, 34] but also fine features in indirect two-mode interaction via the emitter. An excellent review of the dynamical theory of the model has recently been given by Yoo and Eberly in paper [31]. Nonclassical photon statistics has been numerically shown. However, these authors do not make any comments about it. Some discussions about experimental verifications, being possible due to progress in the realization of a single atom in a cavity [33-37], have been given in reference [31]. We shall study the time behaviour of photon statistics and mode correlations below.

We assume that the emitter is initially on the lowest level 1. In the papers [23] the time-dependent two-mode photon-distribution function \( P(n_1, n_2) \) has been found explicitly and rigorously. It reads

\[ P(n_1, n_2) = P(n_1, n_2) R_{1n_1n_2}(t) + P(n_1 + 1, n_2 - 1) R_{2n_1+1n_2-1}(t) + P(n_1 + 1, n_2) [1 - R_{1n_1+1n_2}(t) - R_{2n_1+1n_2}(t)], \]

(3)

where

\[ R_{1n_1n_2}(t) = 1 - \frac{4 g_1^2 g_2^2 n_1(n_2 + 1)}{[g_1^2 n_1 + g_2^2(n_2 + 1)]^2} \sin^2 \left[ \frac{t}{2} \sqrt{g_1^2 n_1 + g_2^2(n_2 + 1)} \right] \]

(4)

\[ R_{2n_1+1n_2}(t) = \frac{4 g_1^2 g_2^2 n_1(n_2 + 1)}{[g_1^2 n_1 + g_2^2(n_2 + 1)]^2} \sin^2 \left[ \frac{t}{2} \sqrt{g_1^2 n_1 + g_2^2(n_2 + 1)} \right], \]

and the function \( P(n_1, n_2) = P_{e=0}(n_1, n_2) \) is the initial photon distribution. In general case, for an arbitrary initial field state described by a density matrix \( \hat{\rho}_F \) we have

\[ P(n_1, n_2) = \langle n_2, n_1 | \hat{\rho}_F | n_1, n_2 \rangle_F. \]

(5)

Starting from equations (3, 5), we shall examine photon statistics of the modes in the remainder of this paper.
3. Normally ordered variances of the numbers of the photons in the modes.

From equation (3) one easily finds the normally ordered variance $V_{[S]}$ of the photon number in the signal mode (mode 2)

$$V_{[S]}(t) = \langle \hat{a}_2^* \hat{a}_2(t) \rangle - \langle \hat{a}_2^2(t) \rangle^2$$

$$= \sum_{n_1, n_2} P(n_1, n_2) (n_2^2 - n_2) - \left\{ \sum_{n_1, n_2} P(n_1, n_2) n_2 \right\}^2$$

$$= V_{[S]}(0) + 2 \sum_{n_1, n_2} P(n_1, n_2) R_{2n_1n_2}(t) (n_2 - \bar{n}_2) - \left\{ \sum_{n_1, n_2} P(n_1, n_2) R_{2n_1n_2}(t) \right\}^2.$$  \ (6)

The expression of the corresponding quantity $V_{[P]}$ for the pump mode (mode 1) is found from equation (3) to be

$$V_{[P]}(t) = \langle \hat{a}_1^* \hat{a}_1(t) \rangle - \langle \hat{a}_1^2(t) \rangle^2$$

$$= \sum_{n_1, n_2} P(n_1, n_2) (n_1^2 - n_1) - \left\{ \sum_{n_1, n_2} P(n_1, n_2) n_1 \right\}^2$$

$$= V_{[P]}(0) + 2 \sum_{n_1, n_2} P(n_1, n_2) R_{1n_1n_2}(t) (n_1 - \bar{n}_1) - \left\{ \sum_{n_1, n_2} P(n_1, n_2) R_{1n_1n_2}(t) \right\}^2 + 1.$$ \ (7)

Here,

$$\bar{n}_1 = \sum_{n_1, n_2} P(n_1, n_2) n_1$$

and

$$\bar{n}_2 = \sum_{n_1, n_2} P(n_1, n_2) n_2$$

denote the mean numbers of the initial photons in the pump and signal modes, respectively.

As is well known, the quantities $V_{[S]}$ and $V_{[P]}$ characterize the photon statistical properties of the modes. They are proportional to the excess coincidence counting rates to be measured in a Hanbury Brown and Twiss-type [24, 25] experiment. The sign plus or minus or $V_{[S]}$ (or $V_{[P]}$) shows the photon statistics of the signal (or pump) mode is super- or sub-Poissonian and indicates, therefore, whether bunching (+) or antibunching (−) occurs [21, 22]. Now, we consider some consequences of the expressions (6) and (7).

First of all note that if the signal mode 2 is initially in the vacuum state, i.e. $P(n_1, n_2) = \delta_{n_20} P(n_1)$, from (6) we have

$$V_{[S]}(t) = - \left\{ \sum_{n_1} P(n_1) R_{2n_10}(t) \right\}^2 \leq 0.$$ \ (8)

This implies that the signal light field generated from the initial vacuum by arbitrary light pumping has sub-Poissonian photon statistics and, hence, exhibits photon antibunching for all times $t > 0$. In figure 2 we plot $V_{[S]}$ versus time $g_0 t$ for the case of coherent pumping with

$$g_1 = g_2 = g_0, \quad P(n_1, n_2) = \delta_{n_20} P(n_1) P(n_2) = \exp(-\bar{n}_1) \bar{n}_1^{n_1} / n_1!,$$

and $\bar{n}_1 = 12.$ The figure shows distinctly the Rabi non linear oscillations, collapse and revival of their envelopes (see [26] and below).

Let us now assume that both the signal and pump modes are initially in Glauber states $|z_1\rangle$ and $|z_2\rangle$, respectively. This means, the initial distribution $P(n_1, n_2)$ of the photon numbers in the modes is given
Substituting (9) into (6) and (7), we perform a numerical examination for $V_{[S]}$ and $V_{[P]}$. In figure 3 we plot these quantities as functions of time for $g_2 = 2g_0$, $\bar{n}_2 = 9$, $\bar{n}_1 = 4$. For very short times ($0 < g_0 t < 0.25$), the value of $V_{[S]}$ is positive, whilst the value of $V_{[P]}$ is negative. As time goes on ($g_0 t < 3$), we observe oscillations of both the values and their corresponding signs (see Fig. 3a). From the figure it is clear that sub-Poissonian photon statistics and antibunching in the modes occur for some finite intervals. Such a time behaviour of photon statistics is connected with the so-called Rabi oscillations [21, 22, 26]. For longer times ($g_0 t > 3$), $V_{[S]}$ and $V_{[P]}$ reach the steady values. This corresponds to the Cummings collapse region [26]. The negative steady value $V_{[S]}^0$ predicts antibunching in the signal mode. The stationary behaviour of the variances continues until the appearance of revivals [26], see figure 3b, where they start oscillating again. Thus, we see that in the considered case, when $g_0^2 \bar{n}_2/g_1^2 \bar{n}_1 = 9 \gg 1$ antibunching in the signal mode takes place both in the Rabi oscillation region and during the collapse and revival. An analytic approximate description of the above-obtained numerical results can be given in the following.

Due to the well-known properties of the two-dimension Poissonian distribution (9)

$$n_1 P(n_1, n_2) = \bar{n}_1 P(n_1 - 1, n_2),$$
$$n_2 P(n_1, n_2) = \bar{n}_2 P(n_1, n_2 - 1),$$

from equations (6) and (7) we have

$$V_{[S]}(t) = 2 \bar{n}_2 \left[ R_{2n_1n_2+1}(t) - R_{2n_1n_2}(t) \right] - \left[ R_{2n_1n_2}(t) \right]^2,$$
$$V_{[P]}(t) = 2 \bar{n}_1 \left[ R_{1n_1+1n_2}(t) - R_{1n_1n_2}(t) \right] - \left[ R_{1n_1n_2}(t) \right]^2 + 1.$$

Here, for convenience the notation $\overline{(...)} \equiv \sum_{n_1, n_2} P(n_1, n_2) (...) \text{ has been introduced.}$

In the case $\bar{n}_1, \bar{n}_2 \gg 1$ the summations over $n_1$ and $n_2$ in (11) can be performed approximately by using saddle-point techniques [26] if one notices that the weighting factor $P(n_1, n_2)$ will peak at $(\bar{n}_1, \bar{n}_2)$ with relatively narrow dispersion. This means that the expressions in (11) can be changed to integrals of fast oscillating functions which after trivial transformations can be written as

$$V_{[S]}(t) = \frac{g_1^2 g_2^2 \bar{n}_1 \bar{n}_2}{(W^2 + g_2^2)^2} \left[ 4 F(t; 2 g_2^2) - F(2 t; 2 g_2^2) \right] - \frac{\bar{n}_2 + 1}{W^4} \left[ 4 F(t; g_2^2) - F(2 t; g_2^2) \right] - \frac{4}{W^4} \left[ 4 F(t; g_2^2) - F(2 t; g_2^2) \right]^2,$$

$$V_{[P]}(t) = \frac{g_1^2 \bar{n}_1^2}{W^2} \left[ 4 g_2^2(\bar{n}_2 + 1) F(t; g_2^2) + g_1^2 \bar{n}_1 F(2 t; g_2^2) \right] - \frac{g_1^2 \bar{n}_1(\bar{n}_1 + 1)}{(W^2 + g_2^2)^2} \left[ 4 g_2^2(\bar{n}_2 + 1) F(t; g_2^2 + g_1^2) + g_1^2(\bar{n}_1 + 1) F(2 t; g_1^2 + g_2^2) \right] - \left\{ 1 - \frac{g_1^2 \bar{n}_1}{2 W^4} \left[ 4 g_2^2(\bar{n}_2 + 1) F(t; g_2^2) + g_1^2 \bar{n}_1 F(2 t; g_2^2) \right] \right\}^2 + 1,$$

where $W \equiv \sqrt{g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)}$ and

$$F(t; g^2) \equiv 1 - \int_0^\infty dx_1 \int_0^\infty dx_2 \mathcal{S}(x_1, x_2) \cos \left[ t \sqrt{g_1^2 x_1 + g_2^2 x_2 + g^2} \right].$$

with (see [26])

$$\mathcal{S}(x_1, x_2) = (4 \pi^2 x_1 x_2)^{-1/2} \exp[-\bar{n}_1 - \bar{n}_2 + x_1 + x_2 - x_1 \ln(x_1/\bar{n}_1) - x_2 \ln(x_2/\bar{n}_2)].$$

Note that the function $F(t; g^2)$ approaches unity in the limit $t \to \infty$. Then we can easily find from equations (12)
analytical approximate expressions for the steady values $V_0^{[s]}$ and $V_0^{[p]}$ of the variances. They read

$$V_0^{[s]} = 3 g_1^2 g_2^2 \bar{n}_1 \bar{n}_2 \left\{ \frac{\bar{n}_2 + 2}{[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 2)]^2} - \frac{\bar{n}_2 + 1}{[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)]^2} \right\} - \frac{9 g_1^4 g_2^4 \bar{n}_2^2(\bar{n}_2 + 1)^2}{4[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)]^4}.$$  

(15)

$$V_0^{[p]} = g_1^2 \bar{n}_1 \left\{ \frac{g_1^2 \bar{n}_1 + 4 g_2^2(\bar{n}_2 + 1)}{[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)]^2} - (\bar{n}_1 + 1) \frac{g_1^2(\bar{n}_1 + 1) + 4 g_2^2(\bar{n}_2 + 1)}{[g_1^2(\bar{n}_1 + 1) + g_2^2(\bar{n}_2 + 1)]^2} \right\} - \frac{[g_1^4 \bar{n}_1^2 + 2 g_2^4(\bar{n}_2 + 1)^2]^2}{4[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)]^4} + 1.$$  

We plot the quantity $V_0^{[s]}$ as a function of $\bar{n}_2$ in figure 4a (for $\bar{n}_1 = 10$, $g_1 = g_2 = g_0$) and as a function of $\bar{n}_1$ in figure 4b (for $\bar{n}_2 = 0, 10$, $g_1 = g_2 = g_0$). From the figures we see that $V_0^{[s]}$ is negative for very small values $\bar{n}_2$ ($\bar{n}_2 = 0$) and for large values $\bar{n}_2$ as compared with $\bar{n}_1$ ($g_2^2 \bar{n}_2 \gg g_1^2 \bar{n}_1$). By using the first equation in (15) we can easily get the estimate

$$V_0^{[s]} \simeq -\frac{3 g_1^2 \bar{n}_1}{g_2^2 \bar{n}_2} < 0$$  

(16)

for the case $g_2^2 \bar{n}_2 \gg g_1^2 \bar{n}_1$. On the other hand, for
the case when the initial state of the signal mode field is vacuum or near-vacuum, i.e. \( \bar{n}_2 \approx 0 \), and when \( \bar{n}_1 > 1 \), we find straight forwardly from equation (11) the evaluation

\[
V_{0}^0 = -\frac{9 g_1^2 g_2^2 \bar{n}_1^2}{4(g_1^2 \bar{n}_1 + g_2^2)^2} < 0
\]  

(17)

(which is also in compliance with the first equation in (15) in the limit \( \bar{n}_2 \to 0 \)). The inequalities in equations (16) and (17) mean that photon statistics of the signal mode field in the steady regime will be sub-Poissonian if either \( g_1^2 \bar{n}_2 \gg g_1^2 \bar{n}_1 (\bar{n}_1, \bar{n}_2 \geq 1) \) or \( \bar{n}_2 \approx 0, \bar{n}_1 \gg 1 \). This confirms the numerical results obtained above, see figures 2, 3 and 4.

For short times and large initial photon numbers \( \bar{n}_1, \bar{n}_2 \) by using the saddle-point method for evaluating the integral (13), one can get

\[
F(t; g^2) \approx 1 - \exp \left[ -\frac{(g_1^2 \bar{n}_1 + g_2^2 \bar{n}_2) t^2}{8(g_1^2 \bar{n}_1 + g_2^2 \bar{n}_2 + g_0^2)} \right] \times \\
\times \cos \left[ t \sqrt{g_1^2 \bar{n}_1 + g_2^2 \bar{n}_2 + g_0^2} \right].
\]  

(18)

The right-hand side of equation (18) behaves like damped cosine oscillations which are terminated by the Gaussian envelope \( \exp[-t^2/(2\tau_c^2)] \). Here the collapse time \( \tau_c \) is defined as

\[
f(t; g^2) \equiv \left[ 1 + \left( \frac{\bar{n}_1 + \bar{n}_2}{4W_0^2} g_0^4 t^2 \right) \right]^{-1/4},
\]

\[
\psi(t; g^2) \equiv 2(\bar{n}_1 + \bar{n}_2) \sin^2 \frac{g_0^2 t}{4W_0} \left[ 1 + \left( \frac{\bar{n}_1 + \bar{n}_2}{4W_0^2} g_0^4 t^2 \right) \right]^{-1},
\]

\[
\varphi(t; g^2) \equiv W_0 t + (\bar{n}_1 + \bar{n}_2) \sin \frac{g_0^2 t}{2W_0} - \frac{g_0^2 t}{2W_0} (\bar{n}_1 + \bar{n}_2) - \frac{1}{2} \arctan \left( \frac{\bar{n}_1 + \bar{n}_2}{4W_0^2 g_0^2 t} \right)
\]

has been introduced, and

\[
W_0 \equiv \sqrt{g_0^2(\bar{n}_1 + \bar{n}_2) + g_0^2}.
\]

The approximate expression (21) is rather good for all times [26]. It indicates the periodicity of the revivals. The interval \( T_R \) between the revivals of the oscillation envelope of \( F(t; g^2) \) can be found from the second equation in (22) to be

\[
T_R(g^2) = (4 \pi/g_0^2) W_0(g^2).
\]


More interesting is the cross correlation between the two modes. Its magnitude can be characterized by

\[
V_{\text{cross}}(t) \equiv \langle \hat{a}_1^\dagger (t) \hat{a}_2^\dagger (t) \hat{a}_2 (t) \hat{a}_1 (t) \rangle - \langle \hat{a}_1^\dagger (t) \hat{a}_1 (t) \rangle \langle \hat{a}_2^\dagger (t) \hat{a}_2 (t) \rangle =
\]

\[
= \sum_{n_1,n_2} P(n_1, n_2) n_1 n_2 - \left\{ \sum_{n_1,n_2} P(n_1, n_2) n_1 \right\} \left\{ \sum_{n_1,n_2} P(n_1, n_2) n_2 \right\}.
\]

(25)


The quantity \( V_{\text{cross}} \) is proportional to the excess coincidence counting rate to be measured in an experiment of the Hanbury Brown-Twiss-type with two different light beams [22, 27]. We speak of anticorrelation, when \( V_{\text{cross}} \) becomes negative. Anticorrelation has been observed in light scattering from nonspherical particles in dilute solution [27]. We consider now the time behaviour of \( V_{\text{cross}} \) in the model (1).
Utilizing equation (3) and the definition (25) we can get

\[ V_{\text{cross}}(t) = V_{\text{cross}}(0) + \sum_{n_1, n_2} P(n_1, n_2) \left\{ (n_1 - \bar{n}_1) R_{2n_1n_2}(t) + (n_2 - \bar{n}_2) R_{1n_1n_2}(t) \right\} \]

\[ - \left\{ \sum_{n_1, n_2} P(n_1, n_2) R_{1n_1n_2}(t) \right\} \left\{ \sum_{n_1, n_2} P(n_1, n_2) R_{2n_1n_2}(t) \right\}. \]  

(26)

Equation (26) together with equations (4) allow us to describe the time evolution of the cross correlation in the general case of the initial field state (5). Let us assume that the initial photon distribution \( P(n_1, n_2) \) is given by (9). This means that the signal and pump fields are coherent and statistically independent at the beginning of the interaction. Then, due to the properties (10), we find from (26) the result

\[ V_{\text{cross}}(t) = \bar{n}_1 \left[ R_{2n_1+1n_2}(t) - R_{2n_1n_2}(t) \right] + \bar{n}_2 \left[ R_{1n_1n_2+1}(t) - R_{1n_1n_2}(t) \right] - \bar{R}_{1n_1n_2}(t) R_{2n_1n_2}(t). \]  

(27)

Here (...) indicates, as in section 3, averaging over the distribution (9).

A numerical computation of the formula (27) has been performed for the case, for example, when \( g_1 = 2 g_2 = 2 g_0, \bar{n}_1 = 12, \bar{n}_2 = 3 \). The results are plotted in figure 5. We see from the figure that anticorrelation occurs for some finite intervals of times in the Rabi oscillation region, and during the steady regime. The successive presences of correlation and anticorrelation in the Rabi oscillation region are noted. For large photon numbers \( \bar{n}_1, \bar{n}_2 \) by using the approximate formulae

\[ R_{2n_1n_2}(t) \approx \frac{g_1^2 g_2^2 \bar{n}_1(\bar{n}_2 + 1)}{2[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)]^2} \times \{ 4 F(t; g_1^2) - F(2 t; g_2^2) \}, \]

\[ R_{2n_1+1n_2}(t) \approx \frac{g_1^2 g_2^2(\bar{n}_1 + 1)(\bar{n}_2 + 1)}{2[g_1^2(\bar{n}_1 + 1) + g_2^2(\bar{n}_2 + 1)]^2} \times \{ 4 F(t; g_1^2 + g_2^2) - F(2 t; g_1^2 + g_2^2) \}, \]

\[ R_{1n_1n_2}(t) \approx 1 - \frac{4 g_1^2 g_2^2 \bar{n}_1(\bar{n}_2 + 1) F(t; g_1^2 + g_2^2)}{2[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)]^2}, \]

\[ R_{1n_1n_2+1}(t) \approx 1 - \frac{4 g_1^2 g_2^2 \bar{n}_1(\bar{n}_2 + 2) F(t; 2 g_2^2) + g_1^2 \bar{n}_1^2 F(2 t; 2 g_1^2)}{2[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 2)]^2}, \]

(28)

obtained from (4), we can apply the same analytical description we have used in the previous section to (27). The collapse and revival of \( V_{\text{cross}} \) are easily described by utilizing the approximate expressions (18) and (21). Due to that the function \( F(t; g_1^2) \) approaches unity in the collapse region, see (18), and in the region of infinitely large times, see (21), the steady value of \( V_{\text{cross}} \) is quickly found to be

\[ V_{\text{cross}}^{\infty} = \frac{1}{2} \frac{1}{g_1^2 \bar{n}_1 \bar{n}_2} \left\{ \frac{g_1^2 \bar{n}_1 + 4 g_2^2(\bar{n}_2 + 1)}{[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)]^2} - \frac{g_1^2 \bar{n}_1 + 4 g_2^2(\bar{n}_2 + 2)}{[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 2)]^2} \right\} + \]

\[ + \frac{3}{2} \frac{1}{g_1^2 g_2^2 \bar{n}_1(\bar{n}_2 + 1)} \left\{ \frac{g_1^2(\bar{n}_1 + 1) + g_2^2(\bar{n}_2 + 1)}{[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)]^2} - \frac{\bar{n}_1}{g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)^2} \right\} \]

\[ - \frac{3}{4} \frac{1}{g_1^2 g_2^2 \bar{n}_1(\bar{n}_2 + 1)} \left\{ \frac{g_1^2 \bar{n}_1^2 + 2 g_2^2(\bar{n}_2 + 1)^2}{[g_1^2 \bar{n}_1 + g_2^2(\bar{n}_2 + 1)]^4} \right\}. \]

(29)

Fig. 5. — Time evolution of the cross-correlation function \( V_{\text{cross}}(t) \). Both the fields of the modes are initially in the Glauber coherent states with \( \bar{n}_1 = 12, \bar{n}_2 = 3, g_1 = 2 g_2 = 2 g_0 \). Time is measured in units of \( 1/g_0 \).
Hence, for the case \( g^2_1 \tilde{n}_1 \gg g^2_2 \tilde{n}_2 \) one gets
\[
V_{\text{cross}}^0 \approx -\frac{13}{4} \frac{g^2_1}{g^2_2} \frac{\tilde{n}_1}{\tilde{n}_2} < 0. \tag{30}
\]
The inequality in equation (30) indicates the anti-correlation between the signal and pump modes in the steady regime. This analytical prediction is in compliance with the numerical result obtained above, see figure 5 (where \( g^2_1 \tilde{n}_1 / g^2_2 \tilde{n}_2 = 16 \)). In the opposite limit case \( g^2_1 \tilde{n}_1 \ll g^2_2 \tilde{n}_2 \) one has
\[
V_{\text{cross}}^0 \approx 2 \frac{g^2_1}{g^2_2} \frac{\tilde{n}_1}{\tilde{n}_2} > 0 \tag{31}
\]
that indicates the correlation between the modes.

Finally, for very short times \( t \approx 0 \) we find from equations (27) and (4) the formula
\[
V_{\text{cross}}(t) \approx \frac{1}{12} \frac{g^2_1}{g^2_2} \frac{\tilde{n}_1}{\tilde{n}_2} t^4, \tag{32}
\]
which describes the appearance of the correlation between the modes due to the interaction with the emitter, see figure 5.

5. Conclusions.

In this paper we have studied the characteristics of the photon statistical properties in a soluble three-level plus two-mode model. The normally ordered variances of the photon numbers and the cross-correlation function have been calculated. It has been shown that the states of the signal mode field produced from its initial vacuum are sub-Poissonian for all times. This production of sub-Poissonian states does not depend upon the initial state of the pump mode field.

The quantum collapse and revival behaviour of the photon statistical characteristics in the lossless model considered here have been noted.

For the case when both the signal-mode and pump-mode fields are initially in the Glauber coherent states the time behaviour of the photon-number variances and of the cross correlation between the modes has been numerically examined and analytically approximated. The theoretical investigations show that under suitable conditions the occurrence of photon antibunching and the presence of anti-correlation between the modes are potentially possible in the system. It has been established that

(i) In the region of short times (the Rabi oscillation region) photon statistics of the signal-mode field is sub-Poissonian for some finite intervals;
(ii) Photon statistics of the signal-mode field in the steady regime will be sub-Poissonian if either \( g^2_1 \tilde{n}_1 \gg g^2_2 \tilde{n}_2 \) \((\tilde{n}_1, \tilde{n}_2 \gg 1)\) or \( \tilde{n}_2 \approx 0 \) \((\tilde{n}_1 \gg 1)\);
(iii) Due to the interaction with the emitter the correlations between the modes come into existence. The anticorrelation between the modes will occur in the steady regime if \( g^2_1 \tilde{n}_1 \gg g^2_2 \tilde{n}_2 \) \((\tilde{n}_1, \tilde{n}_2 \gg 1)\).

All the results have been obtained without using perturbation theory and decorrelation approximations.

References


