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Long wavelength dynamics of free smectic films

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Résumé. — On étudie la dynamique d’un film mononucléaire smectique A dans la limite des grandes longueurs d’onde. On trouve l’équation hydrodynamique non linéaire en l’absence de dissipation par la technique des crochets de Poisson.

Abstract. — The long wavelength dynamics of monomolecular smectic-A films is studied. Nonlinear nondissipative hydrodynamic equations are derived by means of the Poisson brackets technique. The dissipative terms are constructed, the spectrum of long wavelength modes of the film is found. The transverse sound proves to be anomalously weakly attenuated. The fluctuation contribution to the spectrum is calculated on the basis of the diagram technique.

1. Recently thin, practically monomolecular films of smectic liquid crystals have been obtained and investigated [1-4]. In this connection long wavelength dynamics of such systems is of great interest. Then it should be born in mind that the films under study are freely suspended, which provides a possibility for transverse motion. As will be shown below, transverse oscillations of the film cause strong fluctuation effects in the long wavelength dynamics of the film. Below for simplicity we shall deal with smectic-A films.

The hydrodynamical variables describing a three-dimensional smectic-A are the density of mass $\rho$, the density of entropy $S$, the momentum density $j$ as well as the smectic variable $W$. The latter implies that the equation $W = \text{Const.}$ sets a position of a molecular layer so that the vector $\mathbf{W}$ determines the direction of the normal to the layer. Nonlinear nondissipative hydrodynamic equations for a three-dimensional smectic-A can be constructed by means of the Poisson brackets technique (see [5, 6]). The nonzero Poisson brackets are:

$$\{j_1(r_1), j_2(r_2)\} = \tilde{\rho}(r_1) \nabla \delta(r_1 - r_2)$$

$$\{\tilde{j}(r_1), \tilde{S}(r_2)\} = \tilde{S}(r_1) \nabla \delta(r_1 - r_2)$$

$$\{\tilde{j}(r_1), j_2(r_2)\} = \tilde{j}_1(r_1) \nabla \delta(r_1 - r_2) +$$

$$+ \tilde{j}_2(r_2) \nabla \delta(r_1 - r_2)$$

$$\{\tilde{j}(r_1), W(r_2)\} = - \nabla_i W \delta(r_1 - r_2).$$

Let us now pass over to smectic-A films. The position of a film will be set by the equation $W(r) = 0$. This function can always reduce to the form:

$$W(r) = z - u(x, y).$$

(2)

Thus, the mass, momentum and energy of the film are concentrated on the surface $z = u(x, y)$. To describe the film, instead of the three-dimensional densities $j, \rho, S$, the two-dimensional densities should be introduced:

$$j(x, y) = \int dz \tilde{j}(r); \quad \rho(x, y) = \int dz \tilde{\rho}(r);$$

$$S(x, y) = \int dz \tilde{S}(r).$$

(3)

The energy of the film $E$ can be represented as

$$E = \int dx dy \left\{ \frac{j_1^2 + j_2^2}{2 \rho} + \sqrt{g} \rho \partial \sigma \right\}.$$

(4)

Here the designations are introduced:

$$\sigma = \frac{S}{\rho}; \quad \rho_0 = \frac{\rho}{\sqrt{g}}; \quad g = 1 + (\nabla u)^2.$$

(5)

Formule (4), (5) contain the two-dimensional vectors with components along the $x, y$-axes. Here and in the following they are designated by Greek indices.
We consider a monolayer of smectic-A therefore we do not deal with the variables associated with smectic ordering (actually existing in the direction orthogonal to the layers). In fact it is with this smectic ordering that a possibility of the existence of a freely standing layer is accounted for. A monolayer of ordinary isotropic liquid cannot exist in virtue of the surface tension. Note that in our formalism there is a trace of the three-dimensional smectic variable, i.e., the displacement vector $u$.

The Poisson brackets for the variables describing smectic-A films can be found from (1) by integrating over $z$. Bearing in mind the definitions (3), we get the following nonzero Poisson brackets:

$$\{ f_1(r_1), u(r_2) \} = \delta(r_1 - r_2)$$
$$\{ f_2(r_1), \rho(r_2) \} = \rho(r_2) \nabla_u \delta(r_1 - r_2)$$
$$\{ f_3(r_1), \sigma(r_2) \} = -\nabla_u \sigma \delta(r_1 - r_2)$$
$$\{ f_4(r_1), f_2(r_2) \} = J_{22}(r_2) \nabla_u \delta(r_1 - r_2)$$
$$\{ f_4(r_1), f_4(r_2) \} = J_{44}(r_2) \nabla_u \delta(r_1 - r_2) + \int u(r_2) \nabla_u \delta(r_1 - r_2).$$

(6)

These formulae, unlike (1), contain a two-dimensional radius vector $r = (x, y)$.

Now we write out the nonlinear nondissipative equations for the smectic-A film:

$$\frac{\partial u}{\partial t} = \{ E, u \} = v_z - v_u \nabla_u u;$$

(7)

$$\frac{\partial \rho}{\partial t} = \{ E, \rho \} = -\nabla_u J_2;$$

(8)

$$\frac{\partial \sigma}{\partial t} = \{ E, \sigma \} = -v_u \nabla_u \sigma;$$

(9)

$$\frac{\partial J_2}{\partial t} = \{ E, J_2 \} = -\nabla_u(v_u J_2) - \nabla_u \left\{ \frac{\nabla \mu}{\sqrt{g}} \left( \frac{\rho}{\rho_0} \frac{\partial \mu}{\partial \rho_0} - \varepsilon \right) \right\};$$

(10)

$$\frac{\partial \rho}{\partial t} = \{ E, \rho \} = -\nabla_u(v_u \rho_2) - \nabla_u \left\{ \frac{\nabla \mu}{\sqrt{g}} \left( \frac{\rho}{\rho_0} \frac{\partial \mu}{\partial \rho_0} - \varepsilon \right) \right\} + \nabla_u \left\{ \frac{\nabla \mu}{\sqrt{g}} \left( \frac{\rho}{\rho_0} \frac{\partial \mu}{\partial \rho_0} - \varepsilon \right) \right\}. $$

(11)

Here the velocity $v_u = j_u/\rho$. Note that $\frac{\partial \rho}{\partial t}$ is determined by the divergence of a symmetric tensor, hence follows the conservation law for the $z$-component of the momentum. Remembering that the axes $x, y, z$ are chosen arbitrarily, we come to the conclusion that all the components of the momentum are conserved.

2. Now we write out the dissipative forces due to viscosity, remembering that these forces are concentrated on the surface $z = u(x, y)$. The general form of the dissipative term in the equation for fluctuations of the three-dimensional momentum density $\delta j_\mu/\partial t$ is:

$$- \nabla \frac{\sqrt{g}}{\sqrt{g}} \delta(z - u) \eta_{iklm} \nabla_u v_m. $$

(12)

The structure of the tensor $\eta_{iklm}$ can be found if one makes use of the Onsager symmetry of the stress tensor, as well as of the fact that (12) involves derivatives of the velocity $v$ only along the smectic film surface. As a result:

$$\eta_{iklm} = \eta_1 \delta_{im} \delta_{j_l}^{\perp} + \eta_2 (\delta_{il}^{\perp} \delta_{jm}^{\perp} + \delta_{im}^{\perp} \delta_{jl}^{\perp}).$$

(13)

Here the velocity coefficients $\eta_1, \eta_2$ are functions of $\mu, \sigma$; so that

$$\delta_{il}^{\perp} = \delta_{il} - \nabla_i W \nabla_l W/|\nabla W|^2;$$

$$\delta_{ij}^{\perp} = \frac{g - 1}{g}; \quad \delta_{ii}^{\perp} = \frac{\nabla_i u}{g};$$

$$\delta_{a b}^{\perp} = \frac{\nabla_a u \nabla_b u}{g}. $$

(14)

Now integrating (12) over $z$ and remembering that $v$ depends only on $x, y$, we get

$$\frac{\partial \rho}{\partial t} - \{ E, J_2 \} = \nabla_u \left\{ \frac{\sqrt{g}}{\sqrt{g}} \eta_{i a b m} \nabla_u v_m \right\}. $$

(15)

Dissipative terms in the equation for the density of mass $\rho$ and the displacement vector $u$ are absent, therefore they should be added to the equation for the specific entropy $\sigma$. It can be easily done if one takes into account that $g$ serves as the determinant of the metric tensor:

$$\frac{\partial \sigma}{\partial t} - \{ E, \sigma \} = \frac{1}{\rho} \nabla_u (\sqrt{g} \kappa \delta_{a b}^{\perp} \nabla_u T) + \frac{R}{\rho_0 T}. $$

(16)

Here $T = \frac{1}{\rho_0} \frac{\partial E}{\partial \sigma}$ is the temperature and the heat conductivity coefficient $\kappa$ is a function of $\rho_0$. Requiring that the energy conservation law should hold, we find the dissipation function

$$R = \eta_{i a b m} \nabla_u v_i \nabla_u v_m + \kappa \delta_{a b}^{\perp} \nabla_u T \nabla_u T. $$

(17)

For $R$ to be defined positively, it is required that

$$\kappa > 0; \quad \eta_2 > 0; \quad \eta_1 + \eta_2 > 0. $$

(18)

Let us assume that in equilibrium the film is suspended in the $x, y$-plane, i.e. $u = 0$. Linearizing the system of equations (7-11), (15), (16) with respect to $j_a, u$ and with respect to fluctuations of $\rho, \sigma$ from the equilibrium values, we get the following linear spectrum of the
Here $k$ is a wave vector, $\omega_1$ is frequency
The quantity $C_1$ is the velocity of the longitudinal
sound where $p$ and the component $j_1$, longitudinal to
the wave vector, oscillate, whereas $C_2$ is the velocity of
the transverse sound where $j_2$ and $u$ oscillate and $\omega_1$
and $\omega_2$ describe the diffusive modes, associated with $\sigma$
and with the component $j_3$, transversal to the wave
vector, respectively.

It should be emphasized that (in the quadratic
approximation) in the dissipative function (17) there
is no contribution due to Physically this is
accounted for by the fact that under the uniform
rotation of the film (layer) around, say, the axis $y$,
dissipation is absent, although under these conditions
$V_x V_z$ is non-zero. On the other hand, this leads to the
absence of damping of the transversal sound $\sim k^2$
in the dispersion law in (19). Therefore the anoma-
lously weak damping of the transversal sound is due
to the rotational invariance of the system.

3. The basic feature of the spectrum under study
is the absence of the transverse sound damping which
brings about strong dynamical effects considerably
modifying the character of attenuation, described in
(19). To take into account fluctuation effects, we shall
employ the technique developed in [7, 8]. In these
works the generating functional has been constructed;
this functional makes it possible to calculate fluctua-
tion corrections as a perturbation theory series,
standardly represented by Feynman diagrams. Here
we shall not need the additional Fermi fields intro-
duced in [7, 8].

The quadratic part of the action generated by the
nondissipative equations (7-11) is : (see details in [7, 8])

$$I_1 = \int dt \int dx \left\{ \rho \left( \frac{\partial u}{\partial t} + j_1 \right) + \rho \frac{\partial \delta \sigma}{\partial t} + \frac{\delta \sigma}{\partial t} \right\} +$$

$$+ \left( \frac{\partial \delta \rho}{\partial t} + V \delta \rho \right) + \rho \frac{\delta j_1}{\partial t} + \rho C_1^2 \nabla^2 u +$$

$$+ \rho \frac{\delta j_2}{\partial t} + C_1^2 \nabla \delta \rho \right\}. \quad (21)$$

Here $\delta \sigma$, $\delta \rho$ are deviations from the equilibrium
values, $\rho$ are supplementary Bose fields, conjugate to
the appropriate hydrodynamical variables. The bare
dissipative contributions from (17) are not taken into
account in (21) since, as will be clear below, due to
nonlinear interaction of modes larger (in the long
wavelength limit) dissipative terms appear fluctuatio-
ally. On the same grounds nonlinear dissipative
effects, involving a larger power of the wave vector,
are also neglected.

We shall also need the cubic part of the effective
action generated by equations (7-11) :

$$I_2 = \int dt \int dx \left\{ \rho \left( \frac{\partial u}{\partial t} + j_1 \right) + \frac{\partial \delta \sigma}{\partial t} + \frac{\partial j_1}{\partial t} +$$

$$+ \frac{\partial \delta \rho}{\partial t} + \nabla \delta \rho \right\} \left( V \delta \rho + \frac{\partial \delta \rho}{\partial t} + C_1^2 \nabla^2 u \right). \quad (22)$$

Expression (21) (in which we neglected the dissipative
terms for reasons discussed above) describes non-
interacting modes with the spectrum (19). The contribu-
tions (22), (23) describe their interaction.

The presence of these terms leads to the fluctuational
contribution to the quadratic part of the effective
action. We consider this contribution in Appendix.

As is shown in the Appendix, in virtue of the fluc-
tuation corrections due to the interaction (22), (23) the
action involves the following term

$$I_4 = \int dt \int dx \left\{ \gamma \frac{k^3}{\rho C_1^2} u + \mu \rho \nabla u \nabla \rho \right\} \left( \frac{k^3}{\rho C_1^2} u +$$

$$+ \frac{\partial \delta \rho}{\partial t} + \nabla \delta \rho \right\}. \quad (24)$$

Here $\gamma$, $\mu$ are constants, $k$ is the operator which in the
Fourier representation reduces to the multiplication
by the wave vector module. Now using the explicit
expressions (21), (24), we find for pair correlation functions of the Fourier components with the frequency $\omega$ and wave vector $k$

$$\langle u^* \rangle = -\frac{\omega}{\omega^2 - C_1^2 k^2 + i\omega(\gamma + \mu) k^2} \equiv G(\omega, k) \quad (25)$$

$$\langle uu^* \rangle = \frac{2 T(\gamma + \mu) \omega^2 k}{\rho C_1^2 (\omega^2 - C_1^2 k^2) + \omega^2 k^2(\gamma + \mu)^2} \equiv D(\omega, k). \quad (26)$$

Here the terms, principal with respect to the wave vector, are represented and it is also taken into account that $\omega^2 \approx C_2^2 k^2$.

The pair correlation function $\langle u^* \rangle$ determines the generalized susceptibility of the system with respect to the external forces $[7, 8]$. Accordingly, the function (25) is analytical in the upper half-plane with respect to $\omega$ and its poles determine the oscillation spectrum which with the necessary accuracy can be represented as

$$\omega = \pm C_1 k - i\frac{1}{2}(\gamma + \mu) k^3. \quad (27)$$

Thus due to fluctuation effects the transverse sound acquires the damping $\sim k^3$. Note that this fluctuation damping is much stronger than regular damping $\sim k^4$ (which is retained since the regular damping $\sim k^2$ is absent). Note also the relation:

$$\int \frac{d\omega}{2\pi} \langle uu^* \rangle = \frac{T}{\rho C_1^2} \frac{k^3}{k^2}. \quad (28)$$

This expression coincides with the one-time correlation function which can be found in the Gaussian approximation if the explicit form of the energy (4) is used. It should be stressed that the main fluctuation contributions to the dispersion law of the transverse mode have been taken into account exactly.

The anomalously weak attenuation of transverse waves accounts for the large value of the corrections to the spectrum of modes associated with fluctuations of $u$. The corresponding corrections to the action are due to the interaction described by the cubic term (23) and the first term in (22). Calculation of the corrections is performed in the Appendix, as a result the following fluctuation contribution to the action is obtained:

$$I_5 = \int dt dx dy \left( p_\alpha \Sigma_{\alpha\beta} j_\beta + i\eta T p_\alpha \text{Re} \Sigma_{\alpha\beta} p_\beta \right). \quad (29)$$

Here $\Sigma_{\alpha\beta}$ is an operator which in the Fourier representation equals:

$$\Sigma_{\alpha\beta} = \frac{T}{12 \rho(\gamma + \mu)^{2/3}} \frac{k^{5/3}}{C_1^{1/3}} \phi_{\alpha\beta} \left( \omega \right) \left( 1 + i \frac{1}{\sqrt{3}} \right). \quad (30)$$

Note that the spectrum of the mode connected with the transversal velocity component now has (see formula (30)) the real part which is of the order of magnitude of the imaginary part. In the first expression in (31) the argument of the function $\phi$ is $C_1/C_n$ in the second expression — zero (remember that the function $\phi$ is set by formula (A.10) and is involved in expression for $\Sigma_{\alpha\beta}$). Thus hydrodynamics of smectic-A films essentially differs from hydrodynamics of isotropic liquids where due to fluctuation corrections renormalization of the kinetic coefficients it realized $[8, 10]$. Note that in the case under study logarithmic corrections even to the spectrum of the diffusive mode associated with the specific entropy oscillations are absent; this spectrum is not renormalized due to fluctuations of $u$.

Everywhere above for the calculation of fluctuational effects we have confined ourselves to the first order perturbation theory. Then we have used the nonlinear vertices due to only the reactive (but not dissipative) part of the hydrodynamic equations. This is due to the fact that the dissipative terms in comparison with the reactive terms are smaller by the parameter $k/A$ ($k$ is a wave vector, $A$-cut-off wave vector). Our calculations reveal that although the fluctuational effects strongly modify the spectral dependence of the damping, they give only small corrections to the real part of the spectrum (for the transversal sound these corrections are as small as $(k/A)^2$, for the longitudinal
Thus in our case the perturbation theory expansion is an expansion over the small parameter $k/\lambda$ and we can confine ourselves to the first non-vanishing term of this expansion. This statement holds for the hydrodynamic equations of isotropic two-dimensional liquids [9, 10].

Note that strictly speaking our results are applicable only for smectic-A films without any ordering in a layer. For smectics-B and -C films with a certain ordering in a layer damping due to defects (see [11]) should be taken into account. However, this mechanism is important only near the « temperature of the transition » due to the breaking of ordering in the layer.

Appendix.

The fluctuation contribution (24) to the action emerges due to corrections generated by the average $\frac{i}{2} \langle I_2 I_2 \rangle + i \langle I_2 I_3 \rangle$ where averaging is performed with the quadratic action. While calculating $I_4$ we can neglect attenuation on the intermediate lines of the corresponding diagrams so that for pair averages of the Fourier components with the frequency $\omega$ and wave vector $k$ it is sufficient to use the following approximation:

\[ \langle u p^\ast \rangle = \frac{\omega}{\omega^2 - C_i^2 k^2 + i0}; \]
\[ \langle u p_{\ast}^\ast \rangle = \frac{i}{\rho(\omega^2 - C_i^2 k^2)} + i0; \]
\[ \langle j_x p_{\ast}^\ast \rangle = - \frac{i \rho C_i^2 k^2}{\omega^2 - C_i^2 k^2 + i0}; \]

\[ \int \frac{d\mathbf{r} d\mathbf{y}}{4 \pi^2 \rho} \left\{ \frac{TC_i}{4 \pi^2 \rho} \int dv d^2q \frac{q_x q_y}{q} \delta(\nu^2 - C_i^2 q^2) q' \frac{1}{\nu^2 - C_i^2 q^2 + i0} + \right. \]
\[ \left. + \frac{TC_i}{4 \pi^2 \rho} \int dv d^2q \frac{q_x q_y}{q^2} (2 q_x q_y - \delta_{xy} q^2) \frac{1}{\nu^2 - C_i^2 q^2 + i0} \times \frac{q'_x q'_y}{q'} \delta(\nu^2 - C_i^2 q'^2) \right\} \nabla y u; \]
\[ \frac{i T^2 C_i}{4 \pi \rho C_i} \int \frac{d\mathbf{r} d\mathbf{y} d\mathbf{p}}{C_i} \int dv d^2q \frac{q_x q_y}{q} \delta(\nu^2 - C_i^2 q^2) \times \frac{q'_x q'_y}{q'} \delta(\nu^2 - C_i^2 q'^2) \mathbf{p}. \]

Here
\[ \nu' = \nu + \omega; \quad \mathbf{q}' = \mathbf{q} + \mathbf{k}; \quad \omega = i \frac{\partial}{\partial t}; \quad k = -i \nabla. \]

The explicit calculation of the integral in (A.6) is complicated. Yet for the case of the transverse sound we are interested in, we can assume $\omega = C_i k$, then it becomes clear that the integral over $v, q$ in (A.6) is $\sim k$, which corresponds to the third term in (24). The real part of (A.5) yields a minor correction $\sim k^3$ to the real part of the spectrum. We shall be interested only in the imaginary part describing the attenuation. To calculation the attenuation of the transverse mode we shall take the imaginary part of (A.5) in accordance with formula

\[ \langle j_x p_{\ast}^\ast \rangle = - \frac{\omega}{\omega^2 - C_i^2 k^2} \frac{k_x k_y}{k^2} - \frac{1}{\omega + i0} \left( \delta_{xy} - \frac{k_x k_y}{k^2} \right); \]
\[ \langle \delta \rho p_{\ast}^\ast \rangle = - k_x (\omega^2 - C_i^2 k^2 + i0)^{-1}. \]
Im \( \frac{1}{x - i0} = \pi \delta(x) \). Transforming this expression we can reduce this integral to the form of (A. 6). The proportionality coefficient then corresponds to the relation of the first and second terms in (24). Similarly, the remaining contributions to \( I_4 \) can be considered.

Now consider the contribution to (29) due to the term \( i \langle I_2 I_3 \rangle \) where the intermediate averages of (25), (26) are involved. This contribution leads to the first term in (29) with the self-energy function:

\[
\Sigma_{\alpha \beta} = - C_4^2 \int \frac{d\omega}{(2\pi)^3} (q_{1\alpha} q_{2\beta} + q_{2\alpha} q_{1\beta} - \delta_{\alpha\beta} q_1 \cdot q_2) \times G(v_1, q_1) D(v_2, q_2) q_{2\gamma} \gamma_{\beta}. \quad (A.7)
\]

Here

\[
v_1 = v + \frac{\omega}{2}; \quad v_2 = v - \frac{\omega}{2}; \quad q_1 = q + \frac{k}{2}; \quad q_2 = q - \frac{k}{2}.
\]

Employing the explicit expressions (25), (26) and their analytical properties and the symmetry of (A. 7), we find, for instance, for the fluctuation damping

\[
\text{Re} \Sigma_{\alpha \beta} = - \frac{C_4^4 \rho}{2 T} \int \frac{d\omega}{(2\pi)^3} (q_{1\alpha} q_{2\beta} + q_{2\alpha} q_{1\beta} - \delta_{\alpha\beta} q_1 \cdot q_2) \times
\]

\[
\times D(v_1, q_1) D(v_2, q_2) (q_{1\gamma} q_{2\delta} - \frac{1}{2} q_1 \cdot q_2 \delta_{\alpha \beta}) \gamma_{\beta} \delta. \quad (A.8)
\]

Note that in the calculation of (A.8) we cannot use the approximation of (A. 2) since this integral is substantially dependent on the form of the transverse sound damping. Inserting the explicit expression (26) into (A.8) we can calculate the integral over the frequency \( \nu \) (which reduces to residues in the poles). The dependence of (A.8) on \( \omega, k \) in the obtained expression in virtue of the inequality \( k \ll q \) should be retained only in the singular denominators. As a result

\[
\Sigma_{\alpha \beta} = - \frac{T}{2 \rho} \int \frac{d\omega}{(2\pi)^3} \left( \frac{n_{\alpha} n_\beta - \frac{1}{2} \delta_{\alpha\beta}}{\gamma + \mu} \right) q^3 - i(\omega - C_4 n \cdot k) \gamma_{\beta} \delta. \quad (A.9)
\]

Now calculating the integral over the module \( q \) we get (30) with the function:

\[
\phi_{\alpha \beta}(\xi) = \int_0^{2\pi} \frac{d\phi}{2\pi} \left( \frac{n_{\alpha} n_\beta - \frac{1}{2} \delta_{\alpha\beta}}{\gamma n_\beta - \frac{1}{2} \delta_{\alpha\beta}} \right) \frac{k_p k_q}{k^2} \quad (A.10)
\]

The second term in (29) emerges from the average \( i/2 \langle I_2 I_3 \rangle \) involving two intermediate D-lines. This term is directly reduced to the form analogous to (A.8). The proportionality coefficient is fixed in (29).

References