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M. Kicińska-Habior, T. Matulewicz

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Thermonuclear rates of the ^{28}Si(p, γ) reaction

M. Kicińska-Habior and T. Matulewicz

Institute of Experimental Physics, University of Warsaw, 00-681 Warsaw, Poland

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Résumé. — Nous avons analysé les données de la réaction ^{28}Si(p, γ) suivant deux versions du modèle direct-semidirect. L'extrapolation des fonctions d'excitation théoriques et le coefficient astrophysique S sont obtenus sans différences significatives entre les deux modèles. L'analyse révèle le rôle dominant du processus non résonnant pour des températures inférieures à $T_9 = 0{,}10$.

Abstract. — The ^{28}Si(p, γ) reaction data have been analysed in terms of two alternative versions of the direct-semidirect model. The extrapolation of the theoretical excitation functions has been made and the astrophysical S-factor has been extracted with no significant differences between the predictions of the models. The analysis reveals the dominant role of the nonresonant process at temperatures less than $T_9 = 0.10$.

1. Introduction.

Thermonuclear reaction rates $N_A \langle \sigma v \rangle$ are the quantities of essential importance for models of stellar evolution. The rate at which the nuclear fuel is burned out at a given temperature is directly related to the nuclear properties of interacting species, namely the total cross section. Direct measurement of cross section values at stellar energies is, at present, an insurmontable experimental problem for very many reactions, mainly those involving charged particles or unstable targets. Extrapolations from experiments carried out at higher energies or pure model calculations have to be made.

A model of direct radiative proton capture reaction, which accounts for the presence of broad single-particle resonances, has been proposed [1] and applied to some (p, γ) reactions in our previous works [2-5]. Since the new approach and the conventional combined direct plus R-matrix model of Rolfs [6] are alternative, the question arises as to the difference between extrapolations of cross sections carried out by means of different methods. In the case of the ^{12}C(p, γ) reaction both methods agree within reported errors [4]. However, in this case the extrapolation was performed only for an energy range relatively small ($\sim$ 100 keV) in comparison with the experimentally measured excitation function. Extrapolation of the cross section of the ^{28}Si(p, γ) reaction, where the experimental data start from $\sim$ 1.3 MeV, would allow comparison of predictive capabilities of both methods in a much more extended energy range.

The thermonuclear rates of the ^{28}Si(p, γ) reaction, although it does not possess primary astrophysical importance, were calculated by many authors [7-9] and they differ by more than one order of magnitude at low temperatures ($T_9 < 4$). Reproduction of the experimental excitation function of this reaction in a broad energy range would allow for a quite secure extrapolation and then reliable values of the reaction rates can be established.

The ^{28}Si(p, γ) reaction data were already analysed within the framework of the new effective potential approach [2, 5]. In the present paper only a few details of those calculations will be given, with more attention devoted to the results of the direct plus R-matrix analysis.

2. Results and discussion.

2.1 EFFECTIVE POTENTIAL APPROACH. — The presence of broad shape resonances causes some problems in the combined direct plus R-matrix analysis scheme of Rolfs [6], especially when the extraction of the spectroscopic factor of the final state is concerned. The effective potential approach [1] overcomes these problems by accounting for the direct contribution and broad shape resonances on the same basis. The novelty of the method depends on using an energy-dependent single-particle potential for the evaluation of the scattering state wave function. The effects of coupling of single-particle continuum states with more complex structures (BSEC) are taken into account, thus allowing reproduction of the experimentally observed widths of resonances. In the description of the excitation function, the semidirect mechanism has to be included as well. Then, the normalization of the theoretical calculations to the experimental cross sections yields the value of the spectroscopic factor. In the case of the ^{28}Si(p, γ) reaction, values $S = 0.36$ and $S = 0.1$ were obtained for the ground and first excited state of the final ^{29}P nucleus, respectively [2, 5].

2.2 DIRECT PLUS *R*-MATRIX APPROACH. — The existing experimental data [5, 10, 11] of the ^{28}Si(p, γ)^{29}P reaction were reanalysed in terms of the conventional approach based on combined direct plus *R*-matrix calculations [6] extended by taking into account the semidirect mechanism [12, 13]. The γ-ray yield curve for the ^{28}Si(p, γ) reaction in the proton energy range of 1.3-3 MeV is dominated by three resonances at E_p = 1 652, 2 090 and 2 800 keV, which were accounted for through appropriate Breit-Wigner amplitudes. Also the resonance at E_p = 3 570 keV was taken into account in the cross section calculations with regard to the interference effects, while the 1 380 keV and 2 285 keV resonances could be neglected because of their small total widths.

The direct capture component in the yield curve due to an E1 transition was calculated by a method proposed by Rolfs [6]. As at the subbarrier energies the conventional direct capture model assumes the hard sphere phase shifts for the entrance waves, the proton-target interaction in the entrance channel was approximated by Coulomb plus hard sphere potentials with the radius parameter r_0 = 1.3 fm. In the final state the interaction was taken as Coulomb plus real Woods-Saxon potential with the same r_0 = 1.3 fm radius parameter and depth adjusted so as to reproduce the experimental binding energy of the captured proton [14]. The values of the spectroscopic factor for the ground and the first excited states were taken equal to 0.36 and 0.1, respectively, i.e. identical to those obtained in our previous modified direct-semidirect (DSD) analysis [2, 5]. The coupling with the giant dipole resonance (GDR) was taken into account *via* an effective charge factor with the same GDR parameters and coupling constants as in ^{27}Al(p, γ)^{28}Si reaction analysis [13]. The resonance parameters (Table I) adjusted to fit to the experimental data are in good agreement with the previous works [8, 11, 15]. It is worth noting that in our analysis the proton energy range has been extended to 3 MeV.

The excitation functions at 0° and 90° for the γ-transition to the ground ($J^\pi = 1/2^+$) and first excited ($J^\pi = 3/2^+$) states of the ^{29}P nucleus are shown in figures 1 and 2 along with the theoretical predictions based on the DSD capture model supplemented by the *R*-matrix treatment of the compound nucleus resonances. The agreement of the DSD plus *R*-matrix calculations with the experimental excitation functions is equally good, as obtained earlier for the modified DSD model [2, 5].

2.3 *S*-FACTOR ESTIMATES. — The total cross section for the proton capture reaction on the ^{28}Si target could be described as a sum of transition probabilities leading to different final states in the ^{29}P nucleus. It was assumed that only the cross sections related to the ground state (major term 90 %) and the first excited state are important in this sum. The contribution coming from transitions to the second excited state ($J^\pi = 5/2^+$, E^x = 1.954 MeV) could be neglected due to its presumably two particle-one hole configuration $(\pi_{s1/2})^2 (\pi_{d5/2})^{-1}$ and relatively small value of the spectroscopic factor. Additionally the effect of the $E_\gamma^3/E_p^{3/2}$ factor in the cross section expression (see e.g. Ref. [6]) reduces its value for final states with higher excitation energies. Thus, the uncertainty related to the above limitation of the sum to only two parts should not exceed 10 %.

The usual way to express charged-particles cross section values at low energies is the so-called astrophysical *S*-factor defined as

$$S(E) = \sigma(E)\, E \exp(2\,\pi\eta) \qquad (1)$$

where η is the Sommerfeld parameter and E is the centre-of-mass energy. In this way the steepest energy dependence of the cross section related to the barrier penetration factor is removed.

Table I. — *Comparison of* ^{28}Si(p, γ) *resonances.*

		Present work			Previous works			
E_p MeV	J_π	Γ keV	$(2J+1)\Gamma_p\Gamma_{\gamma 0}/\Gamma$ eV	$(2J+1)\Gamma_p\Gamma_{\gamma 1}/\Gamma$ eV	Γ keV (a)	$(2J+1)\Gamma_p\Gamma_\gamma/\Gamma$ eV (a)	$(2J+1)\Gamma_p\Gamma_{\gamma 0}/\Gamma$ eV (b)	$(2J+1)\Gamma_p\Gamma_{\gamma 1}/\Gamma$ eV (b)
1.65	$3/2^-$	53	6.96	0.45	52.0 ± 0.8	5.2 ± 0.8	6.06 ± 0.92	0.39 ± 0.06
2.09	$1/2^+$	19	1.05	0.07	15.6 ± 0.6	0.85 ± 0.12	0.98 ± 0.17	0.06 ± 0.01
2.80	$1/2^-$	400	4.1	2.4	400 ± 20	4	4.8 ± 3	0.8 ± 0.6
3.57	$3/2^-$	95	2.1	0.1	95 ± 6			

(a) Reference [14], (b) reference [8].

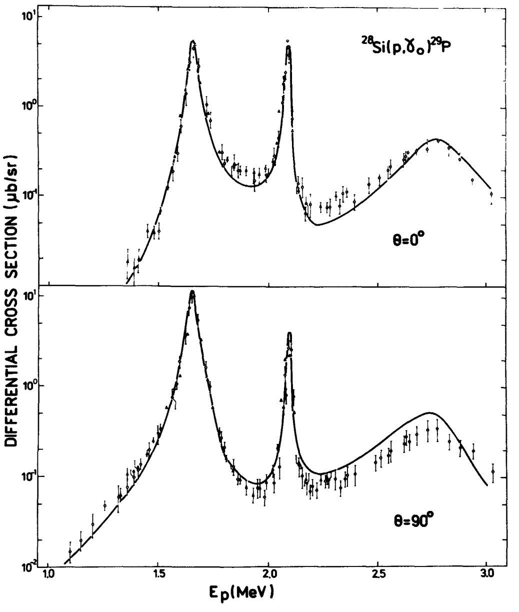

Fig. 1. — Comparison of the experimental excitation functions of the ^{28}Si(p, γ_0)^{29}P reaction with theoretical predictions of the direct plus R-matrix approach. Experimental data from : reference [10]-circles, reference [11]-triangles.

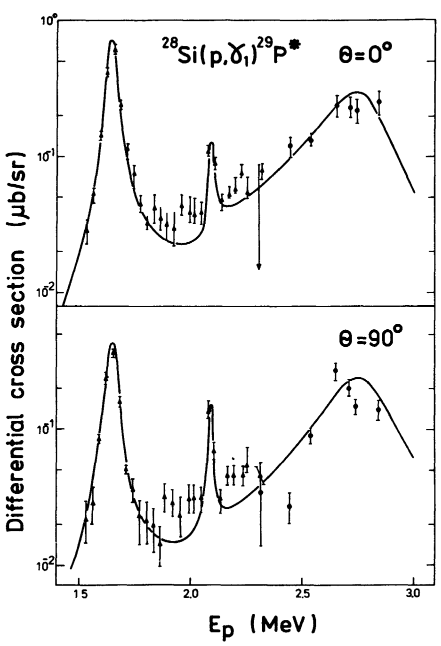

Fig. 2. — Comparison of the experimental excitation functions of the ^{28}Si(p, γ_1)^{29}P reaction with theoretical predictions of the direct plus R-matrix approach. Experimental data from : reference [5]-circles, reference [11]-triangles.

The energy dependence of the S-factor for the ^{28}Si(p, γ)^{29}P reaction leading to the ground state is presented in figure 3 for calculations of both types. The general agreement of the two methods is satisfactorily good also in the extrapolation region, with

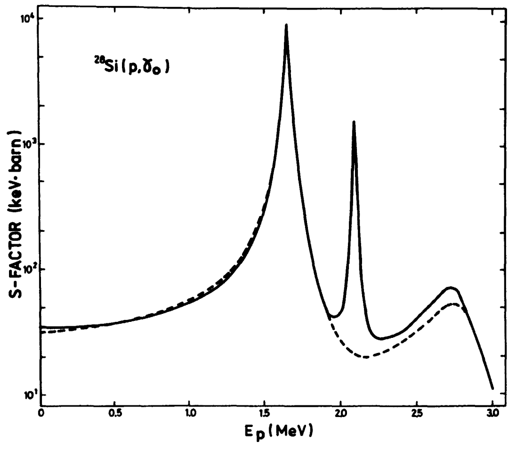

Fig. 3. — Astrophysical S-factor for the ^{28}Si(p, γ_0)^{29}P reaction predicted by the direct plus R-matrix approach (solid line) and by the effective potential approach (dashed line).

the only exception caused by the 2 090 keV resonance ($J^\pi = 1/2^+$), which could not be accounted for in the DSD calculations using effective potential approach (M1 transition).

The total S-factor values, because of their weak energy dependence at energies less than 1 MeV, could be expressed by the power series expansion

$$S(E) = S(0)\left[1 + \frac{S'(0)}{S(0)} E + \frac{1}{2}\frac{S''(0)}{S(0)} E^2 + \cdots\right]. \quad (2)$$

The expansion parameters were calculated by the least squares method for both approaches and are given in table II. Their values are similar, which points to the lack of essential differences between the conventional approach and that using effective potential in predicting cross section values at astrophysical energies. The above result also lends credibility to the previously performed conventional direct capture calculations of the S-factor.

2.4 Thermonuclear reaction rates. — Thermonuclear reaction rates for the ^{28}Si(p, γ) reaction have been predicted by Woosley *et al.* [7] in the framework

Table II. — *Parameters of the power series expansion of the S-factor for the* ^{28}Si(p, γ)^{29}P *reaction at* $E < 1$ MeV.

Method	$S(0)$	$S'(0)$	$S''(0)$
	keV barn	barn	barn keV^{-1}
Conventional calc.	36.9 ± 1.0	− 11.0 ± 3.9 × 10^{-3}	65.0 ± 6.6 × 10^{-6}
Effective potential	39.1 ± 0.4	− 2.6 ± 1.8 × 10^{-3}	52.3 ± 4.1 × 10^{-6}

of the statistical model. However, in the region of low excitation energy and low level density, where the basic assumptions of the statistical model are not fulfilled, sizeable discrepancies have been observed between model predictions and calculations based on experimentally established resonance parameters [8, 9]. Riihonen *et al.* [8] proposed a semiempirical formula to describe the narrow resonances contribution to the thermonuclear reaction rate for $T_9 < 10$ (¹)

$$N_A \langle \sigma v \rangle = [4.40 \times 10^2 \exp(-4.146\, T_9^{-1}) + 7.38 \times 10^3 \exp(-15.474\, T_9^{-1})] \times \times T_9^{-3/2} + 6.14 \times 10^5\, T_9^{-1.4} \exp(-18.302\, T_9^{-1})\ \mathrm{cm^3\ mole^{-1}\ s^{-1}}. \quad (3)$$

This formula, according to reference [8], holds for temperatures greater than $T_9 = 0.3$, while at lower temperatures the direct capture process is expected to play a significant role. Estimation of the S-factor values at $E < 1$ MeV enables us to calculate the thermonuclear reaction rate related to the direct process, according to the nonresonant formula [16]

$$N_A \langle \sigma v \rangle = = N_A (2/M)^{1/2}\, \Delta E_0 (kT)^{-3/2}\, S_{\mathrm{eff}} \exp(-3\, E_0/kT) \quad (4)$$

where N_A is the Avogadro number, M-reduced mass, E_0 and ΔE_0 is the energy and width of the Gamow peak and S_{eff} denotes the effective value of the S-factor (weak function of temperature). Formula (4) yields

$$N_A \langle \sigma v \rangle = \substack{7.57 \\ 6.93} \times 10^8\, T_9^{-2/3} \exp(-24.395\, T_9^{-1/3})\ \mathrm{cm^3\ mole^{-1}\ s^{-1}} \quad (5)$$

where the upper value comes from the effective potential type calculations and the lower from conventional ones. Comparison between different contributions to the total reaction rate is presented in figure 4 along with the statistical model calculations of Woosley *et al.* [7]. As is clearly seen, the direct capture process is dominant at $T_9 < 0.10$, while at higher temperatures it becomes meaningless. At temperatures of the order of $T_9 = 0.1$ and smaller, the calculations of Woosley *et al.* [7] overestimate the reaction rate by as much as 5 orders of magnitude. For reaction rate estimations at $T_9 < 0.15$ in the case of the ^{28}Si(p, γ) reaction a modification is proposed to the formula (3), which consists in including an additional term (5) with an average normalization value of 7.25.

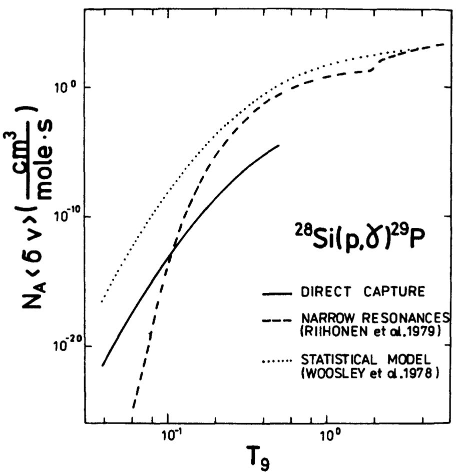

Fig. 4. — Thermonuclear reaction rate of the ^{28}Si(p, γ)^{29}P reaction as a function of temperature. The dashed line represents the narrow resonances contribution according to reference [8], while the solid one describes the nonresonant capture. Statistical model calculations of Woosley *et al.* [7] are shown by the dotted line.

3. Conclusion.

The ^{28}Si(p, γ) reaction data have been analysed in the framework of two alternative versions of the DSD model : the traditional one, which accounts for the resonance contribution within the R-matrix formalism and a new one, which applies the effective potential approach. After obtaining a reasonable agreement of model calculations and experimental data, extrapolations of theoretical excitation functions have been carried out towards the low energy range. Comparison of extrapolated values of both methods points to reasonably good agreement between them. The lack of essential differences between models in predicting cross section values at low energies confirms credibility of S-factor estimations evaluated previously within the conventional approach. The nonresonant process contribution to the stellar reaction rate of the ^{28}Si(p, γ) reaction has been evaluated and found to be the dominant one at temperatures less than $T_9 = 0.10$.

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(¹) Typographical error in reference [8] has been corrected here.

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