Spin dynamics and glassy relaxation on fractals and percolation structures

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Spin dynamics and glassy relaxation on fractals and percolation structures

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Résumé. — Dans le cadre d'une approche de basses températures, on étudie la dynamique des modèles de spin discrets sur les structures fractales et de percolation. On montre, à l'aide d'un argument d'échelle, que la barrière d'énergie pour retourner un amas de s spins croît comme ln s. On propose la loi des valeurs extrêmes comme distribution de probabilité de la barrière d'énergie associée à un amas de percolation. La loi de relaxation trouvée est du type exponentielle étirée, avec un exposant qui dépend de la température. Nos résultats donnent lieu à une formulation naturelle d'une nouvelle hypothèse d'échelle dynamique et sont discutés en relation avec les relaxations dites vitreuses.

Abstract. — Single-spin-flip dynamics of discrete spin models on fractals and percolation structures is studied within the framework of a low temperature approach. Using a scaling theory we show that, in general, the energy barrier for overturning a finite cluster of s spins scales as ln s. The probability distribution of the energy barriers for percolation clusters is argued to be given by the extreme-value distribution. The resulting long time relaxational dynamics so obtained is a stretched exponential with a temperature dependent exponent. Our results lead to a natural formulation of a new dynamic scaling hypothesis and are discussed in relation with the so-called glassy dynamics.

1. Introduction.

In a previous paper [1], a numerical study of the spin dynamics of discrete spin models on diluted lattice has been presented. Within the framework of a low temperature approach, we have argued that single-spin-flip dynamics on finite percolation clusters can be described with thermal energy barriers. In particular, it has been shown that the energy barrier V_s to overturn a finite cluster of ferromagnetic Ising spins scales logarithmically with the cluster size s. More precisely, for percolation clusters at threshold ($p = p_c$), the following result has been obtained :

$$V_s/2J = A.\ln s + B \qquad (1)$$

with $A = 1.058 \pm 0.05$, $B = 0.024$ at two dimensions and $A = 0.97 \pm 0.03$, $B = 0.065$ at three dimensions. Here J refers to the coupling constant between nearest neighbour sites on the cluster. This result is the counterpart of the known power law behaviour on Euclidean lattices of dimension d : $V_s/2J \simeq s^{(d-1)/d}$, and leads to a relaxation rate $\Gamma_s = \Gamma_0 \exp(-V_s/T)$ which is a power of the cluster size. Some direct implications of equation (1) on the single-spin-flip dynamics of individual clusters have been described elsewhere [2]. In this paper we address the questions of the origin, the generality and the practical implications of equation (1). More precisely, we shall show that : i) using renormalization group ideas, equation (1) is actually a natural consequence of the scale invariance of percolation clusters, ii) logarithmic size dependence can be derived directly on fractal lattices, due to some elementary rules for the calculation of the energy barriers, iii) the energy barriers distribution, for percolation clusters of fixed size, is given by the extreme-value distribution, iv) the long-time relaxational dynamics assumes a stretched exponential form with a temperature dependent exponent, v) the standard dynamic scaling hypothesis ($\tau(T) \sim \xi_T^z$) is violated and a scaling law ($\ln \tau(T)$ *vs.* $\ln \xi_T$) emerges naturally from equation (1).

Our results are used to illustrate some basic mechanisms leading to non exponential relaxation behaviours and this in relation with the recent increasing interest for the glassy dynamics.

2. Phenomenological scaling theory.

The renormalization group (RG) picture provides an useful framework and a natural language to discuss equilibrium as well as relaxation phenomena

occurring on many length scales. Following the ideas of reference [3], we assume an RG procedure which forms a block spin from $\lambda^{\bar{d}}$ spins with a length scale change of a factor λ. Here $\bar{d}$ refers to the fractal dimension of the lattice where a set of discrete spins is placed. Let us denote by $J \equiv J_0$ the ferromagnetic interaction scale at the lowest length scale. We assume that the energy scale changes under the action of the RG but the shape of the interaction distribution (if any) is assumed to be invariant. The RG recursion relation for the dimensionless coupling constant : $K = J/T$ ($k_B = 1$) can be written in general in the form [3] :

$$K_{n+1} = K_n . F(K_n) . \tag{2}$$

In the strong-coupling limit $F(K)$ approach a constant and we assume a power law expansion : $F(K) = \lambda^a - \mu . K^{-1} + \cdots$, where λ, μ and a denote constant numbers. An estimation of the energy barrier, for overturning a block of spins, can be obtained from the energy of the domain wall passing across the block. The scale V_n of energy barriers at step n can then be written as [3] :

$$v_{n+1} = v_n + K_n \tag{3}$$

where $v_n \equiv V_n/T$ and $v_0 = 0$.

Equations (2) and (3) are the basic iteration equations. The iteration of equation (2) leads to the following expression for the coupling constant K_l, at length scale $l = \lambda^n$:

$$K_l = l^a K_0 - \mu . (1 - l^a)/(1 - \lambda^a) . \tag{4}$$

Depending on the value of the exponent a, three generic cases are possible.

i) case $a < 0$: equation (4) iterates until $K_l = 0$, defining the correlation length $\xi_T \sim (J_0/T)^\nu$, where $\nu = 1/|a|$. In the critical regime where $K_l \simeq l^a K_0$, the energy barrier is given by $V_l \sim J_0(1 - l^a)/(1 - \lambda^a)$. The largest barrier occurs at $l \sim \xi_T$, giving $V(T) \sim J_0(1 - C.\xi_T^{-1/\nu})$ where C denotes a numerical factor ;

ii) case $a \to 0$: in this case, $K_l = K_0 - \mu . (\ln l/\ln \lambda)$ and the correlation length is given by $\xi_T \sim \exp\left[\frac{J_0}{T}\frac{\ln \lambda}{\mu}\right]$. In the critical regime, the energy barrier is $V_l \sim J_0 \mu \frac{\ln l}{\ln \lambda}$ and the largest thermal barrier is $V(T) \sim J_0^2/T$;

iii) case $a > 0$: there is a phase transition at finite $T_c \simeq J_0(\lambda^a - 1)$ and the correlation length assumes a power law behaviour $\xi_T \sim (T - T_c)^{-\nu}$ with $\nu = 1/a$. In the critical regime, the energy barrier at length scale l is given by $V_l \sim J_0 \frac{\ln l}{\ln \lambda}$, and the largest thermal barrier is $V(T) \sim - J_0 \ln \left| \frac{T - T_c}{T_c} \right|$. For $l \gg \xi_T$, V_l is a power of l :

$$V_l = V(T) + J_0(l^{1/\nu} - \xi_T^{1/\nu})(T_c - T)/T_c .$$

This general analysis is based mainly on equations (2, 3). The validity of equation (2), for all n, is actually supported by the self-similarity of the underlying structure. The size dependence of the energy barriers is therefore controlled by the sign of the exponent a. Among the three possible cases, only case ii) is consistent with the static critical behaviour [4] of discrete spin models on the infinite percolation cluster : $T_c(p_c) = 0$ and $\xi_T \sim \exp\left(\frac{2J}{T} v_p\right)$. Therefore, the energy barrier for a cluster of $s \sim l^{\bar{d}}$ spins scales as $V_s \sim (J_0 \ln s)/\bar{d} v_p$. More generally, case ii) corresponds to barriers of the form given by equation (1) and this can be checked directly on regular fractals. For instance, it is easy to derive equation (2) for linear fractals (e.a. branching or non branching von Koch curves, ...) for both Ising and Potts spins models [5]. In this case $a = 0$ but $A = 0$ in equation (1) (see below). Note that in the case of the Sierpinski gasket, the corresponding iteration relation : $K' = K(1 - e^{-4K}/K + \cdots)$ does not fit the above assumption for the function $F(K)$. However a careful analysis (see below) leads to equation (1) as expected.

It should be noticed that case iii) may occurs on fractal lattices, where a finite T_c exist. This is actually the case of the Sierpinski carpets [6] of fractal dimension $\bar{d} = 1 + \varepsilon$ for which $T_c \sim \varepsilon$ and $\nu = 1/\varepsilon$ at small $\varepsilon \gtrsim 0$. The free energy barrier scales logarithmically with size in the critical regime. However, according to the analysis of case iii), the standard dynamic scaling hypothesis [7] :

$$\tau(T) \sim \exp(V(T)/T) \sim \xi_T^z ,$$

with a constant value for z, holds in this case. A crude estimation of the exponent z can be obtained as follows. For small ε and close to T_c, the spin dynamics will be dominated by the 1D diffusion of domain walls [7]. This is actually the extrapolation from the 1D case where instead of a single kink, we have here a wall. This picture leads to $z = 2 + \varepsilon = \bar{d} + 1$. This prediction for z fits the known result ($z = 2$) at one dimension, but call for a numerical check at $\varepsilon \gtrsim 0$. Furthermore, as will be shown below, this scaling breaks down on a large number of fractal lattices, of finite ramification number where $T_c = 0$.

3. Direct calculation of the energy barriers.

Because of dilation invariance, fractals lend themselves particularly conveniently to scaling approaches. Thanks to some simple composition rules, the size dependence of the energy barriers can be extracted in some cases. In the following, we shall list some of these rules for two-terminal basic units (Fig. 1a) viewed as blobs. For the sake of simplicity, we limit our discussion to Ising spins. The energy barrier V and the domain wall energy barrier W are measured in units $2J$.

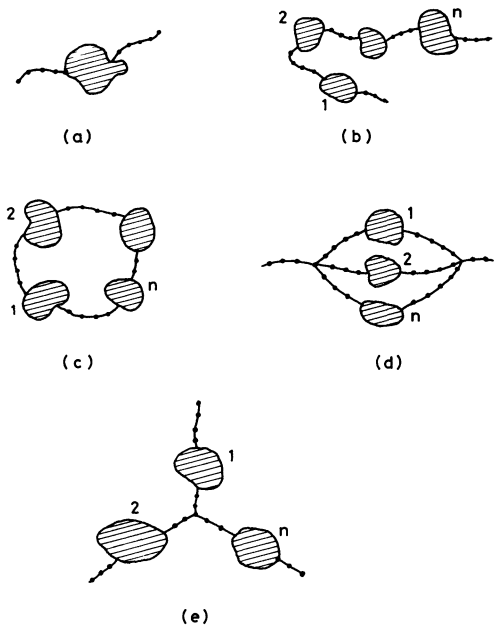

Fig. 1. — Schematic illustration of the composition rules for the calculation of the energy barriers of structures made of blobs. (a) : typical basic unit (two-terminal), (b) and (c) : series, (d) : parallel, (e) : star.

i) Series rule : for n blobs (Fig. 1b) of individual barriers V_i (resp. W_i), the global barriers read $V = \underset{i}{\text{Max}} \{ V_i \}$ and $W = \underset{i}{\text{Max}} \{ W_i \}$ respectively. For a closed ring geometry (Fig. 1c), one gests $V' = V + 1$ and $W' = W + 1$.

ii) Parallel rule (Fig. 1d) : similarly, $V = \text{Max}(V_1, V_2 + 1, ..., V_n + n - 1)$ for $V_1 \geqslant V_2 \geqslant \cdots \geqslant V_n$ and the same result holds for W.

iii) Star rule (Fig. 1e) : for $V_1 \geqslant V_2 \geqslant \cdots \geqslant V_n$, one gets

$$V = \text{Max}(V_1, V_2 + 1, V_3 + 2, ..., V_{n'} + n' - 1)$$

and for $W_1 \geqslant W_2 \geqslant \cdots \geqslant W_n$:

$$W = \text{Max}(W_1, ..., W_2 + 1, ..., W_n + n - 1).$$

Here n' refers to the integer part of $(n + 1)/2$.

Using these rules and their straightforward extension to more complicated blobs (e.a. multi-terminal), one can extract the size dependence of barriers on regular fractal lattices. For instance, the energy barrier for a linear arrangement of identical blobs (Fig. 1b) is independent of the number n of blobs. However, for the regular simplex, shown on figure 2, the series rule in closed ring geometry, leads to the following recursion relation : $V(3\,s) = 2 + V(s)$ for the energy barrier. Thus $V(s) = 2 \ln s / \ln 3$. Such a logarithmic dependence in size holds more generally on regular fractals, obeying a recursion relation $V(\lambda . s) = \mu + V(s)$, which leads to $V(s) = \mu \dfrac{\ln s}{\ln \lambda}$.

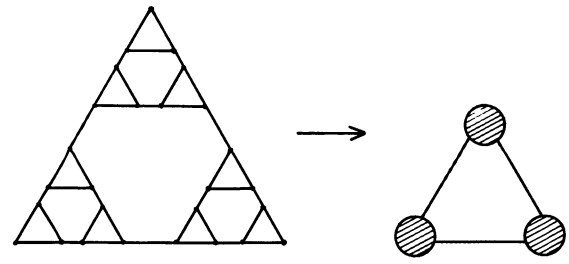

Fig. 2. — Schematic picture of the 2D simplex (modified Sierpinski gasket) at the second state of iteration.

It is important to notice that such a result originates from the exact recursion relation obeyed by $V(s)$. For random fractal lattices, there is no such simple relation and the origin of the result of equation (1) on percolation clusters must be found elsewhere.

4. Energy barriers distribution for percolation clusters.

The result of equation (1) refers actually to the energy barrier for a typical cluster of size s. A more precise analysis would take into account the fact that V and then the relaxation rate depend on the shape of the cluster also. Therefore, for a given size s, there is a probability distribution for the energy barriers and for the sake of clarity we limit our discussion to clusters at threshold ($p = p_c$). The structure of a given cluster is well described by the links-nodes-blobs picture [4, 8] and can be viewed as a network of quasi-one-dimensional string segments (« links »), tying together a set of « nodes ». Each string consists of several sequences of singly-connected bonds, in series with thicker multiply connected regions, or « blobs ». Between two randomly chosen sites, at an Euclidean distance l, on a given cluster, the backbone connecting these two points can be considered as a randomly constructed necklace [8] whose building blocks are blobs of size larger than one (blobs of size one are the singly connected bonds). This picture of topologically linear necklace of strings of blobs, of all possible sizes and shapes, assembled in completely random order, will be used here to calculate the probability distribution of barriers.

Assume that the energy barrier V for a blob of arbitrary size s, $1 \leqslant s < \infty$ is distributed according to an unknown probability density $p(V)$, associated to the probability distribution $P(V)$. It is clear from the series rule that the global barrier V for a cluster of $s \sim l^{\bar{d}}$ sites is actually given by the maximum over $n \sim l^{1/\nu_p}$ independent identically distributed random variables $\{ V_i \}$, $1 \leqslant i \leqslant n$, corresponding to n blobs in a box of size l^d containing $s \sim l^{\bar{d}}$ spins. We shall argue that the most probable value of V as well as its average scale as $\ln n$ for large n. This leads

immediately to $V_s/2\,J \sim \frac{1}{\nu_p \bar{d}} \ln s\,(s \gg 1)$ in agreement with the result of the scaling theory.

To see this, assume that for large n, $V = \text{Max}\,\{V_i\}$ possess a limiting distribution. We know from the asymptotic theory of extreme order statistics [9], that this limiting distribution, after suitable standardization, must be one of just three types, namely : $F_1(x) = \exp(-x^{-\alpha})$ for $x > 0$ $(\alpha > 0)$, $F_2(x) = \exp(-(-x)^{\alpha})$ for $x \leqslant 0$ $(\alpha > 0)$ and $F_2(x) = \exp(-e^{-x})$, $-\infty \leqslant x \leqslant \infty$. Formally, the class of limiting distributions of $P^n(A_n x + B_n)$, where $A_n > 0$ and B_n are suitably chosen constants contains only laws of the type $F_k(x)$, $k = 1, 2, 3$. The limiting distributions $F_k(x)$ are the solutions of the functional equation : $F^n(A_n x + B_n) = F(x)$, $n \geqslant 1$. That is the largest in a sample of n drawn from a distribution with distribution function $F(x)$ must, upon the same standardization as above, itself have the distribution function $F(x)$: stability under Max. The only solutions of the functional equation are respectively of the form $F_1(x)$ to $F_3(x)$ and are associated to $A_n > 1$, $A_n < 1$ and $A_n = 1$ respectively. $F_3(x)$, called also the extreme-value distribution is the only one with an unbounded support. Clearly, $(F_3(x))^n = \exp(-n e^{-x})$ is nothing else than $F_3(x - \ln n)$. Therefore, the distribution function $P_s(V)$ for the properly normalized energy barriers for clusters of size s is given by

$$P_s(V) = \exp(-\exp(-V + \ln s)). \tag{5}$$

Note that the argument leading to equation (5) is mainly based on just two specific ideas : i) the stability of the distribution of V under the action of the operation Max. Such a stability condition is implied by the self-similarity of the blobs structure. ii) The linear topology of necklaces.

Global distribution of barriers is simply deduced from the cluster-sizes distribution n_s. At threshold [10], $n_s \sim s^{-\tau}$ for large s and using equation (1), this leads to

$$\pi(V) \equiv \int_0^\infty s n_s\,\mathrm{d}s.\,\delta(V - V_s) = \lambda \exp(-\lambda V),$$

$$\lambda = (d - \bar{d})/\bar{d}A \tag{6}$$

where we have used $\tau = 1 + d/\bar{d}$ and neglected B in equation (1). The same result for $\pi(V)$ can be obtained from $p_s(V) = \mathrm{d}P_s(V)/\mathrm{d}V$.

The exponential distribution $\pi(V)$ holds also at $p \neq p_c$, up to $V \leqslant V^*$ where V^* corresponds to the cutoff $s^* \sim \xi_p^{\bar{d}}$ in the size distribution n_s.

A straightforward algebra leads to the following power law distribution for the relaxation times $\tau \sim \exp(V/T)$:

$$\pi(\tau) = \lambda T/(\tau/\tau_0)^{1+\lambda T} \tag{7}$$

which exhibits a long-time tail and this particularly at very low temperatures.

5. Relaxational dynamics.

At low temperature the spin dynamics will be dominated by thermally activated process, associated to jump rate scale $\Gamma_s = \Gamma_0 \exp(-V_s/T)$ where Γ_0^{-1} denotes an elementary time scale and V_s given by equation (1). Γ_s depends on s at very low T : $T \ll T^*(s)$ where $T^*(s) \simeq \frac{2J}{\ln s}\bar{d}\nu_p$ is the crossover temperature towards a singular behaviour of Γ_s corresponding to $s \sim \xi_T^{\bar{d}}$. At $T \gg T^*(s)$, $\ln \Gamma_s = -\frac{2J}{T} \ln \xi_T^{\bar{d}} = -\frac{4J^2}{T^2}\nu_p \bar{d}$. In this limit, the relaxation time $\tau(T)$ is independent of s and assumes the following form :

$$\ln \tau(T) = \frac{\bar{d}}{\nu_p}(\ln \xi_T)^2 \tag{8}$$

instead of the usual form of the dynamic scaling [7] : $\tau(T) \sim \xi_T^z$. In the following we shall denote by $\zeta(T) \equiv \frac{2J}{T}\bar{d}$ the « dynamic exponent » describing the size dependence of the relaxation time

$$\tau_s = \tau_0 . s^{\zeta(T)/\bar{d}}. \tag{9}$$

Note that this form for τ_s contrasts sharply with the corresponding expression : $\tau_s = \tau_0 . s . \xi_T$ in the case of 1D chain [11].

Among different relaxational dynamics, let us consider the expressions of the time-dependent correlation function in equilibrium, and the time-dependent magnetization starting from a uniformly magnetized initial state :

$$C(t) = \sum_s s^2 n_s(p) . \int \mathrm{d}V p_s(V)\, e^{-t\Gamma} \equiv g_c(t)\, C(0) \tag{10}$$

$$M(t) = \sum_s s n_s(p) . \int \mathrm{d}V p_s(V)\, e^{-t\Gamma} \equiv g_M(t)\, M(0). \tag{11}$$

Here $n_s(p)$ refers to the probability distribution of clusters size [10] and $\Gamma = \Gamma_0 \exp(-V/T)$ is the relaxation rate for a cluster of size s. The functions $g_c(t)$ and $g_M(t)$ are decay functions normalized to unity at $t = 0$. The characteristic decay times τ_c and τ_M are defined by : $\tau_i = \int_0^\infty g_i(t)\,\mathrm{d}t$, $i = M, c$.

Below threshold $(p < p_c)$, $n_s(p)$ is known to assume the scaling form [10] : $n_s(p) = s^{-\tau} f(\varepsilon s^\sigma)$ where $\tau = 1 + d/\bar{d}$, $\sigma = 1/\nu_p \bar{d}$ and $\varepsilon = p_c - p$. Close to p_c, $\ln f(u) \sim -|u|^{1/\sigma}$, $u \to \infty$ and similar behaviour occurs at $p \gtrsim p_c$. The expressions of τ_c and τ_M are therefore :

$$\tau_c \cong \tau_0 . \xi_p^{\zeta(T)}$$

$$\tau_M \cong \tau_0 . \xi_p^{\zeta(T)} \tag{12}$$

As expected both τ_c and τ_M diverge at $p \to p_c^-$. More interesting is the following form for the scaling of τ_i as implied by equation (12) :

$$\ln \tau_i/\tau_0 = \frac{\bar{d}}{\nu_p} \ln \xi_T \cdot \ln \xi_p \qquad (13)$$

to be contrasted with the 1D result [11] : $\ln \tau_i = \ln \xi_T + \ln \xi_p$. Note that in both cases, this expression for τ_i is only valid at $\xi_T \gg \xi_p$ and crossovers to $2 \ln \xi_T$ at $d = 1$ and $(\ln \xi_T)^2$ at $d \geqslant 2$ for $\xi_T \ll \xi_p$.

The expressions of $g_c(t)$ and $g_M(t)$ are easy to calculate. For instance, at $\xi_T \ll \xi_p$, $\ln g_c(t)$ and $\ln g_M(t)$ fall of linearly in time : $- t/\tau(T)$. In the opposite limit, $\xi_T \gg \xi_p$ the calculation of $g_M(t)$ and $g_c(t)$ can be carried out without difficulties. For instance, $g_c(t)$ is actually a function of the scaling variable t/τ_c. Using a saddlepoint method, the integrals can be developed as an asymptotic expression in t/τ_c, corresponding to the asymptotic regime $t \gg \tau_c$. Then

$$g_c(t) \simeq \exp[-(t/\tau_c)^{1-n}], \quad t \gg \tau_c \qquad (14)$$

where the exponent $1 - n = 1/(1 + \zeta(T)/\bar{d})$ of the stretched exponential decay function is actually a function of temperature. More precisely, $n = n(T) = 1/(1 + T/2J)$ decreases from $n = 1$ at $T \ll 2J$ to $n = 0$ at $T \gg 2J$. This non exponential regime dominates, at $t \gg \tau_c$, the linear behaviour $\ln g_c(t) \sim - t/\tau(T)$ described above. Explicitly, for $\xi_T \gg \xi_p$, two time regimes take place.

a) For $t \gg \tau^*$, $g_c(t)$ is dominated by the purely exponential term :

$$\exp(- t/\tau(T)) ;$$

b) For $\tau_c \ll t \ll \tau^*$, $g_c(t)$ is given by equation (14).

Here $\tau^* = \tau(T)\,[\tau(T)/\tau_c]^{\bar{d}/\zeta(T)}$ denotes the crossover time between the two limiting behaviours. Furthermore, it is easy to see that 1D results [11] are recovered by replacing $\zeta(T) = 1$ and $\bar{d} = 1$ in the above expressions : $\tau_c = \xi_T \xi_p$, $\tau^* = \xi_T^3/\xi_p$. These results hold also for $g_M(t)$, which drops out rapidly : $\ln g_M(t) \sim - t/\tau(T)$ up to $t \sim \tau_c$ and then crossovers to a non exponential regime, described by the analogue of equation (14). Moreover, equation (14) holds also at $p \gtrsim p_c$, $T \ll T_c(p)$ where $n_s(p)$ assumes a similar form [10] as above. The crossover fractal-to-Euclidean will be discussed elsewhere.

It should be noticed however that in the previous analysis we have neglected the contribution, to g_c and g_M, of exceptional very large compact (Euclidean) clusters. For these compact clusters $V_s \sim s^{(d-1)/d}$ and using a Lifshitz-like argument, one gets [12] ($p < p_c$, $T < T_c(p = 1)$) :

$$\ln g_M(t) \sim \ln g_c(t) \sim - (\ln t)^{d/(d-1)} \quad \text{at} \quad t \to \infty . \qquad (15)$$

This regime is more slowly than an exponential but faster than a power law and represents the dynamical counterpart of Griffiths singularities. However, we believe that the observation of this regime in real or computer experiments is actually doubtful and calls for a careful analysis [13].

6. Glassy dynamics and concluding remarks.

Our main results were summarized in the introduction. Let us conclude with three comments.

i) The power law expression (Eq. (9)) of the relaxation rate Γ_s, which is implied by the dilation-symmetry, appears as the basic element in our analysis. The first consequence of this result is the formulation of a new dynamic scaling hypothesis : $\ln \tau(T) = \varphi(\ln \xi_T)$. Standard hypothesis appears as a special case : $\varphi(u) = zu$. In the model investigated here, $\varphi(u)$ is a quadratic function (Eq. (8)). Note that such an extension of the dynamic scaling has been suggested [14] previously but without justification. In this respect, it is interesting to notice that relaxation time such as $\tau(T)$ given here : $\ln \tau(T) \sim T^{-2}$ has been observed in a large number of situations [15] (ionic conductors, short range spin glass models, ...). In this line of ideas, it would be very interesting to compare the temperature dependence of Γ_s obtained here with the similar behaviour observed on kinetic coefficients and relaxation times near the glass temperature of glasses. Actually, the plot of $\ln \tau$ *vs.* $1/T$ has a very surprising form [16] : straight line below T_g and a convex curve (parabola ?) at $T \gg T_g$. Is T_g a simple crossover temperature between two relaxation regimes ?

ii) The relaxation time distribution (Eq. (7)) as well as the exponential distribution of barriers (Eq. (6)) are the consequences of dilation symmetry. Because of the generic features of both n_s and Γ_s, we believe that such results are very general and not specific to percolation clusters. Note that equation (7) leads to a « noise spectrum » $1/\omega^{1-\lambda T}$ which is a kind of « $1/f$ » noise but with a temperature dependent spectral exponent.

iii) Our derivation of the stretched exponential law (Eq. (14)), which appears in a large number of glassy materials [17] does not involve any *ad hoc* hypothesis on the dynamics mechanism and this contrasts with all previous attempts [18]. The results reported here show clearly, that non exponential or glassy relaxation can be obtained in at least two ways : i) from a power law expression of relaxation rates Γ_s in addition to a power law distribution n_s of clusters size, ii) from rare « events » which are the non equilibrium counterpart of Griffiths singularities. The mechanism of rare « events » is the analogue of that giving the survival probability of a random walk in the presence of traps [19]. An important difference between i) and ii) comes from the temperature dependence of the exponent n : n is a constant number for ii) whereas $n = n(T)$ in the first case. In this respect,

it is interesting to compare our prediction for $n(T)$ with the available data. Actually, the linear behaviour $1 - n(T) \sim T$ at low T, seems to be observed (see Fig. 3 of Ref. [17]). Such a result is in fact a very general one, resulting from the scaling in T ln t of thermally activated processes.

Note added in proof. — After this manuscript was submitted, two papers related to this work have been published [20, 21]. Though using a completely different argument, the result of reference [20] fully confirms our results for the percolation clusters. Regarding reference [21], there is an agreement with our results for linear fractals. However, the claimed result $z = \bar{d} + 1$ (Sierpinski gasket), which seems to coincide with our prediction for the carpets, must be considered carefully. In fact, real space renormalization group does not provide a reliable method for the study of kinetic Ising models [22] and particularly for the calculation of the exponent z.

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