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Quality factor and boundary-layer attenuation of lower order modes in acoustic cavities

M. Bruneau (*), Ch. Garing (*), and H. Leblond (**)

(*) Laboratoire d'Acoustique, J. E.-C.N.R.S., Université du Maine, B.P. 535, 72017 Le Mans Cedex, France
(**) Société Badin-Crouzet, B.P. 13, 78117 Chateaufort, France

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Résumé. — Certains appareils acoustiques utilisent des cavités remplies de gaz, dont les dimensions sont du même ordre de grandeur que la longueur d'onde et dont les parois sont parfaitement rigides. La connaissance de la réponse acoustique de telles cavités, excitées par des sources mécaniques ou thermiques, nécessite de tenir compte des phénomènes d'amortissement qui, en l'occurrence, sont essentiellement dus aux effets d'absorption visqueux et thermiques dans les couches limites, effets décrits par une impédance acoustique apparente finie dépendant des modes. Dans ce travail, le champ sonore est étudié pour un espace clos parallélépipédique excité par une source ponctuelle, dans le cadre d'une théorie modale. Le facteur de qualité est calculé et ses propriétés sont discutées et comparées aux résultats expérimentaux. L'accord est convenable.

Abstract. — Some acoustic devices make use of closed cavities, filled with gas, the dimensions of which are of the same order of magnitude as the acoustic wavelength and the walls of which are perfectly rigid. To know the acoustic response of such small cavities, driven by a mechanical or a thermodynamical source, one needs to take into account the damping processes. In this case they are essentially due to viscous and thermal effects in the boundary layers and are described as a finite apparent acoustic impedance depending on the modes. In this paper, the sound field is determined in a « rectangular » enclosure excited by a point source and is described by a modal theory. The Q-factor is calculated and its properties are discussed and compared with experimental results. The agreement is quite good.

Introduction.

In the past, a few works have been devoted to the problem of determining the boundary layer attenuation of acoustic modes in *tubes with rigid walls* by using the conservation of energy law (see for example [1]) or, more recently, by solving the boundary problem for the first lower modes [2]; at the same time, the determination of the field distribution inside *cavities*, filled with gas, assuming *classical Neumann or mixed boundary conditions*, has received considerable attention (see for example [3]). But some acoustic devices make use of small cavities, driven by a mechanical or a thermodynamical source, the dimensions of which are of the same order of magnitude as the acoustic wavelength and the walls of which are assumed to be *perfectly rigid*. So, to know the acoustic response of such cavities, one needs to take into account the *boundary layer attenuation which may depend on the field distribution over the wall surfaces according to the mode*. Here, the sound field is determined in a three dimensional rectangular enclosure excited by a point source (a loudspeaker coupled to the cavity through a thin hole), and the viscous and thermal effects in the boundary layers are adequately modelled by a *finite apparent acoustic impedance depending on the incidence angle* of the wave on the wall. The solution appears as a sum of modal terms. *The Q-factor* is calculated and *its properties are discussed* (its values are much higher than those which usually occur in acoustic tubes for example).

The beginning of the paper is concerned with the analysis of the stationary acoustic motion in terms of normal modes of the cavity, assuming a very small damping. The quality factor Q is expanded in a series of these normal modes, and the amplitude of each resonance, in response to a periodic driving source, is given as a function depending on Q. Then, we still consider cavities with rigid walls, but the damping of modes is specifically due to the viscosity and the thermal conductivity in the boundary layers. The viscothermal quality factor corresponding to each mode is expressed as a function of several parameters of the system : the serial numbers of the modes, the dimensions of the cavity, the static pressure and the absolute temperature in the cavity, the coefficient $\gamma = C_p/C_v$, the thermal conductivity coefficient, the viscosity coefficient and the molecular mass of the gas. For axial modes, the theoretical and the experimental results are compared.

1. Normal modes in cavities with small damping.

The cavities considered here are rectangular parallelepipeds and they are of the same order of size as the wavelength, so the modal analysis turns out to be the most fruitful. The walls are assumed locally reacting; we assign a specific acoustic admittance for each frequency of the sound and possibly for each mode (in the cases where the admittance depends on the angle of incidence). The admittance is assumed very small and constant over each wall, consistent with many real situations. In order to make the equations simpler, without restricting the generality of the study, each of the three pairs of walls is assumed to have the same admittance. Let $\psi_{n_1}(x_1)\,\psi_{n_2}(x_2)\,\psi_{n_3}(x_3)$ be the eigenfunctions, corresponding to the eigenvalues $k_{n_1n_2n_3}$. They satisfy the homogeneous problem :

$$\begin{cases} [\Delta + k^2_{n_1n_2n_3}]\,\psi_{n_1}(x_1)\,\psi_{n_2}(x_2)\,\psi_{n_3}(x_3) = 0 \\ \qquad\qquad \text{inside the cavity } (V) \\ (\partial_n + ik\varepsilon_j)\,\psi_{n_1}\,\psi_{n_2}\,\psi_{n_3} = 0 \\ \qquad\qquad \text{on the boundaries } (S_j),\ j = 1, 2, 3 \end{cases} \tag{1}$$

where

$k = \omega/c$ is the wavenumber (ω is the angular frequency of the sound)

$\varepsilon_j = \rho c/Z_j$ the specific acoustic admittance of the two walls $x_j = 0$ and $x_j = l_j$ (ρ is the mass density of the gas and c the speed of sound)

∂_n is the normal derivative pointing out of the cavity.

Let us notice that this problem is slightly different from a self-adjoint problem in which the ε_j are equal to zero. Nevertheless it can be solved approximately because here the ε_j are assumed to be very small. Especially, the eigenfunctions and the eigenvalues of the two problems are close.

The well-known approximate solution can be written as follows (see for example [3]) :

$$\psi_{n_j}(x_j) = a_{n_j}\cos(k_{n_j}x_j + \alpha_{n_j}) \tag{2}$$

where

$$k^2_{n_j} \simeq \left(\frac{n_j\pi}{l_j}\right)^2 - 2k(2 - \delta_{n_j0})\frac{\operatorname{Im}(\varepsilon_j)}{l_j} + 2ik(2 - \delta_{n_j0})\frac{\operatorname{Re}(\varepsilon_j)}{l_j} \tag{3}$$

$$\alpha_{n_j} \simeq -i\frac{k}{k_{n_j}}\varepsilon_j \tag{4}$$

and

$$k^2_{n_1n_2n_3} = \sum_j k^2_{n_j}. \tag{5}$$

The effects of the wall admittances ε_j are to make the resonant frequencies (the real part of $k^2_{n_j}$) slightly different from those corresponding to Neumann boundary conditions, and to introduce the wall losses (the imaginary part of $k^2_{n_j}$). Note that the orthogonality property of the eigenfunctions can be used as a first-order approximation (see for example [4], p. 475).

Throughout the paper, equation (5) is written :

$$k^2_{n_1n_2n_3} = k^{02}_{n_1n_2n_3} + 2ik\sum_j(2 - \delta_{n_j0})\frac{\operatorname{Re}(\varepsilon_j)}{l_j} \tag{6}$$

where

$$k^{02}_{n_1n_2n_3} = \sum_j\left(\frac{n_j\pi}{l_j}\right)^2 - 2k\sum_j(2 - \delta_{n_j0})\frac{\operatorname{Im}(\varepsilon_j)}{l_j} \tag{7}$$

2. The quality factor of the cavities.

If ω is the angular frequency of excitation, Q is defined as ω times the ratio of the time average stored energy to the time average of energy dissipated per time unit :

$$Q = \omega\langle E\rangle/\langle P\rangle. \tag{8}$$

From this definition it is a simple matter to link the Q-factor to the decay constant of the damped oscillations. If the sound is shut off suddenly at $t = 0$ for instance, the stored energy drops off continuously in such a way that its rate of change is equal to the power absorbed by the walls $\langle P\rangle = -\,\mathrm{d}\langle E\rangle/\mathrm{d}t$. The solution of this equation can be written $\langle E\rangle = \langle E\rangle_0\exp(-\omega t/Q)$ or, for the acoustic pressure, $p = p_0\exp(-\omega t/2Q)$. This result shows that the time τ for the amplitude to drop to $1/e$ of its original value can be used as a measure of Q :

$$Q = \omega\tau/2. \tag{9}$$

For each mode $(n_1\, n_2\, n_3)$, a relationship between the Q-factor $Q_{n_1n_2n_3}$ and the properties of the stationary field inside a cavity is derived by substituting in equation (8) the expression for the total outflow of energy per second through the six surfaces S_j of the walls

$$\langle P_{n_1n_2n_3}\rangle = \sum_{j=1}^{6}\iint_{S_j}\frac{1}{2}\frac{|p_{n_1n_2n_3}|^2}{c}\operatorname{Re}(\varepsilon_j)\,\mathrm{d}S_j,$$

where $p_{n_1n_2n_3}$ is the acoustic pressure corresponding to the mode $(n_1\, n_2\, n_3)$, and the expression of the time average of the total stored energy in the whole volume V of the cavity

$$\langle E_{n_1 n_2 n_3} \rangle = = \iiint_V \left[\frac{1}{2} \rho \langle v^2_{n_1 n_2 n_3} \rangle + \frac{1}{2 \rho c^2} \langle p^2_{n_1 n_2 n_3} \rangle \right] \mathrm{d}V$$

where $v_{n_1 n_2 n_3}$ is the acoustic particle velocity obtained from the Euler equation. After some algebraic juggling, this substitution and the use of equations (2) to (5) give, to the second order in ε_j :

$$Q_{n_1 n_2 n_3} = \frac{k^2}{\operatorname{Im}(k^2_{n_1 n_2 n_3})} = = \frac{k}{2 \sum_j (2 - \delta_{n_j 0}) \operatorname{Re}(\varepsilon_j)/l_j} \quad (10)$$

where we assumed that $k \simeq k^0_{n_1 n_2 n_3}$ which is the resonant frequency of the mode $(n_1\ n_2\ n_3)$.

Inserting this expression in the equation (6), we obtain :

$$k^2_{n_1 n_2 n_3} = k^{02}_{n_1 n_2 n_3} + ik^2/Q_{n_1 n_2 n_3} . \quad (11)$$

3. Standing waves and forced motion.

The amplitude of the resonances inside a cavity can be determined in terms of the Q-factor $Q_{n_1 n_2 n_3}$. In order to carry out this analysis, we study the behaviour of a steady-state situation representing the spatial distribution of the radiation from a point source of angular frequency ω at a point $\mathbf{r}_0$ inside the cavity or on its walls. A common procedure for solving this problem is the classical method of eigenfunction expansion. The velocity potential at a point $\mathbf{r}$ produced by a point source of unit strength is the Green function (see [3] for example) :

$$G(\mathbf{r}, \mathbf{r}_0) = \sum_{n_1 n_2 n_3} \frac{\psi_{n_1 n_2 n_3}(\mathbf{r}_0)\, \psi_{n_1 n_2 n_3}(\mathbf{r})}{k^2_{n_1 n_2 n_3} - k^2} . \quad (12)$$

Substituting the expression (11) for the eigenvalues $k_{n_1 n_2 n_3}$, this equation yields :

$$G(\mathbf{r}, \mathbf{r}_0) = \sum_{n_1 n_2 n_3} \frac{\psi_{n_1 n_2 n_3}(\mathbf{r}_0)\, \psi_{n_1 n_2 n_3}(\mathbf{r})}{k^{02}_{n_1 n_2 n_3} - k^2 + ik^2/Q_{n_1 n_2 n_3}} . \quad (13)$$

In this paper, we discuss the response of small cavities, where the resonance peaks are separated from each other. The field has a resonance whenever the wavenumber k is equal to the real part $k^0_{n_1 n_2 n_3}$ of one of the eigenvalues. At a resonance $(k^{02}_{n_1 n_2 n_3} - k^2 = 0)$ the corresponding standing-wave $\psi_{n_1 n_2 n_3}$ predominates, having an amplitude proportional to $Q_{n_1 n_2 n_3}/k^{02}_{n_1 n_2 n_3}$. The ratio of the amplitude of the resonant mode to another mode (labelled $n'_1\ n'_2\ n'_3$) is given by the following approximate expression :

$$\frac{Q_{n_1 n_2 n_3}/k^{02}_{n_1 n_2 n_3}}{1/(k^{02}_{n'_1 n'_2 n'_3} - k^{02}_{n_1 n_2 n_3})} = \left(\frac{k^{02}_{n'_1 n'_2 n'_3}}{k^{02}_{n_1 n_2 n_3}} - 1 \right) Q_{n_1 n_2 n_3} . \quad (14)$$

This result shows that the modes whose eigenvalues do not correspond to the frequency of the source are roughly $Q_{n_1 n_2 n_3}$ times lower in amplitude than the resonant mode $(n_1\ n_2\ n_3)$; assuming a very small damping, the value of $Q_{n_1 n_2 n_3}$ is much greater than one, and the only term we need to consider in the series (12) is the one corresponding to the triplet $n_1\ n_2\ n_3$, neglecting all others (except on the nodal surfaces). Consequently, another well known method (besides the one discussed earlier in this paper) allows us to measure the Q-factor at a resonance. The Q-factor is equal to the ratio of the resonance frequency to the difference between the two driving frequencies for which the square of the amplitude is half that at the maximum (at the resonance frequency). It is a simple matter to show from equation (13) that :

$$Q_{n_1 n_2 n_3} = f_{n_1 n_2 n_3}/\Delta f_{n_1 n_2 n_3} . \quad (15)$$

4. Viscous and thermal damping in rigid cavities : the basic equations.

For cavities, whose walls are considered as perfectly rigid, a finite Q-factor cannot be explained from the ideal fluid-dynamic equations. The processes we have to take into account involve viscosity, thermal conductivity and possibly relaxation. Only the contributions to the Q-factor from losses due to viscosity and heat conduction at the cavity walls are considered here, because the dissipation of acoustic energy outside the boundary layers is usually negligible. Nevertheless, for quite large cavities, the damping effect inside the medium due to relaxation effects must be taken into account in the theoretical studies [5]. In this case, we only have to modify the expression of the specific-heat ratio (γ) in the results given below [4].

The variables describing the dynamical and thermodynamical state of the fluid are the acoustic pressure (p), the particle velocity ($\mathbf{v}$), the fluctuating part of the density (ρ'), the entropy variation (s) and the temperature variation (τ). The parameters which specify the properties and the nature of the fluid are the ambiant values of the pressure (P), of the temperature (T) and of the density (ρ), and the viscosity (μ), the bulk viscosity (η), the coefficient of thermal conductivity (λ), the specific heat coefficients at constant pressure and constant volume (C_p, C_v), the specific heat ratio ($\gamma = C_p/C_v$), the increase in pressure per unit increase in temperature at constant density ($\hat{\beta}$), the fractional decrease in volume per unit increase in pressure at constant temperature (χ_T).

A complete set of linear equations governing small amplitude disturbances includes the Stokes-Navier equations, the conservation of mass equation, the Fourier equation (heat conduction), and the equations showing that the entropy variations and the acoustic part of the density can be expressed as total differentials (regarded as functions of the two independent variables p and τ). Thus, assuming that the acoustic power is radiated by a point source set on a wall, we insert its effects in the boundary layer conditions, and write the equations as follows [3] :

$$\left| \begin{array}{l} \rho\, \partial_t \mathbf{v} = -\nabla p + (\frac{4}{3}\mu + \eta)\nabla(\nabla \cdot \mathbf{v}) - \mu \nabla \wedge (\nabla \wedge \mathbf{v}) \\ \partial_t \rho' + \rho \operatorname{div} \mathbf{v} = 0 \\ \dfrac{\rho}{M} T\, \partial_t s = \lambda\, \Delta\tau \\ s = \dfrac{C_p}{T}\left(\tau - \dfrac{\gamma - 1}{\hat{\beta}\gamma} p\right) \quad \text{and} \quad p = \dfrac{\rho'}{\rho \chi_T} + \hat{\beta}\tau . \end{array} \right. \tag{16}$$

Any disturbances governed by this system of linear equations can be considered as a superposition of acoustic, vorticity and entropy modes. The corresponding acoustic pressure p, rotational velocity $\mathbf{v}_v$ (due to viscosity effects) and entropic temperature τ_h (due to heat conduction effects) satisfy respectively the « wave » equations :

$$\left| \begin{array}{l} \{ \Delta - c^{-2}\partial^2_{tt} + c^{-1}[l_v + (\gamma - 1) l_h] \Delta\, \partial_t \}\, p = 0 \\ [\Delta - (1/cl'_v)\, \partial_t]\, \mathbf{v}_v = 0 \\ [\Delta - (1/cl_h)\, \partial_t]\, \tau_h = 0 \end{array} \right. \tag{17}$$

where the characteristic lengths l_v, l'_v and l_h are defined as follows (c is the velocity of sound) :

$$l_v = (\tfrac{4}{3}\mu + \eta)/\rho c \quad l'_v = \mu/\rho c \quad l_h = \lambda M/\rho c C_p . \tag{18}$$

The complex wave numbers

$$k_v = (1 - i)\sqrt{\omega/2\, cl'_v}$$

and

$$k_h = (1 - i)\sqrt{\omega/2\, cl_h} \tag{19}$$

obtained from the two last equations (17) for a simple harmonic motion with an angular frequency ω, show that the diffusion velocities, given by the real part of the wave numbers, are dispersive and much lower than the velocity of sound, and that the diffusion damping coefficients, given by the opposite of the imaginary part of the wave numbers are also dispersive but much stronger than the acoustic damping. This suggests that the vorticity mode and the entropy mode fields created at the boundaries (see next section) die out rapidly with increasing distance from the boundaries. On the other hand, it is well known [3, 4, 6] that the greatest part of the power loss through viscosity and thermal conductivity occurs within the boundary layers, and since these layers are very thin, we can consider the acoustic behaviour of the medium everywhere outside the boundary layers as being adequately described as a propagational mode, and, consequently, the factor $c^{-1}[l_v + (\gamma - 1) l_h]\, \Delta\partial_t$ in the first equation (17) can be cancelled. In other words, outside the boundary layers, the behaviour of the acoustic mode is the same as in a perfect gas.

5. The boundary conditions at a rigid wall.

The variables of the problem for each mode mentioned in the previous section may be written in the following form (we set the subscripts a, v and h for the acoustic, vorticity and entropy modes respectively) :

$$\mathbf{v} = \mathbf{v}_a + \mathbf{v}_v + \mathbf{v}_h \quad \text{and} \quad \tau = \tau_a + \tau_h \quad (\tau_v = 0) \tag{20}$$

With the requirements that the thermal conductivity of the boundary material is much greater than that of the medium, and that the boundaries are perfectly rigid, we can expect the temperature fluctuation and the particle velocity $\mathbf{v}$ to be nearly equal to zero on the walls. Therefore, the acoustic particle velocity and the acoustical temperature are not equal to zero at the boundaries :

$$\mathbf{v}_a = -\mathbf{v}_v - \mathbf{v}_h \quad \text{and} \quad \tau_a = -\tau_h . \tag{21}$$

This result suggests that one can define a finite apparent acoustic impedance of a rigid wall when viscous and thermal effects are taken into account [6]. The properties of plane-wave reflexion on a plane surface with an angle of incidence θ, assuming the prescriptions described just above, are given by the following expression for the ratio of the normal component $v_{a\perp}$ pointing outside the cavity of the acoustic particle velocity to the acoustic pressure p at the boundary :

$$v_{a\perp}/p = \varepsilon_{vh}/\rho c \tag{22}$$

where ε_{vh} is the « apparent » specific admittance

$$\varepsilon_{vh} = \frac{1 + i}{\sqrt{2}} \sqrt{\frac{\omega}{c}} [\sin^2\theta \sqrt{l'_v} + (\gamma - 1)\sqrt{l_h}] . \tag{23}$$

This result shows the effect of the thermal and shear modes on the boundary conditions for the propagational mode. The viscous part of the apparent specific admittance depends on the angle of incidence because the viscous effects depend on the tangential motion only. Consequently, we have to assign specific acoustic admittances for each mode in a cavity, whereas a usual admittance of the boundary is a property of the wall material which fixes the local acoustic field.

Note that the real and the imaginary parts of ε_{vh} are equal.

For air at atmospheric pressure and room temperature, we have $l'_v \simeq 4.5 \times 10^{-8}$ m and $l_h \simeq 6.1 \times 10^{-8}$ m which means that at a frequency of about 4 300 Hz (corresponding to the experimental study that we carried out) we obtain (for $\theta = \pi/2$) :

$$\operatorname{Re}(\varepsilon_{vh}) = \operatorname{Im}(\varepsilon_{vh}) \simeq 2 \times 10^{-3} .$$

6. The viscothermal quality factor of cavities with perfectly rigid walls.

Insofar as the boundary conditions and the wave propagation can be adequately approximated by the previous considerations, the quality factor is derived from substituting expression (23) in equation (10). This substitution combined with the expression for the incidence angle for the mode $(n_1\, n_2\, n_3)$ on the wall (S_j)

$$\sin^2 \theta_j = (k_{n_i}^2 + k_{n_k}^2)/k_{n_1 n_2 n_3}^2 \simeq \left[\left(\frac{n_i \pi}{l_i}\right)^2 + \left(\frac{n_k \pi}{l_k}\right)^2\right] \Big/ \sum_{\nu} \left(\frac{n_\nu \pi}{l_\nu}\right)^2$$

with (i, j, k) circular permutation (1, 2, 3) leads to

$$\frac{1}{Q_{n_1 n_2 n_3}} = \sqrt{\frac{2}{k}} \sum_j \frac{2 - \delta_{n_j 0}}{l_j} \left[\frac{\left(\frac{n_i \pi}{l_i}\right)^2 + \left(\frac{n_k \pi}{l_k}\right)^2}{\sum_\nu \left(\frac{n_\nu \pi}{l_\nu}\right)^2} \sqrt{l_\nu'} + (\gamma - 1)\sqrt{l_h} \right]. \tag{24}$$

In the special case of an axial mode (0, 0, n), which can be monitored properly, taking into account that $k \simeq n\pi/l_3$, we accordingly obtain

$$\frac{1}{Q_{00n}} = \sqrt{\frac{2}{n\pi}} \frac{1}{l_1 l_2} \left[(l_1 l_3 + l_2 l_3) \sqrt{\frac{l_\nu'}{l_3}} + (l_1 l_3 + l_2 l_3 + 2 l_1 l_2)(\gamma - 1) \sqrt{\frac{l_h}{l_3}} \right]. \tag{25}$$

For air at atmospheric pressure and room temperature, in a cavity having dimensions 2.7 × 3 × 4 cm, one has :

$$Q_{001} = 238 . \tag{26}$$

Making use of the Prandtl number $P_r = \mu C_p/\lambda M$, which is nearly equal to $4\,\gamma/(9\,\gamma - 5)$ (see [7] for example), and assuming the well-known properties of perfect gases,

$$\rho = \frac{PM}{RT}, \qquad c = \sqrt{\frac{\gamma RT}{M}} \qquad \text{and} \qquad C_p = \frac{\gamma}{\gamma - 1} R$$

the expression (25) yields :

$$Q_{00n} = \sqrt{\frac{n\pi}{2}} \frac{l_1 l_2 \sqrt{l_3}}{(l_1 l_3 + l_2 l_3)\sqrt{P_r} + (l_1 l_3 + l_2 l_3 + 2 l_1 l_2)(\gamma - 1)} \left[\frac{\gamma^{3/2}}{\gamma - 1} \cdot \frac{P}{\lambda} \cdot \sqrt{\frac{R}{MT}} \right]^{1/2}. \tag{27}$$

The quality factor corresponding to an axial mode $(00n)$ is expressed as a function of several parameters of the system : the serial numbers of the modes n, the dimensions of the cavity l_i, the static pressure P and the absolute temperature T in the cavity, the thermal conductivity coefficient λ, the coefficient γ (on which P_r depends) and the molecular mass M of the gas. The next section is devoted to the study of the influence of each parameter.

7. Experimental results *versus* theoretical predictions.

Some experiments were carried out with a cavity having dimensions $l_1 = 2.7$ cm, $l_2 = 3.0$ cm and $l_3 = 4.0$ cm, with a length tolerance approximately equal to 0.01 cm. The thickness of the walls was equal to 1 cm. A piezoelectric loudspeaker and two electret microphone cartridges were coupled to the cavity through very thin holes (the diameter of the holes was equal to 1 mm). The acoustic source was set at the centre of the wall $z = 0$ in order to excite only an axial mode (0, 0, n) in the oz direction (Fig. 1). A microphone was set at the centre of the opposite wall

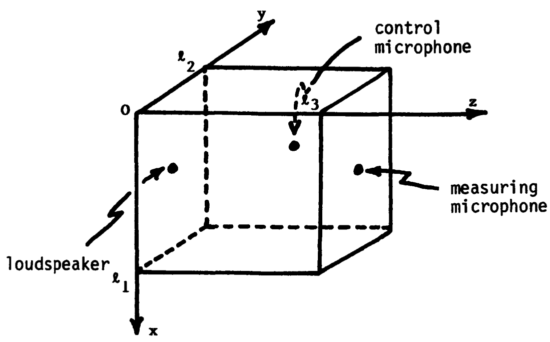

Fig. 1. — The cavity.

$z = l_3$; it supplied the signal to be analysed. The other microphone was set in the middle of the wall $y = 0$ or $y = l_2$ to make sure that the only mode generated was the one of interest.

Two classical methods were used to measure the Q-factor, one making use of equation (9) which connects Q with the characteristic decay time τ, and the other one making use of equation (15) where Q is equal to the resonance frequency divided by bandwidth between half power points. Experimentally, these two methods give nearly the same results, but these were always lower than the theoretical predictions ; the magnitude of the discrepancies is typically of the order of twenty per cent. (These discrepancies can be explained basically by the presence of the holes in the walls of the cavity.) For example, with the cavity afore-mentioned, filled up with air at atmospheric pressure and room temperature, one has :

$$Q_{001})_{\text{th}} = 238 \quad \text{and} \quad Q_{001})_{\text{ex}} = 195 \tag{28}$$

where $Q_{001})_{\text{th}}$ and $Q_{001})_{\text{ex}}$ are respectively the theoretical and experimental results for the mode (001) (nearly 4 300 Hz).

Now, let us compare the measured Q-factor with the results predicted by equation (27) ; the effects of each parameter on the behaviour of Q are successively studied.

7.1 NORMAL MODE (00n). — From the equation (28) we deduce directly $Q_{00m}/Q_{00n} = \sqrt{m/n}$.

The ratio was evaluated for the three values of the frequency, corresponding to the modes (001), (002) and (003) : the results are given in table I.

Table I. — *Ratio of quality factors for several modes.*

	Q_{002}/Q_{001}	Q_{003}/Q_{001}
Theoretical	1.41	1.73
Experimental	1.3	1.9

The theoretical (straightline) and experimental (points) results are shown in figure 2.

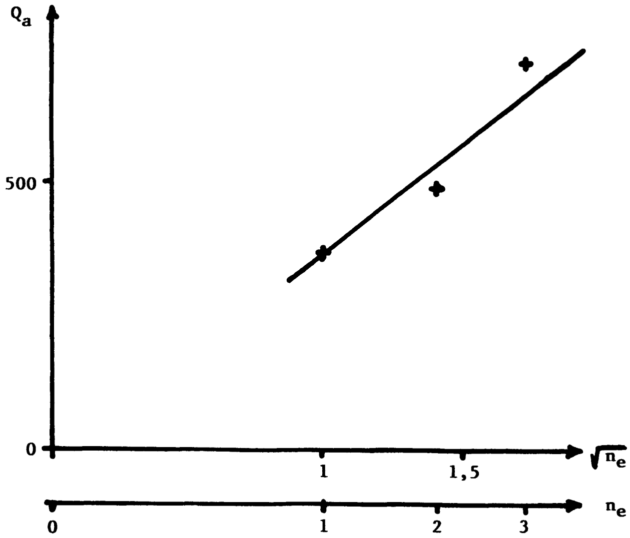

Fig. 2. — The quality factor *versus* the frequency of the first three modes. Theoretical predictions : straight line. Experimental results : points.

7.2 DIMENSIONS OF THE CAVITY. — We observe that formula (27) is a rather complicated function of the dimensions of the cavity. However, if l_1, l_2 and l_3 are nearly equal, Q is roughly proportional to the square root of the length of the edges. We have evaluated the ratio of Q_{001} for a cavity which dimensions are $10 \times 10 \times 12$ (cm) to Q_{001} for the cavity aforementioned ; the theoretical and experimental results are respectively 2.10 and 2.03.

7.3 TEMPERATURE AND STATIC PRESSURE. — The quality factor is inversely proportional to the fourth root of the absolute temperature. Its influence is rather weak, even if the effect of temperature is taken into account in determining the thermal conductivity. Therefore, an extensive experimental study of this effect is not given here.

On the other hand, the quality factor is proportional to the square root of the static pressure P. The cavity was set in an airtight cell. The curves of the quality factor as a function of the square root of the static pressure are shown in figure 3. Measurements were

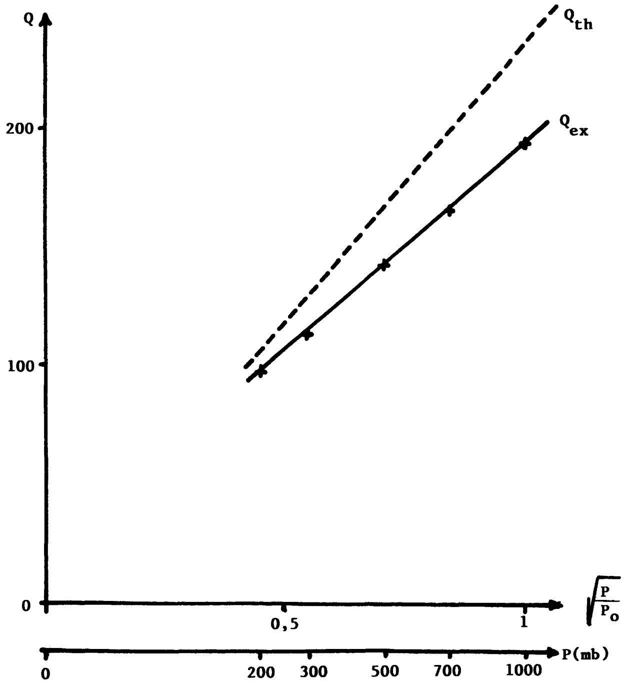

Fig. 3. — Q-factor *versus* the square root of the static pressure (millibars). Theoretical predictions : dotted line. Experimental results : points (on a straight line).

performed from 200 to 1 000 mb (the atmospheric pressure P_0). The discrepancy vanishes as the pressure decreases and the experimental points are on a straight line as predicted by the theory.

7.4 PARAMETERS OF THE GAS. — The effect of the gas inside the cavity depends on three parameters : the heat ratio γ, the coefficient of thermal conductivity λ and the molar mass M. The Q-factor is mainly proportional to $[(\gamma - 1)\,\lambda\sqrt{M}]^{-1/2}$. Experiments were carried out with the small cavity set in the airtight cell at atmospheric pressure and room temperature, for the mode (0, 0, 1), with two gases :

Air $\left(\frac{4}{5}N_2 + \frac{1}{5}O_2\right)$:

$$\gamma = 1.4\,, \quad \lambda = 2.5 \times 10^{-2}\ \text{W/mK}\,,$$
$$M = 29 \times 10^{-3}\ \text{kg/mol}$$

Freon (CF_2Cl_2) :

$$\gamma = 1.14\,, \quad \lambda = 0.85 \times 10^{-2}\ \text{W/mK}\,,$$
$$M = 121 \times 10^{-3}\ \text{kg/mol}\,.$$

The results are given in table II :

Table II. — *The quality factor for two different gases* (*mode* 001).

	Q_{air}	Q_{freon}	$Q_{\text{freon}}/Q_{\text{air}}$
Theoretical	238	563	2.36
Experimental	195	370	1.90

One may note marked discrepancies between theoretical and experimental results. The reason why the experimental value of Q is somewhat lower than the theoretical value for freon is probably because the values of the parameters γ and λ are not very reliable, especially at atmospheric pressure which is not so far from the liquefaction pressure of this gas (5 atm). At the same time, the effect of parasitic losses becomes increasingly more important as the Q-factor increases.

For the other cavity ($10 \times 10 \times 12$ cm^3), with freon at a pressure of one atmosphere, for the mode (001) (630 Hz), the measured magnitude of the Q-factor reaches a quite high value : it is equal to 750.

7.5 CONCLUDING REMARKS. — Although the agreement does not always appear to be perfect, essentially because the experimental results suffer from the effects of parasitic losses, the theory, however, seems accurate, especially in predictives the effect of different kinds of parameters on the variation of the quality factor.

In a subsequent paper, some results will be presented about a rate gyro based on the acoustic response of a small cavity, which makes use of the results given here.

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