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Jumping particle model. Modulation modes and resonant response to a periodic perturbation (*)

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Résumé. Nous discutons la formation de modes de modulation d'une version dissipative du modèle de Fermi et Ulam. La réponse résonnante du système à une perturbation périodique est étudiée expérimentalement et numériquement.

Abstract. The formation of the modulation modes within a dissipative version of the Fermi-Ulam model is discussed. The resonant response of the system to a periodic perturbation is studied both experimentally and numerically.

1. Introduction.

Like most Hamiltonian systems [1] the Fermi-Ulam model displays a very complex structure in its phase space [2]: islands of regular motions are submerged within the sea of chaotic trajectories. A general discussion of the effects which dissipation has on the trajectories of the originally conservative system can be found in our previous paper [3]. Below, we provide a more detailed study of the problem, illustrated with results of both numerical and mechanical simulations of the model.

2. Conservative origin of the stationary $M^{(n,m)}$ modes.

As shown in our previous paper, of infinitely many modes, which may be set within the conservative jumping particle model (JPM) only a few survive the effects of experimentally achievable dissipation $(k \approx 0.85)$. To simplify analysis of the problem, we limit ourselves to only a single mode $M^{(1,3)}$; however, the analysis is quite general and can be applied to any other mode. For the sake of clarity we summarize below some facts and formulae already discussed before [3].

Motion of a particle jumping on a sinusoidally vibrating surface can be in the first approximation described by the area-contracting (Zaslavskij-Rachko [4]) version of the universal mapping:

\[ v_i = kv_{i-1} + \lambda \sin \theta_i, \]
\[ \theta_{i+1} = \theta_i + v_i, \]

in which $v_{i-1}$ and $v_i$ denote the velocities of the particle just before and just after the $i$th collision, $\theta_i$ denotes the moment of time at which the collision takes place, $\lambda$ is the amplitude of the sinusoidally time dependent action of the vibrating surface and $k$ denotes the fraction of the particle's momentum which is preserved during the collision. Any solution of (1) can be conveniently presented graphically within the $(\theta_i, v_i)$ space, where $\theta_i$ denotes the phase at which the $i$th collision takes place i.e. $\theta_i = \theta_{i-1} \mod 2\pi$ (here, the period of the surface vibration is assumed to be $2\pi$). For instance, the $(\pi, 2\pi)$ point within the space represents the particular mode in which the particle jumps between centres of the consecutive surface vibration periods i.e. between those consecutive moments of time at which the surface action $\lambda \sin \theta$ changes its sign from positive to negative:

\[ \theta^{(1)} = \pi + i 2\pi . \]

Any (small) perturbation of the mode leads to one of its infinitely many modulated versions:

\[ \theta^{(1,L)} = \theta^{(1)} + a \sin \left( \frac{2\pi}{L} i + \varphi_0 \right), \]

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in which the period of the modulation depends in the
limit \( a \to 0 \) solely on the amplitude \( \lambda \) of the surface
vibration action:

\[
L(\lambda) = 2 \pi/\arccos(1 - \lambda/2).
\]  

(4)

The modulated modes are represented in the \((\dot{\theta}, \dot{v})\)
phase space by trajectories covering smooth elliptic
orbits surrounding the central point. For \( L \) incommensurate a single trajectory covers densely such an
orbit.

For \( L \) commensurate any trajectory covers only a
finite cycle of points located along such an orbit. However, in this case the whole orbit can be also
densely covered due to the arbitrariness of the phase \( \phi_0 \)
with which such a trajectory may be initiated. Generally, the trajectories located near the \((\pi, 2 \pi)\) points
are unlocked [5].

Figure 1A presents the central part of the JPM phase
space calculated at \( \lambda = 3 \). As seen in the figure, the
period \( L \) of the trajectories in the vicinity of the \((\pi, 2 \pi)\)
centre equals 3; one such trajectory is seen as a cycle
of three points (whose position along the elliptical
orbit they determine is, however, arbitrary).

For larger values of the modulation amplitude \( a \),
the period exceeds 3. Any trajectory of such a period
can be regarded as slowly rotating trajectory of
period 3. Consequently, the trace left in the phase
space after a finite number of iterations consists of
three pieces extending along the elliptic orbit along
which the rotation takes place.

Under even an infinitesimal dissipation the low
amplitude (unlocked) trajectories turn into spirals
converging to a point attractor formed at a \((\pi - \epsilon, 2 \pi)\) point, where :

\[
e(\lambda, k) = \arcsin[(1 - k) 2 \pi/\lambda].
\]  

(5)

In this way dissipation destroys most of the infinitely
many trajectories which, in principle, could be observed
in the conservative case.

As the amplitude \( \lambda \) of the surface action increases the
commensurate trajectory of period 3 leaves its
birthplace (the \((\pi, 2 \pi)\) centre) and moves away from
it. Thus, at e.g. \( \lambda = 3.5 \) it can be found, as seen in
figure 1B, far away from the centre (now surrounded
by trajectories of a period \( L < 3 \)). This time, however,
it is already locked and two equivalent cycles of fixed
points are formed-each surrounded by small islands
of trajectories (in which a secondary modulation is
imposed on the primary one). When the dissipation
is switched on (see Fig. 1C) the secondary modulations
become damped and the trajectories end in cycle
point attractors located near the previous centres
of the islands. As seen in the figure, at \( k = 0.99 \) both
cycles of islands form their own attractors, but at the experimentally achievable \( k \approx 0.85 \) they merge
into a single cycle. Figure 1D shows this case.

Results of the numerical analysis presented above
explain clearly why the \( M^{(1.3)} \) mode verified experi-
mentally in the previous paper has been observed
at a distance from the main bifurcation tree. This
seems to be quite general; any \( M^{(1.D)} \) mode observed
in the presence of finite dissipation must stay away
from the main attractor i.e. the \( M^{(1)} \) mode.

3. Resonant response of the main \( M^{(1)} \) mode to a
periodic perturbation.

If, at a moment of time one perturbs the \( M^{(1)} \) mode
(at \( \lambda_0^{(1)} < \lambda < \lambda_1^{(1)} \), it will come back to its stationary position \( \lambda \) a transient of damped phase oscil-
lations. The characteristic frequency of these oscil-
lations can be conveniently determined experimentally by a technique analogous to that used in most
kinds of stationary spectroscopy. Namely, applying an external periodic perturbation one can record the response of the system versus the frequency $v_{\text{EXT}}$ of the perturbation and determine the position $v_r$ of the maximum (resonant) response. In practice, the perturbation has been introduced via a frequency modulation of the main signal (surface vibration). Figure 2 presents a block diagram of the experimental set-up we used to perform this analysis.

Figure 3 shows storage oscilloscope images of a series of recordings taken at different points of the main branch of the bifurcation tree. As seen in the figure the frequency $v_r$ of the resonant response changes along the tree. Results of a systematic study of the dependence are collected in figure 4.

A number of qualitative observations can be made:

1) near the $\lambda_0^{(1)}$ threshold at which the M(1) ceases to exist the resonant frequency $v_r$ tends to zero, while
2) near the $\lambda_1^{(1)}$ bifurcation point it tends to $v_0/2$, where $v_0$ is the frequency of the main signal,
3) the M(1,3) mode branch starts well above that value of $\lambda$ at which $v_r = v_0/3$.

All the experimentally determined features have been confirmed by a simple numerical analysis. We performed the analysis modifying appropriately the Zaslavskij-Rachko mapping. Namely, we assumed that the phase of the surface action function is sinusoidally modulated so that equation (1) turn into:

$$v_i = kv_{i-1} + \lambda \sin \left[ \theta_i + \gamma \sin \left( 1/2 \; c\theta_i^2 \right) \right]$$

where $\gamma$ denotes the amplitude of the modulation and $c$ determines the rate at which the frequency of the modulation is swept in consecutive steps of the iteration process. In the numerical calculations, the results of which are shown also in figure 4, $\gamma = 10^{-6}$ and $c$ was chosen such that $v_{\text{EXT}} = v_0/2$ was reached after $5 \times 10^3$ iterations.

As seen in the figure, the agreement between the numerical and experimental results is excellent.
It seems worth emphasizing that virtually no parameter was left for fitting: the dissipation factor $k$ was calculated according to a procedure described previously (to provide coincidence of the theoretical $\lambda_i^{(1)}/\lambda_0^{(1)}$ and $A_i^{(1)}/A_0^{(1)}$ ratios), the scale for the $A$ variable was chosen such that $A_i^{(1)} = \lambda_i^{(1)}$, $\gamma$ and $c$ are not essential providing that $\gamma \ll 1$ and $c$ determines the frequency sweep rate comparable to that used in experiment.

A few of the last experimental points lay slightly below the theoretical curve. This is due to a finite amplitude of the perturbation signal: near the bifurcation point it leads to a non-linear response of the system.


The Fermi-Ulam model, like most Hamiltonian systems displays an infinite complexity of its phase space. Fortunately, physical realizations of the model prove to be dissipative — it is the dissipation which destroys most of the modes possible in the conservative system. As a result only a few modes are left at the experimentally achievable levels of the dissipation. Those which survive prove to be resistant to small external perturbations, reacting to them in a damped oscillatory way. The resonant frequency of the oscillatory response changes along the bifurcation tree of a mode in question.

The phenomena described above have been studied both experimentally and numerically. Quantitative agreement between the experimental and numerical results has been found.

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