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Hydrodynamic instabilities in a periodically sheared nematic liquid crystal

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1. Introduction.

Nematic liquids submitted to an elliptical or linear shear present instabilities which have been widely studied [1-6]. We have investigated the exact importance of the ellipticity or of the linearity of the shear motion. This has led us to subject a nematic to very different periodical shear figures of the general form

\[ x(t) = x_1 \cos(\omega t) + x_n \cos(n\omega t) \]
\[ y(t) = y_1 \sin(\omega t) + y_n \sin(n\omega t) \] (1)

(n is a positive integer).

We study the form of the boundary curve (which we shall call threshold) between the different flow regimes occurring in the nematic cell. Moreover, we give a description of new instabilities induced by various shears (« eight » and « flower »-shaped shears). The experimental threshold curves are compared with the theoretical model [7] (hereafter referred to as KRSS) based on an extensive use of symmetry arguments.

1. Introduction.

Nematic liquids submitted to an elliptical or linear shear present instabilities which have been widely studied [1-6]. We have investigated the exact importance of the ellipticity or of the linearity of the shear motion. This has led us to subject a nematic to very different periodical shear figures of the general form

\[ x(t) = x_1 \cos(\omega t) + x_n \cos(n\omega t) \]
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(n is a positive integer).

2. Experimental conditions.

A nematic is held by capillarity between two parallel glass plates. At rest, the nematic is homeotropically oriented by treating the inner faces of the flow cell with lecithin (i.e. the director n is perpendicular to the plates in absence of shear).

The lower plate remains steady; the shear is applied to the upper plate, producing an oscillatory motion in X and Y directions (\(^1\)) (Fig. 1). The thickness d of the cell is controlled by a micrometer acting on the lower plate. The measurement of d is performed interferometrically with an accuracy of a few microns. The parallelism of the plates is better than 0.4 \times 10^{-3} \text{ rad}.

In our experiments we used TNC [8] (mixture of phenyl and biphenylcyclohexan) because of its high chemical stability. TNC presents a nematic phase from -20 °C to +85 °C. During the measurements, the temperature of the nematic was fixed at 25 °C, with an accuracy better than 1 °C, by air circulation in a closed box around the liquid crystal cell.

The thickness d of the nematic sample lies between 50 \mu m and 150 \mu m. The amplitudes \(x_k, y_k\) (\(k = 1, n\)) of the shear motion (1) are chosen so as to verify

\(^{1}\) In the previous experiments [2, 4, 6] the X-motion was applied to the lower plate and the Y-motion to the upper plate. This geometry differs from ours by the presence of other convective terms.
Experimental apparatus: the nematic liquid is sandwiched between two circular glass plates. The upper one can be moved in directions X and Y. The thickness $d$ of the cell ranges between 50 and 150 $\mu$m, the frequency $\omega/2\pi$ of the shear motion between 50 and 300 Hz.

$x_k/d, y_k/d < 1$; the uncertainty on $x_k$ and $y_k$ does not exceed 2 $\mu$m. In order to assure that $\xi < d < \delta$ [3], we work in the frequency range $50$ Hz $< \omega < 300$ Hz (2). The frequency $\omega$ and its multiple $n\omega$ appearing in (1) are generated by the same oscillator using a divider-by-$n$ circuit. A measuring process is performed at constant frequency for increasing values of the shear amplitudes.

Several techniques are used to determine the instability patterns and thresholds:
- direct observation with a polarizing microscope
- diffraction by the grating formed by the convective pattern, using a laser beam incident at right angle on the cell.

3. Instability patterns.

In the following we shall call « eight-shaped » shears, those shear curves which are characterized by $x_n = y_1 = 0$ in equation (1) (see Fig. 2):

$$x(t) = x_1 \cos (\omega t)$$
$$y(t) = y_n \sin (n \omega t)$$

(2)

and « flower-shaped » shears, the curves of type (1) with the restriction $y_1 = s_1 x_1 (s_1 = \pm 1)$ and $y_n = x_n$ (see Fig. 3):

$$x(t) = x_1 \cos (\omega t) + x_n \cos (n \omega t)$$
$$y(t) = s_1 x_1 \sin (\omega t) + x_n \sin (n \omega t).$$

(3)

The shear motion is applied to the upper plate; if the amplitudes $x_1, y_1, x_n, y_n$ are « small », the flow in the liquid cell remains uniform (no convection) [3, 5]. By progressive increase of $x_1, y_1, x_n, y_n$ we observe the following sequence:

- Beyond a first threshold, convective rolls appear in the nematic cell (see Figs. 4a, b, d).
- Above a second threshold, this roll structure gives way to a bidimensional pattern characterized by a square or hexagonal structure (see Fig. 4c). If the amplitudes are further increased, the flow rapidly becomes turbulent.

This instability sequence has always been observed for periodical shear figures of type (1). Moreover, both thresholds have been verified to be continuously and reversibly obtainable.

4. Theoretical model.

The theoretical study KRSS based on extensive use of the symmetries of the problem shows that both thresholds should be of the form:

$$B_n \omega^2 [x_1 y_1 + nx_n y_n]^2 + C_n \omega^2 0((x_n y_n/d^2)^2) = 1$$

(4)
designs terms of order \( m \) in \( \kappa \). The index \( \alpha \) in \( B_\alpha, C_\alpha \) stands for the studied instability, whether the uniform-roll transition \( (B_\kappa, C_\kappa) \) or the roll-square threshold \( (B_{2\alpha}, C_{2\alpha}) \). \( B_\alpha, C_\alpha \) are functions depending

— on the nematic material constants (elasticity and viscosity coefficients, density)

— on \( \psi \), the direction of the symmetry breaking occurring at threshold (for instance \( \psi \) could be the angle between the normal to the rolls and the \( X \)-axis, as pointed out in KRSS)

— on \( \lambda \), the wavelength associated to the structure in the \( XY \)-plane. Note that \( \lambda \) depends on the thickness \( d \) and on \( n \), the ratio of the two frequencies appearing in (1). We do not explicit these dependences here; however this remark should be kept in mind when the data are numerically treated (see § 5).

For certain shear curves given by (1), the term in the brackets of formula (4) may be zero (this happens especially for eight-shaped shears). Thus the thresholds have to be calculated up to \( 0((x_\kappa y_\kappa/d^2)^3) \); in case of eight-shaped shears, they would be written (see KRSS, Eq. (8)):

\[
C_\alpha \omega^3 x_\alpha^4 y_\alpha^2 / d^2 = 1.
\] (5)

The validity of equation (4) has already been tested with elliptical shear experiments (i.e. with \( x_\kappa = y_\kappa = 0 \) in (1)); it reproduces the shape of the threshold with

Fig. 4. — Instability structures occurring in a sheared nematic (for the same thickness \( d = 95 \mu m \) and frequency \( \omega = 2 \pi 100 \text{ Hz} \) : a) Photograph of the roll structure appearing at threshold in an elliptically sheared nematic. b) Roll structure in an eight sheared nematic \( (n = 2) \). c) Square structure in an eight sheared nematic \( (n = 2) \). d) Roll structure in an eight sheared nematic \( (n = 4) \). e) Roll structure in a flower sheared nematic. One can observe here two roll sets with different orientations.
5. Eight-shaped shears.

Some eight-shaped functions are given on figure 2. Applying such a shear motion to the upper glass plate of the cell, we measure the amplitudes \((x_1, y_1)\) for which a transition takes place in the nematic. In the case \(n = 2\), the experimental values for both thresholds (from uniform to roll regime and from roll to square flow) are plotted on figures 5 and 6 for different frequencies and thicknesses. The experimental shape of the boundary curves is well described by the theoretical formula (5). A numerical fit to our data, using a least squares method, gives the following values for \(C_R\) and \(C_{2D}\):

\[
\begin{align*}
&\text{at } d = 95 \mu m, \quad C_R = 2.9 \times 10^{-8} \mu m^{-4} s^2 \\
&\text{and } \quad C_{2D} = 1.4 \times 10^{-8} \mu m^{-4} s^2 \\
&\text{at } d = 135 \mu m, \quad C_R = 1.6 \times 10^{-8} \mu m^{-4} s^2 \\
&\text{and } \quad C_{2D} = 8.3 \times 10^{-9} \mu m^{-4} s^2 .
\end{align*}
\]

Here we had to take into account the fact that \(C_a\) depends on the thickness \(d\) through the wavelength \(\lambda\); so we performed two different fits for \(d = 95 \mu m\) and \(d = 135 \mu m\).

In the case \(n = 4\), the threshold curves seem also to be well described by (5) (see Fig. 7) (we did not perform a systematic study in frequency and thickness). A numerical fit on the data plotted in figure 7 provides the values

\[
\begin{align*}
&\text{at } d = 95 \mu m, \quad C_R = 1.7 \times 10^{-9} \mu m^{-4} s^3 \\
&\text{and } \quad C_{2D} = 5.7 \times 10^{-10} \mu m^{-4} s^2 .
\end{align*}
\]

Comparing these results with those of elliptical shear [4, 6], we are led to make three remarks:

1) In eight-shaped shears, the instability thresholds are reached for lower amplitudes than in elliptical shear, as shown in figure 8. Moreover the direction \(\psi\) of the rolls (angle between the rolls and \(X\)-axis) is nearly along \(X\) and remains constant, in contrast with the elliptical shear where it depends on the ellipticity [2, 3, 4, 6].

2) The wavelength \(\lambda\) of the roll structure in eight-shaped sheared nematics differs notably from the elliptical shear: \(\lambda/d\) is about 1 for elliptical shear [2, 3, 4], but, in the case of eight-shaped shear, it reaches 2 \((n = 2)\) and even 4 \((n = 4)\) (Fig. 4).

3) In a sample submitted to an elliptical shear, the rolls (or the squares) occupy the whole cell. If the nematic is sheared according to an eight, the rolls (or squares) do not appear throughout the cell, but only in a given region of the liquid. The importance
of this region increases as the amplitudes \((x_1, y_n)\) are increased \(^{(3)}\). This can be explained in the following way: if the glass plates are not exactly parallel, the thickness \(d\) varies slightly through the sample. As \(d\) does not appear in (4) \(^{(4)}\), the rolls (the squares) appear uniformly in the whole sample under elliptical shear. But for eight-shaped shearing, the thickness \(d\) is explicitly present in (5). Thus, if \(d\) changes, some regions may remain « subcritical » whereas other places in the liquid have already reached the threshold value for the applied shear amplitudes.

Let us finally notice that by continuously varying the ratio \(n\) between the two applied frequencies \(\omega\) and \(\omega_0\) in the case of eight-shaped shear, we only observe roll or square instabilities when \(n\) is equal to the ratio of two small integers.

6. Flower-shaped shears.

The measurements were made by keeping \(X_4 = Y_4\) constant and by increasing \(y_1 = s_1x_1\) up to the threshold value (\(\omega\) is fixed during the measuring process). The nematic submitted to a flower-shaped shear presents exactly the same instability sequence and shape as for elliptical shears: uniform regime, roll structure (Fig. 4e), square pattern and, finally, turbulent flow. Roughly the thresholds have the shape given on figure 9. The frequency and amplitude dependences are qualitatively described by theoretical formula (4). In the liquid cell we observed many roll sets: the nematic seems to have preserved a degree of freedom for \(\psi\). This could be explained by noticing that, in a flower-shaped shear curve (Fig. 3), the directions determined by \(\psi\) and \(\psi + \frac{2k\pi}{n - s_1}\) (with \(k = 1, 2, ..., n - s_1\)) are fully equivalent from a symmetry point of view.

However, our measurements for this shear geometry are only qualitative; in practice we did not dispose of sufficient precision to guarantee the stiffness of the relative phases appearing in (1) (for instance, no phase shift between \(\cos(\omega t)\) and \(\cos(\omega_0 t)\), ...); so our results should be only considered as qualitative.

7. Conclusions.

Our experiments tend to show that instabilities presenting the same aspect as obtained for simple shear curves (elliptical or linear) may be produced by applying shear figures belonging to a wider class of curves (eight, flowers, ...).

\(^{(3)}\) See for instance figure 4e: nearly the whole sample presents a square pattern, only the upper left corner still shows a roll-shaped instability.

\(^{(4)}\) In case of elliptical shear it has been experimentally verified that \(B_a\) is independent of \(\lambda\) and \(d\) to a first approximation [6].
Fig. 9. — Threshold curve for a flower-shaped shear. The shear is a flower with \( n = 4 \). The amplitudes \( x_4, y_4 \) are kept constant at \( x_4 = y_4 = 11 \text{ \mu m} \) during the experiment and \( x_1, y_1 \) are increased up to the threshold value by keeping \( y_1 = s_1 x_1 \) (\( s_1 = \pm 1 \)). We have plotted the shear amplitude \( x_1 \) at threshold versus the frequency \( \omega/2 \pi \). Curve A, A': (●) uniform-roll threshold; Curve B, B': (■) roll-square threshold.

Moreover our measurements reveal some striking differences between instabilities occurring under elliptical or eight-shaped shears:

- The roll and square instabilities occur for lower shear amplitudes under eight-shaped shears than under elliptical shears.
- The threshold shapes are completely different and so are the rolls.
- The threshold dependence on the thickness \( d \) of the sample is not the same under elliptic and eight-shaped shear.

This leads us to suggest that the instability mechanisms involved in those geometries differ (this view is supported by the importance given, in the instability motor under elliptical shear, to the constant phase lag between the velocity \( v \) and the director \( n \) of a fluid particle [2]; in eight-shaped shears, the relative phase of \( n \) and \( v \) no longer remains constant).

On the other hand, the elliptical shear and flower-shaped shear instabilities seem similar.

Another interesting observation concerns the « degeneracy » of the roll direction in a flower-shaped shear; many roll sets with different orientations may coexist in the nematic cell. This can be compared to the degeneracy occurring in elliptical shear: near to the circular geometry, all roll directions become equally probable; nevertheless in the later case rolls do not appear with different orientations (the structure being rapidly forced to a square pattern [6]).

Comparison between our results and the predictions of the theoretical model KRSS seems very satisfactory. It is noticeable that a method based solely upon symmetry considerations predicts the correct threshold shape for such a wide class of shear figures, without any reference to the instability mechanism.

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