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1. Introduction.
Before the first operation of a Storage Ring Free Electron Laser (S.R.L.), the S.R.L. was thought to be pseudo-continuous except for some transients at laser turn-on. It was most commonly assumed that the laser would reproduce the electron bunch time structure, the laser pulse being somewhat narrower than the electron pulses (~ 1 ns long) but coming with constant height at the Radio Frequency (several megahertz) of the storage ring. The first experimental investigation of the S.R.L. showed that this was not the case (1); there was a long time scale structure on the scale of milliseconds. In section 2 of this paper, I derive a set of two simple equations modelling the S.R.L. macro-temporal behaviour (as opposed to the micro-temporal structure at the radio frequency). In section 3, I describe a semi-analytical solution of these equations predicting the pulsed nature of the S.R.L. In section 4, I consider the possible methods to maximize the peak power and the implications for the average power. In the last section, I discuss in more detail the conditions for the validity of these equations.

2. Theoretical development of the main equations.
It has been predicted for a long time that S.R.L. would saturate due to the inhomogeneous broadening and the accompanying electron bunch lengthening due to the laser-induced energy spread (2).

Let’s define:

$$\Sigma = \sigma_l^2 - \sigma_l^2 \text{ (laser off)}$$ (1)

where $$\sigma_l$$ stands for the RMS relative energy spread. In the following I shall assume perfect cavity tuning, e.g., that the mirror distance and the ring revolution period are such that light and electron pulses are perfectly synchronized from turn to turn. In this case, since a higher gain exists on the centre of the electron bunch than on the tail, the laser pulse width should be significantly narrower than the electron bunch length. The gain responsible for the laser growth will simply be the longitudinal peak gain which I define as $$g_0 f(\Sigma)$$, where $$g_0$$ is the peak laser-off gain and $$f$$ a function describing the gain depression due to the laser induced energy spread satisfying $$f(0) = 1$$. Note
that I include the gain depression due to the electron bunch lengthening in the definition of \( f \). I shall not deal with the laser longitudinal profile evolution but assume this profile to be much shorter than the electron bunch length but not short enough to introduce slippage problems (3), so that the laser pulse is simply amplified as a whole. Defining \( I' \) as the laser pulse energy integrated over the laser micro-pulse length, and neglecting spontaneous emission, one can write the equation describing the evolution of \( I' \):

\[
\frac{dI'}{dt} = \frac{\tau_o f(\Sigma)}{T} - p I'
\]  

(2)

where \( p \) is the total cavity losses between two amplifications occurring every \( T \) seconds, and \( t \) is the time.

In (2), I have assumed that the average gain \( g_0 f(\Sigma) \) only depends on the variance of the energy distribution function; this assumption will be discussed in more detail in section 4.

Not considering anomalous bunch lengthening the equation describing the growth of \( \Sigma \) is:

\[
\frac{d\Sigma_i}{dt} = -\frac{2}{\tau_s} \Sigma_i + h(\Sigma_i) I'.
\]  

(3)

The term \(-\frac{2}{\tau_s} \Sigma\) describes the energy spread decay due to synchrotron radiation (4). The term \( h(\Sigma_i) I'\) describes the «heating» by the laser field. \( h(\Sigma) \) goes to zero for large \( \Sigma \) for two reasons: the larger \( \Sigma \), the fewer electrons are resonant with the laser, and secondly the larger \( \Sigma \), the longer the electron bunch and fewer electrons fall within the envelope of the short laser pulse.

At this point I shall make a new assumption, namely that \( \Sigma \) is close to some stationary values \( \Sigma_0 \) being defined by (2) and (3) setting \( dI'/dt = 0 \) and \( d\Sigma_i/dt = 0 \). I will expand \( f \) about the stationary value \( \Sigma_0 \):

\[
f(\Sigma) = f(\Sigma_0) + (\Sigma - \Sigma_0) f'(\Sigma_0).
\]  

(4)

The function \( h \) will not be expanded (the validity of this will be discussed in section 4). I will also define \( \Sigma, \tau_0 \) and \( I \) by:

\[
\Sigma = \frac{\Sigma}{\Sigma_0}, \quad \tau_0 = \frac{T}{g_0 \Sigma_0 f'(\Sigma_0)}, \quad I = \frac{\tau_s h(\Sigma_0)}{2 \Sigma_0} I'.
\]  

(5)  

(6)  

(7)

Note that \( f'(\Sigma_0) \) is always negative implying \( \tau_0 \) to be positive. Using the new variables defined by (5), (6) and (7), (2) and (3) can be reduced to a simple dimensionless form:

\[
\frac{dI}{dt} = \frac{1 - \Sigma}{\tau_0} I
\]  

(8)

\[
\frac{d\Sigma}{dt} = 2 \frac{I - \Sigma}{\tau_s}
\]  

(9)

\( I \) and \( \Sigma \) are the dimensionless laser intensity and additional squared energy spread; in the following I will simply call them intensity or power and energy spread. \( \tau_0 \) is obviously a characteristic time. In the case the laser is not too far above threshold, \( \tau_0 = \frac{T}{g_0 - p} \) which is the laser rise time starting from the laser-off condition (to be distinguished from slower laser rise time defined in the next section). In most cases \( f' > 0 \) implying \( \tau_0 > \frac{T}{g_0 - p} \).

I note that the set of non linear equations (8) and (9) is very simple, depending only on the dimensionless parameter \( \tau_0/\tau_s \). These equations describe the SRL intensity around the stationary state \( \Sigma = 1 \). The solutions to these equations are discussed in the next section.

3. Solution of the basic equations.

The key questions for this system are: is the equilibrium state \( \Sigma = 1 \) stable? if so, what are the characteristics of the relaxation to equilibrium?

The usual way to study the stability is to look at the evolution of small deviations from the equilibrium state:

\[
I = 1 + i
\]  

(10)

\[
\Sigma = 1 + \sigma
\]  

(11)

with \( i \ll 1 \) and \( \sigma \ll 1 \).

From equations (8) and (9) one calculates:

\[
\frac{d^2 \sigma}{dt^2} + \frac{2}{\tau_s} \frac{d\sigma}{dt} + \frac{2}{\tau_s \tau_0} \sigma = 0
\]  

(12)

In the Orsay experiment (5) \( \tau_s \approx 200 \text{ ms} \) and \( \tau_0 \approx 0.05 \text{ ms} \) implying \( \tau_0/\tau_s \ll 1 \). This last inequality is a general feature of the SRL. The general solution of (12) is:

\[
\sigma = \sigma_0 e^{-t/\tau_0} \cos \left( 2 \pi \frac{t - \tau_0}{T_R} \right)
\]  

(13)

with:

\[
T_R = 2 \pi \sqrt{\frac{\tau_s \tau_0}{2}}.
\]  

(14)

The equilibrium state is stable but the relaxation is very slow and the laser will display a resonant response to any excitation of period \( T_R \). This behaviour was readily observed in the Orsay experiment where one calculates \( T_R = 14 \text{ ms} \) [5].

Now let's consider the relaxation from a state far from equilibrium. In this case, I have solved (8) and (9) numerically using a small computer. Results for \( \Sigma \) and \( I \) are given in figure 1. Initial conditions are \( \Sigma = 0 \) and \( I = 10^{-3}, \tau_0/\tau_s = 0.025 \). The evolution is followed in figure 1 for a total of 5 synchrotron damping times \( 5 \tau_s \). A new feature appears in these curves: the relaxation is not simple but occurs in a pulsed manner,
each pulse is characterized by a sudden increase in energy spread which then relaxes with the time constant $\tau_0/2$. Such pulsed laser start-up has already been reported [6]. One can give an analytical expression for each pulse by neglecting $2r/T_\Sigma$ in equation (9), obtaining:

$$\Sigma = 1 + C \frac{Ct}{2\tau_0}$$

and

$$I = \frac{C^2\tau_s}{2\tau_0 \cosh^2 \frac{Ct}{2\tau_0}}$$

These equations describe a family of symmetrical laser pulses, each member being defined by the free parameter $C$ ranging from 0 to 1. The physical meaning of $C$ is indicated by equation (15):

$$C = 1 - \tau_s/\Sigma_0$$

where $\Sigma_0$ is the energy spread at the beginning of the pulse. Each pulse has the rise time $\tau_s = \tau_0/C = \tau_0/(1 - \Sigma_0)$.

The main conclusion of this section is that, because the laser rise time is very small compared to the energy spread damping time, the laser intensity function $I(t)$ will be very sensitive to any energy spread fluctuations at frequencies close to $2\pi\sqrt{\tau_0/\tau_s}$. A very "low-noise" energy spread and electron bunch length are required to attain continuous operation. In fact, due to the inherent noisiness of energy spread and bunch length, the laser tends to be naturally pulsed as observed at Orsay [1, 5].

4. Increasing the peak power.

It has been seen in section 2 and experimentally verified [1, 5] that, alone, the SRL tends to display a pseudo-periodic behaviour whose spectrum peaks at the frequency $T_R = 2\pi\sqrt{\tau_s\tau_0}$ as characteristic of a weakly damped oscillator. In principle, this situation could be improved in two ways. The first would be to make the laser run c.w. (except for the micro-temporal structure), by controlling the laser gain with a feedback loop. Another possibility, which seems much simpler, is to secure stable periodic pulsed operation by imposing a little external periodic perturbation bigger than any natural internal perturbation of same frequency. This process should be especially efficient close to the resonance frequency $T_R$.

The gain can be modulated by several means to achieve a c.w. or a periodic pulsed laser as discussed above:

a) Transverse translation of the electron beam to modulate the gain via misalignment of the beam with the optical cavity axis. For the Orsay SRL, a 0.5 mm displacement is sufficient to decrease the gain by a factor of 4.

b) Alteration of the storage ring revolution frequency to shift the synchronism of the electron and optical pulses in the cavity. From the Orsay results, 100 Hz of frequency modulation is sufficient to stop the laser.

c) Modulation of the cavity length via a P.Z.T. device. The effect is equivalent to the frequency modulation except for the electron beam which suffers no perturbation. A 50 $\mu$m displacement is sufficient to stop the laser. The two first methods have been used successfully at Orsay to induce stable periodic pulsed laser operation [5].

Continuing this analysis, it can be seen that both the peak power and average power are changed when one externally modulates the laser gain. This can be anticipated from (8) and (9) in the following manner. Once the external modulation is established to periodically pulse the laser, assuming that the modulation maintains the maximum gain during the laser
pulse duration (typically much shorter than the modulation period), one can use \((15)\) and \((16)\) to calculate the pulse power. The evolution of \(E\) between two successive pulses is just the relaxation obtained from \((9)\) with \(I = 0\). Defining \(\Delta\Sigma\) as the incremental energy spread induced by each pulse, and \(T_o\) as the period of the external modulation, \(\Delta\Sigma\) satisfies (see Fig. 2):

\[
(1 + \frac{\Delta\Sigma}{2}) e^{-2\tau_o/\tau_s} = \left( 1 - \frac{\Delta\Sigma}{2} \right) \tag{17}
\]

implying:

\[
\Delta\Sigma = 2 \tanh \left( \frac{T_o}{\tau_s} \right) \tag{18}
\]

From \((15), (16)\) and \((9)\), one calculates the peak power \(I_p\) and the average power \(I_a\):

\[
I_p = \frac{\tau_s}{2\tau_o} \tanh^2 \left( \frac{T_o}{\tau_s} \right) \tag{19}
\]

\[
I_a = \frac{\tau_s}{T_o} \tanh \left( \frac{T_o}{\tau_s} \right) \tag{20}
\]

There are two interesting extreme cases:

a) \(T_o \ll \tau_s\). The \(I_a \approx 1\), from \((19)\) and \(I_p\) goes to zero which means that the laser is no longer pulsed. Equations \((15)\) and \((16)\), and therefore \((19)\) are not valid in this case, and the laser runs c.w. I note that such a high frequency modulation could be an alternative to the feedback method to achieve a c.w. laser.

b) \(T_o \gg \tau_s\). Then \(I_p = \frac{\tau_s}{2\tau_o}\) and \(I_a = \frac{\tau_s}{T_o} \ll 1\).

The laser is highly pulsed and has lost some average power. In that case, the condition \(\Delta\Sigma \ll 1\) fails but, according to section 5, the conclusions remain the same. The peak power in this case is at a maximum and the pulses are clearly non-symmetric.

At the intermediate point, \(\tau_s = T_o\), I find that \(I_p = \tau_s/4\tau_o\) and \(I_a = 0.7\). In this case, the laser operates at nearly the highest possible peak power without any significant loss of average power.

From this analysis, it is apparent that it is possible to have a pulsed SRL with average power in agreement with the so-called « Renieri's » limit for average power [7], but a peak power \(\tau_s/4\tau_o\) higher. This factor reaches 1 000 for the Orsay experiment, although it has not yet been possible to go far above 100[5]. The present limitations of the Orsay experiment are due to anomalous bunch lengthening effects which complicate the attainment of the low frequency modulation required for these large enhancements.

One consequence of the high peak power attainable in the pulsed mode is the possibility of overbunching and therefore the possibility of enhanced harmonic generation and frequency up-conversion [8] directly within the SRL. For example, taking the Orsay experiment with the parameters \(\tau_s = 200\) ms, \(\tau_o = 0.05\) ms, micropulse length = 100 ps (assumed value, not yet directly measured), and an average intra-cavity power = 2.5 W, one calculates the dimensionless peak field [9] \(a = 0.1\) which represents the onset of over-bunching in the optical klystron used in the experiment [10].

5. Modifications to the basic equations.

The pulses described by \((15)\) and \((16)\) are correct as long as \(\Sigma\) stays close to 1 which is the validity condition for \((8)\) and \((9)\). What happens when the energy spread violates this condition? Maximum optical pulse amplitude corresponds (from Eq. (2)) to \(\Sigma \approx \Sigma_{av}\) such that \(g_0f(\Sigma_{av}) = p\). Let's define:

\[
a = (\Sigma - \Sigma_{av}) > 0. \tag{21}
\]

One can write (2):

\[
\frac{dI}{dt} = \frac{I'}{\tau(a)} \tag{22}
\]

with:

\[
\tau(a) = \frac{T}{g_0 \left( a f'(\Sigma_{av}) + \frac{a^2}{2} f''(\Sigma_{av}) \right)} \tag{23}
\]

In most experimental cases \(f' < 0\) and \(f'' > 0\).

Therefore:

\[
|\tau(a)| > |\tau(-a)|. \tag{24}
\]

The shape of the laser pulse is therefore non symmetric, the decay time is longer than the rise time.

In section 1, it was assumed that the functions \(f\) and \(h\) only depend on \(\Sigma\), and not on any higher moment of the energy distribution function. This condition is valid if the gain curve versus electron average energy is only slightly inhomogeneously broadened. A more general treatment would require solution of the Fokker-Planck equation for this distribution function [10].

In section 1, the function \(h\) was not expanded about \(\Sigma_{av}\). Carrying out this expansion, \((9)\) is replaced by:

\[
\frac{d\Sigma}{dt} = -\frac{2}{\tau_s} \Sigma - \frac{2}{\tau_s} I [1 - \lambda (1 - \Sigma)] \tag{24}
\]

with:

\[
\lambda = \frac{\Sigma_{av} h'(\Sigma_{av})}{h(\Sigma_{av})} \tag{25}
\]

Equation \((24)\) obviously simplifies to \((9)\) in the case \(|1 - \Sigma| \ll 1\) which is already the validity condition for \((8)\) and \((9)\). Moreover, in most of experimental situations \(\lambda \ll 1\).

Finally, I have not considered the effect of spontaneous emission. Although its level is far below the average laser level, it sets a lower limit below which the intensity \(I\) cannot decay. In fact, the effect should be
small since this radiation only modifies the tail of the laser pulses.
In summary, the major limitations of this linearized analysis come from the condition \(|1 - \Sigma| \ll 1\). But, note that this condition can be fulfilled even when the laser is highly pulsed.

6. Conclusion.

It is shown that because of the fast laser rise time compared to the energy damping time, the storage ring free electron laser is very sensitive to fluctuations in the energy spread or bunch length close to a characteristic resonant frequency. As a consequence, it will be difficult to achieve continuous laser operation but easy to attain pulsed laser operation with peak power much larger than the so-called « Renieri's limit » valid for the average power. As a consequence of the high peak power of these pulses over-bunching may occur in a SRL.

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