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Non metal-metal transition in Bruggeman optical theory for inhomogeneous media

A. Bittar (*), S. Berthier (**) and J. Lafait

Laboratoire d'Optique des Solides (1),
Université Pierre et Marie Curie, 4, place Jussieu, 75230 Paris Cedex 05, France

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Résumé. — La théorie optique de champ moyen de Bruggeman définissant la constante diélectrique effective \( \varepsilon_e(\omega) \) d'un milieu inhomogène est analysée en terme de transition non métal-métal pour des composés métal-dieléctrique. L'influence des paramètres caractéristiques des constituants du matériau (paramètres des électrons libres et polarisation due aux états liés) sur la partie réelle et la partie imaginaire de \( \varepsilon_e(\omega) \) est étudiée en détails du visible à l'infrarouge dans tout le domaine de concentration volumique en métal. Une interprétation physique est proposée en terme de polarisation due aux états liés, de polarisation interfaciale et de contribution de Drude des électrons de conduction. Les résultats ainsi obtenus sont comparés aux prédictions d'autres théories de milieu effectif : Maxwell Garnett modifiée par Cohen et al., Ping Sheng.

Abstract. — The Bruggeman optical effective medium theory of the dielectric permeability \( \varepsilon_e(\omega) \) of inhomogeneous media is analysed for metal-dielectric compounds in relation to the non metal-metal transition. The influence of the constituent-material parameters (free-electron parameters and bound-state polarization) on both the real and imaginary parts of \( \varepsilon_e(\omega) \) is investigated in detail for the full range of metal filling factors and for optical to far-infrared wavelengths. A physical interpretation is formulated in terms of bound-state polarization, interfacial polarization and Drude conduction-electrons contributions. The results thus obtained are eventually compared to those of other effective medium theories : Maxwell Garnett modified by Cohen et al., Ping Sheng.

1. Introduction.

A scientific approach to the optical properties of inhomogeneous media was begun in the second half of the 19th century. In recent years the optical and electrical properties of these media have been extensively used in many disciplines including physics, chemistry, biology, medicine, astronomy, geology and meteorology. The main application to physics concern presently the development of selective surfaces for the enhancement of the efficiency of the photothermal conversion of solar energy and, also, surface enhanced Raman scattering (SERS).

The aim of the effective medium theories developed since 1850 is to derive an effective complex dielectric constant of these media from the knowledge of the dielectric constant of their components and their volume filling factors. The effective medium approach supposes that each point in the inhomogeneous medium (with macroscopic uniformity) can be associated with a macroscopic dielectric permeability. This condition is fulfilled in the framework of the quasistatic approximation by assuming that the particle radii are much smaller than the wavelength of the light (the metal-dielectric composites verify this condition). The Maxwell Garnett (1904) — MG — [7], Bruggeman (1935) — BR — [1], and Ping Sheng (1980) — PS — [9] theories are relevant to this approach. However they differ in the treatment of the two components in the composite system. The microstructure described by MG consists in grains of one component embedded in the matrix of the other component, whereas in BR and PS the two components are treated in an equivalent manner with different statistics (Fig. 1). There results a self consistent treatment only for the two last theories, predicting a percolation threshold in the physical properties, whereas MG theory only predicts an anomalous absorption.

A case of practical interest arises in cermet materials where one of the constituents is described by a real, frequency-independent permeability \( \varepsilon_c \), while the...
other one has a Drude metal-like dielectric permeability given by:

\[ \varepsilon_e = \varepsilon_m = \varepsilon_{1m} + i\varepsilon_{2m} = P - \frac{\omega_p^2}{\omega(\omega + i/\tau)}. \tag{1} \]

Here \( \omega_p \) is the vacuum plasma frequency of the free electrons, \( \tau \) their characteristic relaxation time and \( P \) the equivalent dielectric constant of the medium in which they are found (polarization term due to bound-state contribution). A similar case has previously been considered by several authors [3-6], namely that for which \( P \) is set equal to unity (corresponding to vacuum), the parameter of interest being then the imaginary part of the effective dielectric permeability \( \varepsilon_{2e} \).

It is the aim of this paper to highlight the influence of the constituent-material parameters (free-electron parameters and bound-state polarization) on both the real and imaginary parts of the dielectric constant of cermet materials, as predicted by BR, for the full range of metal filling factors (especially around the percolation threshold) and for optical to far-infrared wavelengths. The results thus obtained are compared to those due to the Maxwell Garnett theory [7, 8], as well as to the more recently proposed theory by Ping Sheng [9], all three having been applied to cermet materials [8, 10-14].

The model cermet used throughout is that typical of a noble metal-insulator composite. The Drude parameters for metallic inclusions are those of crystalline gold [15], interband contribution not included: \( P = 6.5, h\omega_p = 9.2 \text{ eV}, h/\tau = 0.06 \text{ eV}, \) and the insulator dielectric constant has been set equal to \( \varepsilon_b = 2.82 \), a value representative of several commonly used dielectric materials. We have shown elsewhere [14] that the effective medium theories give a proper account of the dielectric constant of Au-MgO cerments.

2. Infrared behaviour.

In the infrared region the usual starting point is an approximation to the quadratic equation 3 recast in terms of the ratio \( R = \varepsilon_e/\varepsilon_m \) of the effective medium to metal inclusions permeabilities, i.e.:

\[ R^2 + \frac{g - q}{1 - g} R + \frac{\varepsilon_e}{\varepsilon_m} \frac{R(g + q - 1)}{1 - g} - \frac{g}{1 - g} \frac{\varepsilon_e}{\varepsilon_m} = 0. \tag{4} \]

It is seen from the last two terms in the equation that the customary approximation, \( \varepsilon_e \propto \varepsilon_m \), true only for \( q \rightarrow 1 \), neglects complex quantities not necessarily small or unimportant. A more accurate approach is to solve either explicitly or numerically for \( \varepsilon_{1e} \) and \( \varepsilon_{2e} \) at each wavelength. This is the procedure adopted here, as it also has the advantage of showing the wavelength-dependent features in \( \varepsilon_{1e} \) and \( \varepsilon_{2e} \).

2.1 Non metal-metal transition. — Figure 3a and b shows the normalized effective permeabilities \( R_1 = \varepsilon_{1e}/\varepsilon_{1m} \) and \( R_2 = \varepsilon_{2e}/\varepsilon_{2m} \) as a function of metal filling fraction \( q \), for wavelengths between 25 \( \mu \text{m} \) and 50 \( \mu \text{m} \).

The general far-infrared behaviour of the cermet to metal ratio is largely wavelength independent for all but a range of values of \( q \) close to \( g \) (equal to 1/3 in this case). In the limits of low and high \( q \), both \( R_1 \) and \( R_2 \) tend towards straight lines. Deviations from straight lines which occur around \( q = g \) are more pronounced in \( R_1 \) than in \( R_2 \).

As \( \lambda \rightarrow \infty \), for \( q \gg g \), this leads to the already well-known D.C. behaviour predicted by E.M.T. [6].
optical wavelengths, the Bruggeman Theory leads to three distinct regions in \( \epsilon_1 \) and \( \epsilon_2 \), which may be defined as follows:

(i) Well below \( q = g \), \( \epsilon_1 \) is positive, \( R_1 \) is negative and slowly increasing, while \( \epsilon_2 \) and \( R_2 \) are close to zero. This is the non-metal region.

(ii) Well above \( q = g \), both \( \epsilon_1 \) and \( \epsilon_2 \) tend asymptotically towards \( \epsilon_{1m} \) and \( \epsilon_{2m} \). This is the metallic region.

(iii) Around \( q = g \). In this range and starting below \( q = g \), \( \epsilon_2 \) increases rapidly and \( \epsilon_1 \) changes sign from positive to negative. This will be called the optical percolation region, as opposed to the \( \epsilon_2 \) defined D.C. conductivity threshold localized at \( q = g \) [2].
2.2 CRITICAL EXponent. — This latter region corresponds to that for which scaling theory [16] and resistor-network simulations [17] predict a single power-law relation between effective conductivity $\sigma_e$ and filling fraction $q$:

$$\sigma_e \sim (q - q_p)^t$$  \hspace{1cm} (5)

where $q_p$ is the percolation volume fraction and $t$ the critical exponent. Since the optical conductivity may be related to the imaginary part of the dielectric constant, it could be expected that $\varepsilon_{2e}$ would also display a power-law relationship of the type shown in equation 5. It is generally assumed [16, 18] that the E.M.T. predicts a $q_p$ equal to the depolarization factor $(g)$ and, a critical exponent of unity. At optical frequencies, the calculated $\varepsilon_{2e}$ shown in figure 3b does not confirm this; moreover, a strong wavelength dependence appears even for far-infrared wavelengths (100 $\mu$m) which, only in the limit of $\lambda \rightarrow \infty$, tends to the assumed behaviour. Critical exponent and critical volume-fraction calculations at discrete wavelengths yield different values of both $q_p$ and $t$. It is concluded that the $\varepsilon_{2e}$ values do not follow a simple power-law relation. Optical absorption is present over a wide wavelength range before and near the D.C. percolation volume fraction $q_p = g$. The nature of this absorption, which is believed to be independent of the free-electron conduction in the infinite cluster, will be discussed in section 4.

2.3 DRUDE FIT. — To give an explicit account of the metallic behaviour of cermets in the infrared for the metallic region $q > g$ suggested by figure 3, the effective $\varepsilon_{1e}$ and $\varepsilon_{2e}$ values, between 25 $\mu$m and 50 $\mu$m have been simultaneously least-square-fitted to the Drude formula equation 1. For each $q$, a set of effective parameters $\omega_{pe}$, $1/\tau_e$ and $P_e$ are obtained which represent the plasma frequency, the inverse relaxation time and a polarization term (which will be further detailed below) for the cermet in question. The fitting procedure has also been applied over the dielectric range $0 \leq q \leq g$, where $\varepsilon_{2e} \sim 0$. This then reduces the Drude formula to the purely real equation $\varepsilon_{1e} \approx P_e$. In practice, this procedure yields good fits for all values of $q$, except in a narrow range, around $q = g$, situated well inside the percolation region (iii). In the infrared region the cermets are well described by either a free-electron-like or a purely real dielectric permeability, according to whether $q$ is greater or smaller than $g$ ($q \neq g$).

The fitted value of $1/\tau_e$ is found to be constant, independent of either $q$ or $g$ and equal to the value assumed for the metallic inclusions. Size dependence effects can thus be taken into account essentially via a variable relaxation time in the metallic inclusions.

On the contrary, the fitted values of $\omega_{pe}$ and $P_e$ depend both on $q$ and $g$. The resulting functions $\omega_{pe}(q)$ and $P_e(q)$ for three values of $g$ are shown in figure 4. The cermet plasma frequency decreases parabolically from the constituent metal value at $q = 1$, to zero near $q = g$, and remains at this value as $q$ tends to zero (pure dielectric). The relationship between $\omega_{pe}(q)$ and $\omega_p$ is:

$$\omega_{pe}^2 = \frac{q - g}{1 - g} \omega_p^2 \quad \text{for} \quad q > g \quad \text{(6a)}$$

$$\omega_{pe} \approx 0 \quad \text{for} \quad q < g . \quad \text{(6b)}$$

It is to be noted that, in the hypothesis of a constant relaxation time for the cermets, the parabolic relation (6a) may be directly inferred from figure 3b. Equation (6) may be written in terms of the number of « optically active » free electrons per unit volume of cermet, $n_e$, and the corresponding parameters for the constituent metal $n$:

$$n_e(q) = \frac{q - g}{1 - g} n \quad \text{for} \quad q > g \quad \text{(7)}$$

$$n_e(q) \approx 0 \quad \text{for} \quad q < g .$$

The $q$ variation in $\omega_{pe}$ may be thus totally attributed to a change in the effective number of free electrons in the cermet, the optical mass being assumed constant.

The transition from dielectric to metallic behaviour around $q = g$ is not as abrupt as suggested by equations (6) or (7); rather it occurs smoothly over the width of the A.C. percolation region, $\varepsilon_{2e}$ being consistently above the value predicted by equation (6) (dashed curves in figure 3b). The $\varepsilon_{2e}$ values calculated by E.M.T. for this region are directly attributable to the resonance usually confined to the visible spectral region, which extends to $\lambda \rightarrow \infty$ for values of $q$ close to $g$. As $q$ approaches $g$ from either below or above, $\varepsilon_{2e}(q)$ reflects more and more the contribution of the broadening resonance over the insulator or metal.
value. The same phenomenon is the cause of the wavelength-dependence observed in $\varepsilon_{2e}$ in the A.C. percolation region; the larger wavelengths see less of the resonance contribution for a given value of $q$. This is illustrated in figure 3b for wavelengths between 25 and 50 µm.

2.4 THE CERMET POLARIZATION TERM. — The cermet polarization term $P_e(q)$ for $0 \leq q \leq 1$, determined from the fitting procedure, is shown in figure 4 for three values of $q$.

From a value equal to $\varepsilon_e$ at $q = 0$ (pure dielectric), $P_e$ increases rapidly over the cermet dielectric behaviour region, reaching a finite maximum value $P_{e,max}$ at $q = g$. $P_{e,max}$ is found to be proportional to $[(g/(1-g))]^{1/2}$. This may be seen by setting $q = g$ in equation (2). Beyond this, $P_e$ decreases equally sharply and approaches the bulk metal polarization term $P$ for high $q$ values. At both dielectric and metal limits, BR theory reproduces the constituent polarizations correctly. This general behaviour, a sharp peak centred around $q = g$ and the reaching of constituent values at the limits of low and high $q$ is reproduced for all values of $q$, $P$, $\omega_p$ and $1/\tau$.

In the pure metal considered, the polarization term $P$ is the contribution of bound-states to the real part of the dielectric constant at $\lambda = \infty$. Similarly for the dielectric material, $\varepsilon_e$ is due entirely to the contribution of bound states. Therefore, in the cermet, the polarization $P_{ec}$ can also be assumed as arising from bound states alone and may be calculated from the real valued terms, $P$ and $\varepsilon_e$, by straightforward use of the E.M.T. expression:

$$q \frac{P - P_{ec}}{gP + (1-g)P_{ec}} + (1-q) \frac{\varepsilon_e - P_{ec}}{g\varepsilon_e + (1-g)P_{ec}} = 0$$

(any other mean-field theory will lead to the same result).

The function $P_{ec}(q)$ then represents the bound-state contribution at $\lambda = \infty$ to the dielectric constant of the medium in which the cermet electrons travel. This function (Fig. 5) is monotonic in $q$ and is equal to $\varepsilon_e$ or $P$ at $q = 0$ or 1 respectively. One can see that it is markedly different from $P_e$, the total cermet polarization, since the constituent bound-state contributions cannot by themselves account for the full polarization permeability of the cermet.

By application of the additivity principle to the polarizations (on which BR theory is based) a polarization term $P_i$, equal to the difference between $P_e$ and $P_{ec}$ may be defined which is directly attributable to interface polarization in the heterogeneous cermet [19]. This interface polarization $P_i$ is shown in figure 5 for three values of $g$ (note the reduced abscissa scale $q^* = (q - g)/(1-g)$).

Also shown are the three corresponding $\omega_{pe} (q)$ curves which are then identical.

For the dielectric region ($q^* < 0$ in Fig. 5) the interfacial term increases rapidly, reaching a maximum at $q^* = 0$ ($q = g$). The curves for the three depolarization factors $g$ are identical over a large range of $q$ values, differing only near $q^* = 0$. For the whole of the conducting region ($q > 0$), an interfacial polarization still exists. It decreases rapidly from its maximum at $q^* = 0$, reaching zero only at $q^* = 1$. The curves for the three depolarization factors remain distinct for all positive values of $q^*$ less than 1. Only the $g = 0.5$ curve shows symmetry about the $q^* = 0$ axis.

In summary, the infrared behaviour of the cermet over the full cermet range, $0 \leq q \leq 1$, has been shown to be made up of two contributions: a free-carrier component and a supplementary polarization term. The free-carrier component is related to the metallic-constituent behaviour, thus:

$$\varepsilon_e(\omega) = P_{ec}(q) - \frac{\omega_p^2}{\omega(\omega - i/\tau)} \frac{q - g}{1 - g} \qquad q > g$$

$$\varepsilon_e(\omega) = P_{ec}(q) \qquad q < g$$

while the polarization term shows a pronounced maximum at the percolation threshold, is additive with $P_{ec}(q)$ and is non zero over the whole cermet range. This polarization term, whose origin will be further explained below (section 4) may be attributed to interfacial charge effects in the inhomogeneous medium.

3. Visible behaviour.

In the visible range a pronounced structure arises in both $\varepsilon_{1e}$ and $\varepsilon_{2e}$ (Fig. 2) which can be attributed to the excitation of interfacial polarization modes in the
metallic grains. The cermet dielectric permeability can thus no longer entirely be described by a simple Drude relation. The resonance is predominant in \( \varepsilon_e \) (Fig. 2) for \( q \) values < \( g \) (there is only an additional contribution from bound-states polarization). To distinguish clearly the structure for \( q > g \), the free carrier component given by equation (9) has been subtracted out of the effective cermet permeability \( \varepsilon_{e} \) and the resultant \( \Delta \varepsilon_{e} \) curves are shown in figure 6. The resonance varies in intensity, bandwidth and position as a function of filling fraction \( q \) (\( q < g \), Fig. 2; \( q > g \), Fig. 6).

The resonance peak lies for both low and high \( q \) values around 0.45 \( \mu \)m and looks weak and narrow. As the metal fraction approaches \( g \), the percolation threshold for the metal, from low or high values, the peak increases, broadens and moves towards infinite wavelengths. The accompanying increase in \( \Delta \varepsilon_{1e} \) at long wavelengths gives rise to the large polarization term, obtained in the infrared Drude fit, as emphasized before. Both \( \varepsilon_{1e} \) and \( \varepsilon_{2e} \) (Fig. 2) or \( \Delta \varepsilon_{1e} \) and \( \Delta \varepsilon_{2e} \) (Fig. 6) are markedly asymmetric about their turning points, \( \varepsilon_{1e} \) and \( \Delta \varepsilon_{1e} \) showing a cusp around their minimum.

This asymmetric behaviour can be related to the resonance observed in the energy loss function — Im \( 1/\varepsilon_e \) (Fig. 7) which has already been described in detail by Stroud [6]. At high \( q \) values corresponding to low concentrations of dielectric, a narrow peak is present in the energy loss function at a wavelength corresponding to the plasma resonance of the metal. From \( q = 1 - g \), the percolation threshold for the dielectric, and below this value, the peak decreases, broadens and moves towards higher wavelengths. This behaviour can be attributed to the excitation of voids (or dielectric grains) polarization modes [20].

4. Physical interpretation.

The Bruggeman theory detailed above may be given a physical interpretation in so far as it has been shown to model physical phenomena such as percolation and Drude conductivity. However, what is proposed below is derived from the theoretical model only and does not necessarily apply to real systems.

The basis of the physical interpretation is that a dielectric to metal percolation transition occurs in \( \varepsilon_e \) at \( q \approx g \), implying that an infinite metal cluster, spanning the whole sample, is present at this value. From \( q = 0 \) up to the threshold, the metallic grains, which systematically replace dielectric grains in the cermet material, contribute uniquely to the polarization of the medium (both interfacial and bound-state polarization terms increasing with \( q \)) irrespective of the particle shape and thus of the value of the percolation threshold (Fig. 5).

These grains are then isolated and interact only through their dipole (to a first approximation) moments, which give rise to the surface plasmon resonance observed. For a given particle shape (depolarization factor \( g \)), this situation is possible up to a filling factor \( q \) equal to \( g \), at which point an infinite metallic cluster is formed and free-electron like behaviour sets in the cermet.
Above $q = g$, the addition of further metallic atoms can have up to three consequences:

a) Some may form isolated metallic grains, thus increasing both bound-state and interfacial polarization of the cermet.

b) Some may join the infinite cluster. In this case, $P_i$ is invariant, while both bound-state polarization and the number of conduction electrons are increased.

c) Some may connect isolated metal grains to the infinite cluster. Then, the bound-state polarization is increased, the interfacial term decreased, while the number of conduction electrons is increased accordingly.

In fact, for a positive $\Delta q$ variation above $g$, the actual number of cermet conduction electrons $\Delta n^c$ calculated from equation (7) and shown in figure 5 is related to the number of metal electrons added $\Delta n$ by the relation:

$$\Delta n^c = \Delta n \cdot \frac{1}{1 - q_p}.$$  

(10)

This multiplicative factor is $q$ independent and always greater than unity. This fact, along with the decrease in $P_i$, precludes either of the first two processes. Process c) above is the only one compatible with BR theory. It is also interesting to note that in section 2.4 the interface polarization $P_i$ was renormalizable for negative $q$ values, but not for positive $q$ values. This implies that a contribution other than the metallic grain polarization may exist for positive $q$ values ($q > g$). A possible cause for this may be found in the dielectric grains which are present over the whole range of $q$ values ($q \neq 1$). Indeed, dielectric grains in a conducting matrix may be shown to have an interfacial polarization [20] in direct analogy with the metallic grains embedded in a dielectric. The conditions for the existence of this dielectric resonance, dielectric grains surrounded by a conducting matrix, are not easily met for positive $q$ values ($q > g$). Such a resonance is known to appear as a maximum in the $1/\varepsilon_e$ spectrum [21]; this has been shown in section 3 in the energy loss function $(-\text{Im} \varepsilon_e)$ curves in figure 7.

5. Comparison with other effective medium theories.

The discussion will be focused on the ability of the theories to give a proper account of the non metal-metal transition and the resonance generally observed in real systems.

The real part of the effective dielectric permeability predicted by Bruggeman has been seen (Fig. 2a) to be positive for all wavelengths as long as the percolation threshold ($q_p = g = 0.33$ in this case) has not been exceeded. Thus, below percolation, no plasma frequency can be defined for the cermet, which exhibits a typical dielectric behaviour. Above percolation, $\varepsilon_e$ takes negative values and has a noticeable curvature as a function of wavelength; both infrared and visible ranges correspond to metallic behaviour for the effective medium.

In this sense, the BR theory is more powerful than the Maxwell Garnett theory (MG). The MG theory is asymmetric in the constituent materials and predicts only one effective medium behaviour — dielectric or metallic according to the choice of the host — for the whole range of filling fractions. However a plasma frequency defined by $\text{Re} \varepsilon_e = 0$ can be found even when dielectric behaviour is exhibited in the infrared. As suggested by Cohen et al. (CCCA) [8] the percolation problem can be solved by interchanging the host and inclusion parameters at an arbitrary, experimentally determined critical filling fraction $q_p$.

The infrared dielectric constant for the model cermet predicted by CCCA is then simply proportional to the chosen host dielectric constant. Its value below and above the critical fraction is given by:

$$\varepsilon_e^{(\text{CCCA})} \sim \begin{cases} \varepsilon_e + \frac{2q}{1 - q} & \text{for } q < q_s \\ \varepsilon_e - \frac{2q}{3 - q} & \text{for } q > q_s \end{cases}$$  

(11)

showing a discontinuity at $q_s$.

In the visible range, CCCA predicts the MG resonance for the cermet as long as the dielectric is taken as the host and the metal as the inclusion. No resonance in $\varepsilon_e^{(\text{CCCA})}$ occurs for the metal host/dielectric inclusion regime above $q_s$, in contrast with the BR theory.

The symmetrical theory recently proposed by Ping Sheng (PS) [9] is essentially an effective medium theory for coated ellipsoids, type A being material 1 coated by material 2, type B being material 2 coated by material 1. The shapes and occurrence of the coated ellipsoids are included in the theory via the respective axes ratios $r_A$ and $r_B$ and the probability ratios $f_A$ and $f_B$ related to the filling fractions $q_A$ and $q_B$.

Like the BR theory, the PS theory predicts a percolation threshold for the model cermet. However this threshold is only slightly shape-dependent. For $r_A = 1$ (spheres) and $r_B$ ranging from 0.25 to 2.5, the threshold occurs at metal fractions of 0.4 to 0.5.

The infrared Drude fits (analogous to those of section 2.3) to the PS permeability are shown in figure 8. Again, up to the percolation fraction, the polarization increases substantially while both $\omega_p$ and $1/\tau_p$ remain practically zero. Beyond percolation the relaxation time is equal to the metal constituent's one, the polarization decreases and the plasma frequency increases and reaches the metal value at $q_A = 1$. In contrast with the BR theory $\omega_p$ is no longer simply a function of filling fraction $q_A$, but also depends implicitly on the probability factors and particle shapes. No simple relation similar to equation (6) could be found, thus no simple representation similar to figure 4 could be drawn.
The PS visible and near infrared spectra (after subtraction of the infrared fitted Drude component) for the model cermet are shown in figure 9. Due to the Drude fit of \( \varepsilon_r \) values at long wavelengths, \( \Delta \varepsilon_r \) is expected to tend to zero beyond 10 \( \mu \)m. The expected behaviour is not observed after percolation (\( q > 0.5 \)). This discrepancy has no physical meaning. Considering the very high \( \varepsilon_r \) values in this range (metallic behaviour) it corresponds, in fact, to very small relative differences \( \Delta \varepsilon_r / \varepsilon_r < 10^{-3} \). Two resonances are clearly seen in figure 9: the first one, a MG-like narrow peak located around 0.55 \( \mu \)m; the other one, a broader, near-infrared resonance reminiscent of the BR feature. Both peaks increase in amplitude, broaden and shift towards higher wavelengths as the metal fraction reaches 0.5. Beyond this, both decrease steadily and tend to disappear for \( q \) > 0.8. At this value, auxiliary resonances seem to appear at the metal plasma frequency. This resonance behaviour emphasizes that the self consistent PS theory in fact includes some aspects of both the BR and the MG theories. It presents the disadvantage of being difficult to use in practice for the analysis of real data.

6. Conclusion.

We have shown that a physical interpretation may be given to the optical Bruggeman theory for heterogeneous materials. For this the different contributions to the effective dielectric constant have been formulated: bound-state polarization, interfacial polarization and Drude conduction-electron contribution.
We have shown that the Bruggeman expression may be separately applied to the bound-state polarization terms alone. The resulting cermet bound-state polarization has been subtracted from the total predicted effective dielectric permeability and a valid physical model has been found for the terms thus obtained: interfacial polarization in the cermet grains and Drude contributions due to conduction electrons. In the latter case, an analytical expression has been derived for the cermet conduction electrons in terms of the metal component filling fraction and the metal grain depolarization factor $g$.

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