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Instabilities in smectics A submitted to an alternating shear flow.
II. Experimental results

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Résumé. — Nous présentons et discutons les résultats expérimentaux sur le comportement d'un smectique A en géométrie planaire soumis à un cisaillement alternatif, le vecteur vitesse étant normal aux couches.

Ces études portent sur le 4-cyano-4n-octylbiphenyl (8CB). La comparaison des résultats expérimentaux avec nos prédictions théoriques déjà publiées nous permet d'obtenir en particulier l'exposant critique du temps de relaxation du paramètre d'ordre smectique. La transition nématique-smectique apparaît alors être faiblement du premier ordre. On présente une analyse quantitative du mécanisme de relaxation des contraintes au-dessus du seuil d'instabilité.

Abstract. — We present and discuss experimental results on the behaviour of a smectic A placed in a parallel configuration and submitted to an alternating shear flow with velocity normal to the layers.

The smectic A compound is 4-cyano-4n-octylbiphenyl (8CB). These results are compared with previous theoretical predictions. One then obtains the critical exponent for the smectic relaxation time. The nematic to smectic A transition appears to be weakly first order. A quantitative analysis of the stress relaxation mechanism above the instability threshold is presented.

1. Introduction.

We present here new experiments where a special emphasis has been placed on the threshold amplitude versus temperature dependence. The results are compared with the theoretical predictions given in a previous paper [1] hereafter referred as (1).

The main results obtained on (1) were the following

i) two regimes occur at low and high frequencies, and
ii) at low frequency \( \omega \tau_{\text{th}} < 1 \), the critical shear angle is proportional to the square root of the frequency.

The experiments we present were made in the low frequency case \( \sim 10^2 \text{ Hz} \). The results are in a good agreement with our theoretical predictions and confirm our previous theoretical analysis.

As will be shown, one can derive from these results a good evaluation for the relaxation time of the order parameter \( \tau_{\text{F}} \). The temperature dependence of this relaxation time leads to an evaluation of the critical exponent for viscosity. The value found for this critical exponent is in good agreement with the theoretical evaluation made by Brochard [2].

But our results raise two important questions.

a) The critical exponents we find do not agree with those given by other authors. We attribute this discrepancy to the difference in the experimental methods. Most authors use low-frequency methods, and, in this case, the motion of dislocations plays an important role in the dynamic processes which changes the values of the dynamic critical exponents. In our case, as will be shown in section 4, the motions of dislocations, below the instability threshold, are unable to relax the stresses. This explains why we are in the case of a « dislocation-free » mechanism and why our critical exponents agree with the theoretical values.

b) The relaxation mechanism, above the threshold, involves the motion of Grandjean walls, as has been already shown in previous papers [9]. The theoretical analysis made in (1) is valid only below the instability threshold and does not give any prediction on the mechanisms involved above this threshold. Instabilities of this kind often lead to the creation of simple defects, such as dislocations. The question is therefore...
why the relaxation mechanism is due to the motion
of Grandjean walls (made of focal conics) and not
of dislocation walls. This point is discussed in section 5.
Evaluations of friction forces and velocities for dislo-
cation walls and for Grandjean walls are given in
sections 6 and 7, and we show that Grandjean walls
are the only good candidates for the relaxation mecha-
nism.

2. Experimental apparatus.
The experimental apparatus was described in a pre-
vious paper [3]. It is composed of two plates with
planar boundary conditions obtained by an oblique
evaporation of SiO. The lower plate is fixed and the
upper one is connected to a loud speaker which allows
for an alternating motion parallel to the molecular
orientation and normal to the layers. We are here
concerned by experiments performed in the $10^2$ Hz
frequency range. The amplitude of the plate displace-
ment can progressively be increased from 0 to several
microns. An electrostatic gauge, with sensitiveness
about $10^{-2}$ μm, is used to control this displacement.

The two plates are placed in an oven with a tem-
perature regulation better than 0.1 °C which allows
for a very low cooling velocity. The sample must be
cooled very slowly from nematic to smectic A phase
in order to obtain a good orientation.

The smectic A compound is 4-cyano-4n-octyibi-
phenyl (8CB) with formula

$$
N = C - \varphi \quad \varphi - C_8H_{17}
$$

and with the following transition temperatures

$$
C \rightarrow 21.1^\circ \text{Sm A} \quad \frac{T_{NA}}{33.5^\circ} \text{N} \quad \frac{T_{NI}}{40.5^\circ} \text{Is. Liq}.
$$

Cyanobiphenyl has a bilayer structure [4a] with a
molecular length $\approx 22$ Å and a smectic parameter
$\approx 31$ Å [4b].

The transition enthalpy is small (0.16 cal/g) [5]. On
the other hand, the ratio $T_{NA}/T_{NI} = 0.98$ for which
McMillan [6] predicts a first order transition. How-
ever, pretransitional behaviour in the nematic phase
[7] shows that, if the smectic A-nematic transition is
first order, it is very weakly so and that the first-order
transition must occur within 10 mK of $T_c$.

Experiments were performed for

$$
0.9 \text{ K} < \Delta T = T_{NA} - T < 6.8 \text{ K}
$$

i.e. in the hydrodynamic region where M. P. P. [8]
theory holds.

3. Experimental results.
Each experiment is made at fixed frequency. The ampli-
tude is slowly increased until the instability is obtain-
ed : a quasi-periodic pattern of lines appears (Fig. 1)
with the following main characteristics [3].

— These lines are normal to the direction of the
motion. There are constituted by sets of parallel and
tangent ellipses which spontaneously appear in the
bulk.

— These ellipses are elongated in the direction of
the motion and a careful examination shows that there
are two systems of opposite focal conics (with hyper-
bolas pointing in opposite directions).

The patterns obtained in the $10^{-2}$ Hz frequency
range allowed us to give the following interpretation
[9] : these two systems of focal conics form two Grand-
jean walls in the bulk (Fig. 2). The applied strain is
relaxed through a relative vertical motion of these two
walls.

Two important points must be noted here.

a) The threshold value is insensitive to the d.c.
component of the strain (except perhaps for very high
values of this component) and depends only on the
a.c. component. This shows how important is the role
played in the destabilizing mechanism by hydrodyna-
mic effects.

b) No instability is obtained for smectic materials
presenting an A ↔ C phase transition. This justifies
a contrario the use of a « rigid smectic model » in (1) :
for smectic materials presenting an A ↔ C phase tran-

Fig. 1. — A quasi-periodic pattern of lines. The discrete
points are the « heads » of ellipses. The direction of the
shear is normal to these lines.

Fig. 2. — A schematic representation of the focal conics
located in the bulk. The layers are normal to the plates
and the vertical relative motion of the Grandjean walls
relaxes the stress due to the motion of the upper plate.
The perpendicular elasticity modulus, $B_{\perp}$, vanishes at both $N \leftrightarrow A$ and $C \leftrightarrow A$ phase transitions. It remains weaker than $B_{\parallel}$ in the A-phase and the « rigid smectic » model is not valid: the relaxation time for the director is larger than the period of the motion and $n$ must be taken as an independent hydrodynamic variable. This is not the case if the smectic material does not present an $A \leftrightarrow C$ phase transition.

Similar observations have already been published [3]. It was found that the threshold strain $S_{\text{th}}$ satisfied the relation

$$S_{\text{th}}^2 \sim \omega d^2$$

when $d$ was the sample thickness. A value 2.33 had been found for $\alpha$.

The agreement with the theoretical results of (1) was not very good, since it was predicted that the threshold depends on $d$ and $S$ only through their ratio $\theta = d/S$ (tilt angle). Thus a value $\alpha = 2$ was expected.

It was realized at this point that (1) predicted a ratio $\theta^2/\omega$ proportional to $B/\eta$ which has a critical behaviour at the N-A phase transition. The temperature thus appeared to be a very important parameter. The temperature was therefore carefully stabilized and new experimental results were obtained.

A typical variation of $\theta^2/\omega$ versus frequency at fixed temperature is shown in figure 3. On the same figure is shown the theoretical variation of $N$, $M$ versus $M$ where $N_c$ is the critical value of $N$. $N$ and $M$ are two dimensionless numbers introduced in (1):

$$N = \frac{3}{2} \frac{\theta^2 B_{\parallel}}{\omega \eta} = \frac{3}{2} \theta^2/\omega \tau_{\psi},$$

$$M = \rho d^2 \omega/\eta = \omega \tau_{\text{th}}.$$  (3.1)  (3.2)

The two curves appear to be parallel, which corresponds to a value of 2 for $\alpha$.

In figure 4, $\log (\theta^2/\omega)$ is plotted versus $\log (T_{\text{NA}} - T)$. A linear variation is observed and a least-squares fitting gives a slope $-0.992$ in good agreement with a critical exponent 1 for $\tau_{\psi}$: the temperature dependence of the threshold tilt angle versus temperature is thus well represented by the expression

$$\theta^2/\omega = C(T_{\text{NA}} - T)^{-1}$$

with $C = 1.95 \times 10^{-6}$ cgs.

4. Instability threshold dependence.

The analysis performed in (1) introduced two regimes, a low frequency regime where the critical value for $N$, $N_{c}$, does not depend on $M$, and a high frequency regime with $N_{c} \sim M^{-4}$. These two regimes are shown in figure 3 and our results clearly correspond to the first one. This was expected since the limit between the two regimes occur for $M \simeq 1$. In our case, typical values are $d \approx 10^{-2}$ cm, $v \approx 10^2$ Hz, $\rho \approx 1$ g/cm$^3$, $\eta \approx 1$ p, and therefore $M = \rho d^2 \omega/\eta \approx 6 \times 10^{-2} < 1$.

Thus $N_{c} = 3 B_{\parallel} \theta_{\psi} \omega/2 \omega \eta$ can be considered as a constant. ($\theta_{\psi}$ is the critical value of the tilt angle.) $\theta_{\psi}^2/\omega$ is proportional to $B/\eta = \tau_{\psi}^{-1}$ where $\tau_{\psi}$ is the relaxation time for the smectic order parameter introduced by Brochard [2]. We find $\tau_{\psi} \approx 10^{-7}$ s for $T_{\text{NA}} - T = 4.6$ K.

The temperature dependence of $\tau_{\psi}$ has been given in the last section: $\tau_{\psi} \sim (\Delta T)^{-1}$.

Such a critical exponent had been predicted by Brochard on the basis of an analogy with the $^4$He superfluidity transition. On the basis of some experimental results [10], she gave for $\tau_{\psi}$ the following estimate:

$$\tau_{\psi}^{-1} \approx 10^8 \Delta T/T.$$  

Our results give $\tau_{\psi}^{-1} \approx 10^9 (\Delta T/T)$ which is nearly the same order of magnitude.
A critical exponent \( \frac{1}{2} \) for \( \tau \) has already been found by several authors [11]. It must be emphasized that this result involves a critical exponent 0.66 for BV, which is the value obtained by renormalization group calculations, but disagrees with a number of experimental results [7, 12-14].

The value of this critical exponent for BV has been the object of long discussions. It thus appears that, if the value 0.66 is obtained from a dislocation-free model, dislocations play a central role in stress relaxation, at least in static or low frequency experiments, and that their contribution tends to reduce this critical exponent [15].

These considerations are true only if the motion of dislocations along the layers is able to relax strains for \( \theta < \theta_{0c} \). (The motion of dislocations above the threshold is treated in section 6.) We shall now discuss the contribution of this motion to the relaxation process.

We are thus led to consider a system of layers (including edge dislocations) tilted by an angle \( \theta \). Under the tilt, the layers interspacing decreases, leading to a compression that can be relaxed through a motion of dislocations.

Let us now calculate the force exerted on one dislocation. The simplest way to do it is to consider a periodic array of dislocations with spacing \( D \) and Burgers vector \( b \) (Fig. 5). Using the standard form for the elastic energy per unit volume [20]

\[
F = \frac{1}{2} B u^2
\]

where \( u \) is the relative dilation of layers, one finds, for the free-energy density above and under the array [16]

\[
F_1 = \frac{1}{2} B \left( \frac{\theta^2}{2} + \frac{b}{2D} \right)^2
\]

\[
F_2 = \frac{1}{2} B \left( \frac{\theta^2}{2} - \frac{b}{2D} \right)^2.
\]

In a displacement \( \delta z \) of the array, the free-energy variation per unit surface is

\[
\delta F = (F_2 - F_1) \delta z = -\frac{1}{2} B \theta^2 \frac{b}{D} \delta z.
\]

The elastic force per unit surface is therefore

\[
-\frac{\delta F}{\delta z} = B \theta^2 \frac{b}{2D} \text{ and, as the dislocation density is } 1/D, \text{ the force per unit line acting on each dislocation is}
\]

\[
f = B \theta^2 \frac{b}{2}.
\]

The upper limit for velocity is obtained when this force is balanced by the friction force [19] \( \eta K b V_d \), where \( V_d \) is the dislocation velocity, \( K^{-1} \) a molecular length, and \( \eta \) the viscosity. We have then:

\[
V_d = \frac{B \theta^2}{2 \eta K}.
\]

If \( m \) is the order of the dislocation, \( \kappa b = \sqrt{m} \).

Compare now this velocity to that needed for relaxing stresses. The velocity of the upper plate can be written, at the threshold, as \( \omega \theta_{0c} \), which gives, along the layers, a velocity

\[
V = \frac{\omega \theta_{0c}}{\sin \theta} \approx \frac{\omega \theta_{0c}}{\theta_{0c}}.
\]

Thus

\[
\frac{V_d}{V} = \frac{1}{\kappa} \left( \frac{\theta}{\theta_{0c}} \right)^3 \frac{1}{kd} \approx \frac{1}{3} N_c \left( \frac{\theta}{\theta_{0c}} \right)^3 \frac{1}{kd}.
\]

Here \( \theta \) is the actual layer inclination angle \( \theta_1 \). \( \theta_1 \) is very close to \( \theta_{0c} \) except at the onset of instability. Then, with \( d \approx 160 \mu \) and \( \kappa \approx 5 \times 10^6 \text{ cm}^{-1} \)

\[
V_d/V = \frac{1}{3} N_c \frac{1}{kd} \approx 3.2 \times 10^{-5}.
\]

The dislocation velocity is therefore much too low to allow for this relaxation process.

The situation becomes different at the onset of instability. \( \theta_1 \) then increases and takes much higher values than \( \theta_{0c} \). A typical value is the one found above the threshold corresponding to the figure 8 of reference [9]: \( \theta_{0c} = 1.4 \times 10^{-2} \text{ rad, } \theta = 8.6^\circ, d = 160 \mu \).

In this case, \( (\theta/\theta_{0c})^3 \approx 1.2 \times 10^3 \) and \( V_d/V \approx 4 \times 10^{-2} \).

This value is still very low : the dislocation motion is again too slow to allow for this relaxation process. It follows that, in our case, the dislocations can be considered as frozen and that the involved mechanism must be « dislocation-free ». This accounts for the value we have found for a critical exponent of \( \tau \), and consequently of BV.
The situation would be quite different at lower frequencies: \( \Theta_{bc} \) decreases as \( \omega^{1/2} \), and, as a consequence, the ratio \( (\theta/\Theta_{bc})^3 \) increases as \( \omega^{3/2} \). Then, at the onset of the instability, one goes from a « dislocation-free » behaviour to a plastic, where the stresses are relaxed by the motion of the dislocations. In this case, the instability appears for a much higher value of \( \theta \) and the observed threshold is much higher than that predicted by (1). This explains the low-frequency behaviour shown on figure 3.

5. Strain relaxation mechanism.

The relaxation mechanism presents three important characteristics.

a) Ellipses grow in the bulk and not on the surface. This can be understood in the light of the analysis made in (1). The orientational surface constraint on the layers is relaxed in a thin curvature boundary layer. Then the instability occurs in the region where the distortion is maximum, i.e. deep in the bulk.

b) The actual inclination \( \theta_1 \) of layers can be measured from the ellipticity of focal conics. One finds \( \theta_1 \approx 10^\circ \), i.e. a value about ten times higher than the threshold « tilt angle » \( \Theta_{bc} = S_0/d \) where \( S_0 \) is the amplitude of the motion of the upper plate. This discrepancy shows that the layers suffer a very strong distortion at the moment the focal conics are generated.

c) This mechanism involves the motion of focal conics, and not dislocations. As we shall now show, this can be understood if we introduce friction forces which limit the velocity of dislocations.

For this we need to evaluate the elastic force exerted on the dislocations during the a.c. motion.

Let us start from point b). One finds a finite value for \( \theta_1 \), which means that the average inclination angle, above the threshold, is non-zero. This can be understood if the instability process freezes the layers when they are tilted. This can be done in relaxing the layers compression by the induction of a periodic array of fixed surface dislocations and one, or most probably two, splay walls in the bulk [18a]. Outside of these splay walls the layers spacing is normal. The grains boundary themselves can be made either of conic focal (Grandjean wall), or by periodic arrays (with period \( D \)) of pairs of dislocations relaxing the dilation (grain boundaries), as suggested by Kleman (Fig. 6a in Ref. [18b]).

This is, of course, a frozen picture. The relaxation mechanism above the threshold is ensured by an upward-downward motion of the middle plane. Let us consider only the motion of the half lower part of the sample. The median plane \( (z = 0) \) will be assumed to move with velocity \( V/2 \), the lower plate \( (z = -d) \) remains at rest. A grain boundary (or wall) will be located at \( z = -h \) at time \( t = 0 \). The layer, above and under the wall will have an initial tilt \( \theta \) with normal spacing \( a \) (Fig. 6b).

Under the motion of the middle plane, the upper layers tend to tilt more. This results in a compression of the layers which can be relaxed, as we shall now show.

6. Relaxation by a grain boundary.

A schematic picture of a grain boundary is shown on figure 6a. Let \( a \) be the layers spacing. In the absence of dislocations, the layers spacing along x-axis would be \( a/\cos \theta_1 \). This dilation is relaxed through a periodic array of dislocation pairs with period \( D \) and Burgers wave vector \( b \) (along the x-axis). \( D \) and \( b \) are obviously related by

\[
b = D(1 - \cos \theta_1).
\]

In what follows, we shall assume a symmetric geometry with a double grain boundary and consider only the motion of the half lower part of the sample. The median-plane \( (z = 0) \) will be assumed to move with velocity \( V/2 \), the lower plate \( (z = -d/2) \) remains at rest. A grain boundary (or wall) will be located at \( z = -h \) at time \( t = 0 \). The layer, above and under the wall will have an initial tilt \( \theta \) with normal spacing \( a \) (Fig. 6b).

Under the motion of the middle plane, the upper layers tend to tilt more. This results in a compression of the layers which can be relaxed, as we shall now show.

Fig. 6a. — Grain boundary where the stresses are partly relaxed by a periodic array of pair of dislocations.

Fig. 6b. — O is on a dislocation line. COA corresponds to the non-perturbed layer. A displacement \( X \) of the middle plane involves a new tilt \( \psi \). The displacement \( O \to O' \) tends to relax the stresses.
show, by a downward motion of the wall. We shall evaluate the downward force exerted by this mechanism on the wall and, comparing it to the friction force due to the motion of the dislocation pairs, calculate the wall velocity. This velocity will be compared to that needed for the relaxation mechanism.

Let us thus assume that, under the motion of the middle plane, the upper layers suffer a new tilt \( \psi \). This corresponds to a displacement \( AB = X \) of the middle plane. The spacing of the upper layers is now \( (a \cos \psi) \) (instead of \( a \)). Using the standard form for the elastic free energy density \[ f = \frac{B}{2} u^2 + \frac{K_1}{2} (\text{div} \, \mathbf{n})^2 \]
where \( u \) is the relative layers dilation, neglecting the curvature term, and integrating on \( z \) above the wall, one finds, for the elastic free energy per unit surface of the wall

\[ F_s = \frac{B}{2} (1 - \cos \psi)^2 h. \quad (6.2) \]

This energy increase can be released by a motion of the dislocations along the layers corresponding to a downward motion of the wall \(^{1}\).

To this relaxation process corresponds a force \( f_s \) acting on each dislocation pair, and pulling it downwards along the lower layers. We shall now calculate this elastic force.

Let us perform an elementary downward displacement \( dl \) of the wall along the lower layer. Points \( O \) and \( A \) go to \( O' \) and \( A' \) with

\[ |OO'| = dl \]
\[ |AA'| = -dX = 2 \sin \theta \, dl. \quad (6.3) \]

The vertical displacement of the wall is

\[ dh = dl \cos \theta. \quad (6.4) \]

The force \( f_s \) acting on the wall is, per unit surface

\[ f_s = -\frac{dF_s}{dl} \quad (6.5) \]
directed downward along the lower layers.

On differentiating \( F_s \), one finds that \( \psi \) and \( h \) are both functions of \( l \). \( dh/dl \) is drawn from equation (6.4). Let us calculate \( d\psi/dl \). In the triangle \( OAB \), one has

\[ \frac{X}{\sin \psi} = \frac{h}{\cos \theta \cos (\theta + \psi)}. \]

Differentiating this expression and using equations (6.3-4), one finds, after some algebra

\[ \frac{d\psi}{dt} = -\frac{\cos (\theta + \psi) \sin (2 \theta + \psi)}{h}. \quad (6.6) \]

Finally, using equations (6.2-6), (6.5) leads to

\[ f_s = -\frac{B}{2} \left[ 2(1 - \cos \psi) \sin \psi h \frac{d\psi}{dl} + + (1 - \cos \psi)^2 \frac{dh}{dl} \right] \]
or

\[ f_s = + B f(\theta, \psi) \quad (6.7a) \]
where \( f(\theta, \psi) \) is defined as:

\[ f(\theta, \psi) = \frac{1}{2} (1 - \cos \psi) \left[ 2 \sin \psi \cos (\theta + \psi) \times \times \sin (2 \theta + \psi) - (1 - \cos \psi) \cos \theta \right]. \quad (6.7b) \]

This elastic force exerted on the wall is balanced by the friction forces. Friction occurs i) in the motion of the fluid bound to the wall displacement and ii) in the motion of dislocations. It is shown in appendix A that the viscous force due to the fluid motion is directed along the \( x \)-axis. It therefore does not play any role in the wall motion. We have just to take into account the friction force due to motion of dislocations. This friction force has been evaluated by de Gennes \[ ^{17} \]. Its value, per unit length of dislocation, is equal to \( 2 \eta b V_D \) where \( V_D \) is the dislocation velocity and \( \kappa^{-1} = (v_{3} \lambda_{\eta})^{1/2} \) a molecular length (\( \kappa^{-1} \) and \( \lambda \) have the same temperature dependence). \( \eta \) is the smectic viscosity coefficient corresponding to a gliding of layers one on each other. The factor 2 is due to the presence of two dislocations in each pair.

The friction force per unit surface due to the motion of dislocation is therefore

\[ f_V = 2 \eta b V_D/D. \quad (6.8) \]

We want this motion to relax the stresses due to the motion of the middle plane with velocity \( V/2 \). If this is true, one must have

\[ V_D = \left| \frac{dl}{dX} \right| \frac{V}{2} \]
and, equating \( f_s \) to \( f_V \), and using equations (6.1) and (6.3)

\[ B f(\theta, \psi) = 2 \eta b (1 - \cos \theta) V/4 \sin \theta. \quad (6.9) \]
Define now
\[ g(\theta, \psi) = f(\theta, \psi) \sin \theta \quad (6.10) \]

The maximum velocity at threshold is
\[ V = \omega \theta_{\text{oc}} d. \]

At threshold, the dimensionless number \( N_c = 3 \theta_{\text{oc}} B \omega / \eta \) is a constant. Equation (6.9) can be rewritten as
\[ g(\theta, \psi) = \frac{3}{4} \frac{\theta_{\text{oc}}^3}{N_c} \sin \theta d. \quad (6.11) \]

The r.h.s. of equation (6.11) can be easily evaluated. Taking the experimental values corresponding to the figure 8 of reference [9] \( \theta_{\text{oc}} = 1.4 \times 10^{-2} \) rad., \( \theta = 8.6^\circ \), \( d = 160 \) nm, and evaluating \( \kappa^{-1} \approx 20 \AA \), one finds
\[ g(\theta, \psi) \approx 2.2 \times 10^{-2}. \quad (6.12) \]

This equation is satisfied, for \( \theta = 8.6^\circ \), by \( \psi \approx 12^\circ \). This value is obviously much too high, since it is more than ten times higher than \( \theta_{\text{oc}} \) : a new instability would be achieved for a much lower value of \( \psi \).

It is therefore necessary to fix an \textit{a priori} upper limit for \( \psi, \psi_{\text{max}} \). The condition \( \psi_{\text{max}} = \theta_{\text{oc}} \) is probably a little too restrictive, since our calculation was rather crude.

If we set \( \psi_{\text{max}} = 5^\circ \), the maximum value of \( g(\theta, \psi) \) is obtained for \( \psi = \psi_{\text{max}} \) and is only \( 1.5 \times 10^{-3} \). Equation (6.12) cannot be satisfied.

Our conclusion is therefore that the dislocation mobility is far too low to allow the stress relaxation above the threshold.

7. Relaxation by a Grandjean wall

A Grandjean wall is composed of cofocal conics. Let \( a', b', c' \) respectively be the half long-axis, the half short-axis and the interfocal distance of the ellipses.

As shown by many authors [18], a Grandjean wall is, at high distances, strictly equivalent to a grain boundary. The layers tilt, \( \theta \), is equal to the angle between the asymptotes and the x-axis. Each array (along the y-axis) of cofocal systems is equivalent to a dislocation with Burger vector \( b = 2(a' - c') \). On the other hand, for elongated ellipses (i.e. small \( \theta \) angle) the layers perturbation is very weak except in small regions around the edge of the long-axis of the ellipses. The size of these perturbed regions is also of order \( 2(a' - c') \).

A complete calculation of the viscous force bound to the displacement of the focal conics is inextricable. However a crude evaluation can be made using the following argument.

Outside of the small disturbed regions, the distortion of the layers is very weak. Then the friction force is very close to that expected for a grain boundary, if one excepts dislocation motion. This friction force is shown in appendix A to be parallel to the wall.

We are left with the friction force on the perturbed regions. These regions can be assimilated to small spheres. This point will be discussed further. Here again, these spheres will move along the layers, since such a motion involves no permeation and is therefore much easier than a motion normal to the layers [17, 19].

We have now a picture quite similar to that described in section 6. Dislocations are replaced by lines of spheres with radius \( R = (a' - c') \). The distance between neighbour spheres is \( \delta = 2b' \), since, in the Grandjean walls we observe, the ellipses are tangent at the extremities of their small axes.

The calculation of the elastic force per unit surface of the wall, \( f_b \), is identical to that of section 6.

The friction force on each sphere has been shown by de Gennes [17] to be equal to \( 8 \pi \eta RV_s / D \delta \), where \( V_s \) is the sphere velocity and \( \eta \) an effective viscosity. As in section 6, in order to ensure relaxation, the sphere velocity must be
\[ V_s = \frac{d}{dx} \left| \frac{V}{2} \right. \quad (7.1) \]

The friction force per unit surface is now:
\[ f_V = 8 \pi \eta RV_s / D \delta. \quad (7.2) \]

Equating it to the elastic force, one now finds
\[ Bf(\theta, \psi) = 8 \pi \eta R \frac{V}{4 \sin \theta} \frac{1 - \cos \theta}{b \delta} \quad (7.3) \]
and, using \( b = 2R \)
\[ g(\theta, \psi) = \frac{\pi \eta V}{\delta} \quad (7.4) \]

where \( g(\theta, \psi) \) is defined by equation (6.10).

As in section 6, we set \( V = \omega \theta_{\text{oc}} d \), and, using the dimensionless number \( N_c \), we find, at threshold
\[ g(\theta, \psi) = \frac{3 \pi}{2} \frac{\theta_{\text{oc}}^3}{N_c} \frac{d}{\delta}, \quad (7.4) \]

Taking again the experimental values corresponding to the figure 8 of reference [9] \( \theta_{\text{oc}} = 1.4 \times 10^{-2} \) rad., \( \theta = 8.6^\circ \), \( d = 160 \) nm, \( \delta = 20 \) \mu m, \( N_c = 7.5 \), one finds for the r.h.s. of equation (7.4) the value \( 1.4 \times 10^{-5} \).

This value is much smaller than that obtained in section 6. We can therefore expect a small value for \( \psi \), and use a \( \psi \) expansion of \( g(\theta, \psi) \)
\[ g(\theta, \psi) = 2 \cos^2 \theta \times \cos^2 \theta / 2 \times \psi^3 + O(\psi^4) \]
\[ \approx 1.95 \psi^3 \]
which gives
\[ \psi \approx 1.9 \times 10^{-2} \text{ rad.} \approx 1^\circ. \]
This value has the good order of magnitude. It is little too high since \( \psi \) should be less than \( \theta_{0c} \) in order to avoid the onset of a new instability. On the other hand, we expect \( \psi \) (which is the maximum distortion amplitude corresponding to the maximum velocity of the higher plate) to be very close to \( \theta_{0c} \) in order to account for the existence of a double wall: with a single wall, \( V \) and consequently \( g(\theta, \psi) \) would have been multiplied by a factor 2, and \( \psi \) by a factor 1.25. \( \psi \) would thus have exceeded the instability threshold leading to the formation of a second Grandjean wall.

This calculation was based upon two rather crude assumptions i) we have taken the same viscosity coefficient \( \eta \) for the calculation of the critical number \( N_c \) and for the evaluation of the friction force, and ii) we have assimilated the distorted part of focal domains to spheres. This last point is certainly topologically wrong. Layers have a complex shape but are continuous through focal domains. The result is that we have certainly overestimated the friction force, and, as a consequence, underestimated the Grandjean wall velocity. A more correct value of the friction force would have led to a smaller value of \( g(\theta, \psi) \) at threshold, which reinforce our conclusion: the motion of a double Grandjean wall appears to be able to account for the relaxation mechanism.

8. Conclusion.

We have presented here new experimental results that are in good agreement with the analysis of the instability mechanism made in (1). At a given frequency and temperature, the instability threshold occurs for a critical tilt angle \( \theta_{0c} \) and at a given temperature, \( \theta_{0c} \sim \omega^{1/2} \).

These results concern an intermediary frequency range (\( \sim 10^2 \) Hz). We have shown why the analysis made in (1) failed to explain the low-frequency results: this is due to the occurrence of a plastic regime leading to much higher instability thresholds.

At higher frequencies (\( \omega \tau_{el} \sim 1 \)), the \( \theta_{0c}/\omega \) ratio is predicted in (1) to decrease. Our experimental apparatus did not allow us to explore this frequency range.

The temperature dependence of \( \theta_{0c}/\omega \) had allowed us to assert the value 1 to the critical exponent for the relaxation time \( \tau_{\phi} \) of the order parameter. This value is in good agreement with the prediction of Brochard [2] for a dislocation-free model. We have shown that, in our case, dislocations are too slow for relaxing stresses and hence that the elasticity modulus has not to be renormalized.

This critical behaviour has been drawn from a calculation made in the « London approximation » (the amplitude of the smectic order parameter has not been taken as a dynamic variable). Since our experiments were made at constant temperature and far enough from the critical temperature, this approximation seems to be justified.

The mechanism for strain relaxation above the threshold has been analysed. We have shown why it involves focal conics and not dislocations (which are too slow) and why a double Grandjean wall is needed. This opens the way to a more quantitative study of relaxation mechanisms above the threshold. A direct measurement of the force exerted on the upper plate would provide knowledge of the friction force involved in the motion of a focal conic, and thus open a new chapter in the dynamics of defects.

Appendix A.

**VISCOUS FORCE ON A GRAIN BOUNDARY.** — We want to calculate here the viscous force exerted on the motion of a grain boundary (excepting the friction forces of the motion of dislocation).

This force is obviously identical to that exerted in the motion of a fluid through a boundary (with \( v_y = 0 \)).

The geometry is shown on figure 7. The plane \( z = 0 \) is a symmetry plane. The director \( n \) (normal to the layer) makes an angle \( \theta(z) \) with the \( x \)-axis for \( z < 0 \) and an opposite angle

\[
\theta(z) = -\theta(-z) \quad (A.1)
\]

for \( z > 0 \).

The viscous force per unit volume can be written [20]

\[
f_y = g + \text{div} \bar{\sigma}
\]

where \( g \) is the permeation force, normal to the layer. \( g \) only derives from elastic terms. From symmetry considerations, one obviously has

\[
g_y = 0 \\
g_z(z) = -g_z(-z) \\
g_x(z) = g_x(-z)
\]

Fig. 7. — A symmetrical grain boundary whose thickness is \( 2d \). The friction force is parallel to the \( x \)-axis.
After integration on \(z\), \(g\) results in a force parallel to \(x\)-axis.

We shall assume the smectic material to be incompressible. Then the stress tensor \(\sigma^\prime\) has components [20]

\[
\sigma_{zz}^\prime = \alpha_1 \sum \frac{\partial}{\partial x_i} A_{ij} + \alpha_4 A_{zz} + \\
\alpha_{56} \sum \left( n_{x_i} A_{ij} + A_{y_i} n_{x_i} n_{y_i} \right).
\]  

(A.2)

Here \(A_{ij}\) is the usual velocity-gradient symmetric tensor.

\[
A_{ij} = \frac{1}{2} \left( \frac{\partial V_x}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right).
\]  

(A.3)

\(V\) is in the \((x, z)\) plane and depends only on \(z\). From flow conservation, \(V_x\) is a constant, and, from symmetry considerations,

\[
V_x(z) = - V_x(-z).
\]  

(A.4)

Let us now calculate the \(z\)-component of the friction force

\[
f'_{vz} = \frac{d}{dz} \sigma_{zz}^\prime.
\]

Using equations (A.2-3), we obtain

\[
f'_{vz} = \frac{1}{2} \frac{d}{dz} \left( \alpha_1 n_x^2 n_z + \alpha_{56} n_z n_x \frac{dV_x}{dz} \right).
\]

Let us now integrate \(f'_{vz}\) on \(z\). We must integrate it on the whole width of the grain boundary, from \(z = -d\) to \(z = d\), where \(d\) is such as

\[
\left( \frac{d\theta}{dz} \right)_{z=d} = 0
\]  

(A.5)

\(f'_{vz}\) is even in \(z\). One thus gets per unit surface of the grain boundary, a force

\[
F_{vz} = \left[ \alpha_1 n_x^2 n_z + \alpha_{56} n_z n_x \right] \frac{dV_x}{dz} \bigg|_{z=d}.
\]  

(A.6)

Let \(V_0\) and \(\theta_0\) be the fluid velocity and the tilt angle outside the grain boundary. Since \(V_z\) is a constant,

\[
V_z = - V_0 \tan \theta \cos \theta_0
\]

and

\[
\frac{dV_x}{dz} = - V_0 (1 + \tan^2 \theta) \cos \theta_0 \frac{d\theta}{dz}.
\]  

(A.7)

Using now relation (A.5), one finds

\[
\frac{dV_x}{dz} \bigg|_{z=d} = 0.
\]

And, from (A.6), \(F_{vz}\) vanishes.

The friction force is therefore parallel to the \(x\)-axis.

References

