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Optically detected variations of nematic liquid crystal orientation induced by ultrasound

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Résumé. — Nous avons examiné le changement d'orientation des longs axes des molécules du cristal liquide provoqué par les ondes ultrasonores. En nous basant sur l'hypothèse de Nagai, Peters et Candau, selon laquelle les changements d'orientation des longs axes des molécules sont excités par l'écoulement acoustique qui apparaît dans la direction de propagation des ondes ultrasonores, nous avons réalisé l'analyse théorique et les calculs numériques correspondant aux conditions expérimentales de la géométrie d'échantillon. Assurant des conditions de propagation correspondant à des ondes progressives longitudinales, on a réalisé la vérification expérimentale des calculs numériques sur les dépendances des changements d'orientation des longs axes des molécules en fonction de l'intensité des ondes ultrasonores. Les changements d'orientation des longs axes des molécules sont linéaires aux intensités et pour les géométries utilisées dans notre expérience.

Abstract. — The variations in the orientation of the long axis of molecules in nematic liquid crystals induced by an ultrasonic wave have been studied using optical methods. The theoretical and numerical analysis which preceded the experimental examinations was based on the Nagai, Peters and Candau hypothesis assuming the acoustic flow force to be the factor responsible for variations in molecular orientation. The analysis allowed us to determine adequate experimental conditions for the liquid crystal samples situated in the cell of a given geometry with respect to the direction of initial orientation induced by a magnetic field, the direction of an ultrasonic wave and that of a polarized light beam. Our numerical results have been verified experimentally in the range of linear dependence of molecular orientation variations on the intensity of the ultrasonics.

1. Introduction. — In the last few years a new domain of acousto-optics has been developing i.e. light and ultrasound interaction in liquid crystals. Liquid crystals as a mesophase between liquid and solid state characterize themselves by an anisotropy of viscoelastic and optic properties.

An ultrasonic wave propagating in a liquid crystal (in particular a nematic) causes, among other things, a deformation of the orientation field of molecules, which is described by the field of unitary orientation vectors — the « director ». The deformation of the director orientation lies at the base of acoustical effects in liquid crystals.

There exist few theoretical models of the physical mechanism of interaction between an ultrasonic wave and a nematic sample [11, 15, 16, 4, 14, 19]. The same can be said about a light wave [7, 9]. The first theory related to the acousto-optical interactions in nematic liquid crystals was presented by W. Helfrich [8]. According to this theory there exists a threshold value of an ultrasonic wave intensity above which ultrasonics can induce variations of orientation of the long axis of molecules (orientation of a director) of nematic liquid crystals [8].

In the above-mentioned theories as well as in the later ones of Nagai, Peters and Candau [16] and also of Sriapiapan, Hayes and Fang [19], it has been assumed that the director orientation variations in the nematic in which an ultrasonic wave is propagating are influenced by a non-laminar flow of an initially
oriented volume of the nematic induced by the
radiation pressure of the wave.

In the course of the ultrasonic wave and the liquid
crystal interaction the influence of an alternating
acoustic pressure is neglected because orientational
relaxation times are long compared to the period
of the wave and the average value of the alternating
pressure is equal to zero. For a finite ultrasonic
intensity the average value of the acoustic pressure
gives a constant value different from zero which is
called the wave radiation pressure.

In one of the newest theories, that of Dion and
Jacobs [4], it is assumed that the long axes of molecules
of a nematic shift to an orientation which corresponds
to the minimum value of the ultrasonic wave absorp-
tion.

The theories mentioned above have not yet been
verified experimentally in an unambiguous way,
though some trials of such verification were per-
formed [12, 16]. However, it can be seen from the
literature that in most of the cases examined the
conditions assumed in the theories were not suffi-
ciently fulfilled in the experiments.

In this paper the results of detailed theoretical
analysis and experimental examinations of the inter-
action between an ultrasonic wave and a nematic
liquid crystal are presented to verify the theory of
Nagai, Peters and Candau (NPC) [16]. The agreement
with NPC theory seems to be the most conclusive
compared to the other ones from the point of view of
the model assumed for the process of interaction
between ultrasonics and nematic liquid crystal consid-
ered as a strongly attenuating medium. Starting from
the NPC model the theoretical analysis of the inter-
action in given experimental conditions was performed,
in particular for the acoustic field being applied and the
field of flow induced by radiation pressure of it at given
and determined geometry of measuring cell, namely the
thickness and the width of the layer of the nematic
crystal. Also the effect of an external magnetic field
acting on the liquid crystal as an orientation factor
was taken into account.

In the particular case which is considered here the
theoretical considerations and experimental results
for a planar texture are presented. It is worth men-
tioning that also for the case of a homeotropic texture
similar theoretical and experimental relations have
been obtained.

As already mentioned above, the paper of NPC [16]
on the influence of an acoustic pressure flow upon
orientation of liquid crystals was mainly concerned
with the homeotropic texture case.

2. Theoretical. — In figure 1 the shape and dimen-
sions of the experimental cell as well as directions of
the fields applied (i.e. of the orientational external
magnetic one, of the reorientational ultrasonic one
and of the linearly polarized coherent light beam) are presented.

![Fig. 1. A geometry of the sample for numerical calculations. The shape and dimensions of the experimental cell, and directions of the fields applied.](image)

Variations of intensity of the light beam influenced
by changes in liquid crystal orientation were the
subject of the direct observation and measurements.
The fundamental system of nematodynamic equations
was solved for such experimental conditions and
with the following assumptions:

1) the layer of a nematic of the planar texture is
initially homogeneously oriented (with respect to the
magnetic field),

2) the liquid is incompressible i.e. \( \text{div} \, \mathbf{v} = 0 \); \( \mathbf{v} \) : particle velocity (of a volume element),

3) in the first approximation a longitudinal ultrasonic
wave propagates along the direction of the
OZ axis,

4) the processes of flow of the medium being
generated by the acoustic field are of a stationary
character,

5) the dynamical variations in the direction of
the OX axis may be described by a system of trans-
verse modes characterized by the wave number
\( q_m = \frac{m \pi}{L} \) [18], where \( L \) is the width of the liquid
crystal layer along the OX axis, \( m \) : integer,

6) variations in orientation induced by the ultra-
sonic field are small (below the threshold of the
so-called dynamic scattering).

The system consists of the equilibrium equation of
forces and one of the moments of the forces under the
ultrasonic wave equilibrium perturbation of the
medium.

The equations are [11]:

\[
\begin{align*}
\rho \frac{dv_i}{dt} &= F_i + \frac{\partial \sigma_{ij}}{\partial x_j} \\
\rho \frac{dI_i}{dt} &= \tau_i - \varepsilon_{ijk} \sigma_{jk}' + \frac{\partial C_{ij}}{\partial x_j}
\end{align*}
\]

where

- \( \rho \) density of a liquid crystal,
- \( v_i \) \( i \)-th component of a linear velocity of the
  volume element of the nematic,
- \( F_i \) \( i \)-th component of the external force acting
  on the volume element,
- \( \sigma_{ij} \) tensor of stresses in the nematic [11, 17],
- \( i \)-th component of the external force acting
  on the volume element,
\[ l_i \]  \( i \)-th component of the inner momentum of the mass unit of a liquid crystal due to the rotation of the unitary orientation vector,

\[ \tau_i \]  \( i \)-th component of moments of external forces,

\[ \varepsilon_{ijk} \sigma_{jk} \]  \( i \)-th component of moments of viscous stresses,

\[ C_{ij} \]  pair of elastic stresses.

Variables which appear in the motion equations are:

\[ \rho : \text{density of nematic}, \ p : \text{hydrostatic pressure}, \ v : \text{linear velocity of the volume element of the nematic}, \ n : \text{unitary orientation vector-« director »,} \ F : \text{external force,} \ \tau : \text{moment of external forces.} \]

According to the geometry (Fig. 1) along the OZ axis (perpendicularly to the direction of molecules orientation) in the nematic a longitudinal ultrasonic wave propagates with frequency \( \omega \). The perturbation of the medium by the wave causes variations of its density, pressure and temperature as well as variations of the unitary orientation vector in the case of liquid crystal.

Let us introduce the parameter, \( \varepsilon \), which measures the degree of approximation of the local and momentary value of one of the quantities mentioned above which we denote symbolically by \( u \).

The quantity is equal to \( u_0 \) at the thermodynamic equilibrium state \([10]\), \( u \) can be represented with a power series in the form

\[ u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots \]  \hspace{1cm} (3)

where

\[ u_1 \]  variations of physical quantity (elastic wave) described, for instance, by a sinusoidal function of space and time,

\[ u_2 \]  variation due to the second order non-linearity of the physical value described, for instance, by a sinusoidal function of time with a double frequency and space a term being a result of time averaging of the quantity \( u_1 \).

If the ultrasonic wave has a small amplitude then \( \langle u_1 \rangle \approx 0 \) and \( u_1 \gg u_2 \), for an ultrasonic wave of a great amplitude \( u_1 \neq 0 \) and \( u_1^2 \neq 0 \). In the case considered here the quantities represented by \( u_1 \) are acoustic flow and acoustic pressure and \( \langle u_1^2 \rangle \) determines the velocity of acoustic flow or radiation pressure.

In the theoretical model which we assumed for describing the interaction of an ultrasonic wave with a liquid crystal medium the acoustic field is represented by the second order quantities i.e. \( \langle u_1^2 \rangle \neq 0 \). We assume that the orientation state of a molecule has too high an inertia to change during the very short period \( T \) of the ultrasonic wave i.e. \( T \ll \tau \), where \( \tau \) is the orientational relaxation time of the « director ».

The nematic layer in which the ultrasonic wave is propagating(Fig. 1) is a limited layer, thus it is assumed that dynamical variations in the direction of the axis OZ are of this character as in an infinite medium, while the variations along the axis OX (similarly along axis OY) have a transverse mode character with wave number \( q = \pi/L \) as mentioned before.

The external force in the motion equation (1) for nematics is the force of acoustic flow \([9]\) i.e. the change of momentum of an acoustic particle averaged in time, which for an unlimited medium is described by the formula (9)

\[ F = \frac{2 \alpha J}{c} \]  \hspace{1cm} (4)

\( \alpha, c, J \) absorption coefficient, propagation velocity and intensity of an ultrasonic wave, respectively.

For a limited medium one can represent the acoustic flow force distributed along the axis OX as a sum of components (modes) and express it by the Fourier series (as it was done by NPC \([16]\)) in the form

\[ F_z = F_m e^{i q mx} \]  \hspace{1cm} (5)

where

\[ F_m = \frac{2 \alpha J}{m c} \]  \hspace{1cm} (6)

For the sample geometry (see Fig. 1 and the assumption 5) and the directions of the fields (magnetic and ultrasonic) and using assumption 2 as well as reference \([18]\) the variables can be presented in the form:

the total pressure together with components related to the flow

\[ p = p_0 + p(z) e^{i q mx} \]  \hspace{1cm} (6)

\[ \text{Fig. 2. — Dependence of flow velocity component against an ultrasonic wave intensity for several coordinates (numerical results).} \]
components of acoustic flow velocity
\[ v_2 = \left[ \frac{i}{q_m} \frac{d v_2(z)}{dz} e^{i q_m z}, 0, v_2(z) e^{i q_m z} \right] \]  

components of unitary orientation vector
\[ \mathbf{n} = [n_x, n_y, n_z] = [1, 0, \theta(z) e^{i q_m z}] \]

components of acoustic flow force
\[ \mathbf{F} = [0, 0, F_m e^{i q_m z}] . \]

The solutions of the nematodynamical equations were obtained for the following boundary conditions for acoustic flow velocity and unitary orientation vector
\[ v_2(z = 0) = v_2(z = d) = 0 \]
\[ v_2(z = 0) = v_2(z = d) = 0 \]  
\[ \theta(x, z = 0) = \theta(x, z = d) = 0 . \]

The analytical form obtained by the authors (see Appendix A1) for acoustic flow velocity and variations of long axes of molecules induced by an ultrasonic wave propagating in the liquid crystal layer is the following

\[ v_2(z) = \sum_{m=1}^{\infty} \left[ A_m e^{i q_m z} + B_m e^{-i q_m z} + C_m e^{i q_m z} + D_m e^{-i q_m z} - P_m \right] \cos q_m x ; \]  

\[ \theta(x, z) = \sum_{m=1}^{\infty} \frac{2}{mn} \left[ X_m e^{i q_m z} + Y_m e^{-i q_m z} + \frac{(\alpha_2 + l^2 \alpha_3)}{q_m k_{11}(\gamma_1^2 - l^2)} \left( A_m e^{i q_m z} + B_m e^{-i q_m z} \right) + \frac{(\alpha_2 + r^2 \alpha_3)}{q_m k_{11}(\gamma_1^2 - r^2)} \left( C_m e^{i q_m z} + D_m e^{-i q_m z} \right) + \frac{\alpha_3 P_m}{q_m k_{11} \gamma_1} \right] \sin q_m x . \]

The coefficients which appear here in separate components are rather complicated expressions depending on the viscoelastic constants of a liquid crystal, on the absorption and velocity of an ultrasonic wave, on magnetic field intensity (orienting field) and on ultrasonic field intensity (reorienting field) as well as on dimensions of the cell.

In formulae of such complicated form the explicit.

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**Fig. 3.** — Dependence of orientational angle against an ultrasonic wave intensity for several coordinates (numerical results).
dependence of the angle with the thickness of the cell has been lost. However, as is seen from formulae (AI.22) there appears the intensity of a magnetic field which is scaled by the magnetic field intensity at saturation, this intensity being a function of the thickness of the nematic layer. The full expressions determining the coefficients are given in appendix AII.

3. Numerical calculations. — The aim of these calculations was to determine \( v_{2z}(x, z) \), \( \theta(x, z) \) and their distributions in the experimental cell at different points in the \( XZ \) plane and for chosen intensities of magnetic and ultrasonic fields.

The viscoelastic constants for various nematics do not differ very much between themselves, and thus it was convenient to choose for numerical calculations the set of values of viscoelastic constants of MBBA as representative for the whole group of nematics because the values are the most certain in the literature [6].

For the calculations the following values were taken:

\[
\begin{align*}
    k_{11} &= 5.3 \pm 0.5 \times 10^{-7} \text{ dyn}, \\
    k_{33} &= 7.45 \pm 1.1 \times 10^{-7} \text{ dyn}, \\
    \alpha_1 &= 6.5 \pm 4 \text{ cp}, \\
    \eta_a &= 23.8 \pm 0.3 \text{ cp}, \\
    \eta_b &= 103.5 \pm 1.5 \text{ cp}, \\
    \eta_c &= 41.6 \pm 0.7 \text{ cp}
\end{align*}
\]

for elastic and viscosity constants,

0.08-0.80 T for magnetic field

(0.014-1.4). \( 10^4 \) W/m² by steps of 0.001 \( \times 10^4 \) W/m²

for ultrasonic field.

Based on the results for \( v_{2z}(x, z) \) and \( \theta(x, z) \) obtained from numerical calculations for various points at the plane \( XZ \) the dependence of acoustic flow velocity and variations of director orientation of a nematic on the ultrasonic intensity has been found. In the range of chosen values of ultrasonics intensity and for chosen points on the \( XZ \) plane the linear behaviour of \( v_{2z}(J) \) and \( \theta(J) \) has been reported. The results for different points in the \( XZ \) plane are presented in the figures 2 and 3.

In figure 4 the variations of acoustic flow \( v_{2z}(x, z) \) at \( x = \text{const.} \) and orientation deformation \( \theta(x, z) \) at \( z = \text{const.} \) are graphically represented for a given ultrasonic wave intensity in a map of model distribution.

4. Experimental. — Experimental examinations have been performed to verify the numerically calculated behaviour of variations of director orientation induced by a longitudinal ultrasonic wave propagating in the liquid crystal layer with acoustic intensity. It has been done in the range (0.05-0.90) \( \times 10^4 \) W/m².

The method of optical interferometry using a linearly polarized and coherent light beam was adapted, in parallel as well as in conoscopic vision. The experimental set up is shown in figure 5. The main element of the experimental set-up was the experimental cell of special construction to provide a proper transparency for light \( \lambda = 6328 \text{ Å} \) and conditions for longitudinal ultrasonic wave propagation. The ultrasonic field distribution inside the cell was examined using the schlieren method in the \(+1\) diffraction order [1] in water before filling it.

![Fig. 4. — Flow velocity \( v_{2z}(x, z) \) and orientational angle \( \theta(x, z) \) distribution under ultrasonic field (of \( 0.859 \times 10^4 \) W/m²)](image-url)
Fig. 5. — The experimental set-up.

with liquid crystal. Some special care was taken with mounting the transducer to make certain of the homogeneity of the acoustic field and to avoid reflections. Examinations were performed in the near field of the transducer.

The optical part of the set-up detected variations in phase shift between the ordinary and extraordinary beams of the polarized light appearing in a birefringence effect influenced by changes of orientation of molecules inforced by ultrasonics.

Experiments were performed in various liquid crystals and using measuring cells of different constructions.

In this paper only the results obtained for PCB (phenylcyanobiphenyl) (29.5° N 35.0° I) layer (placed in the cell, whose shape and dimensions are given in figure 1), are presented. In the case of PCB liquid crystal the reproducibility of the experimental results was the best, while for the other liquid crystals examined (MBBA, Merck 5, Merck 8, OCB) it was not so good. Below, the results obtained for the following data and conditions are presented: temperature (32 ± 0.01) °C, frequency of ultrasonics 10 MHz, thickness of the liquid crystal layer 1 x 10^{-3} m and 2 x 10^{-3} m, planar texture of the nematic layer formed with an external magnetic field.

5. Results. — Analysing the characteristic variations of optical interference patterns in relation to the ones of the initial state (at zero value of the ultrasonic intensity), the experimental dependence of the orientation of the optical axis of the sample with the ultrasonic wave intensity were determined.

Variations of the long axes of nematic molecules, described by the variations of the unitary orientation vector and observed as the variations of the optical axis induced by the ultrasonic wave, were found to be a linear function of the acoustic intensity within the range of intensities applied, which is in agreement with the mathematical description in the limit of assumed approximation. Figures 6 and 7 exemplify the dependence for two thicknesses of the liquid crystal layer (1 and 2) x 10^{-3} m, respectively.

Fig. 6. — Variation of orientational angle against an ultrasonic wave intensity (experimental results) for the thickness 1 x 10^{-3} m.

Fig. 7. — Variation of orientational angle against an ultrasonic wave intensity (experimental results) for the thickness 2 x 10^{-3} m.
6. Conclusions. — The results obtained experimentally as well as numerically have allowed to formulate the following conclusions:

1. Optically detected variations of the orientation of molecules of planarly oriented nematic liquid crystal sample induced by an ultrasonic field showed a good agreement with that predicted by the NPC theory and the assumptions that the acoustic flow due to radiation pressure is the main factor responsible for the acousto-optic effects.

2. In the range of intensities applied in the experiment the dependence of acoustic flow intensity as well as the dependence of director orientation variations (of long axes of molecules) are linear functions of the ultrasonic wave intensity.

3. These linear variations of the orientation for the planar texture have approached the order of few degrees in the range of applied ultrasonic intensities; there was a satisfactory agreement between the numerical calculations and the experimental results.

4. For greater intensities than reported above (beyond the linear regime) the dependence of orientation of molecules on ultrasonic wave intensity becomes rather complicated. There exists a threshold of linear behaviour which depends on the thickness of the liquid crystal layer. In the experiments the threshold values of intensities were found to be $J_a = 0.90 \times 10^4 \text{ W/m}^2$ for $d = 1 \times 10^{-3} \text{ m}$ and $J_a = 0.50 \times 10^4 \text{ W/m}^2$ for $d = 2 \times 10^{-3} \text{ m}$ respectively. Above the threshold for higher intensities a new turbulent phenomena (dynamic scattering) appears having firstly a chaotic and at still higher intensities a spatially periodic character [9, 14]. Examinations of these effects are being conducted.

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Appendix A. — To solve the complicated non-linear equations of nematodynamics with the definite boundary conditions, we assumed small deformations of initial orientation which allowed us to linearize the equations of motion. The linearized equations for the problem considered for separate components have the form equilibrium equation of forces in $OX$ axis

$$0 = \frac{\partial p}{\partial x} + \frac{\partial \sigma'_{xx}}{\partial x} + \frac{\partial \sigma'_{xz}}{\partial z} \quad (A1.1)$$
equilibrium equation of forces in $OZ$ axis

$$F_m e^{\psi_{mx}} = \frac{\partial p}{\partial z} + \frac{\partial \sigma'_{xz}}{\partial z} + \frac{\partial \sigma'_{zx}}{\partial x} \quad (A1.2)$$

with $\sigma'_{xx}$, $\sigma'_{zx}$, $\sigma'_{zx}$, $\sigma'_{xz}$: components of viscous stress tensor; $p$: acoustical pressure.

The equilibrium equation of moments of forces is

$$0 = \chi_a [(n \cdot H) n \times H]_r - [n \times (\alpha_3 - \alpha_2) N + (\alpha_6 + \alpha_5) A n]_r + [(h_s + h_n + h_t) \times n]_r \quad (A1.3)$$

where $h_s$, $h_n$, $h_t$, components of the molecular field which characterize the elastic equilibrium state.

The components of the viscous stress tensor are the following

$$\sigma'_{xx} = -(\alpha_1 + \alpha_4 + \alpha_5 + \alpha_6) \frac{d^2 v_{2x}(z)}{dz^2} e^{\psi_{mx}}\quad (A1.4)$$

$$\sigma'_{zx} = -\frac{i}{2} \alpha_3 \left( q_m - \frac{1}{q_m} \frac{d^2}{dz^2} \right) v_{2z}(z) e^{\psi_{mx}} + \frac{i}{2} (\alpha_4 + \alpha_6) \left( q_m + \frac{1}{q_m} \frac{d^2}{dz^2} \right) v_{2z}(z) e^{\psi_{mx}}\quad (A1.5)$$

$$\sigma'_{zz} = \alpha_4 \frac{d v_{2z}(z)}{dz} e^{\psi_{mx}}\quad (A1.6)$$

$$\sigma'_{xz} = -\frac{i}{2} \alpha_3 \left( q_m - \frac{1}{q_m} \frac{d^2}{dz^2} \right) v_{2z}(z) e^{\psi_{mx}} + \frac{i}{2} (\alpha_4 + \alpha_6) \left( q_m + \frac{1}{q_m} \frac{d^2}{dz^2} \right) v_{2z}(z) e^{\psi_{mx}}.\quad (A1.7)$$

The respective components in the equation (A1.3) are:

$$\chi_a [(n \cdot H) n \times H]_r = - H^2 (\chi_\parallel - \chi_\perp) \theta(z) e^{\psi_{mx}},\quad (A1.8)$$
After taking the derivative of the stress components (formula (AI.4)-(AI.7)) and acoustic pressure (formula (11)) the equations for the equilibrium of forces are as follows

$$0 = q_m p(z) e^{\eta n} - \frac{i}{2} \frac{\partial}{\partial z} \left( q_m - \frac{d^2}{dz^2} \right) v_{2z}(z) e^{\eta n} - \frac{i}{2} \left( \alpha_4 + \alpha_6 \right) \frac{d}{dz} \left( q_m - \frac{d^2}{dz^2} \right) v_{2z}(z) e^{\eta n}$$

$$- \frac{d}{dz} \left( q_m - \frac{d^2}{dz^2} \right) v_{2z}(z) e^{\eta n} + \frac{1}{2} \left( \alpha_4 + \alpha_6 \right) \frac{d}{dz} \left( q_m - \frac{d^2}{dz^2} \right) v_{2z}(z) e^{\eta n}$$

$$- q_m \left( \frac{\alpha_4 + \alpha_6}{2} \right) \frac{d^2}{dz^2} v_{2z}(z) e^{\eta n}.$$  \hspace{1cm} (AI.11)

$$F_m e^{\eta n} = \frac{dp(z)}{dz} e^{\eta n} + \frac{d^2}{dz^2} v_{2z}(z) e^{\eta n} + q_m \left( \frac{\alpha_4 + \alpha_6}{2} \right) \frac{d^2}{dz^2} v_{2z}(z) e^{\eta n} - q_m \left( \frac{\alpha_4 + \alpha_6}{2} \right) \frac{d^2}{dz^2} v_{2z}(z) e^{\eta n}.$$  \hspace{1cm} (AI.12)

The equation for the equilibrium of the moments of the forces (AI.3) after taking into the Parodi's relation

$$\alpha_2 + \alpha_3 = \alpha_6 - \alpha_5$$  \hspace{1cm} (AI.13)

is the following

$$\kappa_{11} \frac{d^2}{dz^2} \left( \frac{\alpha_2 q_m + \alpha_3}{q_m} \right) v_{2z}(z) e^{\eta n} =$$

$$- \frac{1}{2} \left( \alpha_2 q_m + \alpha_3 \right) \frac{d^2}{dz^2} v_{2z}(z) e^{\eta n} + \left( \chi_{2} - \chi_{1} \right) H^2 \theta(z) e^{\eta n} =$$

$$= - \frac{1}{2} \left( \alpha_2 q_m + \alpha_3 \right) \frac{d^2}{dz^2} v_{2z}(z) e^{\eta n}. \hspace{1cm} (AI.14)$$

The equation system (AI.11) and (AI.12) is then solved by the substitution method. The expression $dp(z)/dz$ obtained from equation (AI.11) after taking its derivative with respect to the $z$-coordinate is introduced to equation (AI.12) which then appears in the form

$$\frac{d^2}{dz^2} v_{2z}(z) e^{\eta n} - q_m \left( \frac{\alpha_4 + \alpha_6}{2} \right) \frac{d^2}{dz^2} v_{2z}(z) e^{\eta n} + q_m \eta_c \eta_c$$

where $\eta_b, \eta_c$ are given respectively by

$$\eta_b = \frac{\alpha_3 + \alpha_4 + \alpha_6}{2},$$

$$\eta_c = \left( - \frac{\alpha_2 + \alpha_4 + \alpha_5}{2} \right). \hspace{1cm} (AI.16)$$

The particular integral of the equation (AI.15) has the form

$$v_{2z}(z) e^{\eta n} = A_m e^{\eta n} + B_m e^{-\eta n} + C_m e^{\eta n} + D_m e^{-\eta n} - \frac{F}{q_m \eta_c} \hspace{1cm} (AI.18)$$

where

$$l = \pm \left[ \left( \alpha_1 + \eta_b + \eta_c \right) + \left( \left( \alpha_1 + \eta_b + \eta_c \right)^2 + 4 \alpha_1 \eta_c \right)^{1/2} \right]$$

$$r = \pm \left[ \left( \alpha_1 + \eta_b + \eta_c \right) - \left( \left( \alpha_1 + \eta_b + \eta_c \right)^2 + 4 \alpha_1 \eta_c \right)^{1/2} \right]. \hspace{1cm} (AI.19)$$

$$\eta_b, \eta_c$$ are calculated from the boundary conditions (see the formula (15)) and their form is given in the appendix All. For the known $z$-component of the acoustic flow velocity (formula
(12)) with equation (AI.14) one can find the expression describing the spatial variations of the orientation of the long axes of nematic molecules under the action of the moments of viscous forces due to second order hydrodynamical flows (acoustic flow) as well as of the moments of magnetic field forces.

The particular equation of the non-homogeneous equation (AI.4) has the form

$$\theta(x, z) = X_m e^{i \theta_{mz}} + Y_m e^{-i \theta_{mz}} - \frac{i(\xi_2 + l^2 \alpha_3)}{q_m k_11 (\gamma_1^2 - l^2)} (A_m e^{i \theta_{mz}} + B_m e^{-i \theta_{mz}}) - \frac{i(\xi_2 + r^2 \alpha_3)}{q_m k_11 (\gamma_1^2 - r^2)} (C_m e^{i \theta_{mz}} + D_m e^{-i \theta_{mz}}) - \frac{i\alpha_2 \gamma_m}{q_m k_11 \gamma_1}$$  \hspace{1cm} (AI.21)

where

$$\gamma = \pm q_m \left[ \frac{k_{33}}{k_{11}} - \frac{\gamma_0}{k_{11}} \left( \frac{H}{q_m} \right)^2 \right]^{1/2} = \pm q_m \gamma_1.$$  \hspace{1cm} (AI.22)

The constants $X_m$, $Y_m$ were determined from the boundary condition (formula (10)) and their form is given in appendix AII.

Appendix AII. — The constants $A_m$, $B_m$, $C_m$, $D_m$, $X_m$ and $Y_m$ are, respectively, equal to:

$$A_m = \frac{r P_m}{\theta_m} \left[ 2l - (l - r) e^{-l+2i} - (l + r) e^{l+2i} - (l - r) e^{-2l+2i} - 2l e^{-2l+2i} \right];$$  \hspace{1cm} (AII.1)

$$B_m = \frac{r P_m}{\theta_m} \left[ 2l - (l + r) e^{2l+2i} - (l - r) e^{2l+2i} - (l + r) e^{-2l+2i} + 2l e^{2l+2i} \right];$$  \hspace{1cm} (AII.2)

$$C_m = \frac{i P_m}{\theta_m} \left[ 2r + (l - r) e^{2l+2i} - (l + r) e^{2l+2i} + (l - r) e^{-2l+2i} - (l + r) e^{-2l+2i} + 2r e^{-2l+2i} \right];$$  \hspace{1cm} (AII.3)

$$D_m = \frac{i P_m}{\theta_m} \left[ 2r + (l + r) e^{2l+2i} - (l - r) e^{2l+2i} + (l + r) e^{-2l+2i} + (l + r) e^{-2l+2i} + 2r e^{-2l+2i} \right];$$  \hspace{1cm} (AII.4)

where

$$\theta_m = 2(r + l)^2 \cosh [(l - r) q_m d] - 2(r - l)^2 \cosh [(l + r) q_m d] - 8lr$$  \hspace{1cm} (AII.5)

$$P_m = F/q_m^2 \eta_0,$$  \hspace{1cm} (AII.6)

$$X_m = \frac{-i}{2 \sinh \gamma d} \left\{ \frac{1}{q_m k_{11}} \left( \frac{\alpha_2 + l^2 \alpha_3}{\gamma_1^2 - l^2} \right) \left[ A_m e^{i \theta_{mz}} - e^{-i \theta_{mz}} \right] + \frac{1}{q_m k_{11}} \left( \frac{\alpha_2 + r^2 \alpha_3}{\gamma_1^2 - r^2} \right) \left[ C_m e^{i \theta_{mz}} - e^{-i \theta_{mz}} \right] \right\} +$$

$$\frac{i}{2 \sinh \gamma d} \left\{ \left( \frac{\alpha_2 + l^2 \alpha_3}{\gamma_1^2 - l^2} \right) \left[ B_m e^{i \theta_{mz}} - e^{-i \theta_{mz}} \right] + \frac{i}{q_m k_{11}} \left( \frac{\alpha_2 + r^2 \alpha_3}{\gamma_1^2 - r^2} \right) \left[ D_m e^{i \theta_{mz}} - e^{-i \theta_{mz}} \right] \right\} +$$

$$Y_m = \frac{i}{2 \sinh \gamma d} \left\{ \left( \frac{\alpha_2 + l^2 \alpha_3}{\gamma_1^2 - l^2} \right) \left[ A_m e^{i \theta_{mz}} - e^{-i \theta_{mz}} \right] + \frac{i}{q_m k_{11}} \left( \frac{\alpha_2 + r^2 \alpha_3}{\gamma_1^2 - r^2} \right) \left[ B_m e^{i \theta_{mz}} - e^{-i \theta_{mz}} \right] \right\} +$$

$$\frac{i}{2 \sinh \gamma d} \left\{ \left( \frac{\alpha_2 + l^2 \alpha_3}{\gamma_1^2 - l^2} \right) \left[ C_m e^{i \theta_{mz}} - e^{-i \theta_{mz}} \right] + \frac{i}{q_m k_{11}} \left( \frac{\alpha_2 + r^2 \alpha_3}{\gamma_1^2 - r^2} \right) \left[ D_m e^{i \theta_{mz}} - e^{-i \theta_{mz}} \right] \right\} + \frac{\alpha_2 \gamma_m}{q_m k_{11} \gamma_1} (1 - e^{-2i \theta_{mz}}).$$  \hspace{1cm} (AII.7)

References


