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Diffusive modes in Rayleigh-Bénard structures

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Résumé. — Nous présentons un ensemble de mesures expérimentales qui permettent de caractériser les propriétés dynamiques des structures convectives de Rayleigh-Bénard lorsque le nombre de Prandtl du fluide est grand et que le nombre de Rayleigh est légèrement supercritique. Nous montrons que ces mesures sont analogues à celles que l'on fait dans les cristaux sur les phonons. Néanmoins nous mettons en évidence le fait que, comme le prévoit la théorie, les modes de phase sont diffusifs dans la convection de Rayleigh-Bénard. Nous déterminons, quand cela est possible, le coefficient de diffusion associé $D_{\parallel}$ ou ses propriétés fondamentales dans le cas de $D_{\perp}$. Nous trouvons que nos résultats sont en accord avec les calculs théoriques.

Abstract. — We present experimental measurements of the dynamical properties of Rayleigh-Bénard convective structures in the case of high Prandtl number fluid and slightly supercritical Rayleigh number. We show that these measurements can be carried out like a phonon study in a crystal. However we demonstrate that, as predicted, the dynamics of phase modes is actually diffusive in Rayleigh-Bénard convection. When possible we determine the associated diffusion coefficient $D_{\parallel}$ or its main characteristic in the case of $D_{\perp}$. We find that these results are in agreement with the theoretical predictions.

1. Introduction. — In the field of the Rayleigh-Bénard instability, some very important works have been devoted to the study of the amplitude of the convection in an infinite structure and to structural instabilities of the pattern. This structure and its properties, when stable, have motivated relatively few works; this has been so, until « stable » structures have turned out to be turbulent in some cases [1]. In fact, the degrees of freedom of the Rayleigh-Bénard structures have been underestimated and this may be understood if we remark that these degrees of freedom are associated, not with the amplitude of the convective rolls, but rather with their position. In this work we have tried to exhibit these various degrees of freedom of a Rayleigh-Bénard structure in the high Prandtl number case. We show that these degrees of freedom may be seen as phase modes, the dynamics of which is actually diffusive. The experimental confirmation of this diffusion is of the first importance, since

this is the starting point of the understanding of defects, which are suspected to be responsible for the turbulence behaviour observed in large containers.

2. Phase diffusion mechanism. — In the Rayleigh-Bénard instability, convective motions are under certain conditions spatially organized in a fairly regular roll structure [3]. As long as the Rayleigh number Ra lies in the vicinity of its critical value, this convective structure may be modelled by a single horizontal Fourier component modulated by an amplitude function [5, 4]. Far from any boundary this means that the velocity, in the middle-height plane, takes the very simple form :

$$\begin{aligned} V_z(x, y, z = d/2) &= V_0 \cos(Kx + \varphi) \\ V_x &= V_y = 0 \quad [3]. \end{aligned} \quad (1)$$

This periodic structure suggests an analogy with a simple crystal or a smectic. This analogy is even more vivid when we consider defects in such a structure; figure 1 presents a usual natural R.B. convective structure : as in ordinary crystals we find dislocations and other structural defects.

Fig. 1. — « Natural » structure observed in a rectangular container ($20d \times 12d$) with a high Prandtl number fluid (130) and at a slightly supercritical Rayleigh number ($Ra/Ra_c = 2$).

With this analogy, phase diffusion is just the equivalent of the dynamics of elastic waves in a crystal : that is, if we slightly disturb the roll positions in a perfect structure, phase diffusion describes the way this structure will respond to the small roll displacement.

Let us follow this analogy to describe more completely the phase diffusion mechanism.

First, the crystal itself : as we have seen in the R.B. case it is a two-dimensional crystal, periodic in the x direction and homogeneous in y direction. It is convenient to consider it in the reciprocal space : only two Bragg peaks at $+K$ and $-K$ in the $\hat{x}$ direction (with $K = 2\pi/\Lambda$, Λ being the wavelength of the structure).

Second, the deformation modes : in a bidimensional crystal we expect four basic modes of deformation, corresponding to the two kinds of polarization, transverse and longitudinal, in both directions $\hat{x}$ and $\hat{y}$. However in the Rayleigh-Bénard structure, the absence of modulation in the y direction leads to the disappearance of modes associated with a polarization along y . Thus, as in the smectics, we only have to consider two modes : one with its wavevector $\mathbf{q}_{\parallel}$ along $\hat{x}$ axis, the other one having its wavevector $\mathbf{q}_{\perp}$ along $\hat{y}$ axis and both with their polarization along x axis. In real space, the first mode corresponds to a periodic compression and expansion of the roll structure (see Fig. 2a) like a sound wave in a fluid, it is a longitudinal mode. The second mode corresponds to a « zig-zag » shape of the roll structure (see Fig. 2b) like the one described by F. H. Busse [6], it is a transverse mode.

Fig. 2. — a) Description of a longitudinal mode in the real and reciprocal space. b) A transverse mode.

Fig. 3. — Instability diagram for an infinite Rayleigh-Bénard structure. The full line parabola constitutes the marginal stability boundary for the convection, the broken line is the zig-zag instability boundary, the dotted parabola is the Eckhaus instability boundary.

We remark that a roll displacement is easily described, in equation (1), by a modulation of the phase factor « φ ». Thus it is natural to speak of « phase modes », for instance longitudinal modes are conveniently modelled by a phase modulation in the x direction :

$$\varphi = \varphi_0 \cos(q_{\parallel} \cdot x). \quad (2)$$

A transverse mode is associated with a phase modulation in the y direction :

$$\varphi = \varphi_0 \cos(q_{\perp} \cdot y). \quad (3)$$

Third, the dynamics of these modes : the analogy with an ordinary crystal breaks down here. The usual propagation equation, for the modes of small wave-vector $\mathbf{q}$, becomes, in the Rayleigh-Bénard case, a diffusion equation (when the Prandtl number is large). From a very simple point of view, at high Prandtl number P , the rolls behave as if the viscosity completely damped their inertial effect, so that the elasticity of the structure is mainly balanced by the viscous effect, leading to a diffusion equation.

This equation has been derived, by Y. Pomeau and P. Manneville, from a model equation [7]. In the Rayleigh-Bénard case they propose :

$$\frac{\partial \varphi}{\partial t} = D_{\parallel} \frac{\partial^2 \varphi}{\partial x^2} + D_{\perp} \frac{\partial^2 \varphi}{\partial y^2} \quad (4)$$

with

$$D_{\parallel} = \left(\frac{\xi_0^2}{\tau_0} \right) \times \left(\frac{\varepsilon - 4.31 \delta^2/k^2}{\varepsilon - 1.44 \delta^2/k^2} \right) \quad (5)$$

$$D_{\perp} = \left(\frac{\xi_0^2}{\tau_0} \right) \times \left(\frac{\delta}{k_c} \right) \quad (6)$$

$\xi_0 = 0.385 d$; $\tau_0 = (d^2/D_{\text{th}}) \times (1.95 P + 1/38.44 P)$; $\delta = k - k_c$; $k_c = 3.117 d^{-1}$.

$D_{\parallel}$ is the diffusion coefficient for longitudinal modes. Its expression contains two factors : the first one (between the first brackets) reflects its dependence upon the fluid under experiment : $\xi_0^2/\tau_0 = 2.92 D_{\text{th}}$. The second contribution is associated with the structure properties : ε and δ . This factor reaches its maximum (1) when $\delta = 0$, then it decreases with increasing $|\delta|$, equals zero on the Eckhaus instability boundaries and is negative beyond [8] (see Fig. 3).

$D_{\perp}$ is the diffusion coefficient for transverse modes. $D_{\perp}$ equals zero when $\delta=0$, it is positive when the roll structure is stable against zig-zag instability and negative in the zig-zag instability region, see figure 3.

An analogous equation has been also derived by E. D. Siggia from the amplitude equation of Newell and Whitehead, see equation (3.2b) in reference [2]. This equation is interesting since it provides a complementary term in $\partial^4 \varphi / \partial y^4$ and also describes the coupling between amplitude and phase field.

Fourth, dispersion relations : we obtain them just by Fourier transforming the diffusion equation (4).

For longitudinal modes we get :

$$i\omega = -D_{\parallel} q_x^2 \quad (7)$$

as for transverse modes

$$i\omega = -D_{\perp} q_y^2. \quad (8)$$

Here we can point out the relationship between phase diffusion and zig-zag instability and Eckhaus instability. The phase diffusion just concerns the very beginning of the dispersion relations, that is, it describes the dynamics of the large wavelength phase modulations. The complete dispersion relation may be found in equations (3.10) of reference [5]. With this dispersion relation, Newell and Whitehead have pointed out the most unstable longitudinal or transverse mode which gives rise respectively to the Eckhaus and zig-zag instability. A very important feature of these instabilities is that precisely at their boundaries, the most unstable mode is such that $q \rightarrow 0$ when $\omega \rightarrow 0$, thus constituting a hydrodynamic mode. This explains why the Eckhaus and zig-zag marginal boundaries just coincide with respectively : $D_{\parallel} = 0$ and $D_{\perp} = 0$. As $D_{\parallel}$ and $D_{\perp}$ measure the stiffness of the roll structure *versus* compression or roll displacement, we may wonder if such perturbations only affect the phase of the structure; in fact, there is a coupling with the amplitude of the roll structure; however in most cases this coupling may be dropped except in some peculiar cases like Eckhaus regions or abrupt phase variations.

3. Experimental set-up. — To study the dynamical properties of R.B. convective structures, it would be convenient to use an infinite R.B. « crystal ». However real experiments are far from that condition and all the experiments described in this paper have been performed with a container of « medium aspect ratio » : $Lx = 20 d$, $Ly = 12 d$, $d = 5 \text{ mm}$.

The fluid layer is bounded by a plexiglass frame of rectangular shape, sandwiched between a polished copper plate, at the bottom, and a sapphire plate, at the top. These two plates are temperature controlled by water circulation, the temperature difference being monitored with an accuracy of $2 \times 10^{-2} \text{ }^{\circ}\text{C}$. Plexiglass sidewalls were chosen since plexiglass has nearly the same thermal conductivity as the silicone oil used for the fluid. This silicone oil has the following physical properties : thermal diffusivity

$$D_{\text{th}} = 7.65 \times 10^{-4} \text{ cm}^2/\text{s},$$

kinetic viscosity $\nu = 10 \text{ cst}$ (at $25 \text{ }^{\circ}\text{C}$), giving a Prandtl number $P = 130$. The sapphire plate offers two interesting features : it is a good thermal conductor [9] but nevertheless it is transparent. Associated with the polished copper plate it is possible to perform optical visualization of the convective pattern by focalization

effects, velocity measurements by laser Doppler anemometry and also structure induction using the method developed by Chen and Whitehead [10]. This method relies on the high sensitivity of the fluid layer to thermal perturbations when the Rayleigh number is in the vicinity of its critical value. In such a situation it is relatively easy to trigger the convection pattern by producing thermal perturbations corresponding to this pattern. This is achieved by illuminating the cell, during a time equal to several times the thermal vertical diffusion time, by a powerful light beam passing through a grid (see Fig. 5). The temperature difference may then be increased to its final value greater than ΔT_c so that the convection grows everywhere in the cell with its pattern, image of the grid. The light source may then be removed so that the time evolution of the induced pattern can be followed. As described in reference [19] our experiments consist of inducing structures with a longitudinal or a transverse phase modulation and in studying the evolution of this modulation; with this study we have determined the dynamics of both modes, and this is the purpose of the two last sections of this paper.

4. Unperturbed structure, natural structure and wave-number selection. — As we have already said, our container is far from infinitely large and the sidewalls have very important effects as we will explain in the transverse mode study. Our first experiments have been performed to evaluate these effects on Rayleigh-Bénard structures. These experiments may be separated into two kinds : natural structures and regular structures induced with various wavevectors $\mathbf{K}$.

We call natural structures, those which appear when we have not used any induction procedure to trigger a pattern, that is to say we have just increased ΔT very slowly ($10^{-3} \times \Delta T_c$ per min.) above ΔT_c . In these conditions the structure is not perfectly regular, there is a tendency of the roll axis to be perpendicular to the largest dimension of the container but many possible structures do exist. Two conditions are always satisfied : the rolls appear such that at and near the sidewalls their axes are perpendicular to these walls and have a relatively uniform width $\lambda/2$ over all the structure. Figure 1 presents a typical natural structure; we see that defects are present in this structure, this is not unreasonable since it is difficult to fulfil these two conditions simultaneously. The exact shape of the pattern changes at random for each new experiment as far as we know. Such structures may be time dependent during the one or two days following their build up but after this evolution, where one dislocation may disappear for example, the structure seems to be stationary, at least on the time scale of a few days with our experimental conditions, a complete description of these structures is given in reference [11]. We think that our results are in complete agreement with those reported by Gollub

in [12]. We emphasize that it is relatively easy to bend rolls in our Rayleigh-Bénard structures.

The transverse mode study requires structures having different structural wavevector $\mathbf{K}$; the induction process allows the preparation of such structures, however this wavevector undergoes a natural evolution with time [8], and this is a problem to consider before studying the transverse modes. The induction process gives the possibility of preparing structures with a prescribed wavevector; this process is successful when $\mathbf{K}$ is not too different from $\mathbf{K}_c$, that is for $\Delta \mathbf{K}/K_c \simeq 25\%$. The mechanism of the wavevector evolution seems to be governed mainly by the rolls near the sidewalls : one roll has to appear or to disappear at the sidewall but never elsewhere, and this point is in agreement with recent calculations [13, 14, 15]. In fact the usual mechanism is somewhat more complicated than the proposed one : instead of creating one roll, the structure finds it easier to build a row of perpendicular small rolls (see Fig. 4) which are far more efficient for wavenumber adjustment since it is a continuous process. Such a process has been analysed in the paragraph 4.4 of reference [14]. This kind of mechanism occurs each time the wavevector is smaller than the natural one. When the wavevector is larger than the natural one, extreme rolls collapse at the sidewall or break into small rolls. It seems that even with our relatively large container the wavevector selection is quite efficient and determines one wavevector by the adjustment possibility given by the small perpendicular rolls. There does exist some wavenumber variation, mainly due to the defects in the structure.

Considering the transverse mode study, this natural evolution of the structural wavevector $\mathbf{K}$ has to be avoided since the dynamics that we are studying depends strongly on the structural wavevector. Thus, we have to maintain this wavevector constant during the experiment in order to get coherent results. In the case where the wavevector is larger than the

Fig. 4. — Sidewall effect : the rows of small rolls perpendicular to the sidewalls are a very efficient mechanism for the wavenumber selection ($Ra/Ra_c = 1.6$).

Fig. 5. — Sketch of the experimental set-up for structure induction, the arrows stand for the powerful light illumination. The grid represented will give a longitudinal mode.

critical one K_c , we use the metastability of those structures. If the grid is perfectly located with respect to the cell sidewalls, the structure remains unchanged during one or two hours, a duration which is sufficiently long to allow the experiment. In the case of a wavevector smaller than K_c we have to use a trick in order to maintain such a wavevector. Since the instability always starts from the sidewall at $x = 0$ and $x = Lx$, we impose a rising flow at each sidewall by a local horizontal temperature gradient, by means of a powerful illumination at the sidewalls during the experiment. This process appears to be convenient in order to maintain the wavevector different from K_c . A sketch of the set-up is given in figure 5.

5. Study of the longitudinal modes. — This study deals with the determination of the dispersion relation $\omega = f(q)$ of these modes. If they actually diffuse, as predicted, this relation would be : $i\omega = D_{||} q^2$ (which is the Fourier transform of the diffusion equation). Although we have already performed this study earlier [16], it is interesting to test the « imposed q technique ». In our previous experiment, a temporal phase modulation was realized by a small periodic oil stream, injected at $x = 0$ in the structure. The propagation of this disturbance was measured and, by a careful study of the attenuation and phaseshift along the x axis, the dispersion relation of these modes has been worked out. We can call this method « imposed ω technique » since the physical parameter, which is controlled, is the frequency « ω » of the stream, and the physical variable that is measured, is, in fact, the wavevector « q » of this propagation. In the present experiment we have done the reverse, so that we can refer to it as the « imposed q technique ». We have used it because it extends obviously to the case of the transverse mode while the « imposed ω technique » does not.

Fig. 6. — Sketch of the experimental process used to maintain a defined wavenumber in a structure.

In order to prepare a structure with a phase modulation of wavevector « q », we have to draw the corresponding grid. This requires considering the problem of the finite dimension of the container and the boundary conditions. This problem is not obvious and it will be one limitation of the method; in the case of an unperturbed structure the velocity field, for relatively small Rayleigh number in an infinite geometry may be conveniently described by equation (1), the corresponding grid will correspond to a transparent area each time V_z equals V_0 , see figure 7a. In these experiments we consider patterns with a rising flow near the sidewalls, that is, we have placed a transparent area in the grid at both ends; this may be justified since the correlation length ξ is quite small for the Rayleigh number of our experiments ($\frac{R - R_c}{R_c} = 0.6$, $\xi = 0.7 d$). (This implies that the

a)

b)

Fig. 7. — a) Longitudinal mode grid, $K = K_c$, $q_{\parallel} = K_c/5$.
b) Transverse mode grid, $K = 1.1 K_c$, $q_{\perp} = K_c/3$.

amplitude of the convection reaches its normal amplitude at a distance of approximately one roll from the wall). The second point to consider is the phase modulation; as the rolls at $x = 0$ and $x = Lx$ cannot move due to the sidewalls, we have chosen to impose $\varphi = 0$ at $x = 0$ and $x = Lx$, that is a node on each sidewall. Thus we have drawn grids corresponding to the following velocity field :

$$V_z = V_0 \cos(Kx + \varphi), \quad \varphi = \varphi_0 \sin q_{\parallel} x \quad (9)$$

with

$$K = N \cdot \frac{\pi}{Lx} \quad \text{and} \quad q_{\parallel} = n \cdot \frac{\pi}{Lx}$$

N is just the number of rolls in the container.

In order to minimize the boundary effects, we have measured the phase evolution as far as possible from the lateral walls of the container. Another condition to fulfil is the slow variation of the phase with x : this is achieved if $q_{\parallel}$ is much smaller than K , that is

if n is small; we have chosen $n = 2$ and $n = 4$ (with $N = 20$ which corresponds to the critical wavevector).

The induction of these two structures is relatively simple since their wavevector $\mathbf{K}$ corresponds to $\mathbf{K}_c$; the focalization images give a good idea of the roll structure, therefore it is possible to appreciate, during the experiment, if the structure is correctly modulated, how it relaxes, or if a structural defect appears. Simultaneously, the velocity measurements, at one point of the structure, give the phase evolution at that point. When V_z is nearly null, its exact value is directly proportional to the phase φ . As long as φ does not vary too much, the velocity recordings reproduce the phase evolution. In order to get a good contrast of the focalization image and sufficiently large velocity variations we have to work with ε large enough (0.6) which may be a difficulty in comparing with the theory.

Fig. 8. — Recording of the velocity relaxation arising just after the induction of a longitudinal modulated structure ($\varepsilon = 0.6$, $K = K_c$, $q_{\parallel} = K_c/10$).

In these experiments the phase evolution has been found to be an exponential relaxation. That means that the phase modulated structure relaxes towards a regular structure of N rolls, with a characteristic time τ_n , see figure 8. This is in complete agreement with the dispersion relation (7): the time evolution, for a structure, modulated with a wavevector $q_{\parallel}$, is given by $e^{i\omega t}$ with $\omega = -iD_{\parallel} q_{\parallel}^2$, this leads to an exponential decay: $e^{-D_{\parallel} q_{\parallel}^2 t}$ with a characteristic time $\tau_n = (D_{\parallel} q_{\parallel}^2)^{-1}$. The two experiments were repeated several times, the average characteristic times they supply are :

$$\tau_{n=2} = 1\,600 \text{ s} \pm 500 \text{ s} \quad \text{and} \quad \tau_{n=4} = 350 \text{ s} \pm 100 \text{ s}.$$

The accuracy of these experiments is not as good as for the first measurements [13]; this is mainly due to the sensitivity of the exponential fits *versus* their base lines. The base line corresponds to the final value of the velocity when the modulation has disappeared. This value is very sensitive to a misfit of the grid with respect to the sidewalls of the cell, a misfit which produces a slow relaxation of structure towards an equilibrium state.

However, these results are in good agreement with the phase diffusion theory : the quadratic behaviour of τ_n versus « q » or « n » appears clearly. The experimental value of the diffusion coefficient

$$D_{\parallel \text{exp}} = 1.7 \pm 0.5 \times 10^{-3} \text{ cm}^2/\text{s}$$

is in good agreement with the theoretical prediction $D_{\parallel \text{th}} = 2.23 \times 10^{-3} \text{ cm}^2/\text{s}$. We have also tried to measure $D_{\parallel}$ for different $\mathbf{K}$ vectors, but we have found no appreciable variations within the accuracy of our measurements. This is consistent with the theory since $D_{\parallel}$ is predicted to be at a maximum when $\mathbf{K} = \mathbf{K}_c$ and with the fact that we cannot change $\mathbf{K}$ very much around $\mathbf{K}_c$. We have also tried to measure $D_{\parallel}$ at lower Rayleigh number : $(R - R_c)/R_c \simeq 0.3$, once more we have found no appreciable variations, within the accuracy of our experiments.

6. Transverse mode study. — As we have already said, this study is only possible using the « q imposed technique » and our aim is to determine the dispersion relation of these modes, predicted to be : $i\omega = D_{\perp} q_{\perp}^2$. In fact this study is more complicated than the longitudinal one, and we have not actually measured the dispersion relation (8). The difficulties arise from the critical dependence of $D_{\perp}$ with the structural wavevector $\mathbf{K}$, and from the effect of the boundary conditions. Thus we have restricted our study to what has appeared to us as the most important feature of these transverse modes ; that is the dependency of $D_{\perp}$ with the structural wavevector $\mathbf{K}$. On the other hand we show that the phase diffusion theory gives the possibility of understanding quantitatively the effect of boundary conditions.

Let us first consider the grid drawing problem. A transverse mode is just a zig-zag roll structure like figure 2b, but we have to fit this zig-zag in our rectangular box. Thus the phase modulation has to cancel in the vicinity of the sidewalls on $x = 0$ and $x = Lx$; this is achieved by multiplying the phase modulation along y by a sinus function which cancels on $x = 0$ and $x = Lx$, that is $\sin \frac{\pi}{Lx} \cdot x$. Another boundary condition came from the observation of the spontaneous structure : the rolls were always perpendicular to the sidewalls, this suggests the choice of $d\varphi/dy = 0$ at $y = 0$ and $y = Ly$.

In our experiments we have used a phase modulation defined by :

$$\varphi(x, y) = \varphi_0 \sin\left(\frac{\pi}{Lx} \cdot x\right) \cdot \sin\left(\frac{\pi}{Ly} \cdot y\right) \cdot \sin(q_{\perp} \cdot y) \quad (10)$$

with

$$V_z = V_0 \cos(Kx + \varphi(x, y))$$

and

$$K = N \cdot \frac{\pi}{Lx} \quad \text{and} \quad q_{\perp} = n \cdot \frac{\pi}{Ly}.$$

A grid corresponding to this pattern with $N = 22$ and $n = 4$ may be seen in figure 7b.

Unfortunately it is easily seen that supplementary modulations imposed by the boundary conditions imply that we are looking for the relaxation of a set of modes instead of a pure transverse mode as described in figure 9.

Fig. 9. — Sketch in the reciprocal space of a pure and an experimental transverse mode.

The temporal evolution of this wave packet is no longer described by the simple dispersion relation (8) and we have to consider the complete dispersion relation $i\omega = D_{\parallel} q_{\parallel}^2 + D_{\perp} q_{\perp}^2$ (11) in order to take into account the effect of the longitudinal component of the wavevectors. On the other hand we would have to consider the time evolution of each of the modes (or more precisely of each couple of modes) since they have not the same dynamics, and combine their evolution to describe the phenomena.

Actually the accuracy of the experiments is not good enough to evidence this behaviour. In fact another difficulty comes from the wavevector $\mathbf{K}$ of the structure : this wavevector has to be varied and we have already said that there is a natural tendency of this wavevector to come back towards its critical value $\mathbf{K}_c$.

Another problem arises with the smallness of $D_{\perp}$, as we will see in the experimental results. This means that the experiments are really only sensitive to small perturbations and a systematic study of the dispersion relation versus $q_{\perp}$ is not possible with our experimental conditions ; the quadratic behaviour with $q_{\perp}$ cannot be checked for example. What we have actually measured is the dependence of $D_{\perp}$ with the structural wavevector $\mathbf{k}$. To do that, we have kept $q_{\perp}$ constant during the experiments and measured the time evolution versus $\mathbf{K}$.

We have found that this time evolution was once more an exponential relaxation (see Fig. 10), or no evolution with time, or an amplification of the modulation. This may be summarized in the following table :

K/K_c	1.1	1	0.9	0.8
	500 s	1 600 s	> 3 600 s	amplification

We see that when $K/K_c = 0.9$, τ becomes very large ; this just means that the modulation does not really

Fig. 10. — Relaxation of a transverse mode when $K = 1.1 K_c$, $q_{\perp} = K_c/3$. The first focalization picture has been taken just after the induction, the second one minute later, the third one minute later.

relax but remains unchanged during at least the experimental time. When $K/K_c = 0.8$, the modulation grows rather than it relaxes, theoretically this amplification would follow an exponential law; however experimentally this modulation saturates relatively quickly and a characteristic time is difficult to determine. These results are in agreement with the « zig-zag » instability analysis of Busse [6]; when $K > K_c$ the structure is stable, when $K < K_c$ the structure is unstable *versus* « zig-zag » instability. In fact we have found that the actual boundary of the « zig-zag » instability is shifted towards a smaller value of the structural wavevector $\mathbf{K}$ ($0.9 K_c$). As we will see, this may be explained by the phase diffusion theory which also explains the dynamics of this instability.

If we consider the modes excited in our experiment,

they have the following form $\begin{pmatrix} \pm \pi/Lx \\ q_{\perp} \pm \pi/Ly \end{pmatrix}$ where the effect of π/Ly can be neglected in a first approximation. The dynamics of the modes $\begin{pmatrix} \pm \pi/Lx \\ q_{\perp} \end{pmatrix}$ is defined by the following expression :

$$i\omega = D_{\parallel} \left(\frac{\pi}{Lx} \right)^2 + D_{\perp} q_{\perp}^2. \quad (12)$$

If we introduce the expression of $D_{\perp}$ and consider that $D_{\parallel}$ is constant for the wavevectors $\mathbf{K}$ that we are studying (this is the case since $D_{\parallel}$ is maximum at $K = K_c$).

We get :

$$i\omega = D_{\parallel} \left[\left(\frac{\pi}{Lx} \right)^2 + \frac{K - K_c}{K_c} \times q_{\perp}^2 \right].$$

The « zig-zag » instability occurs when $i\omega$ becomes negative; then K has to be smaller than K_c in order to balance the positive contribution of $(\pi/Lx)^2$. It is clear then that the « zig-zag » threshold shift is an effect of the finite dimension of the container. The phase diffusion gives here a quantitative value to the stabilizing effect of the sidewall. It is to be noticed that this effect is of importance even in a relatively large container (π/Lx small) since in usual R.B. convection the structural wavevector $\mathbf{K}$ may be very close to K_c .

Neglecting the effect of π/Ly is justified since $q_{\perp}$ is four times larger than π/Ly and also because the effect of π/Lx is magnified by the factor $D_{\parallel}$ which is considerably greater than $D_{\perp}$ when $K \simeq K_c$.

From our measurements it is possible to check the validity of the expression (6). If we plot the characteristic frequency $i\omega$ that is τ^{-1} *versus* the structural wavevector $\mathbf{K}$, we must find a straightline with a slope equal to $D_{\parallel} q_{\perp}^2$. This corresponds to figure 11; obviously this measurement is qualitative since we just display 3 points, but the mean value of this slope divided by $q_{\perp}^2$ gives us a new independent determination of $D_{\parallel}$ which is once more in good agreement with the theoretical value : in this case

$$D_{\parallel \text{exp}} = 1.85 \times 10^{-3} \text{ cm}^2/\text{s} \quad (D_{\parallel \text{theor.}} = 2.23 \times 10^{-3})$$

This means, at least, that the proposed expression of $D_{\perp}$ (6) is qualitatively valid considering its dependency with the structural wavevector $\mathbf{K}$.

7. Conclusion. — In this study, we have shown the diffusive behaviour of the phase variable in Rayleigh-Bénard structures. To achieve this purpose, we have proceeded as a solid state physicist would have done with an ordinary crystal. In some degree, we have measured the equivalent of the sound velocity in the two directions of a Rayleigh-Bénard structure.

Fig. 11. — Characteristic frequency of transverse mode with $q_{\perp} = Kc/3$ versus the structural wavevector.

A priori, such measurements may appear to be completely disconnected with the turbulence occurring in large aspect ratio containers. In fact they are not, since they constitute a first step in the understanding

of the dynamical properties of Rayleigh-Bénard structures. We think that the next step would be a better knowledge of the structural defects present in those structures. Already three studies have been made in that way, the first one deals with a row of dislocations in a R.B. instability [17], the second one concerns the deformation field around a dislocation in a nematic instability [18], the third one is a theoretical determination of the dynamics of dislocations in a Rayleigh-Bénard instability [2]. It is worth noting that the last two studies rely on the phase diffusion equation, in order to explain the deformation field induced by the dislocation.

Certainly, experiments must be carried in that direction however preliminary experiments have shown that it is difficult to study dislocations as they have a tendency to glide and usually in our structures, defects other than dislocations are more frequently encountered. These other types of defects may originate from sidewall effects or from the wavelength selection infinite containers. Such points should be experimentally examined before defects in finite containers can accurately studied.

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