Effective exponents of nematic liquid crystals
Lin Lei

To cite this version:

HAL Id: jpa-00209392
https://hal.archives-ouvertes.fr/jpa-00209392
Submitted on 1 Jan 1982

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
**Effective exponents of nematic liquid crystals**

Lin Lei

Institute of Physics, Chinese Academy of Sciences, Beijing, China


Résumé. — Les « exposants » effectifs $\gamma'$, $\nu'$ et $\Delta'$ sont calculés dans la phase nématicque et des valeurs différentes sont trouvées dans la phase isotrope. Nous proposons des mesures de ces « exposants ».

Abstract. — Effective « exponents » $\gamma'$, $\nu'$ and $\Delta'$ in the nematic phase are calculated and found to be different from their counterparts in the isotropic phase. Measurements of these exponents are proposed.

The nematic-isotropic liquid (N–I) phase transition in liquid crystals is first order in nature [1, 2]. In the I phase near transition point $T_c$, the influence of the fluctuations of the order parameter is more apparent and the mean field approximation has to be improved [3, 4]. However, in the N phase near $T_c$, under certain circumstances the mean field approximation is still useful. For example, this approximation predicts that the critical exponents of the correlation length are not equal to each other on the two sides of $T_c$ [1, 5], which has been confirmed by the nuclear magnetic resonance (NMR) experiments of Dong and Tomchuk [6]. For PAA-d$_6$, reference [5] predicted that in the N phase of range 404 K-409.65 K ( = $T_*$) the apparent exponent of correlation length, $\nu$ (named $\nu$ in Ref. [5]), should be 0.4 which was at odds with the experimental value of 0.66 available in reference [6] at that time. Subsequently, Dong [7] reanalysed his experimental data and obtained $\nu = 0.4$, in agreement with our theoretical value [5].

In view of these developments and the fundamental difficulties in the analysis of NMR data of $\nu$, we here use the Landau-de Gennes (LdG) theory [8] to calculate other exponents $\gamma'$ and $\Delta'$ in the N phase and propose possible experiments to check the results.

1. Theory. — The free energy in the LdG theory is given by

$$F = \frac{A}{2} S^2 - \frac{B}{3} S^3 + \frac{C}{4} S^4 - hS$$

where $S$ is the order parameter, $A = \alpha(T - T^*)$, $h$ proportional to the square of the external field, $a$, $B$, $C$, $T^*$ are positive material parameters, $T^*$ the supercooling temperature. When $h = 0$, by $\partial F/\partial S = 0$ we obtain $S = 0$ (I phase) and the order parameter in the N phase

$$S = S \equiv B/2C + (a/C)^{1/2} (T^* - T)^{1/2}$$

where $T^* = T^* + B^2/4 aC$. When $h \neq 0$, the equation of state $S = S(T, h)$ is given by

$$AS - BS^2 + CS^3 - h = 0.$$  (3)

Differentiating (3) with respect to $h$ we get

$$S' \equiv (\partial S/\partial h)_{h=0} = A - 2BS + 3CS^2$$

and

$$S'' \equiv (\partial^2 S/\partial h^2)_{h=0} = 2(B - 2CS)A^{-3}.$$  (5)

It is easy to show that in the I phase ($S = 0$), $S' = a^{-1}(T - T^*)^{-1}$ and $S'' = 2Ba^{-3}(T - T^*)^{-3}$, giving rise to the well-known critical exponents ($T \rightarrow T^*$) $\gamma = 1$ and $\Delta = 2$ [2-4, 9]. In the N phase, $S = S$ in (4) and (5) and

$$S' = [2a(T_c - T^*)f_1(\overline{\gamma})]^{-1} \sim \bar{\tau}^{-\gamma}$$

$$S'' = -\left(\frac{729}{16\sqrt{2}}\frac{C^3}{B^2}\right) [f_2(\overline{\gamma})]^{-1} \sim \bar{\tau}^{-(\gamma + \Delta)}$$

where

$$T_c = T^* + 2B^2/9aC,$$  (6)

$$f_1(x) \equiv x^2 + (3/\sqrt{8})x,$$

$$f_2(x) \equiv [x^2 + (3/\sqrt{8})x]^3/(x + 1/\sqrt{8})$$

and

$$\bar{\tau} \equiv (T^* - T)/(T_c - T^*).$$

---

*Article published online by EDP Sciences and available at http://dx.doi.org/10.1051/jphys:01982004302025100*
The result $\bar{A} \sim f_1(\bar{r})$ above was first obtained in reference [5]. Since the correlation length in the N phase $\bar{\xi} \sim \bar{A}^{-1/2} \sim \bar{r}^{-\nu'}$, we obtain immediately $\gamma' = 2 \nu'$. Similarly, $\gamma = 2 \nu$.

When $T$ approaches $T^+$ from below, $f_1 \sim \bar{r}^{1/2}$ and $f_2 \sim \bar{r}^{3/2}$. Therefore, $\gamma' = 1/2$, $\nu' = 1/4$, $\Delta' = 1$. Comparing with results in the I phase, one obtains

$$\gamma' = \gamma/2, \quad \nu = \nu/2, \quad \Delta' = \Delta/2.$$ 

All these exponents are mean field values.

In real experimental situations (in the N phase), since the N-I transition is first order, one generally has $T < T_c < T^+$. In other words, the temperature $T$ cannot be sufficiently close to $T^+$ for the above values of $\gamma'$, $\nu'$ and $\Delta'$ to be measured. Yet, in the experimental temperature range, through the slope of the linear fit to the log $S' \sim \log \bar{r}$ curve one can still define an apparent exponent $\bar{\gamma}$ (and similarly $\bar{\nu}$ and $\bar{\Delta}$). The value of these apparent exponents will depend on the exact forms of $f_1$ and $f_2$ and vary according to the temperature range under consideration and the materials involved. Fortunately, if the temperature is measured in terms of $\bar{r}$ (rather than $\bar{r} \equiv (T^+ - T)/T^+$) then by (6) and (7), both $S'$ and $S''$ are functions of $\bar{r}$. Consequently, for different nematics, as long as the temperature range of $\bar{r}$ is the same, one will obtain the same apparent exponents ($\bar{\gamma}$, $\bar{\nu}$, $\bar{\Delta}$). In this sense, apparent exponents of first order transitions in liquid crystals are universal. However, this universality may break down when higher order terms are included in (1).

Alternatively, one may define effective exponents at each point of temperature by

$$\gamma_{\text{eff}} = -\left( \frac{d \ln S'}{d \ln \bar{r}} \right),$$

etc. Then, by (6) and (7),

$$\gamma_{\text{eff}} = \frac{x + 3\sqrt{2}}{x + 3(2\sqrt{2})} = 2 \nu_{\text{eff}}$$ (8)

$$\Delta_{\text{eff}} = \frac{2x + 3\sqrt{8}}{x + 3\sqrt{8}} - \frac{x}{2(x + 1/\sqrt{8})}$$ (9)

where $x \equiv \bar{r}^{1/2}$. Therefore, when $T$ approaches $T^+$ from below, $\gamma_{\text{eff}}$ changes from 1 to 1/2, $\nu_{\text{eff}}$ from 1/2 to 1/4 and $\Delta_{\text{eff}}$ from 3/2 to 1.

2. Calculations. — In figure 1a and figure 2a, the calculated results of $\gamma_{\text{eff}}$ and $\Delta_{\text{eff}}$ from (8) and (9) are plotted, respectively. The point $\bar{r} = 1/8$ is the transition point $T_c$.

In treating the experimental data, and knowing $T_c$ and $T^*$, one may use $T^* = T_c + (T_c - T^*)/8$ to determine $T^*$. The determination of $\gamma_{\text{eff}}$ requires more accurate data (since differentiation with respect to $\bar{r}$ is involved, see definition of $\gamma_{\text{eff}}$ above). Therefore, many experimentalists are still interested in the apparent exponents ($\bar{\gamma}$, etc.) in a certain temperature range.

Fig. 1. — a) Variation of $\gamma_{\text{eff}}$ with reduced temperature $\bar{r}$. b) $\log_{10} f_1$ vs. $\log_{10} \bar{r}$. The dots are calculated results. The slope of the linear fit gives the apparent exponent $\bar{\gamma}$ for a given temperature range.

Fig. 2. — a) Variation of $\Delta_{\text{eff}}$ with reduced temperature $\bar{r}$. b) $\log_{10} f_2$ vs. $\log_{10} \bar{r}$. The dots are calculated results. The slope of the linear fit gives the apparent exponent $\bar{\Delta}$ for a given temperature range.
range. To this end, we plot respectively in figures 1b and 2b, $\log_{10} f_1(\tau)$ and $\log_{10} f_2(-\tau)$ as functions of $\log_{10} \tau$. For the sake of clarity, only selected calculated points (and not the whole curve) represented by dots are presented in figures 1b and 2b. The straight line represents the best linear fit to the theoretical values, the slopes of which give respectively the apparent exponents $\gamma'$ and $\Delta'$. With MBBA in mind, we take $T_0 = 47 ^\circ C$ and $T_0 - T^* = 1 ^\circ C$, the temperature range $37 ^\circ C < T < 47 ^\circ C$ corresponds to $0.125 < \tau < 10.125$. From figures 1b and 2b we obtain approximately $\gamma' = 0.75$ and $\Delta' = 1.2$. Consequently, $\gamma = 0.38$. These theoretical results can be checked experimentally.

Comparing figures 1a and 1b it is obvious that $\gamma_{\text{eff}}$ definitely gives much more information than $\gamma$ as obtained from figure 1b. From figure 1b, one sees that it is not always possible to obtain a good linear fit if the temperature range is too wide. In addition a perfectly straight line in $\log f_1$ vs. $\log \tau$ will correspond to a constant $\gamma_{\text{eff}}$ for the same temperature range. In figure 1a where $\gamma_{\text{eff}}$ does not approach a constant in the range of $\tau$ shown, one may still obtain an approximate constant $\gamma$ for the same range of $\tau$ in figure 1b. Therefore, one must be very careful in discussing the meaning and accuracy of apparent exponents deduced in the way of figure 1b. The same is true for figure 2. These observations are also relevant and should be kept in mind in the discussion of experimental data [2, 10] ($\gamma$, $\Delta$, etc.) in the I phase.

3. Discussion. — In the indirect measurement of $\gamma$ by NMR experiments [6, 7], the analysis of the relaxation time data is known to be complicated and difficult. The main reason is that there is usually more than one relaxation mechanism in liquid crystals and the theoretical description is too complicated to be of sufficient accuracy. In this regard, measurements of $\gamma$ and $\Delta'$ through the quantities $S'$ and $S''$ may be more direct and much easier in comparison. Experimentally, one only has to determine the order parameter $S$ in the nematic phase as functions of $T$ and $h$ (similar to the case in the I phase [10]). And there are quite a number of different ways that $S$ can be measured [11]. The kind of asymmetry of the exponents on the two sides of $T_0$ presented here are characteristic of the first order nature of the N-I transition and are not expected from the tricritical hypothesis [9, 12]. Therefore, experimental determination of these exponents $\gamma$ and $\Delta'$ in the nematic phase will not only provide a check of the LdG model but will also give further tests on the nontricritical nature of the N-I transition.

Acknowledgments. — This work was carried out during the author’s visit at the Laboratoire de Physique des Solides, Université Paris-Sud, Orsay, France Feb.-May, 1980. We thank Jacques Friedel and Georges Durand for useful discussions, and Roland Ribotta for hospitality.

References