Empirical relations for nuclear stopping power

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Résumé. — Nous faisons une étude systématique des valeurs expérimentales obtenues pour la puissance d'arrêt nucléaire. Les données se répartissent en deux groupes : 1) L'un d'entre eux correspond aux mesures de la puissance d'arrêt faites dans la direction de l'axe du faisceau d'ions. Les valeurs ainsi obtenues permettent d'évaluer la vitesse moyenne des ions du faisceau après leur passage au travers de la cible, 2) Les valeurs de l'autre groupe sont obtenues en intégrant la contribution de toutes les particules du faisceau. Ces valeurs fournissent l'énergie nucléaire totale diffusée dans la cible.
Nous proposons un ajustement de ces deux groupes de données expérimentales au moyen de relations analytiques très simples.

Abstract. — We make a systematic analysis of the experimental nuclear stopping power determination. We show that the experimental values are divided into two groups : 1) One of them corresponds to measurements made using a narrow acceptance angle of the detection system along the beam axis. These measurements lead to the evaluation of the mean velocity of an ion beam leaving the target, 2) The other one is obtained by summing the contribution of all the particles scattered in the target. These values give the total nuclear energy dissipated in the target.
We give two simple analytical relations to fit these two groups of data.

Introduction. — The passage of heavy ions though matter is a subject under current investigation. In many experiments, a good knowledge of the energy loss in the target by the incident beam would increase the precision of the measurements. The stopping power dE/dx, defined as the energy lost by the incident particles per unit path length, is the sum of the electronic stopping power, due to inelastic interaction with the target electrons, and the nuclear stopping power induced by elastic collisions between the projectiles and the target nucleus. If the energy per atomic mass unit E/M of the incident particles is high, the nuclear contribution compared to the electronic one is negligible. But at low energy, the electronic effects approach zero while the energy lost by elastic collisions is of importance. So it is necessary to establish relations allowing its evaluation for practical purposes.

The Lindhard, Scharff and Schiott theory (LSS theory) [1] shows that the nuclear stopping power is the same for all elements in a special set of dimensionless coordinates ε and ρ. Using a Thomas-Fermi potential, they found that the reduced nuclear stopping power $d\varepsilon/d\rho$ is given as a function of $\varepsilon^{1/2}$ by a universal approximate curve. The nuclear stopping power is proportional to $d\varepsilon/d\rho$. The ratio factor depends on target and projectile, as is well known and recalled further in this paper (eqs. (4), (5)). The procedure described in the LSS theory has been followed by Biersack [2] and by Wilson *et al.* [3] using more realistic potentials. For example, the average curve obtained by Wilson *et al.* [3] over different potentials is drawn in figure 1.

Analysis of the experimental nuclear stopping power data. — In figure 1, are plotted different experimental values of $d\varepsilon/d\rho$. When necessary, the electronic contribution estimated in the tables of Northcliffe and Schilling [4] has been substracted from the original data. Almost all the experimental values lay below the theoretical curve. Indeed it is now well

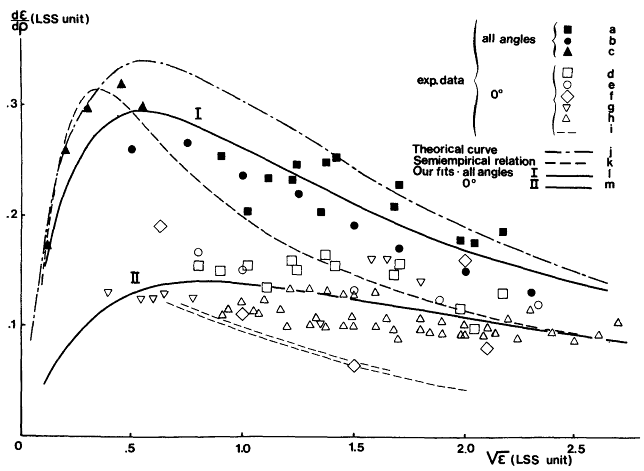

Fig. 1. — Reduced Nuclear Stopping power. *a*) Beauchemin and Drouin [8], *b*) Sidenius [11], *c*) Œztmann *et al.* [12], *d*) Beauchemin and Drouin [8], *e*) Högberg [17], *f*) Carriveau *et al.* [18], *g*) Andersen *et al.* [19], *h*) Grahmann and Kalbitzer [20], *i*) Ormrod *et al.* [21], *j*) Wilson *et al.* [3], *k*) Ziegler [22] from Kalbitzer *et al.* [16], *l*) eq. (1), *m*) eq. (2).

known that the LSS theory overestimates the observed nuclear stopping power [3, 5, 6].

The experimental data are divided into two groups.

The lower data (open points in figure 1) correspond to measurements using a narrow acceptance angle of the detection system along the beam axis. The stopping power thus measured is what we call the stopping power in the forward direction. This value has to be used in the evaluation of the energy reduction of the beam crossing the target. It leads to a good estimation of the mean velocity of an ion beam after a thin target. Indeed it has been proved both theoretically and experimentally that the distribution of the mean energy loss as a function of the angle of scattering is almost parabolic for small angles [7-10]. Moreover for a thin target (we mean by thin target, a target for which the energy loss is small compared to the energy of the beam), the angular spread of the beam after the target is sufficiently small to ensure that the scattering angle of almost all the particles composing the beam, meets the parabola in its flat region. Hence the mean zero degree energy loss is nearly equal to the mean energy loss at $\theta_{1/2}$ ($\theta_{1/2}$ is the angle corresponding to a flux half of the flux at zero degree).

The upper data (full points in figure 1), in better agreement with the LSS theory, are obtained by summing the contribution of all the particles scattered in the target. These measurements are much more difficult to realize and our disposals are only the results of Sidenius [11] and those of Beauchemin and Drouin [8]. Sidenius made the measurements by applying the proportional detector technique and obtained in the aggregate the total nuclear stopping power. Beauchemin and Drouin [8] measured the stopping power at many scattering angles θ and they integrated their values over all angles. The values obtained by Œtzmann *et al.* [12] for very heavy ions in the low energy region and deduced from range measurements may also be regarded as the total nuclear energy loss as the ions are followed until they stop in the target.

The results of Beauchemin and Drouin [8] for argon projectiles on a carbon target confirm the distinction between the two sets of data and clearly show the influence on the energy loss determination of the few particles scattered away from the beam. Hvelplund *et al.* [13] have estimated the nuclear energy loss for particles leaving the target in the forward direction by limiting the integration of the energy loss to particles leaving the target in a narrow cone along the direction of propagation, but their conclusions are only valid for very thin targets and very small angles. More recently a theoretical approach [14] has shown that the ratio between the total energy loss due to multiple scattering (i.e. integrated over all angles) and the zero degree energy loss is approximately equal to 0.7 for thin targets [14] but no application has been made for nuclear stopping power determination.

Fit of the reduced nuclear stopping power. — An attempt to fit the experimental data with an empirical analytical relation was made previously by Kalbitzer *et al.* [16]. But they made no distinction between the forward and all angles measurements. The graph is drawn in figure 1. As can be seen in this figure, there are large differences between the experimental data in each group. This is partly due to the evaluation of the electronic contribution to be substracted from the total measured energy loss and to experimental difficulties as, for example, the determination of the target thickness or of the angle of aperture of the spectrometer in forward measurements. However we consider that those two sets of data cannot be mixed and we propose two new relations which fit the two sets respectively.

All angles

$$\frac{d\varepsilon}{d\rho} = 1.6\,\frac{\ln(1.2\sqrt{\varepsilon} + 1)}{(1.2\sqrt{\varepsilon} + 1)^2}\,. \tag{1}$$

Forward direction

$$\frac{d\varepsilon}{d\rho} = 0.75\,\frac{\ln(0.78\sqrt{\varepsilon} + 1)}{(0.78\sqrt{\varepsilon} + 1)^2}\,. \tag{2}$$

These two relations have been obtained by least square fitting over the corresponding sets of experimental data. The first relation leads to the evaluation of the total nuclear energy lost in the target. The second relation evaluates the stopping power of projectiles leaving a thin target in the forward direction. This quantity has to be used, for example, to evaluate the mean velocity of an ion beam after passing through the target. Let us note that our simple expressions do not exhibit the correct asymptotic behaviour for large ε

$$(d\varepsilon/d\rho \propto (\ln \varepsilon)/2\,\varepsilon\,,\ \varepsilon > 10\ [3, 15, 16])\,.$$

However this has no practical consequence on the evaluation of the stopping power because the nuclear contribution is always negligeable compared to the electronic contribution for large ε.

The graphes of our two relations are drawn in figure 1. Our formulas give the nuclear contribution with enough precision for most practical purposes.

Practical calculations. — To make our paper self-contained, we give here all the formulas used to perform practical calculations.

Let E be the energy of the projectile, in MeV (laboratory system), M, the mass in amu and Z, the atomic number. We use the subscripts p and t for designating the projectile and the target respectively.

The reduced energy ε is given by

$$\varepsilon = AE \quad \text{where} \quad A = aM_t/[Z_p Z_t e^2(M_p + M_t)] \,. \tag{3}$$

The stopping power

$$dE/dx = B \, d\varepsilon/d\rho \tag{4}$$

is expressed in MeV/(mg/cm^2) or in keV/(µg/cm^2). The ratio factor is equal to

$$B = 4 \pi a N^* M_p Z_p Z_t e^2/(M_p + M_t) \quad (\text{MeV}/(\text{mg/cm}^2)) \tag{5}$$

where

$a = a_0 \times 0.8853(Z_p^{2/3} + Z_t^{2/3})^{-1/2}$,

$a_0 = 0.529 \times 10^{-8}$ cm (the Bohr radius),

$e = 3.79 \times 10^{-7}$ (MeV.cm)$^{1/2}$ (the electron charge),

$N^* = 6 \times 10^{20}/M_t$ (the number of atoms per mg of target material).

Conclusion. — From the analysis of the experimental data, we deduce that the experimental values are divided into two groups. One of them gives the nuclear stopping power for projectiles leaving a thin target in the forward direction (leading to the determination of the mean velocity of the ion beam after thin target). The other one gives the total nuclear stopping power (leading to the calculation of the total nuclear energy dissipated in the target).

We propose two new empirical analytical relations for the evaluation of the nuclear stopping power in these two cases. The analytical form of our relations are expressed in view of numerical calculations. We give all the relations needed to perform the complete calculations.

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