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Convective instabilities in nematics caused by an elliptical shear

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Abstract. — A nematic liquid crystal, oriented homeotropically (n/z) between two parallel plates (⊥ z) in relative motion x = x₀ sin ωt on one plate, y = y₀ cos ωt on the other one, exhibits a sequence of convective instabilities for increasing values of a dimensionless parameter N proportional to x₀ y₀ ω. Above a first critical number Nc₁, parallel convection rolls establish through the sample. This article gives a systematic experimental description of this instability which was discussed first by Pieranski and Guyon and analysed in detail by Dubois-Violette and Rothen. For a higher critical number Nc₂, the convective structure changes from rolls to a two dimensional periodic and stationary pattern. For even larger values of the parameter N, the two dimensional structure becomes non stationary and loses long range order symmetry. The melting of this structure is described; possible connection with the loss of ordering of two dimensional systems is suggested.

1. Introduction. — The hydrodynamic behaviour of nematic liquid crystals has received considerable attention over the last few years [1, 2, 3]. A review article [4] by the Orsay group focuses on the characteristic features of some of the remarkable instabilities which are observed in this phase.

In particular the behaviour of the nematics in DC and very low frequency (few Hz) shear flows have been studied by Pieranski and Guyon [5]. Hydrodynamics instabilities develop across the sample if the time required for the growth of the static instability is short in comparison with the period of the shear. These instabilities are caused by first order terms in the velocity field.

Of greater interest for acousto-optic conversion are the second order effects whereby a permanent distortion (which can depend on position) is produced under this influence of the flow. Distortion of an homeotropic nematic by longitudinal acoustic waves were reported in [6]. Helfrich [7] and Nagai and Itsuka [8] discussed these results in terms of the second order forces due to the sound field and also present in an isotropic medium.

Scudieri [9] indicated that a linearly polarized shear at a frequency of several kHz, acting on an homeotropic nematic, induces a convective instability above a critical shear rate. Pieranski and Guyon [10] attempted to reproduce Scudieri’s experiment but at a lower frequency (few hundred Hz). However they could obtain a convective instability only if the polarization of the shear is not strictly linear. The second order effect in this case comes from the joint action of the two out of phase components of the shear.

An attempt has been made to induce a similar instability using a very high frequency (2 MHz) ellip-
tical shear flow [11]. The structures observed at the lowest threshold did not depend crucially on the ellipticity. They were in fact very similar to the observations of Miyano and Shen [12] using linearly polarized surface active waves. The structures had been interpreted in this case as resulting from the streaming caused by the non linear coupling between the different surface modes propagating in the sandwich geometry.

In this article we give a detailed description of the effects of an elliptically polarized shear flow of few hundred Hz applied to an homeotropic nematic sample.

A schematic representation of the cell is given on figure 1. The upper plate motion is parallel to x axis : \( x_0 \sin \omega t \). The lower plate motion is parallel to y axis : \( y_0 \cos \omega t \). The sample thickness \( d \) is of the order of 50 to 200 \( \mu m \). At rest the mean molecular orientation, represented by the director \( \mathbf{n} \), is parallel to the z axis (homeotropic orientation).

![Fig. 1.- The two boundary plates move in quadrature and in perpendicular directions. As shown in the appendix, for the experimental conditions : \( d = 100 \mu m \) and \( \omega = 2 \pi \times 200 \) rad.\(^{-1} \), the shear induced by each plate is uniform; elliptical shear results of the combination.](image)

The experimental set up is described in section 2.

As we increase the shear a sequence of convective instabilities appears in the sample. The first instability, rolls, is discussed in section 3. A two dimensional pattern develops on top of the rolls above a higher threshold (section 4). Finally the transition to spatial chaos is qualitatively described in section 5.

2. Experimental techniques. — We only present the crucial features which are needed for an understanding of the results discussed in the following chapters. A detailed account of the methods used is discussed in [13].

Cell. — The set up is presented schematically in figure 2. Each glass plate is glued to a metallic bar which is attached on its other end to a loudspeaker membrane. The displacements of the indices \( I \), set on each bar, lead to a measurement of the phase and the amplitude of the plate motions. The accuracy on the amplitudes \((x_0, y_0)\) is : \( \pm 1.5 \mu m \). The maximum displacement amplitude which can be obtained for a liquid of viscosity 1 poise is of the order of 40 \( \mu m \) at 200 Hz and 10 \( \mu m \) at 500 Hz.

Sample. — The liquid crystal (L.C.) used is MBBA (p methoxybenzildene p.n. butylanilin) which is nematic at room temperature. The elastic and viscous constants of this classical material are well known [15]. However the variation of properties with time due to the poor chemical stability of the compound strongly limits the accuracy of the comparison of the experimental threshold with theory. The variation of nematic constants with temperature is another source of uncertainty. We estimate a variation of threshold of 3 to 5 \% per degree (which is less than the temperature stability of the laboratory room).

The L.C. is held by capillarity between the plates whose inner faces have been previously treated chemically with lecithin. The anchoring obtained is strong and the orientation of the molecules is uniform through the area of the cell (10 \( \times \) 10 mm).

Parallelism. — The parallelism of the plates is obtained when the centre of the radial flow created by a small decrease of the sample thickness coincides with the centre of the cell. The sensitivity of this method is \( 10^{-3} \) rad. for the angle of the plates which corresponds to a variation of 10 \( \mu m \) across the cell, for a thickness \( d = 100 \mu m \).

Thickness. — The sample thickness \( d \) is measured by interferometry ( conoscopic image) with an accuracy of a few microns.

Observation techniques. — Several techniques are used to characterize the instabilities.

i) Direct observation with a polarizing microscope.
ii) 16 mm movie camera. The speed can be slowed down to two images per second. Such slow speeds are required to study the evolution of the convective
pattern above thresholds. A natural time scale can be constructed from the diffusivity of orientation \( D_0 \) (see appendix eq. (A.4)):

\[
d^2/D_0 \simeq 10 \text{ s} \quad \text{for} \quad d = 100 \mu \text{m}.
\]

This value characterizes the local time to induce a distortion. However the time involved in the overall organization of the convective patterns can be much larger [14].

iii) Diffraction of a laser beam. The spatial modulation of the director in the \((x, y)\) plane can be studied by diffracting a monochromatic light beam parallel to the \(z\) axis. Figure 3a shows the set of equidistant diffracted spots produced by a parallel convective rolls pattern.

![Diffraction pattern observed just above the first threshold of rolls (a) and the « square » one (b).](image)

The diffraction technique is well adapted when the wavelength of the convective cells \( A \approx d \) is small. For \( A = 100 \mu \text{m} \) and a He-Ne laser light \( (\lambda = 0.63 \mu \text{m}) \) the diffraction angle \( \theta \approx 20' \) can be easily resolved.

**Threshold determination.** — In order to detect the instability thresholds we increase the amplitude \( x_0, y_0 \) of the plate displacements by a nearly linear ramp. The slope of the ramp must be small to minimize the error in the threshold determination (the characteristic time for the development of a linear instability diverges at threshold !). The loudspeaker excitation signals are driven by an electronic device which induces step increases of \( x_0, y_0 \) of a fraction of a \( \mu \text{m} \) with time intervals up to 10 s.

The determination of threshold can be done accurately by placing a photocell above the position of a diffracted peak or of the zeroth order central peak and detecting the variation of intensity as the shear increases. Examples of experimental curves are given in figure 4. The accuracy in this measurement is limited by the long time constant of the transition which causes rounding in the detection curves.

![Direct recording of the intensity diffracted in the central peak (order 0), in the first order (order 1), above the roll instability threshold, and in the « square » pattern (order 1'), above the two dimensional pattern instability threshold. The results are obtained in increasing shear (\( y_0 \) increases by 400 A per second).](image)

3. **Roll pattern instability.** — 3.1 **Instability mechanism.** — The behaviour of the L.C. sample for small elliptical shear and above a first linear instability threshold has been first reported in [10]; a detailed theoretical analysis has been given in [16]. We recall here the mechanism.

In the appendix we show that the motions of both plates induce a linear velocity profile across the sample in the experimental range of frequencies and thicknesses used. The resulting velocity field, in the case of the experimental geometry represented on figure 1, can be written as:

\[
t_v^x = \frac{z + d/2}{d} \frac{x_0}{\omega} \cos \omega t \quad t_v^y = \frac{z - d/2}{d} \frac{y_0}{\omega} \sin \omega t \quad t_v^z = 0.
\]

The index «a» indicates an alternating quantity. The flow represented by the equation (1) is a uniform elliptically polarized shear. Using the nematodynamic equations [1, 2, 3] one easily gets the director beha-
viour in this flow. For small plate displacements \((x_0 \text{ and } y_0 \ll d)\), the orientation distortion writes :

\[
\begin{align*}
n_s^a &= -(\gamma_2/\gamma_1) \frac{x_0}{d} \sin \omega t \\
n_s^b &= (\gamma_2/\gamma_1) \frac{y_0}{d} \cos \omega t \\
n_s^c &\approx 1.
\end{align*}
\]

\(\gamma_2\) and \(\gamma_1\) have the dimension of viscosities (the sign of \(\gamma_2/\gamma_1\) is negative).

The director follows the plate displacements and moves on an elliptical cone having the same axis as the elliptical flow.

The experiments (see figure 4) indicate that the first instability is characterized by the continuous and reversible development of a permanent one dimensional periodic structure. Therefore we assume that, at threshold the velocity and director fields are \([17]\) :

\[
\begin{align*}
v &= v^\ast + v^\ast(x, y, z) \\
n &= n^\ast + n^\ast(x, y, z)
\end{align*}
\]

where \(v^\ast\) and \(n^\ast\) are given by equation (1) and equation (2). The infinitesimal quantities \(v^\ast\) and \(n^\ast\), function of position (of the convective geometry), are also taken as independent of time as the convection pattern does not show strong variation at the frequency \(\omega\).

Using the nematodynamic equations \([1, 2, 3]\), the stability of the solution of equations (1) and (2) is analysed in front of the effect of the small perturbation \(n^\ast\) and \(v^\ast\). This solution is found to become unstable above a critical value \(N_{\text{c1}}\) of the dimensionless number \(N\) :

\[
N = (x_0 \ y_0 \ \omega/d^2) (d^2/D_0) = (x_0 \ y_0 \ \omega/D_0) = (x_0 \ y_0 \ \omega/D_0).
\]

The parameter \(N\) can be constructed as an Ericksen number for the problem (see eq. \((A.5)\)) by replacing a shear rate, \(v/d\), by the frequency factor \(\Omega = x_0 \ y_0 \ \omega/d^2\) where the frequency \(\omega\) is weighted by the fractional displacements \(x_0/d\) and \(y_0/d\).

The instability mechanism is obtained through the following destabilizing loop : a distortion \(n^\ast\) create viscous stresses which induce a flow \(v^\ast\) via the coupling of the director and velocity fields. Then this flow can amplify the initial orientation distortion if the diffusive relaxation to a state of rest is slow enough.

In equation (4), \(N\) can be written as :

\[
N = \tau_0/\tau_p
\]

where \(\tau_0 = d^2/D_0\) is a damping time for the relaxation of orientation (see appendix eq. \((A.4)\)) and

\[
\tau_p = (x_0 \ y_0 \ \omega/d^2)^{-1} = 1/\Omega.
\]

The frequency \(\Omega\) (small in front of \(\omega\) in the ranges of validity of the analysis) is related to the time constant for the constructive build up of the instability via the coupling effect of the two shear components on a fluctuation \(n^\ast\). The frequency \(\Omega\) has been measured in an independent experiment reported in \([18]\) : in a similar elliptical shear flow of small enough amplitude \((N < N_{\text{c1}})\) an additional distortion is created by applying an electric field above the critical Freedericksz threshold. The distortion is found to relax by a precession mechanism at a frequency \(\Omega\).

The ratio of two time constants in (5) suggests an interpretation for the critical value of \(N\) : the threshold value is such that, when the time constant for the constructive increase of \(n^\ast\) is shorter than the diffusive decay time, an instability can establish permanently.

3.2 Experimental results. — 3.2.1 Uniform regime. — For small enough \(N\) (eq. (4)), the uniform motion of the director is described by equation (2). This motion can be followed from the slow displacement of the conoscopic image formed by illuminating the sample at a frequency close to \(\omega/2\pi\) : the centre of the image which characterizes the average optical axis of the system moves on an ellipse whose principal axis are those of the shear. If the strobe frequency is smaller than \(\omega/2\pi\) the motion on the ellipse is in the same direction as the shear.

3.2.2 Convection pattern : geometry. — The amplitude of the elliptical motion of the conoscopic image increases with \(N\) up to a critical value where the image distorts and rapidly disappears due to the development of the instability. One must distinguish the general case, \(x_0 \neq y_0\), from the degenerate circular polarization one, \(x_0 = y_0\) where all directions of the \((x, y)\) plane are equivalent.

3.2.2.1 Elliptical shear. — The instability develops as a set of equidistant parallel rolls, the orientation of which in the \(xy\) plane makes an angle \(\psi\) with the \(x\) axis. Figure 5 shows the structures observed for different shear ellipses with the same ellipticity. The orientations of the rolls can be deduced from one of the geometry by simply considering the change of symmetry of the shear ellipse : rotation by + \(\pi/2\) from \(a\) to \(b\), symmetry with respect to the \(x\) axis from \(b\) to \(c\).

The angle is determined from the direction of the diffraction image. Its variation with ellipticity is given on figure 6. The rapid change around the value \(x_0/y_0 = 1\) (circular polarization) corresponds to the passage between figures 5b and 5a.

The wavelength \(A\) of the rolls is determined by microscopic observation. It is the distance between two sharp dark lines in figure 5c. This can be checked from the observation of the defects in the structure : for example the edge dislocation in the lower right hand corner of figure 5c, must correspond to the addition of a pair of rolls in order to insure continuity of the velocity field.
Fig. 5. — The orientation of the rolls as a function of the geometrical characteristics of the ellipse (the ellipticity remaining constant) is measured by the angle \( \psi \). On photo c, we see in the lower right hand corner an edge dislocation of the roll pattern. At the location of this defect two adjacent rolls must disappear.

Fig. 6. — Variation of the direction of the rolls as a function of the ellipticity of the shear. For large ellipticities we notice a nearly constant angle of about 15° between the roll orientation and the small axis of the shear ellipse.

The wavelength of the rolls is plotted as a function of thickness \( d \) on figure 7. A linear relation:

\[
2 \frac{d}{\lambda} = 1.7 \pm 0.2
\]

(6)

is obtained, indicating rolls elongated in the direction perpendicular to the plates. The ratio is independent of frequency between 50 and 600 Hz as expected from the theoretical analysis [16]. It is however larger than the value 1.3 predicted for the same material. In the analysis, the ratio \( 2 \frac{d}{\lambda} \) is given as a function of several material parameters. It would clearly be of interest to check the dependence of \( \lambda \) with these parameters by using a material with a strong variation of viscosity coefficients (as in a nematic near a second order nematic-smectic A transition).

3.2.2.2 Circular threshold. — No preferred direction exists in the centre part of the cell. However, rolls seem to appear slightly before a square pattern establishes on top of it by periodic pinching of the rolls. But the two events are so close even for a very slow increase of \( N \) that it is possible that the former faint rolls structure is a transient structure. This is different from the elliptic case (1) where the two thresholds are well separated (see section 4). We also observe in some regions of the cell the development of hexagonal structures. Again they seem to be transient structures as they disappear after the stabilization of the shear.

3.2.3 Threshold. — The onset of instability is deduced from the reading of the diffracted light intensity as indicated in section 2 and shown in figure 4. In the elliptical case \((x_0/y_0 \approx 10 \text{ at threshold})\) the thresholds deduced from the recording of the continuous decrease of the zeroth order structure and increase of the first order one agree within 10%. A 5\% hysteresis, obtained between increasing and decreasing shears due to the dynamics of the transition, gives the magnitude of the systematic error in the measurement. Similarly a continuous and reversible transition to squares is obtained in the circular polarization case \((x_0/y_0 = 1)\) although the transition is sharper. There has been some interest in the order of the transition of convective instabilities [19] in connection with the symmetry of the structure at threshold. It is well established that the continuous transition observed to rolls can be associated with
the symmetry $v \rightarrow -v$ of the problem ($v$ being the local convective velocity). A similar result is expected to apply for squares (for example in Rayleigh convection with poorly conducting boundaries [20]).

The lowest linear threshold results are indicated as black figures on figure 8 in the case of a thick sample ($d \gg x_0$ and $y_0$). The variation is in rough agreement with the hyperbolic variation ($x_0 y_0 = \text{const.}$) indicated by a solid line on the figure. The three curves of figure 8 correspond to frequencies 500, 280 and 200 Hz and are consistent with a constant value of the product:

$$x_0 y_0 \omega = (1.4 \pm 0.3) \times 10^{-3} \text{ cm}^2 \cdot \text{s}^{-1}$$

in agreement with the form of $N$ (eq. (4)). Using the value $N_C \sim 500$ given for MBBA in [16] and

$$D_0 \sim 10^{-5} \text{ cgs},$$

we get a theoretical estimation of the same order of magnitude: $x_0 y_0 \omega \sim 0.5 \times 10^{-3} \text{ cm}^2 \cdot \text{s}^{-1}$. Occur due to the square geometry obtained when $x_0 = y_0$.

There also exists some quantitative disagreements on the dimensionless quantity $N$ and on the ratio $2d/A$ which may partly be due to the inaccuracy in the evaluation of the material parameters.

There is also a marked disagreement on the value of the angle $\psi$ of the rolls in the elliptic case. A finite value of $\psi (\sim 30^\circ)$ much smaller than the experimental one $\sim 15^\circ$, Fig. 6) has been calculated in [16] from a non linear analysis of the instability. The geometry assumed in this work was that the elliptical shear was applied directly on the boundary plates. In these conditions it can be easily shown [22] that no linear contribution depends on the angle of the rolls with the shear axis. In the different present geometry (sketched in figure 1) the effect of the finite value of $\psi$ enters in a linear analysis.

Convective rolls tend to align along directions which minimize the effects of the velocity gradients as was shown in an independent work using an electrohydrodynamic nematic instability [23]. Such an analysis would also predict an increase of the threshold from the value obtained in reference [16]: the convective mixing has a stabilizing effect. The sense of this correction tends to reduce the difference between experiments and theory in the present case.

4. Two dimensional instability pattern. — We only consider the elliptical case where the rolls are formed above the first threshold $N_{c1}$. The development of a two dimensional structure appears above a distinct second threshold $N_{c2}$ in a reversible and continuous fashion, as seen from the analysis of scattered light (Fig. 4). (The sharpness of the transition is possibly related to the non linear interaction with the pre-existing rolls.)
4.1 GEOMETRY. — The second bifurcation takes place in the following way. Stationary periodic distortion develop along the rolls, in register from one roll to the next leading to a square like pattern. They can also form with a spatial shift from one roll to the next leading to a diamond shaped like pattern (Fig. 10).

Measurements on photographs as well as on the diffracted image (Fig. 3b) show that the convective cells are not squares but diamond shapes with angles of about 80°. However as this is not very different from squares we shall continue to speak of « square » pattern and of 4th fold symmetry.

4.2 THRESHOLD. — The values of thresholds are plotted on figures 8 and 9, as open figures, together with those of \( N_{c2} \). Again, in a first approximation, they are characterized by a constant value of the product \( x_0 y_0 \) even for thin samples.

5. Transition to spatial chaos. — 5.1 OBSERVATIONS. — The structures discussed in the previous chapter are static. However, above a larger critical value of \( N \), characteristic changes of the structures are obtained: the two dimensional crystalline structure gets progressively disorganized for increasing \( N \); it also becomes time dependent in an apparently random fashion. Presently, there is a large interest in the bifurcations that dynamical systems (as in Rayleigh-Bénard convection) undergo between the state of rest and chaos. On the experimental side, the evidences for the loss of long time memory in the turbulent chaotic state come from the temporal dependence of velocity correlation function. It is of large interest to connect the two descriptions.

The following description is based on a film done while increasing progressively the value of \( N \) [24]:

i) Immediately above the threshold \( N_{c2} \), the convective structure is as shown on figure 11 with a coexistence of domains of 4 and 6th fold symmetry. The structure does not change appreciably in a finite range above \( N_{c2} \). However if the sample is kept for a long time at a finite value of \( N \), hexagons tend to be replaced by the square structure. Hexagons are often met together with squares in convective patterns even when there is no asymmetry in the convection geometry as it is the case here. The hexagons can be privileged in particular in time dependent experiments, as was shown recently by Ahlers et al. [25]. The hexagonal domains also play the role of adjustment zones between square crystallites of different orientations. In this range of values of \( N \) some movements of the convective structures around defects or dust particles are seen but they are quite localized.

ii) A markedly different behaviour is observed for larger values of \( N \) (typically 2 \( N_{c1} \)). The motions induced around the defects affect the complete cell by extending along stripes which correspond to the previous hexagonal mismatch zones. The hexagonal structures have melted and the square crystallites move independently of each other in the molten environment. However the ordering within the individual square domains is not markedly decreased despite the compressional and shear elasticity present during their random motions. Thus a small number of active centres appears to drive the chaotic motion. They look fairly similar to the spokes described by Busse [26] in R.B. convection in large cells sufficiently above the first Rayleigh threshold. The importance of such large scale structures in turbulence has been strongly emphasized in the last few years, in particular for 2 dimensional flows, in connection with the concept of large scale intermittency in turbulence [27].

iii) As the shear continues to increase, the size of the square crystallites decreases together with the fraction of the area they occupy in the cell. However the structure within the cells remains practically unchanged as seen in the sequence of photographs of figure 12. Finally, for even larger \( N \), all organized patterns disappear. The fluctuating structure is quite similar to the dynamic scattering that was obtained in electrohydrodynamic experiments in nematics for large applied voltages [4].
Fig. 12. — In increasing shears, one observes the progressive melting of the 2 D structure. The size of the square-like domains decreases progressively but the ordering within any one of them is not appreciably decreased.

5.2 DISCUSSION. — We suggest here the possible connection between the melting observed here and thermodynamic problems of two dimensional ordering which have been widely discussed in the recent years (Refs. [28, 29]). In these descriptions the melting is associated with the unbinding of pairs of defects when the temperature becomes large enough so that the entropy contribution associated with the disordering overcomes the binding energy of pairs. Although there is no free energy for dynamic convective problems (open systems far from equilibrium) it is possible to establish a connection with the thermodynamics ones as was done by Graham, Swift and Hohenberg [30].

Our results in the molten phase are qualitatively very similar to numerical simulations on hard disks by Alder and Wainwright (for example figure 2 of [31] and our figure 12) which showed the existence of a Van der Waals plateau with the coexistence of two phases—ordered and molten—of continuously varying concentration. The agreement suggests a correspondence between the range of interactions in both problems. We expect the convective cells to interact by a fairly local interaction, hard rod like, via the hydrodynamic and elastic couplings as in the experiment of [31].

In the description of [29], the melting can take place in one step due to the melting of bound defects and the simultaneous loss of positional and orientational order. The transition should then be of first order as it seems to be the case in the hard disk experiments. However another interesting possibility is that melting of dislocations pairs is reached at a lower temperature than that of disclinations. In the intermediate range, a liquid crystal like state is obtained where the directional order is retained through the two dimensional lattice. Numerical evidence of the intermediate phase have been obtained in a computer work by Frankel and Mac Tague [32] using a Lennard-Jones potential interaction on a $25 \times 25$ lattice. We are presently studying by direct observation as well as by light diffraction on large single domains of convective structures (300 $\times$ 300) the melting process to see if a one step or two step transition is obtained in our problem.

In addition to providing a large analog model for the study of two dimensional melting, such an experiment can provide a connection with the time dependent chaos mentioned above. A random motion of disordered structures lead to a noisy spectrum of the velocity correlation function. In this context, the presence of some unbound — free — defects within a periodic structure can be considered as a spatial intermittency parameter of the problem. Its effect can lead to time dependent intermittent structures.

Note. — Since this work was completed we have had communications of several independent studies emphasizing the role of geometrical disorder in the understanding of turbulence in convective systems. Pomeau and Manneville [33] have introduced the notion of phase turbulence to characterize the spatial chaos due to the continuous and amplified drift of the period of convective structure. We are presently using this description to study the distortion field around individual defects (edge dislocations) of convective structures in the elliptical shear instability. A recent preprint by Toner and Nelson [34] also suggests the possible analogy between the melting of layered materials in two dimensions and Rayleigh-Bénard like convection pattern. Finally we mention the recent work by Guazzelli who studies, using the same geometry, the nucleation of defects of the convective structure [35] as well as the distortion field around them in connection with the approach of [14] and [33].

Acknowledgments. — We thank L. Strelecki who synthesized the MBBA and prepared the lecithin used in the experiments. We have had discussions on various aspects of the experiment with E. Dubois-
Violette, C. D. Mitescu, P. Pieranski, F. Rothen. One of us (E:G.) wishes to acknowledge interesting comments of S. Alexander and D. Nelson concerning the analysis of the film showing the melting of convective structures.

Appendix. — The purpose of this appendix is to justify the assumptions, made in section 2 on the velocity and director field (eqs. (1) and (2)) and to illustrate the notions of nematodynamics used in the analysis of this work. For small plate displacements \((x_0, y_0 \ll d)\) we only need to consider a linear shear because the components of the velocity field of equation (1) (and the director field for equation (2)) are linearly independent.

We consider the geometry represented on figure 13: at rest, the director field \(n(\ | n \ | = 1)\), which gives the local direction of the optical axis of the positive uniaxial material, is along \(z\). Let us apply a small alternating shear by displacing the upper plate \((z = + d/2)\), the lower one \((z = - d/2)\) being at rest. The shear induces an alternating velocity field \(v_x(z, t)\) and a distortion of the nematic characterized by the component \(n_x(z, t)\). A fundamental property of nematodynamics is the coupling of the director and velocity field. In our case this property leads to the two coupled equations:

\[
\gamma_1 \frac{d^2 n_x}{dz^2} + \kappa_2 \left( \frac{\partial^2 n_x}{\partial z^2} \right) + \alpha_2 \left( \frac{\partial^2 v_x}{\partial z^2} \right) = 0 \quad (A.1)
\]

\[
\rho \frac{d^2 v_x}{dz^2} - \eta_b \left( \frac{\partial^2 v_x}{\partial z^2} \right) - \alpha_2 \left( \frac{\partial^2 n_x}{\partial z^2} \right) = 0 \quad (A.2)
\]

with the boundary conditions:

\[
\begin{align*}
&v_x(z = d/2) = x_0 \omega \cos \omega t \quad (A.3a) \\
&v_x(z = -d/2) = 0 \quad (A.3b)
\end{align*}
\]

\[
\begin{align*}
&n_x(z = \pm d/2) = 0 \quad (A.3c)
\end{align*}
\]

The relation (A.3c) states that the anchoring of the director orientation at the boundary plates is strong. Let us analyse equations (A.1) and (A.2).

![Fig. 13. — Geometry of the linear shear flow experiment used in the appendix. Under our experimental conditions, the director follows the plate motion.](image)

(A.1). — The first equation gives the balance between the torques (along \(y\)) applied to the director \(n\). The first term is a viscous torque which expresses the slowing down of the rotation of \(n\); \(\gamma_1\) is a rotational viscosity. The second one gives the restoring action of elasticity which tends to keep an uniform alignment. We use a one elastic constant description \((K \sim 5 \times 10^{-7} \text{ dyn/cm})\); this simplification has been found satisfactory in the semi-quantitative descriptions of nematic instabilities. The last term is destabilizing and expresses the effect of the shear on \(n\), governed by the constant \(\alpha_2\) \((\alpha_2 \sim \gamma_1 \sim 0.5 \text{ poise})\).

(A.2). — The second equation expresses the conservation of momentum. Here, \(\eta_b\) is the value of the viscosity when \(n\) is perpendicular to the plates (it is usually measured by keeping the direction of \(n\) fixed by the application of a strong enough stabilizing magnetic field). The last term gives the back flow hydrodynamic motion which accompanies the rotation of the director [1, 4].

We now look for particular solutions of the equations (A.1) and (A.2), which illustrate the dynamics of nematics:

1. At a time \(t = 0\), the shear is suddenly suppressed. The relaxation of \(n_x(t)\) to zero obeys a diffusion equation (eq. (A.1)) with a diffusivity of orientation

\[
D_0 = K/\gamma_1 \quad (A.4)
\]

\[
D_0 \sim 5 \times 10^{-7}/5 \times 10^{-1} \sim 10^{-6} \text{ cm}^2/\text{s}
\]

for the material used.

This value is typically \(10^6\) times smaller than the kinematic viscosity \((\nu = \eta_b/\rho)\) which is a diffusivity for vorticity.

The slow relaxation of \(n\) controls the hydrodynamic instabilities. Instead of the Reynolds number \((Re = v d/\nu)\) which is a ratio of diffusion time \((d^2/\nu)\) across a distance \(d\) and the time for convection \((d/v)\), the nematic instabilities are often characterized by the Ericksen number:

\[
Er = vd/D_0 = (d^2/D_0)/(d/\nu) \quad (A.5)
\]

2. Now we consider the effect of equation (A.3). Let us suppose first that the shear rate \(s(t)\) is uniform across the cell:

\[
v_x(z, t) = [z + (d/2)] \left( x_0 \omega /d \right) \cos \omega t = \frac{z}{d} \left[ z + (d/2) \right] s(t) \quad (A.6)
\]

Equation (A.1) is written in a normalized form:

\[
(\partial n_x/\partial t) = - \left( \xi^2 /d^2 \right) \left( \partial^2 n_x/\partial z^2 \right) - (\alpha_2/\gamma_1) \omega s \quad (A.7)
\]

with \(\omega = \omega t, \xi = z/d\) and \(\xi = [K/(\gamma_1 \omega)]^{1/2} = (D_0/\omega)^{1/2}\).

The effect of elasticity which tends to keep the alignment parallel to that at the plates is important in two boundary layers of thickness \(\xi\), the orientation penetration depth. In the frequency range of the experiments \((\omega \sim 2 \pi \times 200 \text{ rad. s}^{-1})\), \(\xi \sim 1 \mu\text{m}\) is small compared with the thickness \(d \sim 100 \mu\text{m}\) of the film:

the effect of elasticity is negligible in the bulk of the
film $(\partial^2 n_0^1/\partial z^2 = 0)$. Therefore in equation (A.7), one gets a uniform distortion:

$$n_0^1(t) = -(x_3/\gamma_1)(x_0/d) \sin \omega t \quad (A.8)$$

in phase with the plate motion.

3. Using the shear applied to the plates and given by equation (A.3), what is the velocity profile when the distortion is given by equation (A.8)? Neglecting the elasticity and using the expression of $n_0^1$ in equation (A.2) one gets:

$$p_{\lambda\epsilon} = \eta_h (\partial^2 v_\lambda/\partial z^2) \quad (A.9)$$

where $\eta_h = \eta_b - (x_3^2/\gamma_1)$ is a viscosity renormalized by backflow [15].

The quantity $\delta = (2 \eta_b/\rho \omega)^{1/2}$ is a viscous penetration depth giving the range of penetration of the shear wave in the sample. In the present experiments $\delta$ ($\sim 200 \mu m$) is larger than $d$ and the velocity profile can be taken as a linear function of $z$ given by equation (A.6) as assumed initially.

4. Conclusion. — Considering the results of paragraph 2 and 3 we may say that velocity field and director field are given by equation (A.6) and equation (A.8) if the following double inequality is fulfilled:

$$\xi \ll d \ll \delta \quad (A.10)$$

This is the case in our experiments and the assumptions of the paragraph 3.1 are justified. We emphasize that fulfilling the double inequality (A.10) was made possible because of the relation: $v_{o}/v \ll 1$.

References

[15] RECENT convection experiments on isotropic liquids (WESFRED, J. E. and CROQUETTE, V., *Phys. Rev. Lett.* 45 (1980) 634) show that the dynamics of the convective structure is of diffusive nature, the diffusion time constant being of the same order of magnitude as that for the growth of convection near threshold. In the present experiments it implies an evolution of the structure over time scales of hours for a fixed shear, which is actually observed.

[17] F. ROTHEN has pointed out to us that the decomposition with a time in dependent perturbation was not strictly correct. Together with J. SADIK and E. DUBOIS-VIOLETTE he has considered the different orders in the static and time dependent corrections. Experimentally, it seems extremely difficult to separate (for example from the frequency spectrum analysis of the diffraited intensity) such a direct time contribution of $n^\ast$ and that coming from the sum $n^\ast + n^\prime$.

[22] DREUYS, J. M., Thèse de 3e cycle, chapter 5. This work has been done in collaboration with Mitescu, C. D. staying at ESPCI.
[24] A copy of the film can be borrowed from us at ESPCI.