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Polariton theory of resonant electronic Raman scattering on neutral donor levels in semiconductors with a direct band gap

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Abstract. — The theory is presented for the Raman scattering on bound electrons of neutral donors in the semiconductors with a direct band gap at the incident photon frequency in the resonance with the exciton. The cross-section of the polariton scattering with the transition of the donor electron from the ground state to the first excited state is calculated.

1. Introduction. — Different electronic Raman scattering processes in semiconductors were investigated both theoretically and experimentally by many authors. In the case of the scattering on a gas of mobile carriers (electrons or holes) there may be the scattering with the single particle excitation or the scattering on collective excitations in this gas. The scattering processes of this type were studied widely by Platzman, Tzoar, Wolff et al. [1-13]. The interband scattering processes, in which the initial and final states of electrons or holes belong to different energy bands, were studied in many works [14-17]. The initial and final states of the charge carriers may be also their bound states in the Coulomb field of the donor or acceptor ions. In this case we have the scattering on the impurity levels [18-27]. The role of different external electromagnetic fields on the electronic Raman scattering processes was also studied in details [28-31]. Recently Weisbuch and Ulbrich have investigated experimentally the resonant electronic Raman scattering on neutral donor levels at the energy of the incoming photon near that of the exciton [32]. The present work is devoted to the theoretical investigation of the resonant electronic Raman scattering on neutral donor levels in a two band semiconductor with the direct band gap and the allowed electrical dipole transition from the valence to the conduction band. In the case of the material with a very low concentration of donors the electron-hole pair can exist in their bound states — the excitons. Due to the virtual transition between the photons and the excitons there arises in the semiconductor a new kind of elementary excitations — the polaritons or more precisely the excitonic polaritons discovered by Hopfield [33] and Pekar [34] and studied further in many works [35-42]. As in the case of the resonant Raman scattering, at the photon energy near that of the exciton the polariton effect becomes very important. The polariton theory of the resonant Raman scattering was developed by Mills, Burstein et al. [43] and also by Bendow and Birman [44].

In the present work we apply the method of Bendow and Birman [44] to the study of the resonant electronic Raman scattering on the neutral donor levels at the energy of the incoming or outgoing photon near that of an exciton. We assume that the semiconductor has a direct band gap with the allowed electrical dipole transition and consider only the case of the material with a high purity, when the donor concentration is very small. In this case there will be no screening of the Coulomb interaction between the electrons and the holes, the excitons will exist, and therefore the polariton effect must be taken into account. We shall use the unit system with \( h = c = 1 \).

2. Electromagnetic interaction and polariton scattering. — The existence of the polariton and all the scattering processes considered in this work are the consequences of the electromagnetic interaction in the semiconductor. We begin our study by discussing the
physical implications of different elementary electromagnetic interaction processes.

One type of these electromagnetic interaction processes in the semiconductor is the Coulomb interaction between the charge carriers. Due to this interaction an electron-hole pair can form their bound states as a hydrogen-like (positronium-like) system. We shall show that many scattering processes involving excitons and charge carriers are also the consequences of the Coulomb interaction between the charge carriers (free or bound in the impurity) and the constituents of the excitons.

Another type of the elementary electromagnetic interaction processes is the recombination of an electron-hole pair into a photon or its inverse process — the absorption of a photon followed by the creation of an electron-hole pair. Because of the Coulomb interaction the electron-hole pair can be created in its bound state — the exciton, and we have the photon — exciton transition. Due to this transition the photons and the excitons are no longer the eigenstates of the Hamiltonian of the exciton-photon system: the eigenstates describe a new kind of quasiparticles in the semiconductor — the polaritons, which will be denoted by \( \pi_a(k) \); \( a = 1, 2, \ldots \). Note that besides the existence of the polaritons the second type of the electromagnetic interaction is also the origin of many inelastic collision processes involving electrons, excitons, photons and contributing to the electronic resonant Raman scattering, as we shall see in the sequel.

In studying the mechanism of the collision processes involving two identical particles in the initial and/or the final states — two electrons, one in the donor ion and one in the exciton, it is necessary also to satisfy the requirement of the Fermi statistics: the state vectors must be antisymmetrical under the interchange of two electrons. The physical implications of the Fermi statistics can be interpreted as the contributions of some exchange interaction. The role of this exchange interaction will be also taken into account.

In this work for simplicity we shall consider the photon in a definite polarization state and denote by \( a(k) \) the destruction operator for the photon with the momentum \( k \) and the energy \( \omega(k) \). Due to the electromagnetic interaction this photon can be transformed into different bound states \( Ex_\nu(k) \); \( \nu = 1, 2, 3, \ldots \) of the electron — hole pair — the excitons. Denote by \( b_\nu(k) \) the destruction operators for the exciton in the states \( Ex_\nu(k) \) with the momentum \( k \) and the energy \( E_\nu(k) \), and by \( g_\nu(k) \) the effective coupling constants of the transition between the photon and the excitons \( Ex_\nu(k) \). After diagonalizing the Hamiltonian of the exciton-photon system we obtain the explicit expression for the destruction operators \( C_\nu(k) \) of the polaritons \( \pi_\nu(k) \) [41]

\[
c_\nu(k) = u_\nu(k) a(k) + \sum_\nu v_\nu(k) b_\nu(k) \tag{1}
\]

\[
u_\nu(k) = \left\{ 1 + \sum_\nu \frac{g_\nu(k)^2}{\Omega_\nu(k) - E_\nu(k)} \right\}^{-1/2}
\]

Here the index \( \nu \) denotes the branch of the polariton whose energy \( \Omega_\nu(k) \) is determined by the equation

\[
\Omega_\nu(k) = 1 + \frac{1}{\Omega_\nu(k)} \sum_\nu g_\nu(k)^2 \frac{E_\nu(k) - E_\nu(k)}{E_\nu(k) - E_\nu(k)} \tag{3}
\]

We assume also that there are only one kind of electrons and one kind of holes, i.e. suppose that the conduction and valence bands are nondegenerate. In this case the index \( \nu \) will be the set of the quantum numbers \( \{ n, l, m \} \) of the internal (envelope) wave function \( F_\nu(r) \) of the exciton as a hydrogen-like system. For the effective coupling constant \( g_\nu(k) \) of the exciton-photon transition we have [41, 42]

\[
g_\nu(k) = \frac{e}{\sqrt{2} \epsilon_0 \omega(k)} \frac{\beta(k)}{m} F_\nu(0) \tag{4}
\]

where \( \beta(k) \) is the electrical dipole transition matrix element, \( \epsilon_0 \) is the background dielectric constant.

Denote by \( e_N \) and \( e_{N'} \), the electrons bound in the donor ion,

\[
N = \{ n, l, m \}, \quad N' = \{ n', l', m' \}
\]

and consider the scattering process

\[
\pi_a(k) + e_N \rightarrow \pi_a(k') + e_{N'} \tag{1}
\]

From eq. (1) it follows that the matrix element of the process (I) is a linear combination of the amplitudes of the following collision processes involving bare particles-bare excitons and bare photons:

\[
\gamma(k) + e_N \rightarrow \gamma(k') + e_{N'} \tag{II}
\]

\[
\gamma(k) + e_N \rightarrow Ex_\nu(k') + e_{N'} \tag{IIIa}
\]

\[
Ex_\nu(k) + e_N \rightarrow \gamma(k') + e_{N'} \tag{IIIb}
\]

\[
Ex_\nu(k) + e_N \rightarrow Ex_\nu(k') + e_{N'} \tag{IV}
\]
namely
\[ \langle \pi_x(k) | S | \pi_x(k) \rangle = -2 \pi i \delta [\Omega_x(k') + \varepsilon_x - \Omega_x(k) - \varepsilon_N] \times \]
\[ \times \left\{ u_x(k) u_x(k') \langle \gamma(k') e_N | \mathcal{K}_{\text{int}} | \gamma(k) e_N \rangle + u_x(k') \sum_{\gamma} v_{\gamma x}(k') \langle \gamma(k') e_N | \mathcal{K}_{\text{int}} | \gamma(k) e_N \rangle \right. \]
\[ + u_x(k) \sum_{\gamma'} v_{\gamma' x}(k') \langle \gamma'(k') e_N | \mathcal{K}_{\text{int}} | \gamma(k) e_N \rangle \]
\[ \left. + \sum_{\gamma} \sum_{\gamma'} v_{\gamma x}(k) v_{\gamma' x}(k') \langle \gamma'(k') e_N | \mathcal{K}_{\text{int}} | \gamma(k) e_N \rangle \right\} \] (5)

where \( \varepsilon_N \) is the energy of the electron bound in the donor ion.

3. Collision amplitudes for bare particles. — In this section we calculate the matrix elements of the processes (II), (IIIa), (IIIb) and (IV). Denote by \( f_x(p) \) and \( g_x(p) \) the (envelope) internal wave functions of the excitons \( E_x \) and \( E_x' \) in the \( p \)-space, by \( \phi_N(p) \) and \( \phi_N(p) \) the Fourier transforms of the wave functions of the bound electrons.

Consider first the scattering of the photons on the electrons (process (II)). In the reference [27] two of the authors have shown that at the value of the photon energy near that of the band gap the virtual transition between two bands give main contributions to the matrix element, and we can neglect the contribution of the intraband virtual transitions between the donor levels. The scattering process with the virtual interband transitions can be represented by the Feynman diagrams in figure 1a and 1b. In comparison with the matrix element of the diagram in figure 1a that of the diagram in figure 1b is negligible. Denote by \( k \) and \( k' \) the photon momenta, by \( E_g \) the band gap, by \( m_e \) and \( m_h \) the effective masses of the electron (in the conduction band) and the hole (in the valence band), by \( a_e \) the Bohr radius of the donor electron and by \( a_{\text{exc}} \) that of the exciton. We have

\[ R_{nn}(k', k) \equiv \langle \gamma(k') e_N | \mathcal{K}_{\text{int}} | \gamma(k) e_N \rangle = \frac{1}{(2\pi)^3} \cdot \frac{e^2}{\varepsilon_0 E_g} \cdot \frac{k'^2}{m} \cdot m_e a_e^2 I_{nn} \] (6)

where

\[ I_{nn} = \frac{\int \phi_N(p + k - k')^\ast \phi_N(p) dp}{2 m_e a_e^2 [E_g - \varepsilon_N - \omega(k) - \hbar]} + \frac{m_e}{m_h} \frac{a_e^2}{a_{\text{exc}}^2} p^2 \] (7)

Fig. 1. — Feynman diagrams of process (II). Solid line : electron, dashed line : hole, wavy line : photon.

Since the Hamiltonian is hermitic, in the lowest order the matrix elements of the collision processes (IIIa) and (IIIb) are complex conjugate one with another

\[ \langle \gamma(k') e_N | E_x(k) e_N \rangle = \langle E_x(k) e_N | \mathcal{K}_{\text{int}} | \gamma(k) e_N \rangle^\ast \].

One of them equals

\[ R_{\text{x}n}(k', k) \equiv \langle \gamma(k') e_N | \mathcal{K}_{\text{int}} | E_x(k) e_N \rangle = \frac{1}{2(2\pi)^2} \cdot \frac{e}{\sqrt{\varepsilon_0} \omega(k)} \cdot \frac{3(k')^\ast}{m} \cdot a_{\text{exc}}^{3/2} I_{\text{x}nn} \] (8)

where

\[ I_{\text{x}nn} = \frac{\sqrt{4\pi}}{(2\pi)^{3/2}} \cdot a_{\text{exc}}^{3/2} \int f_x(p) \phi_N(p + k/2)^\ast \phi_N(p + k' - k/2) dp \] (9)

The Feynman diagrams of the processes (IIIa) and (IIIb) are represented in figure 2a and 2b, respectively. For this scattering mechanism the exchange effect plays an important role as this can be seen from figures 2a and 2b.
The inelastic scattering of the excitons process (IV) can take place due to the Coulomb interaction of the charge carriers: the donor electron with the electron or the hole in the exciton. Taking into account both direct and exchange effect we obtain following result for its matrix element:

\[ R_{\nu \nu NN}(\mathbf{k}, \mathbf{k}') \equiv \langle E_{\nu}(\mathbf{k}') e_{N} | \mathcal{H}_{\text{int}} | E_{\nu}(\mathbf{k}) e_{N} \rangle = \sum_{i=\alpha}^{\varepsilon} T_{\nu \nu NN}(\mathbf{k}, \mathbf{k}') \]  

(10)

where

\[ T_{\nu \nu NN}(\mathbf{k}, \mathbf{k}') = \frac{1}{(2\pi)^3} \frac{e^2}{\varepsilon_0} \frac{1}{|\mathbf{k} - \mathbf{k}'|^2} 3_{\nu \nu NN} \]  

(11)

\[ T_{\nu \nu NN}(\mathbf{k}, \mathbf{k}') = \frac{1}{(2\pi)^3} \frac{e^2}{\varepsilon_0} \frac{1}{|\mathbf{k} - \mathbf{k}'|^2} 3_{\nu \nu NN} \]  

(12)

\[ T_{\nu \nu NN}(\mathbf{k}, \mathbf{k}') = \frac{1}{2(2\pi)^3} \frac{e^2}{\varepsilon_0} \beta a_{\text{exc}}^2 3_{\nu \nu NN} \]  

(13)

\[ T_{\nu \nu NN}(\mathbf{k}, \mathbf{k}') = \frac{1}{2(2\pi)^3} \frac{e^2}{\varepsilon_0} \beta a_{\text{exc}}^2 3_{\nu \nu NN} \]  

(14)

\[ T_{\nu \nu NN}(\mathbf{k}, \mathbf{k}') = \frac{1}{2(2\pi)^3} \frac{e^2}{\varepsilon_0} \beta a_{\text{exc}}^2 3_{\nu \nu NN} \]  

(15)

In the formulae (11)-(15) \( \beta = a_{\text{exc}}/a_e \) and the integrals \( 3^\alpha - 3^\varepsilon \) are defined as follows:

\[ 3_{\nu \nu NN} = \frac{1}{(2\pi)^6} \int f_{\nu}(p + \frac{\Delta k}{2}) f_{\nu}(p - \Delta k)^* \varphi_{\nu}(q - \Delta k)^* \varphi_{\nu}(q) \, dp \, dq \]  

(16)

\[ 3_{\nu \nu NN} = -\frac{1}{(2\pi)^6} \int f_{\nu}(p - \frac{\Delta k}{2}) f_{\nu}(p - \Delta k)^* \varphi_{\nu}(q - \Delta k)^* \varphi_{\nu}(q) \, dp \, dq \]  

(17)

\[ 3_{\nu \nu NN} = -\frac{2}{(2\pi)^6} \frac{1}{\beta a_{\text{exc}}^2} \int f_{\nu}(p + \frac{\Delta k}{2}) f_{\nu}(p - \Delta k)^* \varphi_{\nu}(q - \Delta k)^* \varphi_{\nu}(q) \, dp \, dq \]  

(18)

\[ 3_{\nu \nu NN} = \frac{2}{(2\pi)^6} \frac{1}{\beta a_{\text{exc}}^2} \int \frac{f_{\nu}(p)^* f_{\nu}(p) \varphi_{\nu}(p + \frac{k}{2})^* \varphi_{\nu}(p + \frac{k'}{2})}{|p - q + \frac{1}{2} \Delta k|^2} \, dp \, dq \]  

(19)

\[ 3_{\nu \nu NN} = \frac{2}{(2\pi)^6} \frac{1}{\beta a_{\text{exc}}^2} \int \frac{f_{\nu}(p)^* f_{\nu}(p) \varphi_{\nu}(p + \frac{k}{2})^* \varphi_{\nu}(p + \frac{k'}{2})}{|p - q + \frac{1}{2} \Delta k|^2} \, dp \, dq \]  

(20)

where \( \Delta k = k' - k \). The corresponding Feynman diagrams are represented in figures 3a-3e.

From the above results we can calculate the cross-section of the scattering of the polaritons. At a given energy there may not exist or there may exist some polaritons, which belong to different polariton branches characterized by the index \( \alpha \). Denote by \( \sigma_{\alpha \nu}^{x} \) the cross-section of the scattering process (I) in
Fig. 3. — Feynman diagrams of process (IV). Double wavy line: Coulomb interaction. a) Direct Coulomb interaction between the donor electron and the electron in the exciton; b) Direct Coulomb interaction between the donor electron and the hole in the exciton; c) Coulomb interaction between the donor electron and the electron in the exciton together with the exchange; d) and e) Coulomb interaction between the electron and the hole together with the exchange.

which the initial and final polaritons belong to the branches \( \alpha \) and \( \alpha' \), we have

\[
\sigma^{NN'}_{\alpha\alpha'} = 8 \pi^2 \left| M^{NN'}_{\alpha\alpha'}(k, k') \right|^2 \frac{k'^2}{dk} \frac{dk}{d\Omega_z} \tag{21}
\]

where \( k' \) is determined by the equation

\[
\Omega_z(k') + \epsilon_N - \Omega_s(k) - \epsilon_\alpha = 0 \tag{22}
\]

and \( M^{NN'}_{\alpha\alpha'}(k, k') \) is the following sum

\[
M^{NN'}_{\alpha\alpha'}(k, k') = u_s(k) u_s(k') R_{\alpha\alpha'}(k, k') + \sum_v v_{s\alpha}(k) \times \times u_{v\alpha}(k') R_{\alpha\alpha'}(k, k') + \sum_v u_{s\alpha}(k) v_{s\alpha}(k') \times \times R_{\alpha\alpha'}(k, k')^* + \sum_v \sum_{v'} v_{s\alpha}(k) v_{s\alpha}(k') R_{\alpha\alpha'}(k, k'). \tag{23}
\]

If the probability of the presence of the given polariton branch \( \alpha \) in the incoming polariton beam with the given energy is denoted by \( P_\alpha \), then for the total scattering cross-section of the polariton in the beam to the final state with energy satisfying the conservation law we have the expression

\[
\sigma^{NN'} = \sum_{\alpha, \alpha'} P_\alpha \sigma^{NN'}_{\alpha\alpha'}. \tag{24}
\]

4. An example. — As an illustration of the general results obtained above we consider the polariton effect in the resonant electronic Raman scattering on neutral donor levels in GaAs taking into account only the contribution of the discrete exciton states which are the bound states of an electron and a heavy hole. We use following values of the effective masses of the electron and the hole:

\[
m_e = 0.066 \ m; \quad m_h = 0.5 \ m
\]

and take

\[
E_g = 1.5 \ eV; \quad \epsilon_0 = 12.6.
\]

For the dipole transition matrix element we use the experimental date given in reference [45]. From eq. (4) we obtain following values of the effective coupling constants of the photon-exciton transition:

\[
\begin{align*}
g_{1s} &= 8.40 \ \text{meV} \\ g_{2s} &= 2.96 \ \text{meV} \\ g_{3s} &= 1.54 \ \text{meV}
\end{align*}
\]

At the value of the energy of the incoming polariton near that of the band gap \( E_g \) the matrix elements \( R_{NN'}, R_{\alpha\alpha'} \) and \( R_{v'v NN'} \) are almost constant and equal:

\[
\begin{align*}
R_{NN'} &\approx 2.0 \times 10^{-17} I_{NN'} \ \text{cm}^2 \\ R_{\alpha\alpha'} &\approx 1.1 \times 10^{-17} I_{\alpha\alpha'} \ \text{cm}^2 \\ R_{v'v NN'} &\approx 3.8 \times 10^{-17} \sum_{i=a}^{v} \beta_i^2 \ \text{cm}^2
\end{align*}
\]

For the exciton state \( v = v' = 1s \) and for the bound electron states \( N = 1s, N' = 2s \) we have following values

\[
\begin{align*}
I_{NN'} &\approx 0.29 \\ I_{\alpha\alpha'} &\approx -0.68 \\ \sum_i \beta_i^2 I_{v'v NN} &\approx -3.25
\end{align*}
\]

We see that the matrix elements \( R_{NN'}, R_{\alpha\alpha'} \) and \( R_{v'v NN'} \) have the values of the same order. Their contribution to the matrix element of the scattering of the polaritons depend also upon the transformation coefficients \( u_i(k) \) and \( v_i(k) \).

Consider the special case when only one exciton state \( v = 1s \) is taken into account, and the dispersion of the exciton is neglected

\[
E_{1s}(k) \approx E_{1s}(0) = E_{1s}.
\]

In this case there are two polariton branches \( \alpha = 1, 2 \) (Fig. 4) with energies

\[
\Omega_{1,2}(k) = \frac{1}{2} \times \left\{ E_{1s} + \omega(k) \mp \sqrt{(E_{1s} - \omega(k))^2 + 4 g_{1s}^2} \right\}.
\]

(28)
The transformation coefficients are determined by equations

\[
\begin{align*}
\nu_1(k) &= \frac{\theta_{1s1}(k)^2}{1 + \theta_{1s1}(k)^2} \\
\nu_{1s}(k)^2 &= \frac{1}{1 + \theta_{1s1}(k)^2}
\end{align*}
\]

(29)

where

\[
\theta_{1s1}(k) = \frac{1}{g_{1s}} (\Omega_1(k) - E_{1s}) ; \quad \alpha = 1, 2
\]

The polariton scattering matrix element becomes

\[
M_{2s}^{NN}(k, k') = \nu_1(k) \nu_{1s}(k') R_{NN} + \\
+ \nu_{1s}(k) \nu_{2s}(k') R_{1sNN} + \\
+ \nu_{1s}(k) \nu_{1s1}(k') R_{1s1sNN}
\]

(30)

It can be verified that at large values of the polariton momentum \( k \) the coefficients \( \nu_1(k) \) and \( \nu_{1s}(k) \) are very small and the corresponding terms in eq. (30) can be neglected. Now let the initial and final states of the bound electron be the 1s- and 2s-states. Denote by \( \Delta \delta \) the difference of their energies

\[
\Delta \delta = E_{2s} - E_{1s}
\]

(31)

by \( \delta = g_{1s}^2 / E_{1s} \) the gap in the polariton energy spectrum. If the energy of the incoming polariton is smaller than \( E_{1s} \), then there is only the polariton in the first (lower) branch \( \alpha = 1 \), and the total cross-section (24) equals the cross-section \( \sigma_{11} \). For the incoming polariton energies in the range from \( E_{1s} + \delta \) to \( E_{1s} + \delta + \Delta \delta \) there is only the polariton of the second (upper) branch \( \alpha = 2 \) in the incoming polariton beam, but the final polariton must belong to the first (lower) branch; in this case the total cross-section equals \( \sigma_{21} \). Finally, at the incoming polariton energies larger than \( E_{1s} + \delta + \Delta \delta \) both initial and final polaritons belong to the second (upper) branch of the difference between the incoming polariton energy and that of the dispersionless 1s-exciton. The relative contributions of the scattering mechanisms corresponding to the processes (II), (III) and (IV) to the polariton scattering matrix element for \( N = 1s \) and \( N' = 2s \) are represented by the curves in figure 6.

If the dispersion of the exciton is not negligible, then there exist the energy intervals where two polaritons from two different branches have the same energy. This case also can be considered, as it was done in reference [44].
5. Discussion. In this concluding section we discuss on some aspects related to the above results.

First, note that we have studied the case when the initial and final polaritons are the mixing of the photon and the same exciton state \( v = ls \). There exists, however, an infinite number of exciton states with the discrete energies, and it may happen that the initial photon is in the resonance with one exciton state, \( v = 2s \) for example, but the final photon is in the resonance with the other, \( v' = 1s \). We can generalize above results to this case. Now the matrix element (23) becomes

\[
M_{2s}^{1s,2s}(k, k') = u_s(k) u_s(k') R_{1s,2s} + \nonumber \\
+ v_{2s}(k) u_s(k') R_{2s,1s} + \nonumber \\
+ u_s(k) v_{1s}(k') R_{1s,2s} + \nonumber \\
+ v_{2s}(k) v_{1s}(k') R_{2s,1s} .
\]

(32)

More precisely we can use the general expression (23) for the matrix element and eqs. (2) and (3) for the transformation coefficients and the polariton energies, but in the sums over \( v \) and \( v' \) we take only the terms with \( v = 2s \) and \( v' = 1s \).

For the integrals \( I_{NN'} \) and \( J_{VV',NN'} \) we have the following values :

\[
I_{1s,2s}^1 = I_{2s,1s}^1 = -0.68 \\
I_{2s,1s}^2 = I_{2s,2s}^1 = 1.48 \\
J_{1s,2s}^1 = J_{1s,2s}^2 = 1.41 \\
J_{2s,1s}^1 = J_{2s,2s}^1 = 1.76 \\
J_{1s,1s}^{2s} = J_{2s,2s}^{1s} = -2.33 \\
J_{1s,1s}^{2s} = J_{2s,2s}^{1s} = -3.72 \\
J_{2s,1s}^{2s} = J_{2s,1s}^{1s} = 4.61 \\
J_{2s,1s}^{2s} = J_{2s,1s}^{1s} = 1.37 .
\]

Secondly, we have investigated the scattering on bound electrons in a donor ion. In this case the momentum is not conserved, as it is easily seen from the expressions of the scattering matrix elements: eqs. (5), (6), (8) and (10). These matrix elements are nonzero for any momenta \( k \) and \( k' \) of the incoming and outgoing photons, which are related one with another by the energy conservation law

\[
\hat{\epsilon}_N + \omega(k) = \hat{\epsilon}_N' + \omega(k') .
\]

(33)

It is straightforward to modify our results to apply to the case of the scattering on free electrons. In this case the (free) electron wave functions in the \( p \)-space are the \( \delta \)-functions

\[
\varphi_N(p) = \delta(p - q) ; \quad \varphi_N'(p) = \delta(p - q')
\]

(34)

\( q \) and \( q' \) being the momenta of the initial and final electrons. Substituting the expressions (34) for the wave functions into the r.h.s. of eqs. (7), (9), (16)-(20), we obtain the expressions for the scattering matrix elements proportional to the \( \delta \)-function

\[
\delta(q + k - q' - k')
\]

which means that the total momentum is conserved.

Third, from the behaviour of the curves in figure 6 it follows that in the resonance region

\[
| \Omega - E_{1s} | \lesssim 10 \text{ meV}
\]

the exciton-photon mixing is very essential, and the polariton effect is important. In particular, at the energy range

\[
0 \lesssim \Omega - E_{1s} \lesssim 5 \text{ meV}
\]

the amplitude of the scattering of the light is determined completely in terms of the exciton-electron scattering amplitude. For the light with the frequency \( \omega = \Omega \) such that

\[
| \Omega - E_{1s} | \gg 10 \text{ meV}
\]

the mixing between the photon and the exciton is weak, and the scattering of the bare photons (process (II)) is the most important mechanism contributing to the matrix element of the Raman scattering of the polaritons. In order to evaluate the role of the polariton effect we compare the scattering cross-section calculated above for the polaritons with that for the bare photons (see Fig. 5).

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References


[34] PEKAR, S. I., Sov. Phys.-JETP 6 (1958) 785.