Bound states and high field longitudinal magnetoresistance in semiconductors

G. Bastard, C. Lewiner

To cite this version:


HAL Id: jpa-00208739
https://hal.archives-ouvertes.fr/jpa-00208739
Submitted on 1 Jan 1978

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
BOUND STATES AND HIGH FIELD LONGITUDINAL MAGNETORESISTANCE IN SEMICONDUCTORS

G. BASTARD and C. LEWINER

Groupe de Physique des Solides de l'Ecole Normale Supérieure (*),
24, rue Lhomond, 75231 Paris Cedex 05, France
and Université Paris VII, 2, place Jussieu, 75221 Paris Cedex 05, France

(Reçu le 6 juin 1977, accepté le 29 septembre 1977)

Résumé. — Nous calculons la magnétorésistance en régime quantique lorsque les électrons interagissent avec des centres donneurs par un potentiel de contact. L'influence des états liés sur \( \sigma_{zz} \) est prise en compte. Les positions et largeurs des états localisés sont discutées ainsi que les effets d'élargissement par collisions multiples.

Abstract. — The longitudinal magnetoresistance is calculated in the quantum limit in the case of a point like interaction between electrons and donors. We account for the influence of the bound states on \( \sigma_{zz} \). The position and width of donor states are derived, and the effects of collision broadening are discussed.

1. Introduction. — The study of the oscillations of the conductivity tensor is now a popular tool to determine the band structure parameters of semiconductors (effective masses and \( g \) factors...). In the extrinsic regime, when the magnetic field is so strong that only the \( 0^+ \) Landau level is populated, the problem of carriers freezing out unavoidably arises.

The purpose of this paper is to provide within the framework of a very simple model a unified treatment of the energy spectrum (free and bound states) of an electron moving in a quantizing magnetic field and interacting with random donor impurities. We will also qualitatively discuss the several impurities effect which will be shown to destroy the one dimensional nature of the electron motion.

Once the electronic energy spectrum established, we will calculate the longitudinal magnetoresistance and examine the influence of the potential strength and of the freeze out effects on the magnetic field dependence of \( \langle \Delta \rho / \rho \rangle_{zz} \).

The paper will be organized as follows : in section 2, we shall present the model and derive the general formulae for the longitudinal conductivity. Section 3 will be devoted to the problem of bound states in a magnetic field. In section 4, we shall discuss the behaviour of \( \sigma_{zz} \) when \( \mu / k_B T \gg 1 \). Finally, section 5 will deal with the freeze-out effect.

2. Model. — In the following we shall assume a spherical parabolic dispersion relation for the electrons whose unperturbed eigenenergies in the presence of an external magnetic field \( H \parallel z \) will be written :

\[
\varepsilon_n = (n + 1/2) \hbar \omega_c + \frac{\hbar^2 k_z^2}{2 m^*} - s_c \sigma
\]

where \( m^*, \omega_c, s_c \) are the effective mass, cyclotron frequency and spin splitting \( (g^* \neq 0) \) of the carriers at the bottom of the conduction band. The eigenfunctions corresponding to \( \varepsilon_n \) are

\[
\langle r | \varepsilon_n \sigma \rangle = \langle r | nk_z k_x \sigma \rangle =
\]

\[
\frac{1}{L_x L_z} \exp ik_y y \exp ik_z z \varphi_n(x + \lambda^2 k_y) \chi_\sigma(3)
\]

where \( \varphi_n(x + \lambda^2 k_y) \) is the normalized harmonic oscillator function centred at \( x_0 = - \lambda^2 k_y \), \( \lambda \) is the magnetic length \( (\lambda^2 = \hbar c/eH) \) and \( \chi_\sigma \) the spin eigenfunction \( (\sigma = \pm 1/2) \).

Using the results of references [1, 2] we may express the static longitudinal conductivity \( \sigma_{zz} \) as

\[
\sigma_{zz} = \frac{2 e^2}{\pi \Omega} \int_{-\infty}^{+\infty} d\varepsilon f(\varepsilon) \times
\]

\[
\times \text{Tr} \left\{ J_z \text{Im} G(\varepsilon) J_z \text{Im} \left( \frac{d}{d\varepsilon} G(\varepsilon) \right) \right\}
\]

(*) Laboratoire associé au Centre National de la Recherche Scientifique.
where \( j_z \) is the current along the \( z \) direction:

\[
j_z = \frac{\hbar}{m^*} k_z l_m^* \]

\( J_z \) is the total Hamiltonian of an electron interacting with the external field \( H \) and \( N_D \) impurities, which are assumed randomly distributed over the whole crystal (volume \( \Omega \)).

For mathematical convenience the impurity potential will be chosen in the form

\[
V(r) = \sum_{\mathbf{R}_i} V \delta(\mathbf{r} - \mathbf{R}_i)
\]

and \( V \) is related to the scattering length \( f \) by

\[
V = \frac{-2 \pi \hbar^2 f}{m^*} \quad \text{(note that } V \text{ should be taken negative in order to provide free electrons in the conduction band)}.
\]

\( (\sigma_{zz}) \) may be represented by the diagrams shown on figure 1. Ultimately \( \sigma_{zz} \) should be averaged over the random position of the impurities. However, the contact potential defined in equation (6) enables a complete separation of the two branches, which can then be averaged independently. For instance, the graph (b) gives a contribution to \( \sigma_{zz} \) equal to:

\[
\frac{2}{\pi \Omega} \int_{-\infty}^{+\infty} f(\epsilon) d\epsilon \sum_{\sigma = \pm 1/2} \sum_{N,N'} k_+ k_-' \times \sum_{\mathbf{R}_i} V \delta(\mathbf{r} - \mathbf{R}_i) \]

which vanishes by parity because of the lonely factor \( k_+' \). At zero field the same situation holds as \( \langle k | V(r) | k' \rangle \) does not depend on the relative orientation between \( k \) and \( k' \) after averaging over \( \mathbf{R}_i \).

Since we expect \( \sigma_{zz} \) to be proportional to \( \tau \) (where \( \tau \) is the relaxation time) we need to calculate \( G \) up to any power in \( N_D \). This can be done only if we neglect the diagrams with meshing lines (graph c), i.e. in the limit \( \frac{\mu}{\hbar} \ll \epsilon_0 \) or \( \frac{\hbar}{\Omega} \gg 1 \).

Then after averaging

\[
\langle N_{\sigma} | \tilde{G}_{\sigma} | N'_{\sigma'} \rangle = \delta_{N N'} \delta_{\sigma \sigma'} \frac{1}{\epsilon - \epsilon_{\text{as}} + i\eta} \]

\[
\left( \alpha_{\sigma} \right) = \sum_{\mathbf{R}_i} \frac{N_D}{\Omega} \frac{V}{1 - \frac{V}{\Omega} K_{r}^{*}}
\]

\( K_{r}^{0}(\epsilon) = \sum_{\mathbf{R}_i} \frac{1}{\epsilon - \epsilon_{\text{as}} \pm i\eta} = K_{r}^{(1)}(\epsilon) + iK_{r}^{(2)}(\epsilon)
\]

In principle, equations (8-10) should be solved self-consistently, although we shall frequently replace \( K \) by \( K_{0} \) in equation (9), i.e. without impurities we have:

\[
K_{r}^{0}(\epsilon) = \sum_{\mathbf{R}_i} \frac{1}{\epsilon - \epsilon_{\text{as}} \pm i\eta} = K_{r}^{(1)}(\epsilon) + iK_{r}^{(2)}(\epsilon)
\]

The summation over \( n \) diverges for \( K_{r}^{(1/2)}(\epsilon) \) essentially because the conduction band retains an infinite extension in energy. This problem will be discussed in section 3.

Note in equation (8) the spin conservation which is imposed by the scalar nature of the potential: the \( \sigma = \pm 1/2 \) electrons conduct independently. We may rewrite \( \sigma_{zz} \) as

\[
\sigma_{zz} = \frac{e^2 \hbar}{4 \pi \Omega} \sum_{N_{\sigma}} \left( \frac{\hbar k_+^2}{m^*} \right)^2 \int_{-\infty}^{+\infty} \frac{df}{d\epsilon} \sum_{N,N'} \sum_{\mathbf{R}_i} V \delta(\mathbf{r} - \mathbf{R}_i)
\]

In the presence of impurities the number of electrons of spin \( \sigma \) is equal to

\[
N_{\sigma} = -\frac{1}{\pi} \Im \sum_{N} \int_{-\infty}^{+\infty} f(\epsilon) d\epsilon \tilde{G}_{\sigma}(\epsilon)
\]

3. Bound states in a magnetic field. — Before calculating \( \sigma_{zz} \) explicitly with the help of equations (8-13), we will investigate the spectrum of the one electron Hamiltonian \( \mathcal{H} \).

In the presence of an external magnetic field \( H \), an attractive potential \( (V < 0) \) will at least create a bound state with energy \( \epsilon_0 < \epsilon_{\text{as}} \). This general statement is a direct consequence of the quantization of the electron motion perpendicular to \( H \). As a first step, we solve the single impurity problem, i.e. we replace \( K \) by \( K_{0}^{0} \) in equation (9). Then, eventually, multiple scatterings may be included to broaden the localized level obtained in the single impurity model. If there
is a single donor atom in the crystal, the localized level is the solution of:

$$\frac{1}{\nu} = K_{00}(\epsilon) K_{00}^{(2)}(\epsilon) = 0 \quad \epsilon < \epsilon_{0\sigma}$$  \hspace{1cm} (14)

$$K_{00}^{(1)}(\epsilon) = P \sum_{n} \frac{1}{\epsilon - \epsilon_{n\sigma}} = - \frac{m^* \Omega}{2 \pi \hbar^2 \lambda \sqrt{2}} \times$$

$$\times \sum_{p=n}^{n_{\sigma}} \left[ p + 1 - \frac{s_{\sigma}}{\hbar \omega_c} - \frac{\epsilon}{\hbar \omega_c} \right]^{-1/2}$$  \hspace{1cm} (15)

where $n_{\sigma}$ is the smallest integer which exceeds

$$\frac{\epsilon}{\hbar \omega_c} > \frac{1}{2} + \frac{s_{\sigma}}{\hbar \omega_c}.$$

As we have stressed before, $K_0^{(1)}$ is given by a divergent summation which arises from the unboundedness of the unperturbed bandwidth. Of course, in real solids, bands are always finite and we only need to find a plausible description of $K_0^{(1)}$. Such a model should necessarily incorporate two facts:

$$\lim_{\epsilon \to -\infty} K_{00}^{(1)}(\epsilon) = 0$$

$$\lim_{\epsilon \to \epsilon_{0\sigma}} K_{00}^{(1)}(\epsilon) = - \infty.$$

The first assumption follows from the relation

$$K_{00}^{(1)}(\epsilon) = \frac{1}{\pi} \int^{\epsilon_{\text{max}}}_{\epsilon_{0\sigma}} K_{00}^{(2)}(\epsilon') \frac{d\epsilon'}{\epsilon - \epsilon'}$$  \hspace{1cm} (16)

together with $\epsilon'_{\text{max}} < + \infty$ i.e. a finite bandwidth.

The second follows from equation (15) which can be rewritten:

$$[K_{00}^{(1)}(\epsilon)]_{\epsilon < \epsilon_{0\sigma}} = - \frac{m^* \Omega}{2 \pi \hbar^2 \lambda \sqrt{2}} \times$$

$$\times \left[ n + \frac{1}{2} - \frac{s_{\sigma}}{\hbar \omega_c} - \frac{\epsilon}{\hbar \omega_c} \right]^{-1/2} + \alpha_{0\sigma}(\epsilon)$$  \hspace{1cm} (17)

where $\alpha_{0\sigma}(\epsilon)$ is a regular function near

$$\epsilon = \epsilon_{0\sigma} : \alpha_{0\sigma}(\epsilon_{0\sigma}) = \alpha_{0\sigma} \quad \text{and} \quad \alpha_{0\sigma}(- \infty) = 0.$$

As we are mostly interested in shallow donor levels (1 $\epsilon_{0\sigma} - \epsilon_{0\sigma}$) $\ll$ energy range of $\alpha_{0\sigma}(\epsilon)$ we assume $\alpha_{0\sigma}(\epsilon) = \alpha_0$ : our cutting procedure amounts only to renormalizing the potential strength. This does not alter the origin of the bound level which exists for any $\nu$ : even if $\nu \to 0$, $\delta_0$ exists (see Fig. 2a) because $K_{00}^{(1)}(\epsilon_{0\sigma}) = - \infty$. This is in marked contrast to the zero field case where it is well known that the condition $\nu < \nu_{\text{c}} < 0$ should be fulfilled to obtain one bound level. The difference lies in the quantization of the perpendicular motion which already localizes the electron in two directions leading to the one-dimensional nature of the impurity problem.

![Fig. 2. Sketch of the energy dependence of $K_{00}^{(1)}$ (solid line) and $K_{00}^{(2)}$ (dashed line) : a) $H \neq 0$, b) $H = 0$.](image-url)

Note also that the unperturbed spectrum (equation (1)) consists of Landau sub-bands repeating themselves each $\hbar \omega_c$. Then, in contrast to the zero field case, we will not obtain a single donor level below the conduction band edges, but as many impurity states as the number of Landau levels enclosed in the unperturbed bandwidth.

Let us now describe a simple model which will help to derive an explicit expression for the cut off factors $\alpha_{0\sigma}$. To retain a tractable algebra we shall describe the finite bandwidth assuming

$$\rho_{0\sigma}(\epsilon) = \frac{K_{00}^{(2)}}{\pi} = \frac{m^* \Omega}{2 \pi \hbar^2 \lambda \sqrt{2}} \sum_{p=0}^{n_{\sigma}-1} \left[ \frac{\epsilon}{\hbar \omega_c} - p - \frac{1}{2} + \frac{s_{\sigma}}{\hbar \omega_c} \right]^{-1/2} \epsilon_{0\sigma} < \epsilon < \Delta$$  \hspace{1cm} (18)

$$\rho_{0\sigma}(\epsilon) = 0 \quad \epsilon \geq \Delta$$

The first assumption follows from the relation

$$K_{00}^{(1)}(\epsilon) = \frac{1}{\pi} \int^{\epsilon_{\text{max}}}_{\epsilon_{0\sigma}} K_{00}^{(2)}(\epsilon') \frac{d\epsilon'}{\epsilon - \epsilon'}$$  \hspace{1cm} (16)

together with $\epsilon'_{\text{max}} < + \infty$ i.e. a finite bandwidth.

The second follows from equation (15) which can be rewritten:

$$[K_{00}^{(1)}(\epsilon)]_{\epsilon < \epsilon_{0\sigma}} = - \frac{m^* \Omega}{2 \pi \hbar^2 \lambda \sqrt{2}} \times$$

$$\times \left[ n + \frac{1}{2} - \frac{s_{\sigma}}{\hbar \omega_c} - \frac{\epsilon}{\hbar \omega_c} \right]^{-1/2} + \alpha_{0\sigma}(\epsilon)$$  \hspace{1cm} (17)

where $\alpha_{0\sigma}(\epsilon)$ is a regular function near

$$\epsilon = \epsilon_{0\sigma} : \alpha_{0\sigma}(\epsilon_{0\sigma}) = \alpha_{0\sigma} \quad \text{and} \quad \alpha_{0\sigma}(- \infty) = 0.$$

As we are mostly interested in shallow donor levels (1 $\epsilon_{0\sigma} - \epsilon_{0\sigma}$) $\ll$ energy range of $\alpha_{0\sigma}(\epsilon)$ we assume $\alpha_{0\sigma}(\epsilon) = \alpha_0$ : our cutting procedure amounts only to renormalizing the potential strength. This does not alter the origin of the bound level which exists for any $\nu$ : even if $\nu \to 0$, $\delta_0$ exists (see Fig. 2a) because $K_{00}^{(1)}(\epsilon_{0\sigma}) = - \infty$. This is in marked contrast to the zero field case where it is well known that the condition $\nu < \nu_{\text{c}} < 0$ should be fulfilled to obtain one bound level. The difference lies in the quantization of the perpendicular motion which already localizes the electron in two directions leading to the one-dimensional nature of the impurity problem.

![Fig. 2. Sketch of the energy dependence of $K_{00}^{(1)}$ (solid line) and $K_{00}^{(2)}$ (dashed line) : a) $H \neq 0$, b) $H = 0$.](image-url)

Note also that the unperturbed spectrum (equation (1)) consists of Landau sub-bands repeating themselves each $\hbar \omega_c$. Then, in contrast to the zero field case, we will not obtain a single donor level below the conduction band edges, but as many impurity states as the number of Landau levels enclosed in the unperturbed bandwidth.

Let us now describe a simple model which will help to derive an explicit expression for the cut off factors $\alpha_{0\sigma}$. To retain a tractable algebra we shall describe the finite bandwidth assuming

$$\rho_{0\sigma}(\epsilon) = \frac{K_{00}^{(2)}}{\pi} = \frac{m^* \Omega}{2 \pi \hbar^2 \lambda \sqrt{2}} \sum_{p=0}^{n_{\sigma}-1} \left[ \frac{\epsilon}{\hbar \omega_c} - p - \frac{1}{2} + \frac{s_{\sigma}}{\hbar \omega_c} \right]^{-1/2} \epsilon_{0\sigma} < \epsilon < \Delta$$  \hspace{1cm} (18)

$$\rho_{0\sigma}(\epsilon) = 0 \quad \epsilon \geq \Delta$$
i.e. we abruptly cut off the band at $d$ and our results are expected to be meaningful only if $\varepsilon \ll \Delta$ (the actual case in semiconductor physics). Using equations (16, 18), we obtain

$$K_{\alpha}^{(1)}(\varepsilon) = \frac{m^* \Omega}{2 \pi^2 \hbar^2 \lambda \sqrt{2}} \left\{ P \int_{\varepsilon_{\alpha \varepsilon}}^{d} \frac{d\varepsilon'}{\sqrt{\hbar \omega_c - \varepsilon'}} + \sum_{p=\alpha \varepsilon + 1}^{\alpha} P \int_{\varepsilon_{p \varepsilon}}^{d} \frac{d\varepsilon'}{\sqrt{\hbar \omega_c - \varepsilon'}} \right\}$$

(19)

in which we have isolated the first term which diverges when $\varepsilon$ approaches $\varepsilon_{\alpha \varepsilon}$ from below and where $P$ stands for the Cauchy principal value. To evaluate the remaining contribution we will crudely approximate the summation by an integral. Then

$$K_{\alpha}^{(1)}(\varepsilon) = - \frac{m^* \Omega}{\pi^2 \hbar^2 \lambda \sqrt{2}} \left\{ \sqrt{\frac{\hbar \omega_c}{\varepsilon_{\alpha \varepsilon} - \varepsilon}} \arctg \frac{\Delta - \varepsilon_{\alpha \varepsilon}}{\hbar \omega_c} + 2 \sqrt{\frac{\Delta - \varepsilon_{1 \varepsilon}}{\hbar \omega_c}} - 2 \sqrt{\frac{\varepsilon_{\alpha 1 \varepsilon} - \varepsilon}{\hbar \omega_c}} \arctg \frac{\Delta - \varepsilon_{1 \varepsilon}}{\varepsilon_{1 \varepsilon} - \varepsilon} \right\}$$

(20a)

if $\varepsilon < \varepsilon_{\alpha \varepsilon}$

$$K_{\alpha}^{(1)}(\varepsilon) = - \frac{m^* \Omega}{\pi^2 \hbar^2 \lambda \sqrt{2}} \left\{ \sqrt{\frac{\hbar \omega_c}{\varepsilon_{\alpha \varepsilon} - \varepsilon}} \arctg \frac{\Delta - \varepsilon_{\alpha \varepsilon}}{\hbar \omega_c} + 2 \sqrt{\frac{\Delta - \varepsilon_{1 \varepsilon}}{\hbar \omega_c}} - 2 \sqrt{\frac{\varepsilon_{\alpha 1 \varepsilon} - \varepsilon}{\hbar \omega_c}} \times \arctg \frac{\Delta - \varepsilon_{1 \varepsilon}}{\varepsilon_{1 \varepsilon} - \varepsilon} \right\}$$

(20b)

These results may be compared with other approaches [4, 5] which essentially keep $\Delta$ infinite but replace the $\delta$ shape of the impurity potential by a well with a finite range. Qualitatively both approaches are equivalent.

The two cases $\varepsilon \lesssim \varepsilon_{\alpha \varepsilon}$ will be discussed separately. In the former case ($\varepsilon < \varepsilon_{\alpha \varepsilon}$), a perfectly localized state will be obtained from equation (14) and a finite level width may be achieved only by accounting for collision broadening on several impurities. In the latter case ($\varepsilon > \varepsilon_{\alpha \varepsilon}$), even for a single impurity, the unperturbed density of states will broaden the resonant donor level attached to each Landau sub-band. These levels are then expected to fade away for large $n$.

3.1 ISOLATED SHALLOW DONOR LEVEL IN THE GAP. 

If $\varepsilon < \varepsilon_{\alpha \varepsilon}$ the binding equation reads

$$\frac{1}{V} = K_{0 \varepsilon}^{(1)}(\varepsilon) \Omega$$

As $K_{0 \varepsilon}^{(1)}(\varepsilon) < 0$ if $-\infty < \varepsilon < \varepsilon_{\alpha \varepsilon}$, we recover the expected result that only the donor potential (i.e. $V < 0$) may create an impurity state below the conduction band edge. Moreover, the existence of this localized level is independent of the potential strength which contrasts with the case $H = 0$. Indeed, when $H = 0$, the same approximation as used in equation (18) is

$$\rho_{0 \varepsilon}(\varepsilon) = \frac{\Omega m^*}{2 \pi^2 \hbar^2} \sqrt{\frac{2 m^* \varepsilon}{\hbar^2}} \quad \text{if} \quad 0 \leq \varepsilon < \Delta$$

$$\rho_{0 \varepsilon}(\varepsilon) = 0 \quad \text{otherwise}$$

which leads to

$$K_{0 \varepsilon}^{(1)}(\varepsilon) = - \frac{\Omega m^*}{\pi^2 \hbar^2} \sqrt{\frac{2 m^*}{\hbar^2}} \times \left[ \frac{\sqrt{\Delta - \varepsilon}}{\Delta - \varepsilon} \arctg \frac{\Delta}{\varepsilon} \right] \quad \varepsilon \leq 0$$

(22)

$$K_{0 \varepsilon}^{(1)}(\varepsilon) = - \frac{\Omega m^*}{\pi^2 \hbar^2} \sqrt{\frac{2 m^*}{\hbar^2}} \times \left[ \frac{\sqrt{\Delta - \varepsilon}}{\Delta - \varepsilon} \arctg \frac{\Delta}{\varepsilon} \log \frac{\sqrt{\Delta + \sqrt{\varepsilon}}}{\sqrt{\Delta - \sqrt{\varepsilon}}} \right] \quad \varepsilon \geq 0$$

$K_{0 \varepsilon}^{(1)}$ is sketched in figure 2b.

Near $\varepsilon = 0$, $K_{0 \varepsilon}^{(1)}(\varepsilon) \approx - \frac{\Omega m^*}{\pi^2 \hbar^2} \sqrt{\frac{2 m^* \Delta}{\hbar^2}}$ which leads to the existence of a critical scattering length $f_{\text{crit}}$ below which no bound state may exist. A localized level appears at $\varepsilon_0 < 0$ only if

$$f = - \frac{m^* V}{2 \pi^2 \hbar^2} > f_{\text{crit}}$$

(23)

$$f_{\text{crit}}^{-1} = 2 q_0 = 2 \sqrt{\frac{2 m^* \Delta}{\pi^2 \hbar^2}}$$

For shallow donor level ($|\varepsilon_D| \ll \Delta$) the binding energy $r_0$ is

$$r_0 = - \varepsilon_D = \frac{\hbar^2}{2 m^*(f^*)^2}, \quad f^* = \frac{f}{2 f q_0 - 1}$$

(24)
i.e. the finite bandwidth only renormalizes the potential strength.

A similar situation holds in a finite magnetic field: if we are interested in shallow levels, $f$ and $K_{0s}^{(1)}$ may be replaced by the renormalized quantities $\tilde{f}$ and $\tilde{K}_{0s}^{(1)}$ defined by:

$$\tilde{K}_{0s}^{(1)} = -\frac{m^*}{2\pi \sqrt{2h^2}} \lambda \left[ \sqrt{\frac{\hbar \omega_c}{\varepsilon_{0s} - \varepsilon}} - 2 \sqrt{\frac{\varepsilon_{0s} - \varepsilon}{\hbar \omega_c}} \right] \frac{1}{f} = \frac{1}{f} - 2 \eta_0. \quad (25)$$

From now on we shall only use $\tilde{K}$ and $\tilde{f}$. If we define the dimensionless unit $\eta$ such that $\varepsilon = \varepsilon_{0s} - \eta \hbar \omega_c$, the bound state is the solution of

$$\frac{\lambda \sqrt{2}}{f} = \eta^{1/2} - 2(1 + \eta)^{1/2}. \quad (26)$$

It is interesting to notice that both $\tilde{f} \leq 0$ give admissible solutions (see Fig. 3). $\tilde{f} > 0$ corresponds to bound states whose activation energies vanish at zero field, whereas $\tilde{f} < 0$ describes states with non zero binding energy when $H = 0$. Note also that both cases correspond to donor potential ($V$ and $K_{0s}^{(1)}$ are negative).

At the border $1/f = 0$ we obtain a localized level whose binding energy depends linearly on the magnetic field strength (by definition $r_\sigma = \varepsilon_{0s} - \varepsilon_\Omega$):

$$r_\sigma = \left( \frac{\sqrt{2} - 1}{2} \right) \frac{\hbar \omega_c}{\lambda}. \quad (27)$$

If $\tilde{f} > 0$ ($r(H = 0) = 0$) we find

$$r_\sigma \approx \frac{1}{2} \left( \frac{\tilde{f}}{\lambda} \right)^2 \hbar \omega_c \quad \text{if} \quad \frac{\tilde{f}}{\lambda} \ll 1$$

$$r_\sigma \approx \left( \frac{\sqrt{2} - 1}{2} \right) \hbar \omega_c - \frac{\hbar \omega_c}{2 \tilde{f} \sqrt{\lambda^2 - 1}} \quad \text{if} \quad \frac{\tilde{f}}{\lambda} \gg 1. \quad (28)$$

If $\tilde{f} < 0$ ($r(H = 0) = r_0$)

$$r_\sigma \approx r_0 + \frac{1}{2} \left( \frac{\tilde{f}}{\lambda} \right)^2 \hbar \omega_c \quad \text{if} \quad \left| \frac{\tilde{f}}{\lambda} \right| \ll 1$$

$$r_\sigma \approx \left( \frac{\sqrt{2} - 1}{2} \right) \hbar \omega_c + \frac{\hbar \omega_c}{2 \tilde{f} \sqrt{\lambda^2 - 1}} \quad \text{if} \quad \left| \frac{\tilde{f}}{\lambda} \right| \gg 1. \quad (29)$$

The behaviours corresponding to these three cases are sketched in figure 4. As seen in equation (29), our model fails to describe the weak field behaviour ($|\tilde{f}/\lambda| \ll 1$) of $r_\sigma(H)$ when $r_0 \neq 0$. In such a case, one expects $r_\sigma - r_0 < 0$ (diamagnetic shift) which could be obtained only by calculating more carefully the summation in equation (19). Note that in narrow gap semiconductors, the magnetic field range of diamagnetic shift is extremely narrow (for instance in InSb with $r_0 < 1$ meV, $\hbar \omega_c/r_0 > 10$ if $H \approx 10$ kG).

FIG. 3. — Graphical resolution of equation (26).

FIG. 4. — Magnetic field dependence of the binding energy in the three cases discussed in section 3.1.
3.2 Deep Impurity Level. — In this case (rather unrealistic in practice) there is no sense in using \( K_{0a}^{(1)} \) and \( f ; K_{0a}^{(1)} \approx 1/|e| \) when \( e \to -\infty \) and we find a bound state whose activation energy \( r \) is almost independent of \( H \):

\[
r \approx r_0 \approx \frac{2}{3} f q_0 A .
\]

3.3 Isolated Shallow Donor Level in the Conduction Band. — Owing to the Landau quantization the situation described in 3.1 reappears periodically: each Landau sub-band carries an impurity level. To calculate the position of these levels we have, as in 3.1, to solve the equation \( V^{-1} = K_{0a}/\Omega \) with \( K_{0a}^{(2)} \neq 0 \). The donor levels appearing in the band are then broadened by their interaction with the conduction band continuum. Denoting by \( E_{D} \) the complex energy of the impurity level attached to the \((n, \sigma)\) Landau sub-band and using standard procedures we find:

\[
E_{D}^{(n, \sigma)} = E_{D}^{(n, \sigma)} + s_{n \sigma} - i\Gamma_{n \sigma} = v_{n \sigma} - r_{n \sigma} + s_{n \sigma} - i\Gamma_{n \sigma}
\]

(30)

\[
\rho^{-1} = \frac{K_{0a}^{(1)}}{\Omega} \left( E_{D}^{(n, \sigma)} \right)
\]

(31)

\[
\Gamma_{n \sigma} = - \frac{K_{0a}^{(2)}(E_{D}^{(n, \sigma)})}{\left[ \frac{dK_{0a}^{(1)}}{dE} \right]_{E = E_{D}^{(n, \sigma)}}}
\]

(32)

\[
s_{n \sigma} = \Gamma_{n \sigma} \left[ \frac{dK_{0a}^{(2)}}{dE} \frac{dK_{0a}^{(1)}}{dE} \right]_{E = E_{D}^{(n, \sigma)}} .
\]

(33)

In a strong magnetic field \(|\lambda f| < 1\), the three situations discussed in 3.1 lead to identical results:

\[
r_{n \sigma} \approx \left( \frac{\sqrt{2} - 1}{2} \right) \hbar \omega_c + 0 \left( \frac{1}{f} \right)
\]

(34)

\[
\frac{\Gamma_{n \sigma}}{r_{n \sigma}} \approx \frac{1}{2} \sqrt{1 + \sqrt{2}} \sum_{p=0}^{n-1} \left[ n - p - \left( \frac{\sqrt{2} - 1}{2} \right) \right]^{-1/2}
\]

(35)

\[
\frac{s_{n \sigma}}{r_{n \sigma}} \approx \frac{1}{16} \sum_{p=0}^{n-1} \left[ n - p - \left( \frac{\sqrt{2} - 1}{2} \right) \right]^{-3/2} \times \sum_{p=0}^{n-1} \left[ n - p - \left( \frac{\sqrt{2} - 1}{2} \right) \right]^{-3/2}
\]

(36)

From equation (35) we obtain:

\[
\frac{\Gamma_{1 \sigma}}{r_{1 \sigma}} \approx 0.9 , \quad \frac{\Gamma_{2 \sigma}}{r_{2 \sigma}} \approx 1.5 .
\]

This implies that only the impurity level attached to the first Landau level will produce a marked resonance in the conduction band. As seen in equation (36), \( s_{n} \) increases with \( n \): the effective activation energy decreases with \( n \). As is observed experimentally [6], our model predicts the occurrence of a doublet structure for the cyclotron resonance absorption, the low-field component corresponding to the impurity cyclotron resonance, and the other peak arising from the usual free electron cyclotron resonance (see Fig. 5).

Note however that the binding energies calculated from equations (34, 36) are too large compared to the experiments. This is due to the assumption of a contact like potential.

\[\text{FIG. 5.} \quad \text{Resonant transitions obtained in the vicinity of the cyclotron resonant frequency. I.C.R. and C.R. denote the impurity cyclotron resonance and the free electron cyclotron resonance respectively.}\]

3.4 Density of States near the Bound States: Inclusion of Higher Order in \( N_{D} \). — To calculate the density of states \( \delta \rho_{e}(e) \) brought out by the impurity, we need to go one step further in the resolution of the equations (8-10) i.e. to use

\[
K'(e) = \sum_{n} \frac{1}{e - E_{n \sigma} - i\rho}
\]

\[
t'(e) = \frac{N_{D}}{\Omega} \frac{1}{1 - V K_{0a}}
\]

(37)

\[
\delta \rho_{e}(e) = \frac{1}{\pi} \left[ K_{e}^{(2)} - K_{0a}^{(2)} \right] .
\]

For the donor level in the forbidden gap \( K_{0a}^{(2)} = 0 \) and \( \delta \rho_{e}(e) \) reduces to

\[
\delta \rho_{e}(e) = \frac{m^2 \Omega}{2 \pi^2 \sqrt{2} \hbar^2} \sum_{p=0}^{n-1} \left( \frac{e - E_{p}^{(1)}(e) - E_{p}}{\hbar \omega_c} \right)^{-1/2} .
\]

(38)

For a given \( e \), \( \delta \rho_{e}(e) \) is the sum of \( v_{e} \) terms which are non zero in the energy range \([y_{p \sigma}, E_{p}]\) where \( y_{p \sigma} \) is the solution of (see Fig. 6)

\[
y_{p \sigma} - E_{p}^{(1)}(e) = E_{p} , \quad p \leq v_{e} - 1
\]

\( v_{e} \) being the smallest integer which exceeds

\[
\frac{e - E_{p}^{(1)}(e) + s_{e} \sigma}{\hbar \omega_c} - \frac{1}{2} .
\]
The \( p \)th term of the summation behaves like 
\((e - \gamma_{pe})^{-1/2}\) near \( \gamma_{pe} \) and like 
\((\varepsilon_0^p - e)^{1/2}\) near \( \varepsilon_0^p \). 
Then the impurity band has a width \( \delta^p = \varepsilon_0^p - \gamma_{pe} \). 
Between \( \gamma_{0a} \) and \( \varepsilon_0^p \), \( \delta \rho_0(e) \) displays an infinite number 
of integrable singularities each time \( e = \gamma_{pe} \). They 
are related to the use of \( G_{0e} \) in equation (37). Note 
also that \( \delta \rho_0(e) \to \infty \) when \( e \) approaches \( \varepsilon_0^p \) from 
below but vanishes identically between \( \varepsilon_0^p \) and \( \gamma_{0a} \) : 
a finite gap survives between the broadened impurity 
level and the conduction band (Fig. 7). A shortcoming 
of this description is of course the strange behaviour 
of \( \delta \rho_0(e) \) in the vicinity of \( \varepsilon_0^p \). It is related to 
the use of a first iterative solution of the self-consistent 
equations (8-10) and it would disappear if we were 
able to explicitly include \( K_0^{(2)} \) instead of \( K_0^{(1)} \) in 
equation (37). Qualitatively, we would obtain an asymmetric 
peak centred at \( \varepsilon_0^p \) with a tail extending towards 
negative \( (e - \varepsilon_0^p) \).

An estimation of \( \delta^p \) can still be given. We find 
(if \( r_0 = 0 \))

\[
\delta^p = \frac{1}{r_0} \left\{ \frac{\hbar}{\sqrt{r_0}} \right\} \left[ 1 - 2 \sqrt{\frac{\hbar}{\lambda}} \right] \left( \frac{\hbar}{\lambda} \right)^{1/2}
\]

in weak field \((\lambda/f > 1)\)

\[
\frac{x}{r_0} \approx \frac{x}{2(\sqrt{2} - 1)^{3/2}} \quad \text{if} \quad \lambda/f \ll 1
\]

where

\[
x = \frac{2^{5/2} \pi N_D \lambda^3}{\Omega}
\]

with \( N_D/\Omega \sim 10^{15} \text{cm}^{-3}, \lambda \sim 100 \text{Å}, \delta^p/r_0 \approx 3 \times 10^{-2} \) 
which means that the impurity band is very narrow.

The total number of states drawn from the conduction 
band to the impurity level is

\[
N_\sigma = \int_{-\infty}^{\gamma_{0a}} \delta \rho_0(e) \, de = -\int_{\gamma_{0a}}^{\varepsilon_0^p} \delta \rho_0(e) \, de
\]

To the lowest order in \( N_0 \)

\[
\delta \rho_0 = \frac{1}{\pi} \text{Tr} \text{Im} \left( G_{0e} T_{0e} \right) =
\]

\[
= \frac{N_0}{\pi \Omega} \frac{d}{de} \text{Arctg} \left( -\frac{VK_{0e}^{(2)}}{1 - VK_{0e}^{(1)}} \right). \quad (40)
\]

Special attention should be paid to the behaviour of 
\( K_{0e}^{(1)} \) and \( K_{0e}^{(2)} \) near the band edges. We find \( N_\sigma = N_D \): 
the total number of states drawn from the band to the 
impurity levels in the gap is 2 \( N_D \), i.e. each donor state 
may trap one electron with two possible spin states 
\( \sigma = \pm 1/2 \).

3.5 Critical Scattering Length Induced by Collision Broadening. — In 3.1 we have shown 
that in the limit of infinite dilution of impurity, there
always exists a donor level below the conduction band edge. This fact arose essentially from the divergent behaviour of \( K_{10}^{(1)} \) near \( e_{0\sigma} \). Accounting for collisions with several impurities will broaden \( K_{10}^{(1)} \) which will display a finite minimum in the vicinity of \( e_{0\sigma} \). In turn, this will induce the existence of a minimal scattering length \( f_m^{(\min)} \) below which no bound states will exist. To evaluate \( f_m^{(\min)} \), we go back to the self-consistent equations (8-10). The approximation used in 3.3 is not sufficient here because \( t_{10}^{(1)}(\varepsilon) = 0 \) if \( \varepsilon < e_{0\sigma} \) and the divergency of \( K_{10}^{(1)} \) is shifted but still exists. We shall then proceed as follows. Suppose \( f = f_m^{(\min)} \), the bound state has a very small binding energy

\[ e_{\sigma}^{(D)} = e_{0\sigma} - \eta \hbar \omega_c , \quad \eta \ll 1 . \]

At the pole \( e_{\sigma}^{(D)} \), the self-consistent equations read

\[ t_{10}^{(1)} = \frac{N_D}{\Omega} \times \frac{\bar{v}_m^{-1} - \bar{R}_{10}^{(1)}}{\bar{R}_{10}^{(2)}} = 0 \]

\[ t_{10}^{(2)} = \frac{N_D}{\Omega} \times \frac{1}{\bar{R}_{10}^{(2)}} \times \frac{\bar{v}_m^{-1} - \bar{R}_{10}^{(1)}}{\bar{R}_{10}^{(2)}} = \frac{N_D}{\bar{R}_{10}^{(2)}} \]

\[ \bar{R}_{10}^{(2)} = \frac{m^* \Omega}{4 \pi h^2 \lambda} \times \]

\[ \times \sum_{n=0}^{\infty} \left\{ - \eta - n + \sqrt{\eta^2 + (n + \eta)^2} \right\}^{1/2} \]

\[ \bar{v}_m^{-1} = \frac{\bar{R}_{10}^{(1)}}{\bar{R}_{10}^{(2)}} = \frac{m^* \Omega}{4 \pi h^2 \lambda} \left\{ \frac{\eta + \sqrt{\eta^2 + \eta^2}}{\eta^2 + \eta^2} \right\} \]

where [7]

\[ a = \frac{t_{20}^{(2)}}{\hbar \omega_c} . \]

Solving equations (42, 43) leads to a relation between \( a \) and \( \eta \). As the bound state has a very small binding energy, we may neglect \( \eta(f = f_m^{(\min)}) \). Then equations (42, 43) give

\[ \frac{1}{a} = \left( \frac{t_{20}^{(1)}}{4 \pi N_D \lambda^2} \right)^2 \]

and we find from equations (44, 45)

\[ f_m^{(\min)} = \frac{8 \pi N_D \lambda^4}{\Omega} \]

if \( \lambda \sim 100 \text{Å} \), \( N_D/\Omega = 10^{15} \text{cm}^{-3} \), \( f_m^{(\min)} \sim 2.5 \text{Å} \) which means a rather weak effective potential.

From equation (46) we note that \( f_m^{(\min)} \) is non-negligible only if the magnetic length \( \lambda \) is of the same order as the mean distance between donors. The collision broadening acts to relax the localization of the electron in the plane perpendicular to \( H \), restoring a situation similar to the zero-field case. It is then expected to lead to a non-vanishing \( f_m^{(\min)} \) as we have found in 3.1 equation (23).

Figure 8 shows a sketch of \( K_{10}^{(1)} \) and \( K_{10}^{(2)} \) near \( e_{0\sigma} \). If \( f > f_m^{(\min)} \), it seems that two bound states may exist. As usual the resonance occurring closer to \( e_{0\sigma} \) is overdamped owing to the large value of \( K_{10}^{(2)} \).

![Figure 8](image_url) 

**FIG. 8. — Energy dependence of \( K_{10}^{(1)} \) and \( K_{10}^{(2)} \).**
which is equivalent to equation (4.28a) of reference [1] and which leads for weak potential to a correction $\delta \sigma_{zz} \propto N_D^{1/3}$.

As

$$\tau_s(\mu) = \frac{\hbar}{2 t_s^2(\mu)} \quad \text{and} \quad t_s^2(\mu) \propto K_0^2(\mu).$$

we recover immediately the well known oscillations of the conductivity which take place whenever $\epsilon_{ex} = \mu$. The shape and amplitude of these oscillations have been discussed in reference [7]. Let us however investigate in closer detail the $0^-$ oscillation, which is generally assumed to be absent [8]. To the lowest order in $f$ (Born approximation) the longitudinal conductivity is written ($f = \tilde{f}$)

$$\sigma_{zz} = \frac{N e^2 \tau_+ (\mu)}{\Omega m^*} \quad \text{if} \quad \epsilon_{0^+} \leq \mu \leq \epsilon_{0^-},$$

$$\sigma_{zz} = \frac{e^2}{m^* \Omega} \left[ N_+ \tau_+ (\mu) + N_- \tau_- (\mu) \right] \quad \text{if} \quad \epsilon_{0^-} \leq \mu \leq \epsilon_{1^+},$$

with

$$\tau_s(\mu) = \frac{m^* \Omega \lambda}{2 \pi \sqrt{2 \hbar f^2 N_D}} \left( \frac{\mu}{\hbar \omega_c} - \frac{1}{2} + \frac{s_x}{2 \hbar \omega_c} \right)^{1/2}.$$  \hspace{1cm} (50)

if

$$\epsilon_{0^+} \leq \mu \leq \epsilon_{1^+},$$

$$\tau_-(\mu) = \frac{m^* \Omega \lambda}{2 \pi \sqrt{2 \hbar f^2 N_D}} \left( \frac{\mu}{\hbar \omega_c} - \frac{1}{2} - \frac{s_x}{2 \hbar \omega_c} \right)^{1/2}.$$  \hspace{1cm} (51)

if

$$\epsilon_{0^-} \leq \mu \leq \epsilon_{1^+}.$$  \hspace{1cm} (52)

As pointed out by Efros [8], contrarily to the cases $\mu = \epsilon_{+n}$, $n \geq 1$, $\tau_+(\mu)$ exhibits no divergence when $\mu$ crosses $0^-$ if the scattering potential is spin independent. In that sense, there is no $0^-$ oscillation. However, populating the $0^-$ level gives rise to a discontinuous slope of $\sigma_{zz}$ because equations (51) and (52) are different and also because $\mu(H)$ has an angular point near $0^-$ (Fig. 9) which enhances this discontinuity. $(\Delta \rho/\rho_{zz})$ is plotted versus $\mu_0/\hbar \omega_c$ on figure 10; it exhibits a kink when $\mu = \epsilon_{0^-}$ of weaker amplitude than the other oscillations. The relative order of magnitude of the anomalies are however in rough accordance with experiments [9] (1).

Let us now focus our attention on the quantum limit case, i.e. we assume $\epsilon_{0^+} < \mu < \epsilon_{0^-}$ and a constant number of free electrons. Strictly speaking, $\mu(H)$ should be calculated from the self consistent equations (8-10). In order to derive analytical results, we shall calculate $\mu(H)$ with the help of the unperturbed propagator $G_{0\sigma}$. In doing so we will describe only qualitatively the correction $\delta \sigma_{zz}$ (case of a strong potential). Otherwise this procedure will be self consistent in the case of a weak potential (i.e. Born approximation for the $t$ matrix and $N$ proportional to $N^{(2)}_D$). With these assumptions

$$\mu(H) = \epsilon_{0^+} + \frac{16}{9} \left( \frac{\mu_0}{\hbar \omega_c} \right)^3 \hbar \omega_c,$$  \hspace{1cm} (53)

(1) However, these experiments show a temperature dependance of the oscillations amplitude which is absent of our theory.
where \( \mu_0 = \hbar^2 K_F^2 / 2 m^* \) is the Fermi level at zero
field. We find

\[
\tau_+(\mu) = \frac{m^* \Omega}{6 \pi \hbar N_D} \left( \frac{2 \mu_0}{\hbar \omega_c} \right)^{3/2} \left\{ \frac{9}{4} \left( \frac{\hbar \omega_c}{2 \mu_0} \right)^2 + \left[ 1 - \frac{\hbar}{\lambda \sqrt{2}} \left[ 1 - \frac{1}{9} \left( \frac{\mu_0}{\hbar \omega_c} \right)^3 \right]^{-1/2} \right] \right\} .
\]

(54)

i) Weak potential: \( f/03BB, 1, f = \hat{f} \).

Within the Born approximation we may forget about \( \delta \sigma_{zz} \). \( \tau_+(\mu) \) reduces to

\[
\tau_+(\mu) = \frac{m^* \Omega}{6 \pi \hbar N_D} \frac{\lambda}{\hat{f}} \left( \frac{2 \mu_0}{\hbar \omega_c} \right)^{3/2}
\]

(55)

i.e. \( \tau_+ \propto H^{-2} \) and \( \rho_{zz} \propto H^2 \), the longitudinal magnetoresistance increases monotonically (Fig. 10). This result agrees with Kubo’s calculations [10] and it has a very simple interpretation: the collision time of an electron with \( \sigma = +1/2 \) whose energy is \( \mu \) and momenta \( k_x, k_z \) is related to the transition probability to a scattered state \( k_x', k_z' \) by

\[
\frac{1}{\tau_+(\mu)} = \sum_{k_x' k_z'} W(k_x, k_z, k_x', k_z') \left( 1 - \frac{k_z'}{k_z} \right).
\]

For a weak potential \( W \) is given by the Fermi golden rule

\[
W(k_x, k_z, k_x', k_z') = \frac{2 \pi}{\hbar} \frac{N_D}{\Omega} \left| \langle 0 k_x' k_z' | V(r) | 0 k_x, k_z \rangle \right|^2 \delta(e_x' - \mu)
\]

which in the case of a contact potential leads to

\[
\frac{1}{\tau_+(\mu)} = \frac{N_D}{\Omega} \frac{\pi^2}{\hbar^2} \left[ \frac{m^* \omega_b}{\hbar \omega_c} \right] \left( \frac{\hbar \omega_c}{2 \mu_0} \right)
\]

(56)

in accordance with (55).

When \( H \) increases and \( N \) is kept constant, the electron velocity along \( H \), \( v_x(\mu) \) decreases because \( \mu - e_0 \) decreases, as does \( \tau(\mu) \) and

\[
\frac{\Delta \rho}{\rho} = \frac{3}{8} \left( \frac{\hbar \omega_c}{\mu_0} \right)^2 - 1 .
\]

(57)

ii) Strong potential:

\[
\hat{f} / \lambda \gg 1 \quad \text{and} \quad \frac{9}{16} \left( \frac{\hbar \omega_c}{2 \mu_0} \right)^3 \gg 1 .
\]

In this limit

\[
\tau_+(\mu) \approx \frac{3}{8} \frac{m^* \Omega}{\pi \hbar N_D} \frac{\hat{f}}{\lambda} \left( \frac{\hbar \omega_c}{2 \mu_0} \right)^{3/2}
\]

(58)

\( \tau_+(\mu) \) no longer depends on the strength of the potential. Also \( \delta \sigma_{zz} \) should be taken into account. Finally

\[
\sigma_{zz} = \frac{N e^2}{m^*} \tau(\mu) \left( \frac{1}{2} + \frac{N D}{2 N} \right)
\]

(59)

As seen from equations (58, 59), \( \tau_+(H) \) and \( \sigma_{zz}(H) \) increase with \( H \). Then in a strong field one may obtain a negative magnetoresistance which will increase in magnitude. Note that this behaviour is exactly the opposite of the one obtained when the potential is weak.

5. Non degenerate statistics, bound states and freeze out effect. — As we noticed before, \( \sigma_{zz} \) exhibits completely different behaviours depending on which approximation is used to perform the calculations. Still, in strong fields equations (57, 59) become inaccurate, the reason being the freezing out of free electrons into the impurity sites which leads to an exponential decrease of \( \sigma_{zz} \) with \( H \). In section 3, we have already calculated the position and the width of the impurity band located in the gap, and shown that the whole band is very narrow. In order to simplify the algebra, we shall assume that the donor levels are perfectly localized. At zero field each donor level is two-fold degenerate \( (\sigma = \pm 1/2) \). As we are interested in the quantum limit behaviour of \( \sigma_{zz} \), only the \( \sigma = +1/2 \) electrons will come into play and the spin degeneracy will be lifted. If

\[
\frac{e_0 - e_0^D}{k_B T} \gtrsim 1
\]

the Fermi level no longer stays within the \( 0^+ \) Landau level, but is located between \( e_0^D \) and \( e_0 \). The assumption of degenerate statistics fails and we should switch to Boltzmann statistics. The equation for \( \sigma_{zz} \) now reads:

\[
\sigma_{zz} = - \frac{e^2}{4 \pi^2} \frac{\omega_b(m^*)^{1/2}}{H_2^2} \sum_{\sigma = \pm 1/2} \sum_{n = 0}^{\infty} \int_{-\infty}^{\infty} \frac{df}{de} \tau_+(e) \times
\]

\[
\times \left[ \left( \frac{\Delta_1 e^D}{2} + \frac{(\Delta_1 e + e_{0D})}{2} \right)^{1/2} - \left( \frac{\Delta_1 e - e_{0D}}{2} \right)^{1/2} \right]^{1/2}
\]

\[
\times \left( \frac{\Delta_1 e^D}{2} + \frac{(\Delta_1 e + e_{0D})}{2} \right)^{1/2} \times \left( \frac{\Delta_1 e^D}{2} + \frac{(\Delta_1 e + e_{0D})}{2} \right)^{1/2}
\]

\[
- \left( \frac{\Delta_1 e - e_{0D}}{2} \right)^{1/2} .
\]

(60)

In equation (60), the integration over \( e \) runs from \(-\infty\) to \( \Delta \). This means that the conductivity includes from the very beginning the free states \( (e > e_{0D}) \) and the bound states. The latter never came into play in case 4 because \( \mu > e_{0D}, \epsilon^D < e_{0D} \) and

\[
\frac{df}{de} = - \delta(e - \mu) .
\]

Also in the case of freeze out phenomena the bound electrons, although very numerous, give a negligible
contribution to $\sigma_{zz}$ as $\tau \propto 1/K^{(2)}(e)$ and $K^{(2)}(e)$ is very large (even infinite, if the localized levels are perfectly sharp). For these reasons, we limit the integration in equation (31) to the energy range $[\varepsilon_0, \Delta]$. Also, as the freeze out effect will overwhelmingly influence the conductivity through the exhaustion of free carriers, we will treat the free electron-impurity scattering to the lowest order in $V$ (Born approximation), in which case equation (60) reduces to

$$\sigma_{zz} = \frac{e^2}{m^* \Omega} \sum_n \langle \tau_n \rangle$$  \hspace{1cm} (61)

where $n_\sigma$ is the number of free electrons (with energy $\varepsilon > \varepsilon_0$) in the conduction band and

$$\langle \tau_n \rangle = \sum_{n=0}^{\infty} \exp(-\beta n\varepsilon) \int_0^{\infty} e^{-\frac{\beta \varepsilon}{2} (\varepsilon + \varepsilon_0)} d\varepsilon$$

$$\tau_n(e) = \frac{\hbar \Omega^2}{2 N_D V^2 K^{(2)}(e)}$$ and $\beta = \frac{1}{k_B T}$. \hspace{1cm} (62)

If the donor levels are perfectly sharp, the number of free carriers may be calculated according to the neutrality equation

$$\sum_n n_\sigma + \langle N_D \rangle = N_D$$ \hspace{1cm} (63)

where $\langle N_D \rangle$ is the average number of occupied donor sites at temperature $T$. In the quantum limit ($\hbar \omega_c > k_B T$), all the free electrons are in the 0$^+$ Landau level and

$$n = N_D \varepsilon^{(a-a_0^+)}$$

$$\langle N_D \rangle = \frac{N_D}{1 + \exp\beta (\varepsilon - \mu)}.$$ \hspace{1cm} (64)

From equations (63, 64), there comes

$$\exp(-\beta n_\sigma - \mu) = -\frac{1}{2} e^{-\beta \mu} +$$

$$+ \frac{1}{2} \sqrt{e^{-2\beta \mu} + \frac{4 N_D}{N_0} e^{-\beta \mu}} \approx \sqrt{\frac{N_D}{N_0}} e^{-\beta \mu / 2}.$$ \hspace{1cm} (65)

Then

$$n \approx \sqrt{N_D N_0} e^{-\beta \mu / 2}$$

and

$$\sigma_{zz} \approx \frac{e^2 m^* k_B T}{4 \pi^2 \hbar^2} \sqrt{\frac{\Omega^2}{N_0 N_D}} e^{-\beta \mu / 2},$$

where $r$ is the binding energy, $r$ is an increasing function of $H$, then $n$ and $\sigma_{zz}$ decrease exponentially. Note that a more exact treatment of the scattering (i.e. not limited to the Born approximation) would not have changed equation (65) sensitively: the behaviour of $\sigma_{zz}$ is clearly dominated by the freeze out of the carriers on to the donor sites.

6. Conclusion. — We have presented a unified treatment of the bound and free states of an electron gas interacting with donor point-like scatterers. Our results remain qualitative owing to the over-simplified form of the impurity potential. In the case of dilute impurities we have shown that an attractive potential creates a localized state independently of the potential strength. We have obtained the binding energies and the width of the localized and resonant donor levels. The effect of nearby impurities on the electron localization has been qualitatively discussed and we have given an estimation of the critical potential strength below which no bound state exists.

In the intrinsic regime, the magnetic field dependence of the longitudinal conductivity has been shown to be extremely sensitive to the potential strength. Despite the very crude form of the donor potential used in our calculations, we believe to have shown the difficulty of extracting from magnetoresistivity measurements reliable informations on the impurity potential.

In the extrinsic regime, the freezing out of the carriers onto the impurity sites leads to an exponential decrease of the carrier concentration which overshadows the weak magnetic field dependence of the relaxation time.

References