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To cite this version:
A.P. Jain. Evidence of narrow surface absorption from the optical model studies of S and p-wave strength functions. Journal de Physique, 1975, 36 (5), pp.335-341. <10.1051/jphys:01975003605033500>. <jpa-00208259>
EVIDENCE OF NARROW SURFACE ABSORPTION FROM THE OPTICAL MODEL
STUDIES OF S AND P-WAVE STRENGTH FUNCTIONS

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(Reçu le 4 décembre 1974, accepté le 17 janvier 1975)

Résumé. — L'effet de la largeur du potentiel d'absorption de surface dans le modèle optique sphérique et déformé a été analysé en vue d'expliquer l'anomalie connue sur les fonctions densité d'onde « S » au voisinage du nombre de masse A ~ 100. Une réduction de la largeur b d'un facteur 2 par rapport à la valeur conventionnellement utilisée permet d'expliquer tous les résultats expérimentaux, depuis A = 40 jusqu'à 250, pour les fonctions densité d'ondes « S » et « P » et pour le rayon R'. La valeur 1.35 fm pour r0, dans la formule du rayon R = r0 A^{1/3}, permet d'ajuster de manière satisfaisante les données relatives aux ondes « S ». La loi en A^{1/3} pour le rayon s'avère suffisante avec un modèle vibro-rotationnel.

Abstract. — The effect of the width of the surface absorption potential in the spherical and deformed optical model is analysed with a view to explaining the long standing anomaly of the S-wave strength functions near A ~ 100. A reduction in the value of the width parameter b by a factor of two from that conventionally used has been able to explain the entire data, from A = 40 to 250, for S and P-wave strength functions and the radius parameter R'. The value 1.35 fm for r0, in the radius formula R = r0 A^{1/3}, has been found to give a satisfactory fit to the S-wave data. The A^{1/3} power law for radii is found to be sufficient when vibrational-rotational model is used.

1. Introduction. — The optical model has been very successful in explaining the interactions of low energy nucleons with medium and heavy nuclei. The model has also been able to explain many of the experimental data in the high energy region up to about 50-100 MeV. It is now well believed that the surface absorption predominates at low particle energies while at higher energies, the volume absorption is more important. This is in good qualitative agreement with the earlier many body calculations [1-6], using the average of two body interactions in nuclear matter. Lately the optical model has also been increasingly used for the applied problems of correlating and estimating neutron cross-sections for use in the design of fast reactors [7].

A considerable amount of S-wave strength function data is now available for A = 40 to 250, from studies of capture and total neutron cross-sections in the resolved resonance region below about 30 keV and from average cross-sections in the 100 keV range. In a previous publication [8], the then available data, for the entire region of A = 40 to 250 was analysed for a detailed fit with the optical model, in order to investigate the shape of the imaginary part of the potential, the diffuseness of the nuclear surface, the validity of the A^{1/3} law for radii and some properties of the area sum rule. Until then several other publications [9-12] had also appeared on the same subject but essentially all had been restricted to a study of part of the region of atomic weights; for example, either for spherical (3 S-resonance) or the rotational (4 S-resonance) nuclei. The need for a simultaneous study of 3 S and 4 S region, with the same set of optical model parameters, was stressed for the first time in the previous publication [8] in order to obtain unambiguous informations from such comparisons. It was shown that although good fit could be obtained near the 3 S and 4 S peaks, the fit to the valley in between was still too high by a factor of 4 to 5. In these calculations vibrations were included for the spherical nuclei, while the effect of rotations were considered for the deformed nuclei. No further studies have been made, since then, to understand the anomaly of the model in the valley. In the present study it is shown that it is possible to fit the valley, along with the 3 S and 4 S peaks, by reducing the width of the imaginary potential by a factor of 2 as compared to that conventionally used. The other properties of the model are also examined in the light of an improved fit. It may

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be stressed that in the low energy neutron cross-sections (below a few keV) only a single partial wave, namely the $L = 0$ S-wave, is involved, in contrast to the high energy (few MeV or more) cross-sections, where several partial waves contribute. The strength functions data can thus be considered as primary; the precise values of various optical model parameters derived from a fit to this data can then be used to obtain improved fits to the cross-sections in the high energy region. In fact there are certain parameters in the model of which the sensitivity to the data is good for strength functions, but is rather poor for the high energy cross-sections. Moreover, because of a considerable number of parameters in the model, it is generally difficult to obtain a unique set from a fit to the high energy cross-section data. On the other hand, because of its simplicity, the strength function data do allow one to use a unique set of parameters. Therefore it is considered important to find precise information for the optical model from a detailed fit to the strength function data.

2. Description of the model. — In our entire analysis a computer code due to Raynal [14] has been used. In this code it is possible to calculate the phase shifts and the cross-section for spherical nuclei without the inclusion of vibrations and also with vibrations, i.e. coupling the ground state of the nucleus with its first excited state. In the 4 S region the coupled channel calculations have been performed for the deformed nuclei, using the same code, but with the inclusion of permanent deformation parameter $\beta$. The real part is assumed to be of the standard Saxon-Woods type which is given by $- V_0 \left[ 1 + \exp \left( \frac{r - R}{a} \right) \right]^{-1}$; the imaginary part is the pure derivative type surface absorption, given by

$$4 W \exp \left( \frac{r - R}{b} \right) \left[ 1 + \exp \left( \frac{r - R}{b} \right) \right]^{-2}$$

and the spin orbit coupling is assumed to be of Thomas form

$$V_{so} \left( \frac{\hbar}{\mu_e c} \right)^2 \sigma \cdot \frac{1}{r} \frac{d}{dr} q(r),$$

where

$$q(r) = \left[ 1 + \exp \left( \frac{r - R}{a} \right) \right]^{-1}.$$

In these expressions $R$ is the nuclear radius, $a$ is the diffuseness of the nuclear surface and $b$ describes the width of the imaginary potential. No calculations have been performed for the volume absorption potential as it was shown in the previous publication [8] that is was not possible to obtain a good fit to the data, using this type of potential. By utilising the same set of parameters which have been used earlier [8], the new code has been found to give results which are in agreement with those obtained earlier, utilising the computer codes due to Buck and Perey and that of Auerbach.

3. The effect of the shape of the imaginary part of the potential on the S-wave strength functions. —

3.1 Spherical optical model. — The optical model calculations allow one to obtain the real and imaginary part of the phase shifts and from these the absorption and the scattering cross-sections are evaluated. For low energy neutrons, the absorption cross-section represents the compound nucleus cross-section $\sigma_e$, while the scattering cross-section is called the shape elastic $\sigma_{el}$. The average compound nucleus cross-section for the S-wave neutrons at low energies is given by

$$\sigma^0_0 = 2 \pi^2 \lambda^2 \sqrt{E_{cm}} \left< \Gamma^0_0 \right>/D (1 - \pi \sqrt{E_{cm}} \left< \Gamma^0_0 \right>/D),$$

where $\lambda$ is the wave length of the neutron corresponding to an incident energy $E_{cm}$, $\left< \Gamma^0_0 \right>$ is the average neutron width at 1 eV and $D$ is the average level spacing. All model calculations have been performed for a neutron energy of 1 keV and the strength functions are deduced using eq. (1), from the calculated absorption cross-section.

In the previous calculations [8], the width of the surface absorption potential of derivative type was taken to be 0.4 fm which is also the value generally in use. All the other parameters of the potential were varied suitably in order to obtain a good fit to the S-wave strength function data, but the width of the imaginary potential was kept fixed. It was believed at that time that it is the volume integral of the imaginary potential that matters, rather than its detailed shape. In fact it is generally believed to be so even now for cross-sections in the MeV energy range. In 1955 Thomas [15] had obtained an analytic expression for the cross-section, using a surface absorption potential, from which it can be seen that the above assumption does not seem to be correct. In these calculations an unusual shape (delta function) of the absorption potential was used; the real part of the potential was chosen to be of square well type. In order to see the effect of the shape of the absorption potential on the S-wave strength functions, for the commonly used Saxon-Woods shape, the results of the present calculations are shown in figure 1 for various values of the width parameter $b$. Generally, the value of the strength function at the peak is proportional to $1/W*$ and that the valley varies as $W^*$, where $W^*$ is the integrated absorption potential. Therefore the peak to valley ratio can be altered suitably with the change of $W^*$. However a change in $W^*$ also affects the width of the peak. In order to see the effect of the parameter $b$ alone (with the width of the 3 S resonance fixed), the integrated intensity of the absorption potential is roughly kept to be the same for all the curves. The parameters used for curve 1 are the same ($b = 0.4$) as that used in figure 3 of the previous work.
The calculated S-wave neutron strength functions as a function of the atomic weight for various values of the width parameter $b$ in the surface absorption potential. The values of $b$ are 0.4, 0.2 and 0.1 fm for curves 1, 2 and 3; the values of $W$ being 5.44, 13.3 and 27 MeV and that for $V_o$ are 52.0, 52.3 and 52.7 MeV respectively. For all the three curves, the values of $r_o$, in $R = r_o A^{1/3}$ and that for the diffuseness parameter $a$ are assumed to be 1.25 and 0.52 fm. In the curve 4, the imaginary potential is centred 0.5 fm outside the real potential; all the other parameters are the same as for curve 3.

Moldauer [13] suggested centering the absorption potential 0.5 f outside the nuclear surface which was called by the name fringe absorption. In figure 1, curve 4 shows the results of the present calculations of such an effect. All the parameters for curve 4 are the same as for curve 3, except for an outward shift of the absorption potential by 0.5 f with respect to the real potential. It is to be noted that:

1) It reduces considerably the value at the peak and at the valley, and
2) It produces a sizeable shift of the valley with rather small changes in peak position. Such effects can as well be produced by a decrease of the diffuseness $a$ of the nuclear surface. Since the inclusion of the fringe absorption adds another parameter, while already enough parameters exist in the model, most of the calculations, described below, have been made without fringe absorption.

Moldauer attempted to fit the 3 S region of S-wave strength functions data and the valley near $A \sim 100$, by simultaneously varying all parameters of the optical model, including the width parameter $b$ of the absorption potential and the fringe absorption $c$. It is not clear from his work whether the improved fit near the valley is a result of a reduction in the value of $b$, or due to the inclusion of the fringe absorption. Moreover, in his attempt to understand the role of parameter $b$, the effect of changing $b$ was studied by keeping the peak value of the imaginary potential $W$ fixed. This, of course, implies a change in the total integrated strength of the absorption potential, which is known to directly affect the values at the peak and in the valley. In our study, as in figure 1, $b$ is varied while keeping the integrated strength of the absorption potential fixed, which demonstrates uniquely the role of parameter $b$ alone.

In figure 2 the results of the present attempt to fit the data of S-wave strength functions for the spherical nuclei are shown. The data has been taken from the recent compilations by Lynn [16]. Several considerations were kept in mind for varying the various parameters in order to obtain a good fit to the data. These are: 1) The peak positions depend on $V_o r_o^2$, where $r_o = R/A^{1/3}$ is the reduced radius of the real potential, which is assumed to be the same for both the real and imaginary part, 2) The integrated value of the absorption potential is chosen close to the value used in figure 1, 3) The peak value depends on the diffuseness $a$ and the integrated value of $W$, 4) The peak to valley ratio can be changed with changes in the parameter $b$ (with appropriate changes in $W$) and 5) For a fixed $V_o r_o^2$, the fit in the region of the valley depends on $b$ and $r_1$, where $R = r_o A^{1/3} + r_1$. With these restrictions, it was possible to arrive at a set of parameters which provide a good fit to the data, with initial trials of as few as 10 different sets. In figure 2, the width parameter $b$ is equal to 0.2 for curve 1 and 0.1 for curve 2. Curve 3...
3.2 ROTATIONAL-VIBRATIONAL OPTICAL MODEL. —

The nuclei with atomic weights less than 150 are known to be spherical, as they do not exhibit any quadrupole moment. However most of the nuclei in the range of $A = 150$ to 190 and above 220 exhibit well-established rotational bands, built on the ground state, showing permanent deformations. The effect of such a deformation on the strength function was first pointed out by Chase, Wilets and Edmonds [11], who showed, using a trapezoidal potential well, that a single 4S spherical model state at $A \sim 160$ is split into two, due to the deformation parameter $\beta$, which is in agreement with the observed data. For the spherical nuclei, the vibrational bands, built on the ground state, are now well known. Few years ago Buck and Perey [12] showed the importance of the inclusion of such effects in the optical model for the calculations of S and P-wave strength functions and for cross-sections in the MeV energy range. Therefore, for a proper comparison of the data with the theory, it is necessary to include both effects in the optical model calculations.

In our previous publication [8] such calculations have been made with a fixed value of 0.4 for the width parameter $b$ of the absorption potential for the entire range of $A = 40$ to $A = 250$. Figure 3 shows the results of the present attempt to fit the data with a vibrational-rotational model for various values of $b$. The curve 2 is for $b = 0.2$, while curve 3 is for $b = 0.1$. For the sake of comparison, the results of the earlier calculations with $b = 0.4$ are shown in curve 1 (curve 3, Fig. 4 of ref. [8]), which has also been reproduced in the recent book on the Theory of Neutron Resonance Reactions by Lynn [16]. In all these calculations the deformation parameter $\beta_1$ for each nucleus has been taken from the recent compilations of Lobner Vetter, and Honig [18] for the rotational nuclei. For the spherical nuclei, the r.m.s. values of the deformation parameter have been taken from the review of Stelson and Grodzins [19]. Many calculations for various different sets of parameters were made, with guidelines, as mentioned earlier in this work, before acceptable fits were obtained. It is observed that the value of the strength function in the valley is considerably changed with a change in the value of the parameter $b$, without seriously affecting the fit near the peaks. In fact curve 3 ($b = 0.1$) passes even below the average value of the experimental data in the valley. Evidently the curve 2 with $b = 0.2$ gives a satisfactory fit to the data, simultaneously in the peaks and in the valley. Curve 4, in the same figure, shows a slightly improved fit in the valley. In this calculation the imaginary potential has been centered $\frac{1}{2}$ fm outside the real potential. It was necessary to change slightly the real part of the potential and considerably the diffuseness parameter ($a = 0.68$, as compared to $a = 0.52$ of curve 3), before a satisfactory fit could
be obtained. The value of the width parameter $b$ for the absorption potential was chosen to be 0.2 fm which is the same as for the curve 2, for which equal radii of the real and imaginary potential have been assumed. However the effect of including the additional parameter, fringe absorption, in curve 4 does not seem to make any substantial improvement to the fit over that obtained in curve 2, which uses no fringe absorption. Thus the improvement in the fit to the data, even in the rotational-vibrational model, is mainly a result of the reduced width of the imaginary potential.

It may be mentioned that in all the calculations of figure 3, the imaginary part of the potential is taken to be non-deformed. The use of the deformed $W$ could not provide a satisfactory detailed fit to the data. The calculated values of the strength function for each nucleus, in actual practice, fluctuate slightly around the curves shown in figure 3, due to the fluctuations in the $\beta$ values and in the energies of the first excited states. However, for the sake of clarity, a smooth curve drawn through these values is only shown in the figure. It is satisfying that the long standing anomaly of the S-wave strength function data in the valley is resolved by a decrease in the value of $b$ from 0.4 to 0.2.

In recent years, with improved experimental techniques, the strength function data for higher angular momentum, particularly for P-wave, has become available for comparison with the optical model. It is comparatively difficult to measure accurate values of P-wave strength functions because of the admixture of large effects due to other partial waves and of S-wave potential scattering in the measurement of the average total cross-sections. Besides, various workers in past have used different formalisms between cross-sections and strength functions [17] in their analysis, which have resulted in the values of P-wave strength functions differing by as much as a factor of 2 to 4, even near the peak of the 3 P resonance ($A \sim 94$). Several other workers have used average capture cross-sections for extracting P-wave strength functions. It was pointed out by Jain [17] and also recently by Lynn [16] (p. 288) that no reliable quantitative information about P-wave strength functions can be obtained from such measurements because of the theoretical complexity in connecting the average capture cross-section with the strength functions. There are insufficient data for the P-wave strength functions, at the present time and one cannot make a precise comparison with the calculations based on the model parameters which have provided a good fit to the S-wave data in this work. For future comparison, a calculated curve for P-wave, with parameters the same as used in curve 4 of figure 3, is shown in figure 4. The few of the P-wave strength functions values, which have been estimated by Lynn [16] p. 290 to be comparatively better, are also shown in this figure. The model is in good agreement with the data, both near the 3 P and 4 P size resonances at $A \sim 94$ and $A \sim 224$ respectively. Recently Newstead [20] reported accurate values of the P-wave strength functions for $^{165}$Ho and $^{239}$Pu from measurements of average total cross-sections in the 50 keV to 1 MeV range of neutron energy. The experimental values of $1.63 \pm 0.25$ and $2.33 \pm 0.35$ in units of $10^{-4}$ for $^{165}$Ho and $^{239}$Pu are in good agreement with the model predictions of 1.66 and 2.30 respectively.

It may be remarked that in all these measurements a value of $r_0 = 1.35$ fm (in the radius parameter $R = r_0 A^{1/3}$) was assumed for deducing P-wave strength functions from the measured cross-sections. At present the gross value of $r_0$ is uncertain and varies between 1.25 to 1.35. Since the potential scattering ($\sim 4 \pi R^2$) constitutes a large part in the measured total cross-sections, such an uncertainty is likely to cause considerable error in the strength function values. Therefore the existing values of the P-wave strength functions may not be very reliable for accurate comparison with the model. In the region of $A \sim 94$, the S-wave strength function has a minimum, while P-wave has a maximum. Until now it has not been possible to fit the two together with the same set of model parameters. The present work in which the fit for the two has been obtained for the same set of model parameters, in particular, in the range of $A \sim 100$ and for other nuclei, should be viewed with considerable interest. It now offers the possibility of extending such calculations for higher partial waves and for comparison of cross-sections in the MeV energy range.

4. The potential scattering radius $R'$. — The potential scattering cross-section of a neutron with the nucleus at low energies is given by $4 \pi R'^2$. In a hard sphere model, $R'$ is equal to the radius $R$ of the
nucleus. In actual case the neutron wave penetrates into the nucleus, as is evident from the observed maxima in the S and P-wave strength functions. Consequently $R'$ differs considerably from the hard sphere value $R$.

The average scattering or shape elastic cross-section for S-wave at low energies can be written as:

$$\sigma_{se}^0 = \frac{4 \pi}{100} R'^2 + (\pi \times 0.22676 \times (10^4)^2),$$

where $\sigma_{se}^0$ is in barns and $R'$ is in fermis. Both the terms in this expression are independent of the energy of the neutron. Jain [17] and Lynn [16] (p. 271) have pointed out that the second term in this expression arises as a result of multi-level averaging and should be included in this analysis. Until now most of the model calculations and the measurements of $R'$ have been made by neglecting the contribution of the second term.

However it may be pointed out that the correction to the radius $R'$ due to this term is large near the maximum of the S-wave strength functions and can be as much as 35% in a few cases. Figure 5 shows the data [16] and the results of the present calculations for $R'$. The model parameters are the same as those used in figure 4 for the P-wave strength functions. The good agreement of the model with the data is evident.

Thus the same set of model parameters have been able to fit the entire data for low energy neutron scattering (from $A = 40$ to $A = 250$), namely the S and P-wave strength functions and the radius parameters $R'$.

5. Conclusion. — It is shown, using a vibrational-rotational optical model, that it is possible to fit the entire data for the low energy neutron scattering, in particular the deep S-wave minimum around $A = 100$, which has been an anomaly for the last 10 years or so. In order to arrive at these results it has been necessary to reduce the width of the surface peaked imaginary potential by a factor of two from that conventionally used. The same set of model parameters also provide a good agreement with the data for the radius parameter $R'$ and the P-wave strength functions.

The recommended parameters for future comparison of the model with the data are, thus, those of curve 2 or curve 4 of figure 3, which cannot be distinguished from the present data. The parameters of curve 2 in figure 3, for example, are $V_0 = 42$, $W = 15$, $V_{so} = 8$ MeV, $a = 0.52$, $b = 0.2$ and

$$R = 1.35 A^{1/3} \, \text{fm}.$$
the imaginary potential), in order to explain the experimental results. It will be interesting to recalculate \( W_1 \), using \( W_0 \) from the present calculations which have been able to explain the average values in this mass range.

Several authors in past have represented the radius \( R \) as \( R = r_0 A^{1/3} + r_i \), in the optical model studies of cross-sections and strength functions \([13, 25]\). A constant density of nuclear matter, however, requires \( r_i \) to be zero. In the earlier studies \([8]\) it was shown that the S-wave strength function and \( R' \) data can be very useful in determining \( r_0 \) and \( r_1 \). The relative positions of 3 S and 4 S peaks in the strength function depend on the choice of \( r_1 \), while \( r_0 \) is determined from a fit to the \( R' \) data. From a fit to the total S-wave data of strength functions and \( R' \), a value of \( r_0 \) of \( 1.35 \pm 0.03 \) is found with no need for the inclusion of the parameter \( r_1 \). The zero value for \( r_1 \) is important, as apart from satisfying the criteria of constant density, it reduces one parameter in the optical model. The ambiguities in the fitting of the model with the data, in general, arise due to its many parameters.

References

[22] **TELLIER, H. and NEWSTEAD, C. M., Int. conf. on neutron cross-sections and technology (Knoxville, 1971) 680.