Prebreakdown events in liquid nitrogen

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PREBREAKDOWN EVENTS IN LIQUID NITROGEN (*)

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Résumé. — On a montré antérieurement qu'un petit volume d'azote liquide devant une fine pointe métallique peut être soumis à un champ électrique très intense sans que se produise aucun signe précurseur de rupture ou d'injection de charge.

Par la suite, l'influence du rayon de courbure de la pointe sur le champ critique a été étudiée, et elle est décrite dans le présent article. On a également analysé l'influence de la distance pointe-plan sur les tensions d'amorçage et de désamorçage ultérieures. A la lumière de considérations théoriques, les résultats montrent que le courant injecté est nettement plus élevé qu'il ne le serait s'il résultait d'une injection par effet tunnel. En fait, la nature du courant injecté dépend de plusieurs paramètres, mais la variation quadratique du courant avec la tension appliquée constitue une propriété commune à toutes les caractéristiques courant-tension, propriété qui peut être interprétée par une limitation du courant par charge d'espace.

L'observation microinterférométrique d'une pointe injectante a révélé la présence de petites bulles, dont le mouvement et la mobilité apparente font l'objet d'une discussion.

Abstract. — It was shown previously that a small volume of liquid nitrogen in front of a sharp metal tip can withstand an extremely high field without any sign of breakdown or charge injection. Subsequently, the influence of the tip radius on the critical field has been investigated and is described here. Also, by monitoring the injection, it has been possible to analyse the influence of the injected charge on further onset and cut-off voltages. In the light of theoretical considerations, the results show that the injected current is definitely larger than it would be if it resulted from tunnel injection. In fact, the nature of the injected current depends upon various parameters ; however, a common feature to all current-voltage characteristics is a quadratic variation of the current with the applied voltage, which may be attributed to space-charge limited motion of carriers. Microscopic observation of a tip under injection has revealed the presence of small bubbles. The motion of these bubbles and their influence on the apparent mobility of the injected charges are discussed.

PART I. — ONSET AND CUT-OFF OF CHARGE INJECTION

1. **Introduction.** — It was shown previously [1]-[3] that a very small volume of pure liquid nitrogen in front of a sharp metal tip can withstand a high voltage without any sign of injection or breakdown, and that discharge inception appears abruptly above a certain voltage subsequently called *onset voltage*, which corresponds to an unusually high field at the tip apex, two orders of magnitude larger than the dielectric strength of usual dielectrics. It was also shown that injection stops abruptly if the geometric field at the tip decreased, either by lowering the applied voltage below a value which is subsequently called *cut-off voltage*, or by sufficient increase of the tip radius resulting from discharge erosion.

This very peculiar behavior is made possible by the combination of two favorable circumstances :

a) the volume under high stress, of the order of r^3 where r is the tip radius, is so small that the probability of presence in it of free charge carriers is negligible ;

b) the liquid being cold (77 K), the probability of thermal ionization, of the order of p^4 where p is the probability at ambiant temperature, is also negligible.

Consequently, no free charge carriers being available in the volume under high stress, the field there

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may be raised up to the level where molecular breakdown (i. e. breakdown by field ionization) will eventually take place. The abrupt character of the transition had escaped the scrutiny of previous workers [4] who had indeed noticed an *instability region* at the lower end of their $I.V$ characteristics, but were apparently confused by the fact that the onset of injection erodes the tip, thereby flattening the transition. By operating with a very clean liquid (i. e. free of solid particles), and carefully raising the voltage, the current — if it exists — stays smaller than the sensitivity of the electrometers, until the instabilities mentioned above start appearing. They are in fact random bursts of current, dying away if the voltage is kept constant.

Conversely, if the voltage is carefully lowered or maintained constant slightly above the onset value, an abrupt and irreversible cut-off occurs rapidly, and the current falls down do an unmeasurable value.

In this paper, we wish to present a comprehensive description of the transition processes, and in particular of the variation of the onset and cut-off fields with the tip radius and the tip-plane distance. We show that the nature of the transition gradually changes from a molecular breakdown process operative with very sharp tips, to volume-controlled process — possibly involving a cooperative breakdown — operative with large tips.

Furthermore, unexplained findings concerning the past history of the test cell and the possibility of stable charge layers on the plane electrode are also described.

2. **Variation of the onset voltage with the tip radius.** — 2.1 EXPERIMENTAL CONDITIONS (Fig. 1). — They are basically the same as described in our previous publications : the test cell consists essentially of a tip-plane electrode system in a vacuum-proof glass

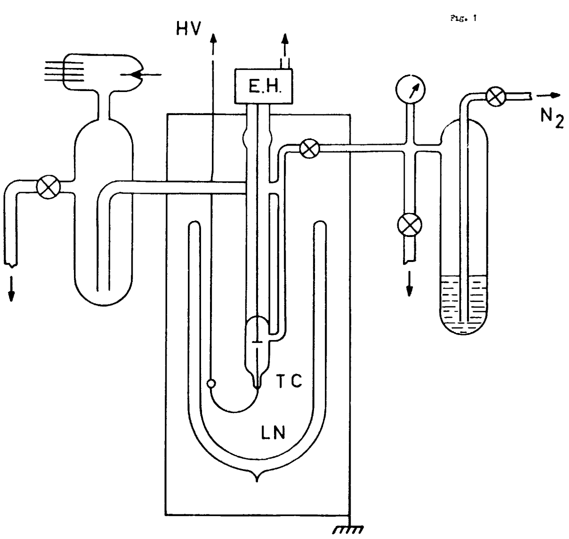

FIG. 1. — Experimental set-up showing the test cell (TC) in the bottom part of the air cable going to the electrometer head (EH). The test cell represented in this schematic diagram is fixed, but cells with variable tip-plane distance have been used in this work.

cell, and the nitrogen gas of *ultrapure* grade (99998) was supplied by l'*Air liquide*. Current-voltage characteristics in high vacuo and in a very low pressure of nitrogen permitted indirect estimates of tip radii smaller than the resolution of our *JEOL Superscope* electron microscope. A new cell in which the tip-plane distance can be monitored externally during a run was assembled and used mainly with tips of radius measurable by electromicroscopy ($r \geqslant 1\,000$ Å). The data given by both types of cells with tips of average radii overlapped satisfactorily.

2.2 VARIATION OF THE FIRST « ONSET FIELD » WITH TIP RADIUS. — The results are presented in figure 2. All the data obtained in 45 runs with a negative tip and 20 runs with a positive tip are within the respective shaded areas. They show that the onset field first decreases with increasing tip radius, slowly at first and then, for radii larger than 1 000 Å, more rapidly. In fact, the data are not as badly scattered as may appear from the figure. For small tip radii,

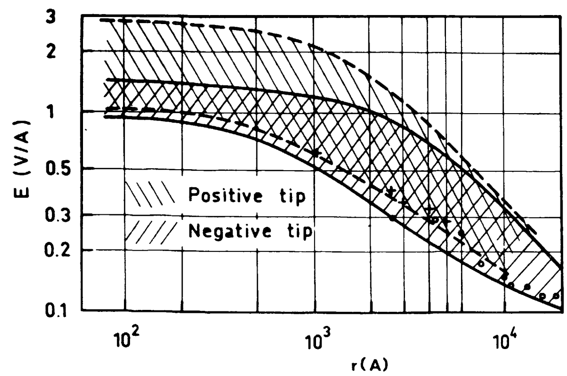

FIG. 2. — Onset field as a function of tip radius. The shaded areas include data obtained from 45 negative tips and 20 positive tips. The points and the crosses correspond to successive states of a same tip.

some uncertainty originates from the indirect estimation of the tip radius. For larger tip radii, the onset field E is derived from direct measurement of the onset voltage V by the relation $E = 2\,V\,[r \ln 4\,a/r]^{-1}$ where r is the tip radius and a the tip-plane distance. The above formula is a good approximation provided that the plane electrode is assumed infinite.

The shaded areas in figure 2 include data from three cells of different size, tips of different metals (W and Ni), and tip-plane distances from 3 to 11 mm, and also nitrogen gas from various supplies. In order to show the scatter of data obtained under very similar conditions with a tungsten tip at various stages of erosion, we have drawn on the figure the corresponding data points (1).

(1) The average first onset field with a positive tip is slightly higher than with a negative tip. Field-emission might be assumed to cause the polarity effect, but the unreasonably high onset fields observed with sharp positive tips (more than 2 V/Å) is more likely related to the presence of residual water, since tiny ice crystals, being negatively charged in liquid nitrogen, might form a thin ice cap on the tip.

The presence on the tip of nitrogen solidified by the high electrostrictive pressure has been invoked to explain the high onset field and its sudden occurrence. However, it is unlikely that any solid cap can withstand (without destruction) the energy dissipation and the induced fluid motion occurring during injection ; this is why we have studied the cut-off as well as the onset events.

2.3 STUDY OF THE CUT-OFF. — The variation of the cut-off field with the tip-plane distance and the tip radius has been investigated under the experimental conditions sketched in the following diagram.

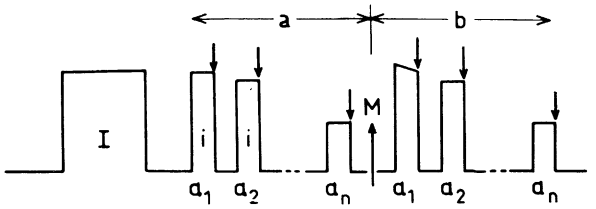

FIG. 3. — Experimental histogram : *I* injection maintained for several minutes to increase the tip radius by erosion ; *i* very short injection leaving tip radius practically undisturbed ; *M* end of run. The tip is dismounted, and remounted after electromicrographic measurement of its radius. New bath is condensed.

The whole sequence of this diagram is made with the same tip and roughly constant radius, at various distances (a_1, a_2, ..., a_n) from the plane. The arrows indicate the cut-off events. Data are first taken in the bath after erosion (« a » data). Then, the liquid is boiled out of the cell, the tip is carefully dismounted, analysed by electromicroscopy and assembled. A new bath is then condensed, new data (« b » data) is taken with great care to maintain a strict minimum of injection required for the measurements. Then, strong injection is deliberately maintained for several minutes to increase the tip radius, by erosion, a new series of measurements is made, as described above, on the same tip with its modified radius, and so on... until the radius becomes so big that injection cannot start with the available voltage supply. Typical results are given on figure 4. They can be summarized as follows :

1) The cut-off fields measured in a sequence are 5 to 10 % lower than the onset fields, which are themselves somewhat lower than the *first onset field*. However, the data for the cut-off field are less scattered than for the onset field.

2) For a given tip-plane distance, the cut-off field varies inversely with the tip radius, following the same type of behavior as the first onset field discussed in paragraph 2.2.

3) For a given tip radius, the cut-off field increases with the tip-plane distance. However, this dependence, relatively weak for the « b » data obtained with a new bath, is strikingly enhanced for the « a » data obtained with a bath where strong injection has occurred.

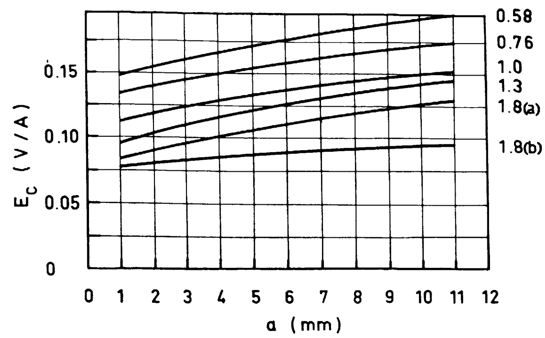

FIG. 4. — Variation of the cut-off field with tip-plane distance. The numbers indicate tip radius in μm

for 1.8 μm a = after injection
b = before injection

(Both « a » and « b » curves are drawn only for r = 1.8 μm, but similar data is available for all radii).

It is worth mentioning that the optical reflectivity of the metal plane becomes blurred when injection starts, and stays so until the liquid is removed. Reference will be made later on to this qualitative observation.

3. **Tentative interpretation of the data.** — 3.1 ONSET FIELD FOR VERY SHARP TIPS. — The onset field, of the order of 1 V/Å observed with very sharp tips ($r \leqslant$ 1 000 Å) and a wide range of tip-plane distances presumably corresponds to molecular breakdown events, as mentioned in the introduction.

The tiny volume of fluid undergoing an electric stress comparable to the maximum stress at the tip apex, combined with the low temperature of the fluid, may account for the unusual field value, owing to the small overall probability of presence of a free charge in the volume where it could start an avalanche. The fact that fields of the order of 1 V/Å (10^8 V/cm) may exist at the apex, even with a negatively biased tip, without extracting a measurable current requires a comment. In fact, the theory of field emission from a metal into vacuo predicts that a dense electronic current should escape the negatively biased metal electrode by tunnel effect through the potential barrier if the field at its surface reaches 10^7 V/cm (0.1 V/Å). To give an idea of the expected current density, a field of only 0.25 V/Å extracts from a tungsten surface a density of the order of 1 A/cm^2, hence a current of the order of 10^{-8} A from an injecting area of 1 μm^2.

Consequently, current injection below the onset is prevented here by one or both of the following processes :

a) nitrogen in contact with the metal, raising its work function with respect to the vacuum value (4.5 eV for tungsten) ;

b) electrons trapped at the interface, blocking further injection (in fact, unnoticed current bursts may easily saturate the emitter).

3.2 ONSET AND CUT-OFF FIELDS FOR LARGER TIP RADIUS. — The onset and cut-off fields decrease when the tip radius is larger than 1 000 Å. This indicates that the critical fields are then no longer an intrinsic property of the fluid. An interesting property common to all the data in a sequence of increasing tip radii appears if one plots the electric field distribution $E(x)$ on the tip axis in front of the apex, using the formula [5]

$$E(x) = (1 + 2\,x/r)^{-1}\,E_0 \tag{1}$$

for all the critical field values E_0 of a sequence. One comes out with a family of curves which practically merge together between $x = 2\,000$ Å and $x = 4\,000$ Å, with a field value between 0.08 and 0.13 V/Å. Consequently, it appears that a field of the order of 0.1 V/Å, hence one order of magnitude smaller than the molecular field, becomes critical for injection if applied to a sufficient amount of fluid. Specifically, the product of a critical field of the order of 0.1 V/Å by a critical dimension of the order of 3 000 Å, in other words a critical voltage drop of the order of 300 V, is more relevant than the maximum field at the apex. In fact, the electrohydrodynamic instability criterion developped by the Grenoble group [6] predicts for liquid nitrogen a value of the same order. Although the agreement might be fortuitous, it is worth mentioning.

In the intermediate radius range, the variation of the onset field with the tip radius of the critical field follows, roughly, a $r^{-0.5}$ law, reminiscent of gas breakdown and suggesting that bubbles are involved in the onset process. On the other hand, the existence of a critical voltage of a few hundred volts suggests that hydrodynamic instability is the primary initiating process, and that bubbles appear by cavitation.

3.3 EFFECT OF TIP-PLANE DISTANCE ON THE CRITICAL FIELD. — It was shown (Fig. 3) that, at least for large tip radii, the critical field increases with the tip-plane distance. This dependence, slightly underestimated here since the field was calculated without accounting for the finite size of the plane, is enhanced if a sufficient charge has been injected in the liquid.

Most of the experimental findings, including the modified reflectivity of the plane surface after injection, are qualitatively consistent with the assumption that some of the injected charge, trapped on the plane surface, decreases the field at the tip.

Assuming, as an over-simplified model, that a 1 cm^2 plane collects all the charge injected by a current of 10^{-8} A during 10 s (typical experimental conditions after onset), the resulting charge density of 10^{-3} $C.m^{-2}$ on the plane produces a field of the order of 0.01 V/Å (10^6 V/cm) which might not only affect the onset and cut-off conditions at the tip, but even cause local discharges at the plane surface. Of course, a direct observation of the discharge inception with and without previous injection would help to confirm the model.

PART II. — STUDY OF THE INJECTION

1. **Injection.** — In the first part of this paper, the conditions for onset and cut-off of charge injection were analysed. Now, we deal exclusively with the injection process occurring above the onset voltage, and will discuss the nature of the current injection and associated processes such as bubble injection and liquid motion in the vicinity of the tip.

It was shown previously [3] that the injected current usually relaxes at an average frequency increasing with the applied voltage, and that the current, undetectable below the threshold voltage, suddenly jumps to about 10^{-8} A at the onset voltage, then increases roughly as the square of the voltage. This quadratic variation was tentatively interpreted in terms of space-charge-limited injection.

In fact, according the crudest possible form of this theory, valid under some approximation for spherical electrodes, the relation between the injected current density I and the applied voltage V is of the form

$$I = \frac{3\,\Omega\varepsilon\mu}{8\,a}(V - V_0)^2 \tag{2}$$

where Ω is the solid angle of injection,
ε is the permittivity of the fluid,
μ is the charge mobility in the fluid,
a is the tip-plane distance, and
V_0 is a constant.

This very crude, spherical approximation has been extended to the less trivial, more realistic case where the emitter is a hyperboloid of revolution [5]. The result of this improved theory is similar to that of the spherical approximation, except that the constant V_0 is now of the form

$$V_0 = \frac{E_p\,r}{2}\ln\frac{4\,a}{r} \tag{3}$$

where E_p is the value of the field at the tip apex (see Appendix).

2. **Presentation of the current-voltage characteristics.** — More than 80 characteristics with a negative tip and 100 with a positive tip have been obtained and analysed. The tip-plane distance was varied between 1 and 11 mm, and tip radii up to 2.7 μm have been used.

All the current-voltage characteristics, plotted as $\sqrt{I}$ versus V, display a linear variation in agreement with the theory. A typical characteristics is schematized on figure 5, on which the quantities V_0 and p, which we shall deal with later, appear clearly.

As expected from the theory, the slope p varies

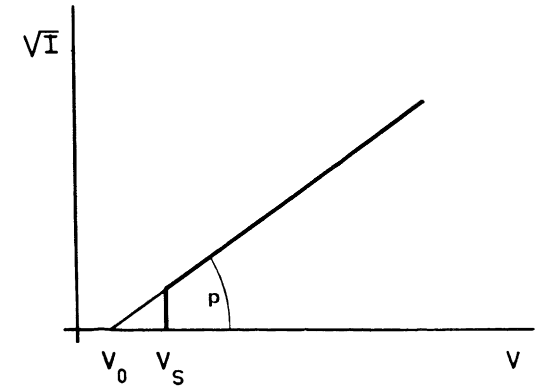

FIG. 5. — Typical $\sqrt{I}$, V characteristic. V_0 is the zero-current extrapolated voltage. V_s is the threshold voltage. p is the slope of the line.

mainly with the tip-plane distance a, in an approximately reciprocal manner, but it also increases slightly with r/a.

For reasons which are explained in the Appendix, we have plotted on figure 6 the square p^2 of the slope p

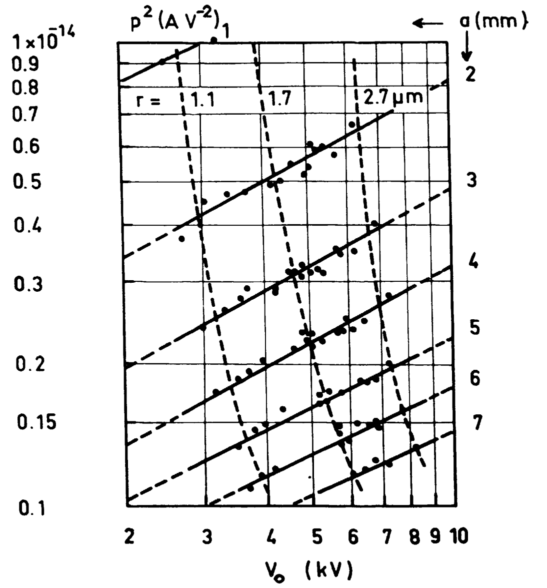

FIG. 6. — p^2 (p being the slope of the characteristics of Fig. 5) vs. V_0 for various tip-plane distances. In all the data of this figure, the tip is negative, but similar data with positive tips is available.

of the linear characteristics versus the intercept voltage V_0, for various values of a. Hence, on this figure summarizing a large amount of experimental results, a current-voltage characteristic is represented by its two parameters V_0 and p^2. Eq. (3) (or A.9 of the Appendix) is of particular interest, since it may be used to obtain the tip field E_p, once the dimensions a and r and the intercept voltage V_0 are known. This has been done for all the data of figure 6, with the remarkable result that E_p is practically constant. It increases slightly with increasing tip-plane distance, from 5.75 MV/cm^{-1} for $a = 1$ mm to 7 MV/cm^{-1} for $a = 7$ mm (Fig. 7) but in any case, it is one order of magnitude smaller than the onset field deduced

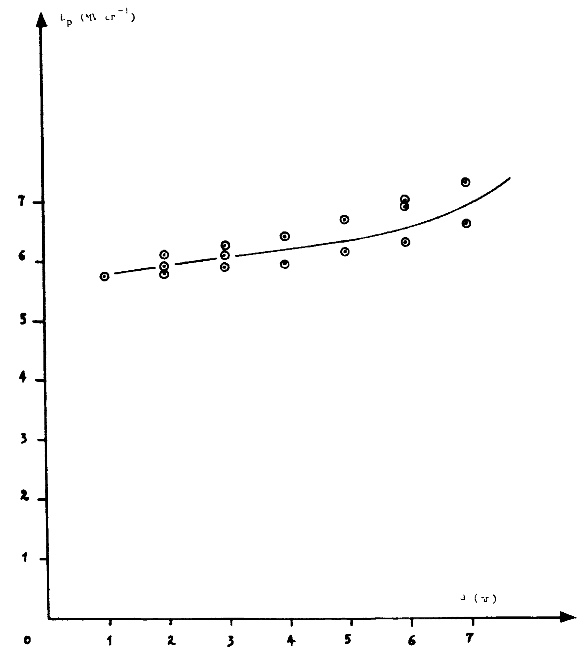

FIG. 7. — Variation of the tip field E_p with the tip-plane distance a.

from the measurement of the onset voltage, as described in part I.

This is a striking confirmation of the fact that, once injection has started, the current is always space-charge-limited.

Figure 8 shows V_0 as a function of the tip radius r for 5 values of the tip-plane distance; the curve is calculated for $E_p = 6$ MV/cm^{-1}. The agreement is quite good in view of all the approximations of the theory.

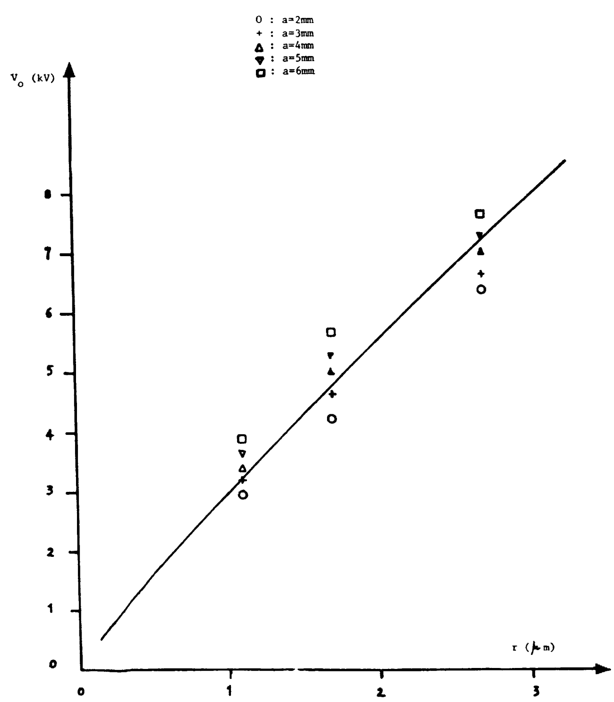

FIG. 8. — V_0 versus r. The continuous curve gives the theoritical variation for

$E_p = 6$ MV/cm^{-1} and $a = 5$ mm from (A.9).

Finally, a rather severe test of the theory is possible by rewriting eq. (A.8) under the form :

$$ap^2 \simeq 4\,\varepsilon\mu\left(1 + 1.6\,\frac{r}{2\,a}\ln\frac{2\,a}{r}\right). \qquad (4)$$

One sees that by plotting the product ap^2 for all the characteristics versus the dimensionless number

$$x = \rho \ln \frac{1}{\rho} \qquad \text{where} \qquad \rho = \frac{r}{2\,a},$$

one should obtain a single straight line intercepting the ap^2 axis at the value $4\,\varepsilon\mu$. This has been done, but the agreement is not quantitative. Since a typical value of ρ, for $r = 1.7$ µm and $a = 4$ mm, is of the order of 1.8×10^{-3}, one can neglect the value of x with respect to unity to find a typical charge mobility of the order of $10^{-3}\ \mathrm{cm}^2/\mathrm{V}^{-1}/\mathrm{s}^{-1}$, which is quite reasonable, and the fact that the product ap^2 varies more steeply than predicted by the theory must be ascribed to the hydrodynamic drag due to fluid motion.

3. **Optical observation of the prebreakdown events.** — A microinterferometer has been assembled in collaboration with the Institut d'Optique, in order to observe the neighborhood of the tip during injection and in particular to check some previous observations of the light emission by the injecting tip. The optics has a resolution of the order of 1 µm, a field depth of 0.1 mm, and interference contrast. A *chronolite* light source provides a series of very short flashes separated by variable intervals as short as 2 µs. With this instrument, it is possible to detect small changes in the refractive index, and to estimate the size, the velocity (using two flashes) and the acceleration (with three flashes) small bubbles within a millimeter distance from the tip.

Simultaneously with the observation of the optical disturbances, the injection current itself was analysed by means of a Tektronix oscilloscope, and a correlation between the disturbances in the fluid and the current fluctuations was sought for. These observations were made in view of clarifying the interaction between the motion of charges, that of the fluid and that of the bubbles in the fluid.

The main preliminary results of the above observations can be summarized as follows.

When the onset voltage for a sharp negative tip is reached, a steady current can be observed, and a short, shaded line appears in front of the tip. This line presumably consists of unresolved bubbles which condense in the liquid after moving over a distance of a few hundred microns. When erosion has sufficiently increased the tip radius, the current usually displays relaxations, and bubbles can be resolved. Their size is widely distributed, but their average radius does not depend on the applied voltage, which affects only their rate of appearance. Figure 9 represents such bubbles being ejected from a tip at -11 kV. The double exposure, with a time interval of 46 µs, allows an accurate measurement of the longitudinal bubble velocity.

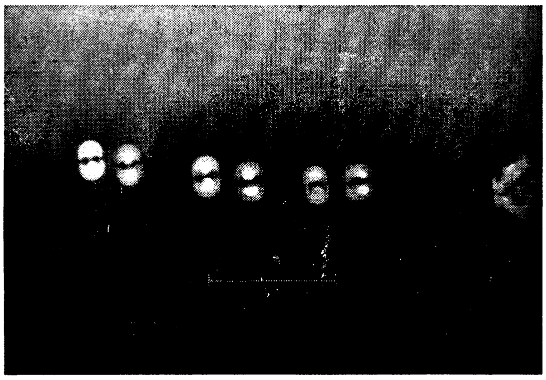

FIG. 9. — Photograph of bubbles moving in front of the tip at -11 kV. Each object on the picture is split vertically by double beam interferometry and horizontally by a delay of 46 µs between the two flashes. The tip is at the right side of the picture, and the white segment corresponds to 200 µm.

Figure 10 represents the variation of the measured velocity of the bubbles as a function of the applied voltage for three values of the distance a. The data is widely scattered, mostly because the bubble radii are widely spread. When this manuscript was being prepared, it had not been possible yet to establish the dependence of bubble velocity on bubble radius ; hence, each point on figure 10 represents the average of 15 to 20 measurements on bubbles of different sizes.

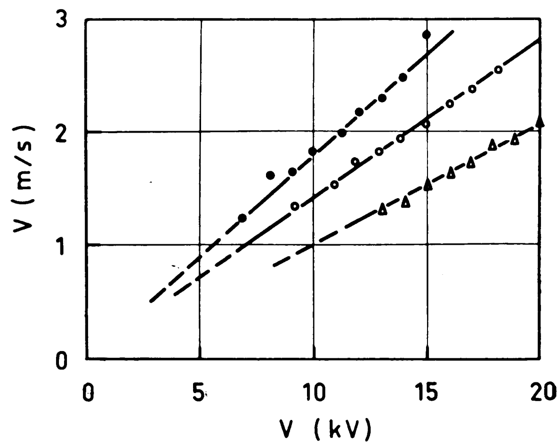

FIG. 10. — Average value of the X-component of velocity of the bubbles in m/s versus applied voltage V, for various tip-plane distances.

• $a = 2.5$ mm , ○ $a = 4.5$ mm , △ $a = 5.5$ mm .

Keeping in mind that the velocity of the fluid varies as the square root of its kinetic energy, the results of figure 10, including the dependence on tip-plane distance, display a profound analogy with the current-voltage characteristics.

Nevertheless, it is only after obtaining sufficiently reliable data relating bubble velocity and bubble radius that one can hope to find the relative velocity of the bubbles with respect to the moving fluid, and, therefore to solve the problem of the hydrodynamic mobility of the charges. This will be the subject of another publication.

4. **Conclusion.** — This work has unveiled new aspects of some very high field phenomena in liquid nitrogen. It has shown that, if the volume under stress, in front of a sharp metal tip, is very small (say, smaller than 1 μm^3), molecular breakdown — unlike the usual cooperative one — can be observed. Sequences of molecular breakdown events suddenly raise the average current density at the tip from an extremely small value (thermal ionization does not occur at 77 K) to a value of nearly 1 A/cm^2. They also produce microbubbles of such small radius that no further discharge can take place in them before they collapse by condensation. With increasing tip radius (hence increasing applied voltage to maintain injection), discontinuous discharges and larger bubbles usually appear. These bubbles might result from hydrodynamic cavitation in the mechanically unstable fluid.

As soon as current sets in, it is limited by its own space-charge, as shown by the linear dependence of $\sqrt{I}$ with the applied voltage, and confirmed by the fact that the actual tip field during injection, obtained by fitting the experimental data to the theory of space-charge-limited injection, is considerably lower than the geometric field calculated in the absence of space-charge distortion from the electrode dimensions and the applied voltage.

Several important questions raised by this work remain more or less open, and we mention below the three major ones.

— There is no detailed, reliable correlation between current relaxation and bubble injection. The current may or may not relax when bubbles are being injected. In fact, the change of injection regime from steady to relaxing does not seem to affect the average current. This might result from the fact that the current density at the tip becomes so high, above the onset voltage, that space-charge limitation dominates throughout and oversimplifies the situation.

— Previous injection in the cell has been shown (part I) to modify the onset conditions. This behavior might be consistent with the presence, on the plane electrode, of a densely charged layer, which could be related to the modified appearance of this electrode after injection has occurred. The hypothesis of a charged layer on the plane is also consistent with the decaying voltage observed after removing the applied voltage, but its present investigation is far from exhaustive. It should be worth trying to see, with proper techniques, whether some sort of electret might be formed in our experiments.

— Finally, the most important question is that of the relative velocity of the injected charges, the bubbles and the moving fluid. Since the well known work of Ostroumov in Russia [7] and of the Grenoble group [6], this question is the subject of considerable interest. Although our experimental cell with highly divergent field is not ideal for studying this sort of problem, the possibility of accurate measurement of bubble velocity, combined with the fairly accurate mobility data deduced from the current-voltage characteristics with our theory of the space-charge limited injection constitutes a powerful tool, which will be used when more information is available on the relation between the size and the velocity of the bubbles.

APPENDIX

Derivation of eq. 3. — The eq. (3) giving V_0 results from the theory of the space-charge-limited current injected by a metal tip of hyperboloidal profile, which is developed in reference [5]. It is shown there that the characteristics are obtained by integration of the axial field over the tip-plane distance.

Using the notation of reference [5], which appears clearly in figure A.1, the integration of E has to be performed between $v = 0$ and $v = 1 - \rho$, where $v = X/a$ and $\rho = r/2\,a$.

The quantity $E(v)$ given in eq. (83) of reference [5] is the square root of the sum of two terms, one being proportional to the tip field E_p at $X = 0$, and the other to the square root of the quantity A, which, from eq. (79) of reference [5], is proportional to the injected current.

Analytic integration of $E(v)$ is impossible, but a fair approximation of the integral can be obtained

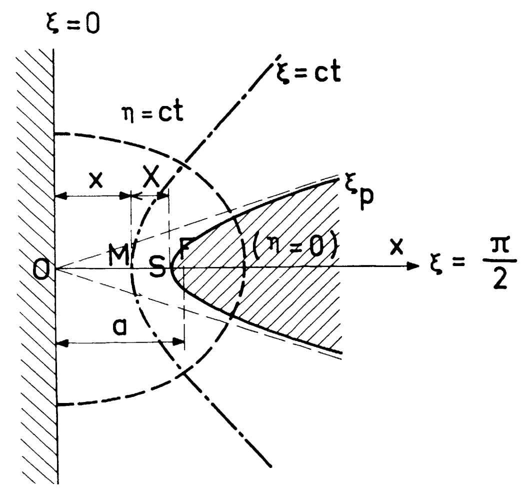

FIG. A.1. — Representation of a point-plane configuration with the various quantities involved.
Note that $x + X + r/2 = a$ or $u + v + \rho = 1$.

if one uses the fact that, for any positive number α and β,

$$\frac{1}{\sqrt{2}} \leqslant \frac{\sqrt{\alpha + \beta}}{\sqrt{\alpha} + \sqrt{\beta}} \leqslant 1 . \tag{A.1}$$

This allows us to separate the square roots in the integrand and to obtain the current-voltage relationship under the form

$$\frac{V}{a} = E_{\mathrm{p}} \, \rho \, \frac{2 - \rho}{2} \ln \frac{2 - \rho}{\rho} + \lambda \sqrt{I} \tag{A.2}$$

with

$$\lambda = \left(\frac{2}{\pi a \varepsilon \mu} \right)^{1/2} \int_0^{1-\rho} \frac{[\rho(2-\rho) + (1-\rho)\, v - v^2/3]^{1/2}}{(v+\rho)\,(2-\rho-v)} \times \sqrt{v}\, \mathrm{d}v . \tag{A.3}$$

Further approximations can be made. In eq. (A.2), since $\rho \ll 2$, the first term of the right hand side reduces to $E_{\mathrm{p}} \, \rho \ln 2/\rho$. The integral in the second term, explicitely written in (A.3), can also be approximated by neglecting ρ everywhere except in the factor $(v + \rho)$ of the denominator, which should be left unchanged to avoid divergence at the lower limit of integration. If the integral is called J, then we can write with a good accuracy

$$J \simeq \int_0^{1-\rho} \frac{(1 - v/3)^{1/2} \, v}{(v + \rho)\,(2 - v)} \, \mathrm{d}v \tag{A.4}$$

and since v is always less than unity, the numerator of the integrand can be expanded to its first power in v, giving J an integrable form :

$$J \simeq \int_0^{1-\rho} \frac{(1 - v/6)\, v \, \mathrm{d}v}{(v + \rho)\,(2 - v)}$$

$$\simeq 0.624 - \frac{\rho}{2} \ln \frac{1}{\rho} . \tag{A.5}$$

Finally, the characteristics, using (A.2) and (A.5), take the form

$$\frac{V}{a} = E_{\mathrm{p}} \, \rho \ln \frac{2}{\rho} + \left(\frac{2\, I}{\pi a \varepsilon \mu} \right)^{1/2} \times \left(0.624 - \frac{\rho}{2} \ln \frac{1}{\rho} \right) . \tag{A.6}$$

From eq. (A.6), one can see that the current-voltage characteristics, expressed as $\sqrt{I}$ versus V, can be written under the form :

$$\boxed{\sqrt{I} = p(V - V_0)} \, . \tag{A.7}$$

This represents, in the $\sqrt{I}$, V plot, a straight line of slope given by

$$p = 2.01 \left(\frac{\varepsilon \mu}{a} \right)^{1/2} \left(1 - 0.8 \, \rho \ln \frac{1}{\rho} \right)^{-1}$$

or, approximately, due to the fact that the logarithmic term is always small with respect to unity :

$$p \simeq 2.01 \left(\frac{\varepsilon \mu}{a} \right)^{1/2} \left(1 + 0.8 \, \rho \ln \frac{1}{\rho} \right) . \tag{A.8}$$

This line intercepts the V axis at

$$V_0 = \frac{1}{2} E_{\mathrm{p}} \, r \ln \frac{4\, a}{r} . \tag{A.9}$$

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