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INFLUENCE OF REALISTIC DEUTERON WAVE FUNCTIONS ON NEUTRAL PION PHOTOPRODUCTION CROSS SECTIONS

K. SRINIVASA RAO

Matscience, The Institute of Mathematical Sciences, Madras 600020, India

R. PARTHASARATHY and V. DEVANATHAN

Department of Nuclear Physics, University of Madras, Madras 600025, India

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Résumé. — On utilise des fonctions d'onde réalistes pour calculer la section efficace différentielle de photoproduction cohérente du pion neutre sur le deutérium, au voisinage de la première résonance, à transfert de quantité de mouvement constant. Une comparaison est faite avec les nouveaux résultats expérimentaux d'Orsay.

Abstract. — The differential cross-section for neutral pion photoproduction from deuterium, near the first resonance region, at fixed momentum transfer, is studied using realistic deuteron wave functions and the results are compared with recent experimental data from Orsay.

1. **Introduction.** — Recently [1], [2] the differential cross section for the coherent photoproduction of the π^0 meson from deuterium, in the first resonance region, has been measured at constant momentum transfers. Since this coherent deuteron process can provide important information on the deuteron structure and hence on the nucleon-nucleon potential, we study the influence of the realistic Hamada-Johnston [3] (HJ) and Reid [4] deuteron wave functions on the neutral pion photoproduction cross section in the impulse approximation using the amplitudes of Chew *et al.* [5] and compare our results with those obtained using the conventional Hulthén wave function for the deuteron.

2. **Deuteron wave functions.** — By slightly modifying the Hamada-Johnston potential (MHJ), Humberston and Wallace [6] have obtained deuteron wave functions which give the correct deuteron binding energy of 2.226 MeV, while Reid [4] has used local and static phenomenological nucleon-nucleon (infinitely) hard-core (RHC) and (Yukawa) soft-core (RSC) potentials to fit Yale and Livermore phase parameters and low-energy two-nucleon data. In table I we summarize the properties of the deuteron predicted by HJ, MHJ, RHC and RSC deuteron wave functions along with the corresponding exper-

imental values. The wave functions in all the four cases are normalized as :

$$\int_{x_0}^{\infty} [u^2(x) + w^2(x)] dx = 1, \quad (1)$$

where $x = \mu r$ with $\mu = 0.7 \text{ fm}^{-1}$ and $u(x)$ and $w(x)$ are respectively the S- and D-state radial wave functions of the deuteron. It should be noted that while the HJ and MHJ wave functions have a hard-core radius of $x_0 = 0.343$, the RHC wave function has $x_0 = 0.384$.

The conventionally used Hulthén (S-state) deuteron wave function is :

$$u(r) = \left[\frac{\alpha}{2\pi(1 - \alpha\rho_1)} \right]^{1/2} (e^{-\alpha r} - e^{-\beta r}) \quad (2)$$

with $\alpha^2 = M\varepsilon$, ε being the binding energy of the deuteron and ρ_1 the effective triplet scattering range (= 1.74 fm) is given by the relation :

$$\frac{3}{\beta} \simeq \rho_1 \left(1 + \frac{4}{9} \alpha\rho_1 \right). \quad (3)$$

3. **Calculation of cross section.** — In an earlier paper, Ananthanarayanan and one of us (KSR) [7] studied the effects of the inclusion of the D-state

TABLE I
Deuteron properties

Potential	Effective range (fm)	B. E. (MeV)	Q_2 (fm ²)	P_D (%)	A_D/A_s
—	—	—	—	—	—
HJ	1.77	2.269	0.285	6.97	0.026 56
MHJ	1.749	2.226	0.284 5	6.953	0.026 42
RHC	1.724	2.224 64	0.277	6.497	0.025 9
RSC	1.720	2.224 60	0.279 64	6.469 6	0.026 223
Experimental	1.726	2.224 52	0.278	4.0-7.0	0.026

admixture and hard-core radius in the ground state of the deuteron on the differential cross section for $\gamma d \rightarrow d\pi^0$, in the impulse approximation using the amplitudes of Chew *et al.* [5] at an incident photon energy of 280 MeV. In that study, phenomenological deuteron wave functions constructed by Hulthén and Sugawara [8a] assuming suitable functional forms containing several parameters were used. Those parameters were adjusted to fit the then existing empirical information on the neutron-proton system by Hedin and Conde [8b].

The main result of reference [7] is that among the following radial integrals :

$$F_{SS} = \int_{x_0}^{\infty} u^2(x) j_0\left(\frac{1}{2} kx\right) dx, \quad (4)$$

$$F_{SD} = \int_{x_0}^{\infty} u(x) j_2\left(\frac{1}{2} kx\right) w(x) dx, \quad (5)$$

and

$$F_{DD} = \int_{x_0}^{\infty} w^2(x) j_l\left(\frac{1}{2} kx\right) dx \quad (l = 0, 2, 4), \quad (6)$$

which are functions of the momentum transfer k , F_{SD} and F_{DD} are at least an order of magnitude smaller

when compared to F_{SS} , so that the terms in the matrix element containing F_{SD} and F_{DD} can be neglected. Thus, it was deemed fit to take the effect of the D-state admixture through the normalization (1) and scale of the S-state wave function only.

In table II, we give the values of F_{SS} , F_{SD} and F_{DD} (we omit the negligible values of the F_{DD} integrals corresponding to $l = 2$ and 4), for a set of values of k , obtained for RHC and MHJ deuteron wave functions only, since we find the differences between RHC and RSC on the one hand and between MHJ and HJ on the other are negligible. From table II, we notice that our conclusion drawn in reference [7] is also valid for realistic deuteron wave functions.

The differential cross section for $\gamma d \rightarrow d\pi^0$ can be written as :

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \mu \mu_0 |Q|^2 \quad (7)$$

where $\mu = |\mu|$ and μ_0 are the momentum and energy of the out-going pion and $|Q|^2$ the square of the matrix element, whose explicit form is given by eq. (2.24) of reference [7], is directly proportional to $|F_{SS}|^2$. Therefore, the cross section is expected to

TABLE II

Nuclear form factors for the deuteron as a function of the momentum transfer k , in the centre of momentum system

k (fm ⁻¹)	RHC deuteron wave functions			MHJ deuteron wave functions		
	F_{SS}	F_{SD}	F_{DD}	F_{SS}	F_{SD}	F_{DD}
—	—	—	—	—	—	—
0.197 6	0.909 9	0.001 8	0.036 8	0.894 1	0.001 3	0.068 4
0.405 7	0.844 9	0.007 1	0.035 1	0.827 8	0.005 2	0.066 5
0.724 9	0.698 4	0.018 0	0.030 8	0.680 5	0.014 0	0.061 5
1.039	0.548 9	0.031 5	0.025 5	0.532 4	0.022 9	0.055 2
1.326	0.430 4	0.040 9	0.020 5	0.415 9	0.029 8	0.048 7
1.575	0.344 3	0.047 4	0.016 3	0.331 5	0.034 4	0.043 1
1.683	0.311 6	0.049 5	0.014 5	0.299 7	0.035 9	0.040 7
1.777	0.284 9	0.051 2	0.013 1	0.273 8	0.037 3	0.038 6
1.928	0.246 9	0.053 4	0.010 9	0.236 9	0.038 2	0.035 4
2.018	0.225 9	0.055 5	0.009 6	0.216 4	0.039 5	0.033 5
2.049	0.219 1	0.054 0	0.009 1	0.209 8	0.039 5	0.032 9

be very sensitive to the nature and choice of the S-state deuteron wave function.

4. Results and discussion. — As the recoil energy is less in the centre of momentum system, the static model calculation for the differential cross section is made in that system. There is no free parameter in this model.

In figures 1, 2 and 3, the differential cross sections for momentum transfers 240 MeV/c, 280 MeV/c and 350 MeV/c, respectively, are plotted as a function of the laboratory photon energy. The results obtained with the RHC and MHJ deuteron wave functions are shown along with those obtained using the Hulthén wave function for the deuteron. The experimental data are from Orsay 1972 [1], Stanford 1962 [9] and Glasgow 1966 [10]. There is a remarkable disagreement between the early Stanford 1962 data — the only cross sections available till recently — and the Orsay 1972 data in the first resonance region which give a smooth extension to the low energy data from Glasgow 1966, at the momentum transfer of 350 MeV/c (Fig. 3). Our theory is in good agreement with the recent data.

In figure 4 we compare our results obtained using the MHJ deuteron wave function $\theta_{\pi}^{c.m.} = 90^\circ$ with the data from Stanford 1962 [9], Stanford 1963 [11], Glasgow 1966 [10] and Bonn 1971 [2]. The measured differential cross sections have been divided by the form factor calculated by Hadjioannou [12], as in Hieber *et al.* [2], to eliminate the steep decrease of the cross section with increasing momentum transfer. The excitation curve at $\theta_{\pi}^{c.m.} = 90^\circ$ determined from

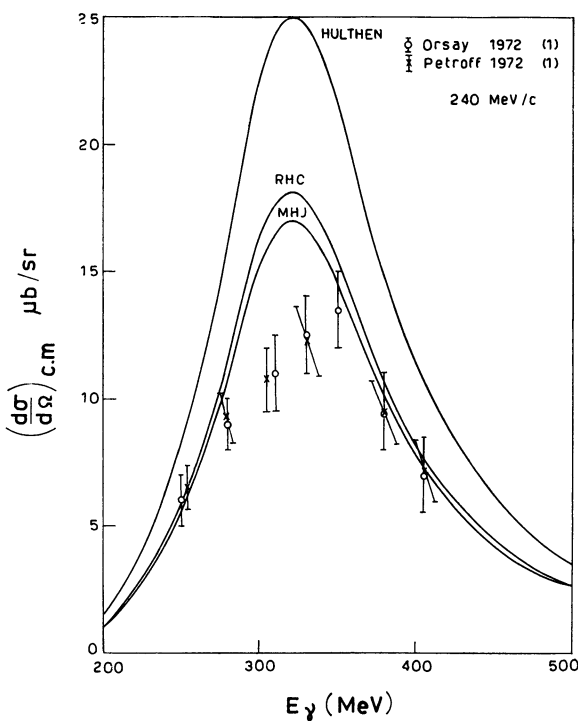


FIG. 1. — Differential cross section at 240 MeV/c deuteron momentum transfer.

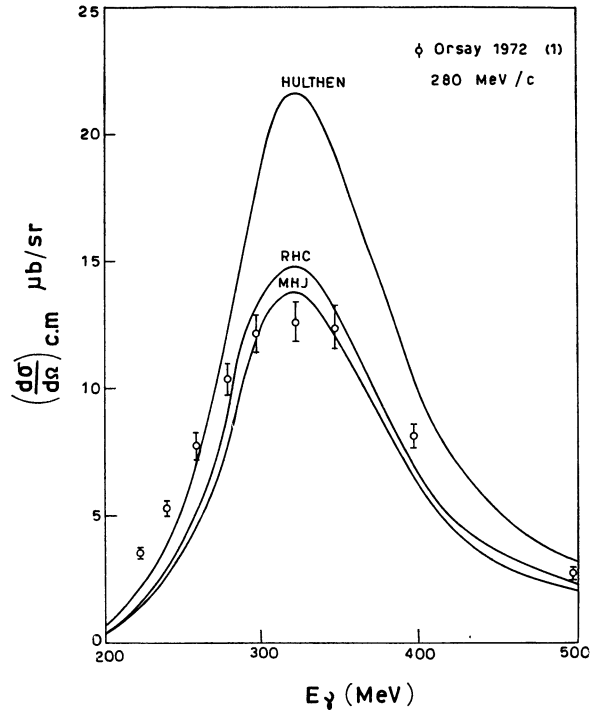


FIG. 2. — Differential cross section at 280 MeV/c deuteron momentum transfer.

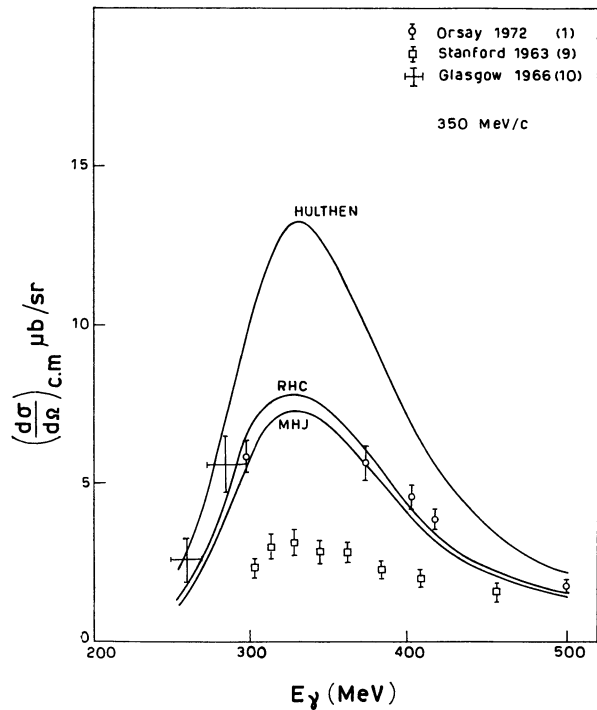


FIG. 3. — Differential cross section at 350 MeV/c deuteron momentum transfer.

the reaction $\gamma d \rightarrow d\pi^0$, taking into consideration only the magnetic dipole excitation and the impulse approximation, has been drawn (dotted line) for the sake of comparison [2].

From the analysis of Pazderskii [13], we find that the corrections due to multiple scattering, to the general impulse approximation for π^0 photo-

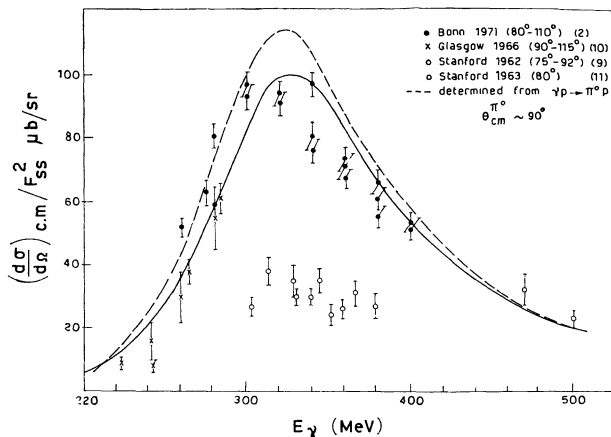


FIG. 4. — Differential cross section at $\theta_{\pi^m}^c = 90^\circ$.

production on deuteron, decrease the differential cross section by 25-30 % for 285-345 MeV photon energies. But we notice, from the nature of the angular distribution of the cross section, that these corrections are negligible at backward angles (where momentum transfer is ≥ 240 MeV/c) since the values of the cross section itself are small. Further, in an analogous situation, Lazard and Marić [14], find the agreement between the impulse approximation calculations and experiment satisfactory for fixed momentum transfers, up to $q \sim 2.4 \text{ fm}^{-1}$ while the rescattering corrections to the cross section are rather insignificant for small momentum transfers ($q < 2.4 \text{ fm}^{-1}$). Therefore, in the present study, since we are concerned only with the differential cross sections at fixed momentum transfers of 240 MeV/c, 280 MeV/c and 350 MeV/c (1.22 fm^{-1} , 1.42 fm^{-1} and 1.78 fm^{-1} , respectively), we do not make an effort to correct for multiple scattering effects.

From figures 1, 2, 3 and 4 we find good agreement between our theoretical curves and the recent data from Orsay 1972 and Bonn 1971. The fits obtained

with the most correct HJ model, viz. MHJ with the correct deuteron binding energy, are slightly better than those obtained with the RHC model. At 350 MeV/c (Fig. 3) we find that our no parameter, static model fit to the Orsay 1972 data is much better than that obtained by Bouquet *et al.* [1] who used a fully covariant model developed by Schiff and Tran Thanh Van [15] to evaluate the π^0 photoproduction amplitude [16].

In figure 4 we find a close agreement between our results obtained for $(d\sigma/d\Omega)/|F_{SS}|^2$ in the impulse approximation for $\gamma d \rightarrow d\pi^0$ using the amplitude of Chew *et al.* [5] and the results for $\gamma p \rightarrow p\pi^0$, which clearly exhibits the validity of the impulse approximation.

In general, we find the agreement between the impulse approximation calculations and experiment rather satisfactory for the small fixed momentum transfers considered here, $q < 1.8 \text{ fm}^{-1}$.

Finally, we find that the results obtained for the neutral pion photoproduction cross section with Hamada-Johnston and Reid realistic deuteron wave functions are comparable, as is the case with shell model calculations [17] and elastic electron-deuteron scattering at low q^2 [18] with these realistic potentials.

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