Incoherent and chaotic bunching effects
C. Bénard

To cite this version:
C. Bénard. Incoherent and chaotic bunching effects. Journal de Physique, 1972, 33 (11-12), pp.1027-1036. <10.1051/jphys:019720033011-120102700>. <jpa-00207328>

HAL Id: jpa-00207328
https://hal.archives-ouvertes.fr/jpa-00207328
Submitted on 1 Jan 1972

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
INCOHERENT AND CHAOTIC BUNCHING EFFECTS

C. BÉNARD

Laboratoire d'Etude des Phénomènes Aléatoires Bâtiment 210, Université de Paris-Sud, 91405 Orsay, France

(Reçu le 18 juillet 1972)

The problem we are dealing with in this article is the origin of the chaotic bunching effect. The chaotic bunching effect has been first observed by Hanbury-Brown and Twiss in 1956 [1]. It has been later the object of more precise experiments, on non resonant lines of mercury [2], [3]. The chaotic bunching effect is the following: the second order coincidence probability density (cpd), that is to say, the probability for detecting a photon at a given point and at a given time and another one at another point at a distance \(a\) and at the same time, \(P_{2}^{CB}[a]\), is such that

\[
P_{2}^{CB}[a = 0] = 2 \frac{P_{2}^{CB}[a]}{P_{2}^{CB}[\alpha \to \infty]}.
\]

A typical curve of \(P_{2}^{CB}[a]\) in terms of \(a/\xi_{c}\), where \(\xi_{c}\) is the coherence length of the considered field, is given in figure 1.

The bunching effect can be interpreted by saying that the detections of two photons in a chaotic electromagnetic field are correlated events. When one photon is detected, the probability for detecting another photon in the same point is twice as large as the probability for detecting another photon at a distance.

The origin of this effect is still not clear. In fact, some authors [4], [5] think that it is only due to the symmetry of photons (their property of being bosons); some others [6], [7] think that the interactions in the source must be taken into account to explain the observed correlations between the detected photons. Such contradictory explanations come from the fact that these authors consider a free field and don't analyse the emission process in a definite chaotic source; thus they cannot relate the bunching effect observed in the emitted field with the properties of the source. The interpretation of the characteristics of a field by the properties of the source has been done for laser sources but not for such usual sources as chaotic sources.

In this paper, we would like to clarify our understanding of chaotic bunching effect.

As the problem is rather broad and difficult, we shall focus our attention, for simplicity, on a definite
type of chaotic sources. We know that low intensity
and narrow line sources such as mercury lamps or
hydrogen lamps looked at for a given ray of their
spectrum, are generally considered as chaotic. But
this has been actually proved in very few cases. In
fact, D. B. Scarl has shown experimentally [2],
with good precision, that the non resonant line 7 3S-6 3P1 (4 358 Å) of a low pressure (∼ 2 torr in
presence of argon gas) mercury lamp can be con-
considered as chaotic : the second order cpd verifies (1).
The same experiment has been done on the non
resonant line 7 3S-6 3P2 (5 461 Å) of mercury [3],
but it is not so accurate. We shall rely on the expe-
riments [2] of D. B. Scarl to admit that the type of
source he used is actually chaotic. In the following,
when we speak of chaotic sources, it must be understood
that it is this type of sources that is referred to.

Our purpose is to understand how the emission
process in these sources can produce a chaotic
field. To do so, we propose a very simple model of
source, which we call incoherent source, and test
whether such a source produces a chaotic field or not. From the comparison between the theoretical
model of an incoherent source and the definite type
of sources we are considering here, we draw several
conclusions concerning the chaotic bunching effect
and point out several features of the chaotic sources.

The present study is thoroughly theoretical. But
the comparison between our incoherent model and
a chaotic source is likely to be worked out experi-
mentally.

Our description of both the source and the field
is entirely quantic. The field is described in the wave-packet formalism [5], [8], [9], [10]. It consists of
associating with every particle of the field a state | \phi_j \rangle
of the Hilbert space H and building by symmetri-
ization the state n(\{ r_j \}) of the Fock space, describ-
ing n particles of the field. If the states | \phi_j \rangle
were orthogonal, this construction would be nothing but
the usual construction of the Fock space from the
one-particle-state-space H. But the states are not
assumed to be orthogonal. In fact, they are defined
in the following way : every | \phi_j \rangle is deduced from
a given state | \phi \rangle by a translation in space charac-
terized by a vector r_j. The states | \phi_j \rangle are one-particle
wave-packets : the impulsion of a particle in such a state is not defined completely. The variables
r_j are random. Their distribution law depends on
the considered source.

We would like to underline here two important
features of the wave-packet formalism :

a) the fact that the wave-packets are non-orthogonal
cannot be neglected. On the contrary it is at the root
of the difficulty in describing a chaotic source,

b) if certain conditions are fulfilled, it is easy to
relate the properties of a field described in the wave-
packet formalism (characterized by the state | \phi \rangle
and the distribution law of the r_j) with the emission
process of the source. It is the case for the incoherent
source model that is proposed here. We can deduce
the density matrix of the electromagnetic field emitted
by an incoherent source from the properties of this
source. Thus it is possible to compare the incoherent
and chaotic density matrices on one hand and the
incoherent and chaotic emission processes on the
other.

In section I, a short review of the definition and
properties of the wave-packet formalism is given.
For more details, one should look at references [5],
[8], [9], [10]. In section II, we give the conditions under
which the description of a field using the wave-
packet formalism provides straightforward informa-
tions about the emitting source. In section III, we
sum up the main results concerning the chaotic
field. In particular, in the case of low intensity fields,
where the probability for having more than two
particles in the field can be neglected, we show how
the chaotic bunching effect can be deduced from the
microscopic description of a two-particle-system.
Our reason for studying low intensity fields very
closely is twofold :

1) The typical chaotic sources that we consider
here produce low intensity fields.

2) This case provides a simple understanding of
the phenomena, without going through the difficulties
of the wave-packet formalism.

In section IV, we define an incoherent source and
give the density matrix of an incoherent field, in
the wave-packet formalism. The incoherent source
model (or model I) is roughly the following : the
source consists of a great number of two level atoms
excited to their upper state by stochastically inde-
pendent pulses and emitting photons spontaneously
without interacting with one another. The emitted
photons are described by wave-packets. The exciting
pulses are short enough to be considered as punctual
in time (their width in time is much smaller than the
time of the upper state of the atoms). And each
pulse is assumed to excite only one atom.

We analyze in particular the case of low intensity
fields and explain the incoherent bunching effect
with the help of a two-particle-system. In section V
we show that the incoherent and chaotic density
matrices are not identical. In particular, in the case
of low intensity fields, we show that the difference
between incoherent and chaotic bunching effects is
large enough to be checked experimentally.

We suggest a way of building an incoherent source,
to realize the experiment.

In section VI, we study the typical chaotic source
that we have chosen and analyze how we should
modify the model I to describe such a chaotic source.

I. Wave-packet formalism. — We recall here the
main properties of the wave-packet formalism (WP
formalism) which is used to describe a boson or fermion field. These properties have already been given elsewhere [11], [10].

**ONE PARTICLE WAVE-PACKET.** Let \( | \Phi \rangle \) be a one particle wave-packet, defined in the one particle state space \( H \). If the volume \( V \), where the particle is confined, is finite
\[
| \Phi \rangle = \sum_k f(k) \, a^+_k \, | 0 \rangle ,
\]
(1.1)

where \( a^+_k \) is the creation operator in mode \( k \).

Let now \( | \Phi_j \rangle \) be the ket deduced from \( | \Phi \rangle \) by a translation of \( r_j \) in the configuration space,
\[
| \Phi_j \rangle = \exp \left( -\frac{iP \cdot r_j}{\hbar} \right) | \Phi \rangle = \sum_k f(k) \exp(-2\,i\,nk \cdot r_j) \, a^+_k \, | 0 \rangle ,
\]
(1.3)

where \( P \) is the impulsion operator of the particle and \( \hbar = \hbar/2 \pi \).

The function \( W_{n}[\{ r_j \}] \) is introduced for \( | n(\{ r_j \}) \rangle \) to be normalized
\[
W_{n}[\{ r_j \}] = \left( \prod_{j=1}^n A(\alpha_j) A^+(\alpha_j) \right) \left( 0 \right) = \sum \lambda _{\alpha_j} \prod_{j=1}^n \langle \alpha_j | \Phi \rangle \langle \Phi \rangle _{\alpha_j} ,
\]
(1.11)

where the symbol \( \sum \lambda _{\alpha_j} \) indicates a sum of all the permutations of \( n \) elements, each permutation being multiplied by \( \lambda _{\alpha_j} \) where \( r \) is the order of the permutation ; \( \varepsilon \) is 1 for bosons and \( -1 \) for fermions.

As the WP are non orthogonal, \( \langle \Phi_j | \Phi \rangle \neq 0 \) for \( j \neq i \). \( W_{n}[\{ r_j \}] \) is different from one.

To emphasize the importance of this function \( W_{n}[\{ r_j \}] \), let us consider the simple case of two bosons described by two exponential WP \( \psi_i(a, t) \) and \( \psi_j(a, t) \) given by
\[
\psi_{i(j)}(a, t) = \sqrt{\Gamma_b} \exp \left[ -\left( \frac{I_b}{2} + iE_0 \right) \left( t - \frac{a - r_{i(j)}}{c} \right) \right] \times \left( \frac{c}{2} \right) Y(t - \frac{a - r_{i(j)}}{c}) .
\]
(1.12)

\( E_0 \) is the mean energy of the particles and \( \Gamma_b \) the energy uncertainty. The function \( Y(...) \) is the Heaviside function.

The normalized two-particles WP is

\[
\psi_{i(j)}(a_1, a_2, t) = \sqrt{\frac{\Gamma_b}{2}} \exp \left[ -\left( \frac{I_b}{2} + iE_0 \right) \left( t - \frac{a_i - r_{i(j)}}{c} \right) \right] \times Y(t - \frac{a_1 - r_{i(j)}}{c}) Y(t - \frac{a_2 - r_{i(j)}}{c}) .
\]
(1.13)

The normalization function
\[
W_{2}[r_i, r_j] = 1 + \exp \left( -\frac{I_b}{2} \right) | r_i - r_j | .
\]
(1.14)

It varies from one to two, according to the values of \( r_i \) and \( r_j \),

\section*{OPERATOR IN THE WP FORMALISM.}

We have shown elsewhere [9], [10] that, if very broad conditions are fulfilled, the set of all the \( | n(\{ r_j \}) \rangle \) defined for a given \( | \Phi \rangle \), for all \( n \) and all \( r_j \) in the volume \( V \) where the system is supposed to be confined, generates the Fock space in a unique way. As a consequence the
operators of the Fock space are written in a unique way on such a set of $| n(\{ r_j \}) \rangle$ [10].

In particular the density matrix $\rho$ is written in the wave-packet formalism as

$$\rho = \sum_{n=0}^{+\infty} \sum_{n'=-\infty}^{+\infty} \chi \int_{V_{+\infty}} | n(\{ r_j \}) \rangle \langle n(\{ r_j \}) | \left| n'(\{ r'_j \}) \rangle \langle n'(\{ r'_j \}) \right| d r_j d r'_j.$$  \hspace{1cm} (I.15)

If there is a set of $| n(\{ r_j \}) \rangle$ for which the density matrix $\rho$ has a diagonal form

$$\rho = \sum_{n=0}^{+\infty} \rho_n = \sum_{n=0}^{+\infty} \chi \int_{V_{+\infty}} \frac{F_n(\{ r_j \})}{\chi^n} \times | n(\{ r_j \}) \rangle \langle n(\{ r_j \}) | \prod_{j=1}^{n} d r_j ,$$  \hspace{1cm} (I.16)

the system can be considered as the statistical mixture of systems with a given number of particles, each of them being described by a vector $| \Phi_j \rangle$ deduced from a given $| \Phi \rangle$.

The probability density for having $n$ particles, each of them being described by a state $| \Phi_j \rangle$, is

$$J_n[\{ r_j \}] = \frac{n^\dagger F_n[\{ r_j \}]}{\chi^n}.$$  \hspace{1cm} (I.17)

The stochastic point process [12], [9] $\delta_k(V)$ constituted by the random distribution of the $r_j$ in $V$, is entirely defined by the set of all these probabilities which are called exclusive probability densities (epd). They are the probability densities for having one event in $r_1$, another in $r_2$, ..., another in $r_n$ and not any other in $V$. We shall use this interpretation of diagonal density matrices later on.

II. Description of the source with the WP formalism. — We establish the conditions that allow a simple connection between the properties of the emission process in the source and those of the emitted field, described in the WP formalism. The field is considered at a large distance from the source and thus assumed to be free. For simplicity, we focus our attention on photon fields emitted by atomic sources. But most of the conditions set here may be extended to other cases.

The first condition deals with the definition of WP; we admit that for a WP to describe a particle it is necessary that its length be much greater than the mean wave-length associated with the particle. This leads to

**CONDITION I** (quasimonochromaticity condition)

$$\Delta E \ll E_0 ,$$  \hspace{1cm} (II.1)

where $E_0$ is the mean energy of the particle, and $\Delta E$ its energy uncertainty.

Let us give here some definitions about quasimonochromatic wave-packets [13], [10].

The width in time of the WP is the coherence time, $\tau_c$. We have

$$\Delta E = h \tau_c^{-1} .$$  \hspace{1cm} (II.2)

The coherence length $l_c$ is

$$l_c = v \tau_c ,$$  \hspace{1cm} (II.3)

where $v$ is the group velocity of the particles (c, in the photon case).

The product of the widths of the WP in the two directions normal to the propagation direction is the coherence area $\sigma$.

The second condition allows the description of the emitted field by a spatial random distribution of WP in volume $V$; the density matrix must be interpretable in terms of probabilities. This yields:

**CONDITION II.** — A vector $| \Phi \rangle$ exists such that the density matrix describing the field is diagonal on the set of $| n(\{ r_j \}) \rangle$ built on $| \Phi \rangle$.

The two following conditions allow the identification between time and space stochastic distributions of WP. To go from a time distribution to a space distribution is very important for us; it allows us to go from the emission process in the source, which is defined in time, to the spatial description of the field, at a given time, given by the WP formalism.

The distribution of points $r_j$ in $V$ at a given time must be identified to a distribution of time instants $t_j$, at a given point, in an interval $T$. Thus $V$ must be reduced to a one dimensional domain $L$.

This yields:

**CONDITION III.** — We consider a parallel beam with a section of the order of the coherence area.

**CONDITION IV.** — For the series of WP to be the same in space and time, the wave-packet spreading must be negligible.

This condition is always realized for a photon field.

For the identical WP $| \Phi_j \rangle$ to describe the state of the particles of the field, it is necessary that the distortions of the WP at the boundaries of $L$ might be neglected.

This entails:

**CONDITION V.**

$$L \gg l_c , \quad (T \gg \tau_c) .$$  \hspace{1cm} (II.4)

Moreover, for simplicity, we set

**CONDITION VI.** — The field is stationary in time.

The stationarity in space follows from this condition and the preceding ones.

**CONDITION VI.** — The field is stationary in time.

The stationarity in space follows from this condition and the preceding ones.

Condition VI is not necessary to allow the identification between the WP distributions in time, at a
given point of space, and in space, at a given time. But it enables us to speak about the spatial pp \( \beta_j(L) \) constituted by the \( r_j \) without specifying any time, and about the temporal pp \( \beta_j(T) \), constituted by the \( r_j \), without specifying any point in space. According to conditions I to V, these two processes are identical.

We won’t study here the problem of the detection of the field [10]. But it must be noticed that the preceding conditions yield the identity of spatial measurements and temporal measurements. For instance the spatial second-order-coincidence-measurement, constituted by the detections of two photons at the same time at two points distant from \( a \), gives the same result as that given by the temporal second-order-coincidence-measurement which consists in detecting two photons at the same point at two time instants distant from \( t = a/c \).

Generally, the description of a field by a temporal distribution of stochastic identical WP, which is allowed by conditions I-VI, does not give a clear insight into the physical processes in the source. Indeed the shape of the WP as well as the characteristics of \( \beta_j(T) \) express various properties of the source indiscriminately:

- way of exciting the atoms;
- different types of interaction between atoms;
- moreover, if we create the parallel beam by an optical means instead of only considering a very narrow solid angle in the emission of the source, both the correlations in a given direction of emission and in different directions influence \( \beta_j(T) \).

We get rid of the latter difficulty by assuming that the source is observed in a very narrow solid angle. It is the case in the coincidence experiments we refer to [2], [3], in this article. Moreover, we assume that the photons are observed for a given polarization.

By these hypotheses we limit our study of the correlations in the emission process of the source to the correlations in given direction and polarization.

We are now going to set three extra conditions which allow a simple interpretation of the WP formalism.

**CONDITION VII.** — Every atom of the source is considered as punctual and localized at a point \( X_j \). It is excited, at a stochastic time instant \( \theta_j \), by a very narrow pulse.

From the physical point of view, a pulse is considered as very narrow if its width in time is much smaller than the lifetime of the excited level of the atom. In the case of spontaneous emission [14], this yields that the shape of the emitted WP does not depend on the shape of the exciting pulse. When the atoms are excited with photons, this type of excitation is generally called broadline excitation [15], [10].

For simplicity, the points \( X_j \) are assumed to be fixed. This restriction is not a basic one and could be removed [10].

Let us point out that a given pulse is supposed to excite given atoms of the source. To understand this point clearly, let us assume that the source is constituted by two identical two level atoms. The levels are \( |a) \) for the ground state, and \( |b) \) for the excited state. An exciting photon arrives on these atoms, initially in state \( |a) \). The only possibility is that it put either atom 1 or atom 2 in state \( |b) \). Just after excitation, the state of the source is \( |a) |b) \), which means one of the indistinguishable states \( |1 a) |2 b) \) or \( |1 b) |2 a) \). We eliminate a possible uncertainty, in the excitation of the atoms, which would be described by a state

\[
\frac{1}{\sqrt{2}} \left[ |1, a) | 2, b) + |1, b) | 2, a) \right]
\]

(II.5)

for instance. This state would describe a situation where the two atoms can see the incident photon. Such an uncertainty plays an important role when the atoms are very close to one another (distance of the order of the wave-length), as it is the case in superradiance or subradiance [10], [16]. Such phenomena are not considered here: we deal with low density sources.

These assumptions allow the definition of the stochastic variable

\[
r_j = X_j - c\theta_j ,
\]

(II.6)

describing the excitation of atom \( j \).

**CONDITION VIII.** — Every atom evolves independently after its excitation.

This means that we don’t take into account interactions between the atoms through the emitted field.

**CONDITION IX.** — Every atom is a two-level-atom and emits a WP \( |\Phi_j) \) given by (I.3) where \( r_j \) is defined by (II.6). The upper level is \( |b) \), the lower level is the groundstate \( |a) \).

Conditions VII and VIII entail that the WP \( |\Phi_j) \) is emitted by spontaneous emission. Thus

\[
|\Phi_j) \equiv |\psi_j) \]

(II.7)

where \( |\psi_j) \) is the ket defined by (I.12) and \( \Gamma_b \) the energy uncertainty of the upper level [13], [10].

Conditions I to IX allow the identification of the one-particle WP of a generating set \( |n(\{ r_j) \) \) of the WP formalism with the one-particle WP assumed to be emitted by the source.

The construction of the diagonal density matrix of the emitted radiation

\[
\rho = \sum_{n=0}^{+\infty} \frac{1}{n!} \times \int_{L^n} J_n(\{ r_j) \) \ n(\{ r_j) \) \ (r_j) \ \prod_{j=1}^{n} dr_j ,
\]

(II.8)
where the $| m(r_j) \rangle$ are built on the WP $| \Phi_j \rangle \equiv | \psi_j \rangle$ and the epd, $J_n[\{ r_j \}]$ are derived from the statistical assumptions on $\theta_j$.

These conditions I to IX are fulfilled by the incoherent source model proposed in section IV.

III. Chaotic boson field. — By definition, a free boson field is chaotic if its density matrix is $[6], [17]$

$$\rho = \rho_c = \sum_{n_k} \prod_k \frac{< n_k >}{(1 + < n_k >)^{1 + n_k}} \times$$

$$\times | \{ n_k \} \rangle \langle \{ n_k \} |. \quad (III.1)$$

$| \{ n_k \} \rangle$ is a state where there are $n_k$ photons in mode $k$. $< n_k >$ is the mean photon number in mode $k$.

$$\sum_{\{ n_k \}}$$ indicates a double summation. First, for a given set $\{ k \} = \{ k_1, ..., k_p \}$ of different wave-vectors, we sum over all the different ordered sets of positive integers $\{ n_k \} = n_1, ..., n_p$. Secondly, we sum over all the different sets $\{ k \}$.

This operator $\rho_c$ can be written in the WP formalism. R. J. Glauber has shown that it can be written in a diagonal form $[6]$. We have shown more recently that this form is unique $[10], [18]$. For a given energy spectrum, defined by $< n_k >$, there is only one set $| m(r_j) \rangle$, defined by the one-particle WP $| \Phi \rangle$ it is built on, for which the chaotic density $\rho_c$ is diagonal. The ket $| \Phi \rangle$ is defined by the relation

$$| f(k) |^2 \sum_k \frac{< n_k >}{1 + < n_k >} = \frac{< n_k >}{1 + < n_k >}$$

for any $k$, \quad (III.2)

where $f(k)$ is the function introduced in (I.1).

The density matrix $\rho_c$ is written

$$\rho_c = \sum_{n=0}^{+\infty} c_n \int_{L^n} W_n[\{ r_j \}] | m(r_j) \rangle \langle m(r_j) | \times$$

$$\times \prod_{j=1}^{n} \frac{dr_j}{L} \quad (III.3)$$

where

$$c_n = \frac{1}{n!} \left( \sum_k \frac{< n_k >}{1 + < n_k >} \right)^n \prod_k (1 + < n_k >)^{-1} =$$

$$= \frac{Z^n}{n!} \prod_k (1 + < n_k >)^{-1}, \quad (III.4)$$

and $W_n[\{ r_j \}]$ the normalization function defined in (I.11).

Eq. (I.8) (III.3) (III.4) yield

$$\rho_c = \sum_{n=0}^{+\infty} \frac{Z^n}{n! L^n} \prod_k (1 + < n_k >)^{-1} \int_{L^n} \times$$

$$\times \prod_{j=1}^{n} A^*(r_j) | 0 \rangle \prod_{j=1}^{n} A(r_j) \prod_{j=1}^{n} dr_j. \quad (III.5)$$

The epd $J_n[\{ r_j \}]$ of the pp $F_n(L)$ are, from (I.17) and (III.3),

$$J_n[\{ r_j \}] = \frac{n_1}{L^n} W_n[\{ r_j \}] =$$

$$= \prod_k (1 + < n_k >)^{-1} \int_{L^n} \times$$

$$\times \prod_{j=1}^{n} Z \prod_{j=1}^{n} (r_j - r_{ja}). \quad (III.6)$$

$J_n[\{ r_j \}]$ depends on the differences $(r_j - r_{ja})$, and thus $F_n(L)$ is stationary in space, a result that could be expected. Moreover $J_n[\{ r_j \}]$ depends on the $r_j$'s in the same way as $W_n[\{ r_j \}]$ does. Hence $J_n[\{ r_j \}]$ is not a constant, and the points $r_j$ are not independent from one another. Let us for simplicity consider the case $n = 2$. The WP are given by (I.12), $W_2[r_1, r_2] \text{(I.14) }$ : it can be said that there is some sort of a bunching effect in the distribution of points $r_1$ and $r_2$. (The $r_j$'s are related to the emission process of the photons, and should not be confused with detection points of the photons.) Up to now, it is naturally impossible to give a physical interpretation of this result. Indeed we don't know whether a chaotic source fulfills conditions I to IX (in particular conditions VII, VIII, IX) or not. But we can analyze how this result comes out, from a mathematical point of view.

By going through all the calculations leading from (III.1) to (III.3) $[10]$, we see that, if a diagonal operator in the mode representation, $\Theta$, has a diagonal form in the WP formalism, we necessarily have

$$F_n[\{ r_j \}] = c_n W_n[\{ r_j \}], \quad (III.7)$$

Moreover, for this diagonal form to actually exist, the operator $\Theta$ must verify

$$\{ n_k | \Theta | n_k \} = c_k n! \prod_k | f(k) |^{2n_k}, \quad (III.8)$$

where $f(k)$ fulfills (I.1).

Thus the bunching of the $r_j$'s express the fact that $\rho_c$ is diagonal in the mode representation and that its diagonal terms verify (III.8).

It is interesting to notice, on (III.4) and (III.8), that, in the low intensity case, $< n_k > \ll 1$, and thus

$$c_n \approx \frac{1}{n!} \left( \sum < n_k > \right)^n = \frac{1}{n!} < N >^n, \quad (III.4') \quad$$

$$\{ n_k | \rho_c | n_k \} \approx < N >^n \prod_k | f(k) |^{2n_k}. \quad (III.9)$$
Eq. (III.9) means that, in the low intensity chaotic case, the photons are in given modes $k$ independently from one another, with probability $< N > |f(k)|^2$. Hence, in the low intensity case, the bunching of the $r_j$'s express this independence.

For a clear understanding of this result, let us study the bunching effect on two photons emitted by two atoms independently. This is the case of low intensity fields. The probability for the two atoms to emit more than two photons in $L$ is negligible. Every photon is emitted in mode $k$ by an atom with probability $1 \frac{N}{f(k)}$. Conditions III to VI are supposed to be fulfilled.

The state of every emitted photon has the form

$$\eta(a) = \frac{1}{\sqrt{L}} \exp(2i\alpha + \varphi),$$ (III.10)

where the $\varphi$'s are independent random variables, equally distributed in $[0, 2\pi]$.

Photons are detected at two points $a_1$ and $a_2$ [10].

Their wave-function, at these points, is for every trial (every emission of two photons)

$$\eta_{k_1k_2}(a_1, a_2) = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{n_{k_1}! n_{k_2}!}} \times$$

$$\times \frac{1}{L} \left[ \exp(2i\alpha_a + \varphi_1) \exp(2i\alpha_b) + \varphi_2) + \exp(2i\alpha_a + \varphi_2) \exp(2i\alpha_b + \varphi_2) \right].$$ (III.11)

The second-order spatial cpd $P_{2}^{CB}[a_1, a_2]$ is obtained by performing an infinite number of two photon detections at $a_1$ and $a_2$. It is related to $\eta_{k_1k_2}(a_1, a_2)$ by [10]

$$P_{2}^{CB}[a_1, a_2] = 2 < | \eta_{k_1k_2}(a_1, a_2) |^2 >$$ (III.12)

where $< \ldots >$ indicates an ensemble average on $k_1, k_2, \varphi_1, \varphi_2$.

Formula (III.12) is nothing but the usual expression of a presence probability in wave-mechanics.

With our hypotheses, (III.12) gives

$$P_{2}^{CB}[a_1, a_2] = 2 < \sum_{k_1, k_2} |f(k_1)|^2 |f(k_2)|^2 \eta_{k_1k_2}(a_1, a_2) >$$

$$\times \frac{1}{2L^2} \left( 2 + \exp(2i\alpha_a + \varphi_1) \exp(2i\alpha_b + \varphi_2) \right).$$ (III.13)

It must be noticed on (III.13) that the distributions of $\varphi_1$, and $\varphi_2$ don't play any role in the value of $P_{2}^{CB}[a_1, a_2]$ [19].

We have [20]

$$P_{2}^{CB}[a_1, a_2] = \frac{2}{L^2} \left[ \sum_{k_1} |f(k_1)|^2 \sum_{k_2} |f(k_2)|^2 + \sum_{k_1} |f(k_1)|^2 \right] \exp(2i\alpha_a - \alpha_b)$$

$$\times \frac{1}{2} \sum_{k_2} |f(k_2)|^2 \exp(2i\alpha_b - \alpha_a).$$ (III.14)

Let us introduce

$$\Phi_i(a) = \frac{1}{\sqrt{L}} \sum_k f(k) \exp(2i\alpha a + \varphi_i) = \frac{1}{\sqrt{L}} \sum_k f(k) \exp(2i\alpha a - r_i),$$ \hspace{1em} i = 1, 2. (III.15)

Eq. (III.14) can be rewritten

$$P_{2}^{CB}[a_1, a_2] = 2 < \Phi_i(a_1) |^2 | \Phi_i(a_2) |^2 + \Phi_i(a_1) \Phi_i^*(a_2) \Phi_i^*(a_2) \Phi_i^*(a_1) >,$$ (III.16)

where the ensemble average is taken over $\varphi_1$ and $\varphi_2$, independent and equally distributed in $[0, 2\pi]$, or on $r_1$, $r_2$, independent and equally distributed in $L$.

Introducing

$$m_{12}(a_1, a_2) = \frac{1}{\sqrt{2}} \left[ \Phi_i(a_1) \Phi_i(a_2) + \Phi_i(a_2) \Phi_i(a_1) \right] = \sqrt{W_2}[r_1, r_2] \Phi_i(a_1, a_2),$$ (III.17)

where $\Phi_{i2}(a_1, a_2)$ is the normalized two-particle WP, we can write

$$P_{2}^{CB}[a_1, a_2] = 2 < \Phi_{i2}(a_1, a_2) |^2 >$$ (III.18)

where the ensemble average is taken on $r_1$, $r_2$, with a distribution $W_2(r_1, r_2) \times L^{-2}$.

The result obtained here, in a simple case, shows how a distribution of independent photons in the...
modes of the field leads to a distribution of non-independent WP in space.

Let us now give the final expression of \( P^\text{BB}[a_1, a_2] \),

\[
P^\text{BB}[a_1, a_2] = \frac{2}{L^2} \left[ 1 + |\gamma(a_1 - a_2)|^2 \right],
\]

where \( \gamma(a_1 - a_2) \) is defined by (I.7).

By setting \([8, 10]\)

\[
2 L^{-2} = \Omega^2,
\]

which allows to consider \( \Omega \) as the linear density of photons, we finally obtain

\[
P^\text{BB}[a_1, a_2] = \Omega^2 \left[ 1 + |\gamma(a_1 - a_2)|^2 \right],
\]

which is the well-known chaotic bunching effect, with a maximum 2 for \( a_1 = a_2 \).

IV. Incoherent source model (model I). — An incoherent source, by definition, emits a field described by the density matrix

\[
\rho_I = \sum_{n=0}^{+\infty} \frac{J_n}{n!} \int_{L^n} \times \left[ n\{r_j\} \right] \prod_{j=1}^{n} dr_j,
\]

where the epd \( J_n \) of the pp \( \beta(L) \), define a Poisson process of linear density \( \Omega \)

\[
J_n = e^{-\Omega L} \Omega^n.
\]

A simple interpretation of model I can be given by assuming that conditions I to IX are fulfilled. Under such conditions, it describes the field emitted by atoms excited at independent time instants and emitting spontaneously without interacting with one another. Every atom emits a WP [15, 10]

\[
|\Phi_j\rangle = |\psi_j\rangle,
\]

where \( (\tau(t)|\psi_j\rangle \) is given by (I.12).

Let us consider the low intensity case. The probability for having more than two photons in the field is negligible. Thus the second order cpd \( P^2[a_1, a_2] \) is given by

\[
P^2[a_1, a_2] = 2 < |\psi_{12}(a_1, a_2)|^2 >,
\]

where

\[
\psi_{12}(a_1, a_2) = \left( \sqrt{2} W_2[r_1, r_2] \right)^{-1} \times \left[ |\psi(a_1)\psi(a_2)| + |\psi(a_1)\psi(a_2)| \right],
\]

\( \psi_{12}(r_1, r_2) \) being given by (I.12).

The symbol \( < \cdots > \) indicates an ensemble average over the independent \( r_1 \) and \( r_2 \) equally distributed in \( L \).

By performing the calculations, we derive the following expression of \( P^2[a_1, a_2] \) from (IV.4) and (I.12)

\[
P^2[a_1, a_2] = \Omega^2 \left[ \frac{1}{2} + \exp(-\Gamma_b |a_1 - a_2|) \right] \left[ 2 \log 2 - \frac{1}{2} - \frac{1}{2} \log (1 + \exp(+\Gamma_b |a_1 - a_2|)) \right] + \frac{1}{2} \exp(+\Gamma_b |a_1 - a_2|) \left[ 1 + \exp(-\Gamma_b |a_1 - a_2|) \right].
\]

In other words, a chaotic source cannot be described as an assembly of atoms excited at independent time instants and emitting WP spontaneously without interacting with one another.

In the low intensity case, the difference between chaotic and incoherent fields appears clearly on figure 1. The maximum bunching effect is 1.4 in the incoherent case, 2 in the chaotic one. By comparing formulae (III.12) and (IV.4), it can be seen that this difference comes from the presence of the normalization function \( W_2[r_1, r_2] \) which appears in the incoherent case, because we work with non-orthogonal WP, and does not appear in the chaotic case where the mode representation is used. Hence, even for low intensities, when the probability of having two photons at a smaller distance than \( \ell \) is weak and thus the probability for \( W_2[r_1, r_2] \) to be very different from one is weak too, the incoherent and chaotic fields cannot be identified. It is in the very low intensity case only, when the problem reduces to a one-particle problem, that the two fields may be considered as identical.

It would be interesting to compare the chaotic and incoherent sources experimentally. To realize such an experiment, we should solve two problems:

— Is the difference between the chaotic and inco-
The answer to the first question is yes. Indeed, the accuracy of the coincidence experiments realized by D. B. Scarl [2] is of a few per cent. A detailed analysis of these experiments is given in reference [10].

To realize an incoherent source, we must excite an assembly of independent atoms or molecules, with random narrow pulses.

Let us first consider which conditions must be fulfilled for the atoms or molecules of a gas to be independent. The pressure of the atomic or molecular gas must be chosen in an optimal way, high enough for the emitted field to be easily observable, low enough for the interactions to be negligible. A pressure of the order of one Torr fulfills these two conditions. Indeed, in such conditions, the short distance electromagnetic interactions (superradiance) are negligible [21], [10], and the long distance electromagnetic interactions (multiple scattering of the emitted photons [22]) may be neglected by observing a non resonant line [23]. The only interactions that cannot be neglected are atomic (molecular) collisions. But collisions don’t influence correlations between the emitted photons. In fact, in low intensity sources such as the one we are studying here, collisions between excited atoms may be neglected. The only collisions that should be considered are the ones between excited atoms and atoms in the ground state: such collisions modify the energy spectrum of the field but do not introduce any correlation between photons.

Obviously, as the field is assumed to be weak, stimulated emission is negligible: the degeneracy parameter is smaller than $10^{-2}$ [23], [24].

Let us now consider the exciting pulses. Their width in time must be short compared with the lifetime of the upper level of the observed transition. This lifetime may vary from $10^{-9}$ (as in the transition observed by D. B. Scarl [2]) to much larger values ($10^{-5}$ for instance) in molecules. Thus the way of producing the pulses is highly dependent on the type of source chosen. We should use a mode-locked laser if very short (picosecond) pulses are needed. We could obturate randomly a continuous-wave laser, by activating a gate, to obtain nanosecond pulses [25].

Let us emphasize that the pulses must be weak enough for stimulated emission to be negligible. In the case where very short pulses would be needed, this would signify that they must be very strongly attenuated. From this point of view, the second possibility looks better: it could easily give much weaker pulses.

This very short analysis shows that an incoherent source could be built that emits a field intense enough to be detected. The most important problem left is a clever choice of the emitting atoms or molecules.

VI. Conclusions. — The main conclusions that can be drawn are the following.

— Independently emitting atoms produce a field exhibiting a bunching effect (the incoherent bunching effect). The only origin of this effect is the symmetry of photons. This effect is smaller than the chaotic bunching effect.

— A chaotic source cannot be considered as an assembly of independent atoms excited at stochastically independent time instants. This is shown by eq. (III.6). Two different interpretations of this result could be given:

1) Conditions VII, VIII, IX, are fulfilled by a chaotic source. Thus, the exciting time-instants are correlated and their distribution law is given by

$$ J_{n}([r_j]) = \frac{n!}{L^n} c_n W_{n}([r_j]). \quad \text{(III.6)} $$

Obviously, such a conclusion cannot be accepted: the distribution law of the exciting time instants would depend on the characteristics of the atoms.

2) Conditions VII, VIII, IX, are not fulfilled by a chaotic source. This means that either the assumption that every atom is excited during a very short time-interval, or the assumption that the atoms of the source are independent from one another, is not correct to describe the typical chaotic source studied in the present paper. If necessary both assumptions should be modified.

Let us first consider the assumption of independence between atoms. The pressure in the source is of a few torrs. The observed transitions are non-resonant. Thus the analysis of section V (done more completely in ref. 10) can be applied here. The only types of interactions that could correlate the photons are superradiance and stimulated emission. These interactions are very weak. And, as already said in section V, it seems reasonable to neglect them. Nevertheless it could be thought that since the number of two-photon detections realized in a chaotic field is very weak too, because of the low intensity of the field, the effect of these interactions is not negligible. If this were true, the bunching effect of such a source would vary with the pressure. Up to now, no experiment has shown such variations. But as they were not done to verify the presence or absence of variations, they cannot really be considered as proofs. And it would be interesting to verify experimentally, by varying the pressure of a chaotic source, whether the bunching effect is constant or not. If it is, this would be a definite proof that the chaotic bunching effect has nothing to do with the interactions in the source. Its only origin therefore would lie in the excitation process of the atoms.

Let us now consider the excitation of the atoms in a mercury lamp. The atoms are excited by electronic discharges. A detailed theoretical analysis of the
interaction between the atoms and the exciting electric current is extremely difficult to perform at a microscopic level. Hence, it is impossible to check condition VII theoretically.

A theoretical conclusion that could be given to the problem is that condition VII is not fulfilled: the atoms don't emit WP but photons with given impulsions. Indeed, our analysis of the chaotic bunching effect, in section III, has shown that it can easily be understood, from a theoretical standpoint, by assuming that independent photons are emitted in the different modes of the field. But such a conclusion is not satisfactory because it does not give any insight into the excitation process of the source.

Thus, we had rather conclude that some more experimental results are needed to give a definite answer to the question of the origin of bunching effects. In particular, the experimental comparison between a chaotic source, the excitation process of which is not known, and an incoherent source (or eventually other sources), the excitation processes of which are controlled, would be of great interest.

References

[6] GLAUBER (R. J.), In Quantum Optics, edited by S. M. Kay and A. Maitland (Academic Press, London 1970). This author claims that the Bose statistic is not sufficient to explain the chaotic bunching effect but does not propose any explanation of the effect.
[14] COHEN-TANNoudji (C.), Compléments de Mec. quantique (3e cycle de Physique atomique et statistique), edited by the « laboratoire de l'Ecole Normale Supérieure ».
[18] BÉNARD (C.), (to be published).
[20] By following the details of the calculations, we notice that the contribution of the terms where $k_1 = k_2$ is negligible. It is of the order of $l_0/L$. In his calculations, FANO (U.) assumed that these terms were negligible.
[23] These points are more closely discussed in ref. 10. Some more arguments are given in section VI of the present article.