Theory of instabilities of nematics under A. C electric fields : Special effects near the cut off frequency

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THEORY OF INSTABILITIES OF NEMATICS UNDER A. C ELECTRIC FIELDS : SPECIAL EFFECTS NEAR THE CUT OFF FREQUENCY

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Résumé. — Nous étudions numériquement les instabilités électro-hydrodynamiques d'un cristal liquide nématique soumis à un champ électrique. Nous déterminons la transition entre les régimes basse fréquence et haute fréquence dans le cas où le champ appliqué a la forme d'un créneau et dans le cas où il est de forme sinusoïdale. Dans les deux cas, la courbe donnant la tension de seuil en fonction de la fréquence a une forme en S près de la fréquence critique : d'où la possibilité de certains effets d'hystérésis.

Abstract. — We have obtained numerical solutions for the coupled equations describing the charge density and the curvature in a nematic system under oscillating electric fields (rectangular pulses or sine waves). In both cases, the resulting curve of threshold voltage versus frequency has an S shape in the vicinity of the cut off frequency. This should lead to observable hysteresis effects.

I. **Introduction.** — The effects of alternating electric fields on nematic liquid crystals with negative dielectric anisotropy ($\varepsilon_{\parallel} - \varepsilon_{\perp} = \varepsilon_a < 0$) have been described in a preceding paper [1]. This paper discussed the threshold V_c for the onset of a cellular type of hydrodynamic motion. The main equations of the model are summarised in section II. They gave :

a) A conducting regime (at frequencies below a certain cut off ν_c) where the charges oscillate, while the molecular distortions are static.

b) A dielectric regime ($\nu > \nu_c$) where the charges are time independent, but the distortions of the molecular alignment oscillate at the driving frequency.

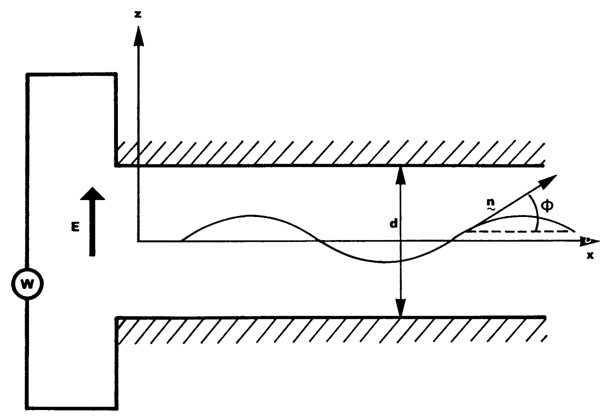

FIG. 1. — A nematic crystal is, at zero electric field, aligned along x direction. An A.C electric field $E(t)$ is applied along z direction. φ is the fluctuation angle of the director around x direction. d is the sample thickness.

The regime (*a*) could be analysed accurately in all the interval $0 < \nu < \nu_c$. On the other hand we were able, to derive the features of regime (*b*) only when ν was significantly larger than ν_c. The aim of the present paper is to bridge this gap ; since the numerical analysis is rather heavy, we begin by a slightly simplified case, where the electric field, instead of being a sine wave, is a succession of rectangular pulses (Fig. 2). We then

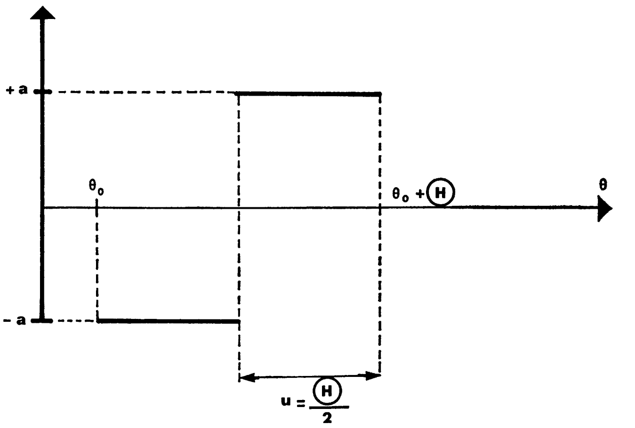

FIG. 2. — Form of the periodic applied field $a(\theta)$. θ is the period.

find an unexpected S shape for the curve of threshold versus frequency (section III). This then induces us to repeat the calculation for the (more usual) case of a sinusoidal field : the results are derived in section IV : they again show the S shape, although less conspicuously.

II. **Electrohydrodynamic equations.** — We consider a nematic slab of thickness d submitted to an alternating field $E(t)$ along Z (see Fig. 1).

If the sample is parallel to the xy plane with the molecules along x in the unperturbed state, we assume that all physical quantities depend only of x not of y or z. We study the stability of the system under pure bending fluctuations (these pure bending modes are the only ones to induce shear flows) where the « director » (preferred direction of the molecules) is in the (x, z) plane making a small angle $\varphi(x, t)$ with the x direction (Fig. 1).

As shown by recent experiments, this approximate treatment is qualitatively correct. The only way we take into account boundaries conditions is to say that the fluctuation wavelenght, in the x direction, cannot be much larger than the sample thickness.

Electrohydrodynamic effects are then described [1] by two coupled equations for the charge density q and the curvature ψ

$$q = q(x, t)$$

$$\psi = \frac{\partial \varphi}{\partial x}(x, t) \, ;$$

these equations are obtained by writing charge balance equation, acceleration and torque equations [1]. They are explicitately

$$\dot{q} + \frac{q}{\tau} + \sigma_H \psi E(t) = 0 \qquad \text{(II.1)}$$

$$\dot{\psi} + \frac{\psi}{T(t)} + \frac{qE(t)}{\eta} = 0 \qquad \text{(II.2)}$$

τ is a dielectric relaxation time defined by :

$$\tau = \frac{\varepsilon_\parallel}{4 \pi \sigma_\parallel}$$

σ_H is an apparent conductivity

$$\sigma_H = \sigma_\parallel \left(\frac{\varepsilon_\perp}{\varepsilon_\parallel} - \frac{\sigma_\perp}{\sigma_\parallel} \right)$$

$\varepsilon_\parallel$, $\varepsilon_\perp$, $\sigma_\parallel$, $\sigma_\perp$ are the dielectric constants and conductivities parallel and perpendicular to the molecules.

$T(t)$ is a molecular relaxation time, depending on the field strength, defined by :

$$\frac{1}{T(t)} = \lambda (E^2(t) + E_0^2) \qquad \text{(II.3)}$$

with

$$\lambda = - \frac{\varepsilon_a}{4 \pi} \frac{\varepsilon_\perp}{\varepsilon_\parallel} \left(\frac{1}{\gamma_1} + \frac{1}{\eta_0} \right)$$

$$\eta_0 = \left(\frac{2 \gamma_1}{\gamma_1 - \gamma_2} \right)^2 \frac{\gamma_1 (2 \alpha_4 + \beta) - \gamma_2^2}{4 \gamma_1}$$

$$\frac{1}{\eta} = \frac{1}{\eta_0} \left(\frac{2 \gamma_1}{\gamma_1 - \gamma_2} \right) - \frac{\varepsilon_a}{\varepsilon_\parallel} \left(\frac{1}{\gamma_1} + \frac{1}{\gamma_0} \right)$$

we use for the viscosity coefficients the notation of [2]. E_0^2 takes into account elastic torque (eventually magnetic torque).

$$E_0^2 = - \frac{4 \pi}{\varepsilon_a} \frac{\varepsilon_\parallel}{\varepsilon_\perp} (K_{33} k^2) \qquad \text{(II.4)}$$

where k is the fluctuation wave vector defined by

$$\varphi(x, t) = \varphi_0 \cos kx$$

K_{33} is the Frank elastic constant corresponding to a bending deformation [3]. Equation (II.1) does not take into account diffusion currents.

They are in fact important only in the high frequency regime where k becomes of the same order as the Debye wave vector q_D defined by $q_D^2 = 1/\tau D_\parallel$ (D : diffusion constant).

Here we are more specially concerned by the transition between low and high frequency regimes : in such a case the wavelengths of interest $2 \pi/k$ are comparable to the sample thickness and much larger than the screening radius. Thus we may omit the diffusion currents for our present purposes (1).

III. **Instability under rectangular field pulses.** — We discuss the case where the applied field $E(t)$ has the form shown on figure 2. This case is exactly soluble. Let us first rewrite the system (II.1, 2) in a symmetrized form and introduce dimensionless parameters.

$$a^2 = E^2 \lambda \tau, \; a_0^2 = E_0^2 \lambda \tau = 2 a_0^2$$

$$p = \omega \tau, \; \theta = \frac{t}{\tau}, \; z^2 = \frac{\sigma_H \tau}{\lambda \eta}$$

then equation (II.1) and (II.2) become :

$$\frac{\partial \varphi_1}{\partial \theta} + \varphi_1 + z a \varphi_2 = 0 \qquad \text{(III.1)}$$

$$\frac{\partial \varphi_2}{\partial \theta} + (a^2 + a_0^2) \varphi_2 + z a \varphi_1 = 0 \qquad \text{(III.2)}$$

with $\varphi_1 = q$, $\varphi_2 = (\sigma_H \eta)^{1/2} \psi$.

It is convenient to rewrite equation (III.1) and (III.2) in the form

$$\frac{\partial}{\partial \theta} (\varphi) = \mathcal{H}(\varphi) \qquad \text{(III.3)}$$

where the vector $(\varphi) = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$ and $\mathcal{H}$ is the matrix defined with use of Pauli matrices

$$\mathcal{H} = - \left(A \, \hat{1} + B \hat{\sigma}_z + z a(t) \hat{\sigma}_x \right) \qquad \text{(III.4)}$$

$\hat{1}$ is the unit matrix

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A = \tfrac{1}{2}(1 + a^2 + a_0^2) \qquad B = \tfrac{1}{2}(1 - a^2 - a_0^2)$$

(1) The effects of the diffusion currents in the high frequency limit will be discussed in a separate paper.

STABILITY OF THE SYSTEM. — $\mathcal{H}(t)$ is periodic and commutes with the evolution operator in one period (or translations operator U) defined by

$$(\varphi(\theta_0 + \Theta)) = U(\theta_0 + \Theta, \theta_0)\,(\varphi(\theta_0)) = (U\varphi)\,(\theta_0)$$

where Θ is the period.

These two operators may be simultanuously diagonalysed. Then we can expand the solutions of the system (III.3) (which form a two dimensional linear space) on the eigenfunctions (W_i) of the operator U

$$(\varphi(\theta)) = C_1(W_1(\theta)) + C_2(W_2(\theta))\,.$$

The eigenfunctions (W_i) of the translation operator U are of the form :

$$(W_i) = \begin{pmatrix} a_i(\theta) \\ b_i(\theta) \end{pmatrix} e^{s_i\theta}\,, \quad \lambda_i = e^{s_i\theta} \qquad \text{(III.5)}$$

where $a_i(\theta)$ and $b_i(\theta)$ are periodic functions of period Θ, λ_1 and λ_2 being the corresponding eigenvalues of the operator U.

The system is instable when

$$\mathcal{R}(s_+) > 0 \; (\mathcal{R} = \text{real part of}) \qquad \text{(III.6)}$$

s_+ is the eigenvalue which possesses the highest real part.

The stability of the system is then determined by knowing the eigenvalues of the translation operator U as functions of the parameters (a and a_0).

EVOLUTION OPERATOR. — The evolution operator $\mathcal{O}$ is defined by :

$$(\varphi(\theta)) = \mathcal{O}(\theta, \theta_0)\,(\varphi(\theta_0))$$

$$\mathcal{O}(\theta, \theta_0) = \mathcal{T}\left(\exp \int_{\theta_0}^{\theta} \mathcal{H}(t')\,dt'\right)$$

$\mathcal{T}$ is the chronological product :

$$\mathcal{T}(\mathcal{H}(t_1)\,\mathcal{H}(t_2)\,...\,\mathcal{H}(t_n)) = \mathcal{H}(t_{\sigma_1})\,\mathcal{H}(t_{\sigma_2})\,...\,\mathcal{H}(t_{\sigma_n})$$

$$t_{\sigma_1} > t_{\sigma_2} \,...\, > t_{\sigma_n}\,.$$

The translation operator U may be decomposed as :

$$U = U_+\,U_- = \mathcal{O}(\theta_0 + \Theta, \theta_0 + u)\;\mathcal{O}(\theta_0 + u, \theta_0)$$

U_+ and U_- are the evolution operators in the two half period $u = \Theta/2 = \pi/p$ (see Fig. 2). They are easely calculated for the present problem because in each half-period $\mathcal{H}(t)$ is constant.

$$U_\pm = \exp u\mathcal{H}_\pm\,, \quad \mathcal{H}_\pm = -\,(A\,\hat{1} + B\hat{\sigma}_z \pm za\hat{\sigma}_x)\,.$$

Using the usual notations, $\mathbf{V}.\boldsymbol{\sigma} = \sum_i V_i.\hat{\sigma}_i$, U may be written :

$$U = \exp(-\,2\,Au)\,T_+.T_-$$

with

$$T_\pm = \exp(\mathbf{V}_\pm.\boldsymbol{\sigma}) \quad \text{and} \quad \mathbf{V}_\pm = \begin{pmatrix} \mp\, zau \\ 0 \\ -\,Bu \end{pmatrix}.$$

The matrices $T_\pm$ are calculated with use of Pauli matrices commutation rules [2]

$$T_\pm = \text{ch}\,(uH)\,\hat{1} + \text{sh}\,(uH)\,\frac{(\mathbf{V}_\pm.\boldsymbol{\sigma})}{\|\,\mathbf{V}_\pm\,\|}$$

where $H = \sqrt{B^2 + z^2\,a^2}$

U is then calculated by application of the identity [3] for evaluating the product $T_+.T_-$.

$$U = \exp(-2\,Au)\left[\text{ch}^2\,(uH)\,\hat{1} + \text{sh}^2(uH)\,\frac{(B^2 - z^2\,a^2)}{H^2}\,\hat{1} + \right.$$
$$\left. -\,\frac{2\,B}{H}\,\text{sh}\,(uH)\,\text{ch}\,(uH)\,\hat{\sigma}_z - \frac{2\,zaB}{H^2}\,\text{sh}^2\,(uH)\;\;\hat{i}\,\hat{\sigma}y\right] \qquad \text{(III.7)}$$

the two eigenvalues λ_+ and λ_- are always real and given by :

$$\lambda_\pm = \exp(-\,2\,Au)\left[\text{ch}\,(2\,uH)\,\frac{B^2}{H^2} + 1 - \frac{B^2}{H^2}\right.$$
$$\left. \pm\,2\left|\frac{B}{H}\right|\text{sh}\,(uH)\left(1 + \frac{B^2}{H^2}\,\text{sh}^2\,(uH)\right)^{1/2}\right].$$

The system is instable when the greatest eigenvalue λ_+ is equal to one. Let us define $F(a, a_0)$ as :

$$F(a, a_0) = \lambda_+ - \exp\,(2\,Au)\,.$$

Then the system is instable if there is one pair (a, a_0) such as $F(a, a_0) > 0$.

THRESHOLD CONDITION. — The threshold field is obtained by determining the greatest domain where the system is instable. This gives the two conditions :

$$F(a, a_0) = 0 \qquad \text{(III.8)}$$

$$G(a, a_0) = \frac{\partial a}{\partial a_0} = 0 \qquad \text{(III.9)}$$

with

$a(a_0)$ minimum for the low and high frequency regime,

$a(a_0)$ maximum for the transition regime.

Equations (III.8) (III.9) have been numerically solved on a computer with the supplementary condition $a_0^2 \geqslant a_{0\text{m}}^2$. This corresponds to the condition that the fluctuation wavelength be smaller than the sample thickness d. Remembering that :

$$a_0^2 = E_0^2\,\lambda\tau \quad \text{and} \quad E_0^2 = \left(\frac{1}{\eta_0} + \frac{1}{\gamma_1}\right)K_{33}\,k^2$$

$a_{0\text{m}}^2$, obtained for $k = \pi/d$, is proportional to $1/d^2$.

(2) $\hat{\sigma}_j\,\hat{\sigma}_k = i\sum_{l=1} \varepsilon_{jkl}\,\hat{\sigma}_l$ where ε_{jkl} is the usual completely antisymmetric tensor with $\varepsilon_{xyz} = 1$.

(3) $(\boldsymbol{\sigma}.\mathbf{C})(\boldsymbol{\sigma}.\mathbf{D}) = (\mathbf{CD} + i\,\boldsymbol{\sigma}.(\mathbf{C} \times \mathbf{D}))$.

Numerical results are shown on figure 3 where the reduced field threshold $a(a_0)$ is plotted versus the reduced frequency p for

$$z^2 = 3.05 \quad \text{and} \quad a_{0\mathrm{m}}^2 = 2.2\ \pi^2 \times 10^{-4}$$

corresponding to a sample thickness $d = 100\ \mu$ in M. B. B. A with $k_{33} \simeq 10^{-6}$ CGS, $\tau = 5 \times 10^{-3}$ s and $\dfrac{1}{\eta_0} + \dfrac{1}{\gamma_1} = 4.4$ CGS.

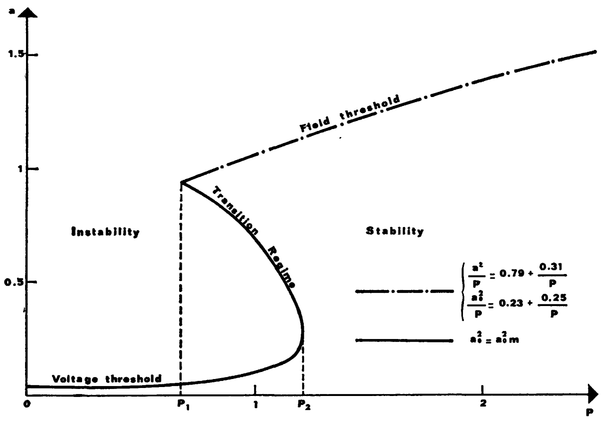

FIG. 3. — Instability threshold reduced field a is plotted versus the reduced frequency p.
for $z^2 = 3.05$ and $a_{0\mathrm{m}}^2 = 2.2\,\pi^2 \times 10^{-4}$ ($d = 100\ \mu$).

Let us note that the diffusion currents are in fact negligible for not to high frequencies. More precisely at the transition : $a_0^2 < 0.8$ which corresponds to

$$\frac{k^2}{q_{\mathrm{D}}^2} < \frac{1}{100}$$

for a sample with $D = \frac{1}{2} \times 10^{-7}$ CGS.

LOW FREQUENCY REGIME. — We are concerned with frequencies such as $p < 1$. Then we shall show that the electric field threshold is small : $a^2 \ll 1$ and $a_0^2 \ll 1$. Then, assuming that

$$a^2 \ll 1, \frac{a^2}{p} \ll 1 \quad \text{and} \quad \frac{a_0^2}{p} \ll 1$$

$B^2/H^2 \simeq 1 - 4\,z^2\,a^2$, the equation (III.8) reduces to :

$$a^2 = \frac{a_0^2\,u(1 + \mathrm{e}^u)}{\left(u(z^2 - 1) - 2\,z^2\right)\mathrm{e}^u + u(z^2 - 1) + 2\,z^2}\,. \qquad \text{(III.10)}$$

The condition (III.9) imposes that $a_0^2 = a_{0\mathrm{m}}^2$. Equation (III.10) then defines a voltage threshold (*ad*) independent of the sample thickness. This is in a very good agreement with the numerical calculations. Let us remark that equation (III.10) leads to a « cut off frequency » $p_{\mathrm{c}} = \pi/u_{\mathrm{c}}$ defined by :

$$\left(u_{\mathrm{c}}(z^2 - 1) - 2\,z^2\right)\mathrm{e}^{u_{\mathrm{c}}} + u_{\mathrm{c}}(z^2 - 1) + 2\,z^2 = 0$$

($p_{\mathrm{c}} = 1.19$ for $z^2 = 3.05$).

The study of the transition regime shows that p_{c} is not a real « cut off » frequency but only defines the upper limit of the voltage threshold branch.

HIGH FREQUENCY REGIME. — We are concerned with frequencies such as $p \gg 1$. Neglecting terms of first order in $1/p$, equations (III.8) and (III.9) lead to :

$$a^2 = C_1 p \quad {a_0}^2 = C_2\,p$$

where C_1 and C_2 are two constants independent of the sample thickness. This regime corresponds to a field threshold independent of the sample thickness.

TRANSITION REGIME. — This regime has only been numerically determined on a computer. The condition (III.9) leads to $a_0^2 = a_{0\mathrm{m}}^2$. The results are shown on figure 3, the value $p_2 = 1.21$ is in a good agreement with p_{c}. There is an interesting interval

$$p_1 < p < p_2 (p_2 - p_1 \sim 0.5)$$

where two types of instabilities can be observed. In this region :

— If we increase the field amplitude, from 0 upwards, at fixed frequency, we find the voltage threshold characteristic of the conducting regime.

— If we start with a high frequency and a suitable field amplitude and then decrease the frequency to reach the regime of interest, we might keep a (metastable) unperturbed state. If we now raised, or decreased, the field amplitude, we would find convection instability. Typically in M. B. B. A. with

$$\tau = 5 \times 10^{-3}\ \mathrm{s}\,,$$

the transition width Δp corresponds to $\Delta\nu = 17$ Hz.

IV. **Instabilities under a sinusoidal field.** — We now consider the case where the applied field $E(t)$ is of the form

$$E(t) = E_M \cos \omega t\,.$$

With use of dimensionless parameters p, θ, z^2, the system (II.1, 2) may be rewritten as :

$$\frac{\partial}{\partial \theta}(\varphi) = \mathcal{K}(\varphi) \qquad \text{(IV.1)}$$

where

$$(\varphi) = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

and

$$\mathcal{K} = -\begin{pmatrix} \dfrac{1}{2\,e^2 \cos(p\theta)} & \dfrac{z^2 \cos(p\theta)}{2(e_0^2 + e^2 \cos^2(p\theta))} \end{pmatrix}$$

$$e^2 = \frac{\lambda E_M^2\,\tau}{2}\,, \quad e_0^2 = \frac{\lambda E_0^2 \tau}{2}\,, \quad \varphi_1 = \frac{q}{\lambda \eta E_M\,\psi_0}\,, \quad \varphi_2 = \frac{\psi}{\psi_0}\,.$$

As shown in section III, the solutions of (IV.1) are of the form :

$$(\varphi(\theta)) = C_1(W_1(\theta)) + C_2(W_2(\theta))$$

where $(W_i(\theta))$ and λ_i are the eigenfunctions and eigenvalues defined by (III.5).

STABILITY OF THE SYSTEM. — The instability condition is still given by (III.6).

The calculation of the translation operator U is much more difficult in the case of an alternating field.

We shall assume that at the threshold

$$\Im(s_+) = 0 \qquad (\Im = \text{imaginary part of}) .$$

This seems to be a quite reasonable assumption : experimental observations [4], [5], [6] have shown that, in all known cases, at threshold, the hydrodynamic cellular motions are infinitely slow. Also for the case of rectangular field pulses we could prove that the eigenvalues were real and the two cases do not appear to be drastically different. With this assumption, the instability threshold is defined by :

$$\Re(s_+(e, e_0)) = 0 . \qquad \text{(IV.3)}$$

This is equivalent to find periodic solutions $(f(\theta))$ of system (IV.1).

THRESHOLD CONDITION. — Expanding the periodic solution $(f(\theta))$ in Fourier series

$$(f(\theta)) = \sum_{n=-\infty}^{+\infty} (f_n)\, e^{inp\theta}$$

we find after some calculations that the threshold field is defined by the implicit equations :

$$\det(M(e, e_0)) = 0 \qquad \text{(IV.4)}$$

$$G(e, e_0) = \frac{\partial e}{\partial e_0} = 0 \qquad \text{(IV.5)}$$

with $e(e_0)$ minimum for the low and high frequency regimes, $e(e_0)$ maximum for the transition regime.

The only non zero elements of the infinite matrix $M(e, e_0)$ are :

$$M_{n,n} = \left[e^2 + 2\,e_0^2 + inp - \frac{z^2\,e^2}{2}\left(\frac{1}{1+i(n+1)\,p} + \frac{1}{1+i(n-1)\,p}\right)\right]$$

$$M_{n,n\pm 2} = \frac{e^2}{2}\left(1 - \frac{z^2}{1+i(n\pm 1)\,p}\right). \qquad \text{(IV.6)}$$

As in section III, equations (IV.4) and (IV.5) have been numericallly solved on a computer with the condition

$$e^2 \geqslant e_{0m}^2 = \frac{a_{0m}^2}{2} .$$

Numerical results are shown on figure 4. We find for low and high frequency regimes the same results as those obtained in [1] (« conduction » and « dielectric » regime).

As in section III, diffusion currents are negligible at the transition where $e_0^2 < 0.8$. This corresponds to

$$\frac{k^2}{q_D^2} < \frac{1}{50}$$

for a sample with $D = \frac{1}{2} \times 10^{-7}$ CGS.

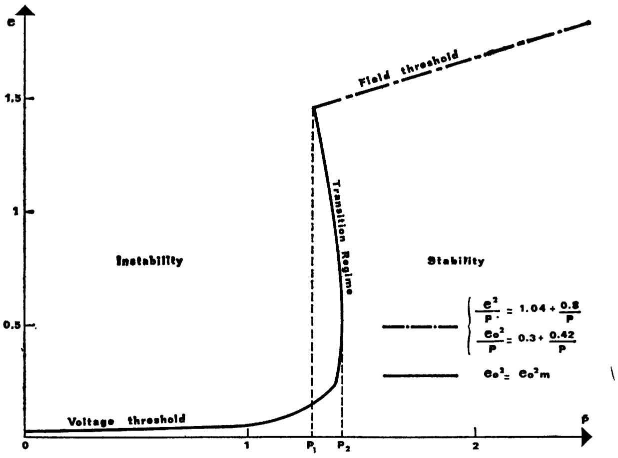

FIG. 4. — Instability threshold reduced field e is plotted versus the reduced frequency p for $z^2 = 3.05$ and $e_{0m}^2 = a_{0m}^2/2 = 1.1\,\Pi^2 \times 10^{-4}$ ($d = 100\ \mu$).

Let us recall first the asymptotic behaviour derived in [1].

LOW FREQUENCY REGIME (« CONDUCTION REGIME »). — This corresponds to frequencies $p \gtrsim 1$ and field such as $e^2 \ll 1$, $e^2/p \ll 1$. Then the threshold is given by :

$$e^2 = \frac{2\,e_0^2(1 + p^2)}{z^2 - (1 + p^2)} . \qquad \text{(IV.7)}$$

Equation (IV.7) defines a cut off frequency p_c :

$$p_c^2 = z^2 - 1$$

($p_c = 1.42$ for $z^2 = 3.05$) .

As in the case of the rectangular field pulses, we shall see that p_c is not a real cut off frequency but only defines the voltage threshold domain.

Equation (IV.5) leads to $e_0^2 = e_{0m}^2 = a_{0m}^2/2$ which is proportional to $1/d^2$

Then equation (IV.7) defines a voltage threshold (ed)

HIGH FREQUENCY REGIME (« DIELECTRIC REGIME » — This corresponds to frequencies $p \gg 1$.

There is a field threshold independent of the sample thickness defined by :

$$\frac{e^2}{p} = C_1 \quad \text{and} \quad \frac{e_0^2}{p} = C_2$$

where C_1 and C_2 are two constants independent of the sample thickness.

TRANSITION REGIME. — The condition (IV.5) leads to :

$$e_0^2 = e_{0m}^2 .$$

The results are shown on figure 4, the value $p_2 = 1.42$ is in a good agreement with p_c.

Here again, the threshold curve has an S shape with a critical interval $\Delta p = p_2 - p_1 = 0.12$. With the same experimental conditions as in section III

$$(\tau = 5 \times 10^{-3} \text{ s}) ,$$

this corresponds to a transition width $\Delta\nu = 4$ Hz. The same qualitative hysteretic features are still expected. To make the interval $\Delta\nu$ larger, it is possible in principle to use doped samples (with ionic impurities) since $\Delta\nu$ is proportional to the conductivity.

V. **Conclusion.** — The stability of a nematic liquid crystal submitted to rectangular field pulses or to a sinewave field is determined by a voltage threshold for low frequencies. But there is a certain interval ($\nu_1 < \nu < \nu_2$ where both thresholds may be observed, the choice between them depending on the detailed method used to change the amplitude or the frequency of the applied field. The interval ν_2-ν_1 is larger in the case of rectangular field pulses but in both cases, hysteresis effects could be seen in principle.

Of course, there may be some difficulties in observing these effects : to reach the metastable static region, we need to decrease the frequency at constant field amplitude. If this decrease is « adiabatic », the process should be successful. But it is not entirely obvious that we will retain the metastable state if the decrease is achieved by an abrupt jump. However, a study of the metastable range appears to be worthwhile : experiments in this direction are currently under way [7].

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References

[1] DUBOIS-VIOLETTE (E.), DE GENNES (P. G.), PARODI (O.), to be published in *J. Physique*.
[2] PARODI (O.), *J. Physique*, 1970, **31**, 581.
[3] FRANK (F. C.), Discussions *Faraday Soc.*, 1958, **25**, 19.
[4] RONDELEZ (F.), Thesis III cycle, Orsay, 1970, see also ref. (5).
[5] Orsay Liquid Crystal Group, *Molecular Crystals and Liquid Crystals*, 1971, **12**, 251.
[6] Orsay Liquid Crystal Group, *Phys. Rev. Let.*, 1970, **25**, 1642.
[7] GALERNE (Y.), Private Comm.