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ON MULTIPLE EXCITATION NEAR THE COULOMB BARRIER IN HEAVY ION SCATTERING

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Résumé. — La méthode semi-classique développée en théorie d'excitation coulombienne est utilisée pour définir les sections efficaces différentielles inélastiques en diffusion, à la barrière coulombienne, d'ions lourds par des noyaux pairs-pairs vibrationnels. La théorie est appliquée à l'interprétation des résultats de Lemberg concernant l'excitation des états 3⁻ des noyaux Sn^{122-124} en diffusion d'ions N^{14} aux énergies de 44,5, 48,5 et 52,5 MeV.

Abstract. — The semi-classical method developed in Coulomb excitation of nuclei is used to describe the differential inelastic scattering cross-section at energies near and above the Coulomb barrier for heavy ion interacting with even-even nuclei possessing vibrational states. The theory is applied to Lemberg's results of 3⁻ states excitation in Sn^{122-124} by N^{14} ions scattering at energies of 44.5, 48.5 and 52.5 MeV.

1. Introduction. — The use of Coulomb excitation to obtain useful information about excited states of nuclei has generally depended on the assumption that the incident particle does not appreciably penetrate the Coulomb barrier and therefore that only electromagnetic interactions need to be considered.

However, even below the Coulomb barrier, the nuclear interaction between the incident and target nuclei may be significant and in some cases cannot be neglected. Modifications of the Coulomb excitation process due to nuclear forces are presented in several experimental results [1]-[5] and are reported in some theoretical works [6]-[15], [20].

In work of Hansen, Nathan and Popov [1], 2+ and 3⁻ states were identified in some rare-earth region nuclei by inelastic α scattering at various incident energies from 14 to 20 MeV. In those papers, the BE₂ values are derived by assuming Coulomb excitation only. The BE₂ values come out correctly with this assumption, but the BE₃ values are three and four times larger than the values deduced in e⁻ scattering.

It has been shown by DWBA calculations [6]-[10], performed with an optical model nuclear potential in addition to the Coulomb potential, that one can have a situation where the lowest 2+ state is excited predominantly by Coulomb excitation whereas there is a significant nuclear contribution to the 3⁻ state. Because of this situation with a bombarding energy of α particles close to the Coulomb barrier applied to the experimental conditions used by Hansen et al., the largeness of their BE₃ values is explained. Recently, the inelastic N^{14} ions scattering by Sn^{116-122-124} target nuclei, has been measured at 180° scattering angle by Lemberg et al. [2] at incident energies of 44.5, 48.5 and 52.5 MeV. It has been shown that the BE₃ values derived by Coulomb excitation theory increases with energy when it pass through the Coulomb barrier (E_B ≈ 50 MeV). These results are presented in Table I.

| Table I |
|-----------------|----|----|----|
| E MeV | 52.5 | 48.5 | 44.5 |
| N^{14} Sn^{122} Lemberg | 0.46 | 0.20 | — |
| Theory | 0.56 | 0.20 | — |
| N^{14} Sn^{124} Lemberg | 0.69 | 0.19 | 0.16 |
| Theory | 0.51 | 0.20 | 0.16 |
| N^{14} Sn^{116} Lemberg | 0.37 | 0.19 | — |

In the present paper, the semi classical method developed by several authors [16]-[17] in Coulomb excitation of nuclei is used to describe the differential inelastic scattering cross sections at energies near the Coulomb barrier for heavy charged particles interacting with an even-even nucleus possessing vibrational states. The results are applied to Lemberg's experiment of inelastic N^{14} ions scattering by Sn^{122-124}.

For α scattering at energies E₀ ≥ E_B, the differential cross-sections has a diffractional structure which evidently cannot be described in a theoretical semi-classical treatment. In such cases, a coupled channel calculation or the first order DWBA approximation...
performed with an optical model nuclear potential in
addition to the Coulomb potential is much suitable.
Such theory has been presented in several theoretical
papers [8], [9], [10], [12], [15]. However, the long
range Coulomb potential requires many partial waves
in the computation of the Coulomb excitation terms.
An easier treatment which introduces a Coulomb
phase correction in the smooth cut-off diffractional
model has been developed [18].

In heavy ion reactions, coupled equations or
DWBA method requires too many partial waves
and the calculations become impractical without aid of
long numerical computation programs and high
speed computers. Fortunately, the excitation by
heavy ions generally affords appreciable simplifications
in the treatment by multiple excitation time dependent
theory. Indeed, in heavy ion scattering, the conditions
of semi-classical approximation are generally well
realized and furthermore, due to the small penetration
in the nuclear forces range, it may be possible to treat
the nuclear interaction as a perturbation to the
Coulomb one. Such approximations can be used
in multiple excitation time dependent theory to facili-
tate the computation of multiple excitation near the
Coulomb barrier in heavy ion scattering.

2. Calculation of excitation probability. — In
the collision of heavy charged particles with a nucleus,
the particle motion in the field $z'z' e^2/r + V^{\text{nucl}}$ can
be described by means of classical mechanics, assuming
$r = r(t)$, if the quasi-classical condition

$$kR \gg 1 \tag{1}$$

is satisfied.

Assuming that, in inelastic scattering, the transfer-
ergy $\Delta E = E_f - E_i$ is small compared with the
incident energy $E$,

$$\frac{\Delta E}{E} \ll 1 \tag{2}$$

we may neglect the change of kinetic energy in the
particle motion. In other words, we shall assume that
in inelastic scattering the trajectory of the charged
particle is undistinguishable from that of an
elastically scattered particle.

If it is assumed, furthermore, that the Coulomb
potential $z'z' e^2/r$ is predominant in the elastic scattering
and if the classical condition (1) $\eta = z'z' e^2/hv \gg 1$
is satisfied, we may identify the trajectory of the
particle exactly with a hyperbolic Rutherford orbit.
The closest approach on a fixed trajectory

$$D_{\min} = \frac{\eta}{k} \left( 1 + \left( \sin \frac{\theta}{2} \right)^{-1} \right) \tag{3}$$

being supposed to stay equal or larger than the range
of nuclear forces, we may also assume that the residual
nuclear and Coulomb interactions act as a perturbation.

When conditions (1), (2), (3) are satisfied, the
differential cross-section for inelastic scattering is

$$d\sigma_{\text{inel}}(\theta) = d\sigma_{\text{el}}(\theta) \cdot P_{\text{it}}(\theta) \tag{4}$$

where

$$d\sigma_{\text{el}}(\theta) = \frac{\eta^2}{4k^2} \sin^{-4} \frac{\theta}{2}$$

and $P_{\text{it}}$ is the probability of nuclear transition

$$P_{\text{it}} = \frac{1}{2I_1 + 1} \sum_{M_i = M_f} |a_{it}|^2 .$$

To define the transition amplitude $a_{it}$, we use the
Tomonaga representation and solve the equation

$$i\hbar \delta \Psi(q, r(t)) = \tilde{H}_{\text{int}}(q, r(t)) \Psi(q, r(t)) \tag{5}$$

with

$$\tilde{H}_{\text{int}} = U_e^* H_{\text{int}}(q, r(t)) U_e$$

and the unitary operator

$$U_e(t, t_0) = \exp \left( \frac{-i}{\hbar} H_e(q) (t - t_0) \right)$$

$H_e(q)$ being the target nucleus hamiltonian and
$H_{\text{int}}(q, r(t))$ the particle-nucleus interaction hamil-
tonian. The state vector $\Psi(q, r(t))$ may be developed in
a linear combination of the basis vector $\Psi_i(q)$
eigenstates of $H_e(q)$.

$$\Psi(q, r(t)) = \sum_i a_i(t) \Psi_i(q) \tag{6}.$$

The solution of equation (5) can be expressed as an
exponential operator according to Magnus or Fer
formulations [19]. With initial conditions

$$\Psi(t_0) = \Psi_i(q),$$

the transition amplitude to the final state $\Psi_i(q)$ at
time $t$ is

$$a_{it}(t) = < \Psi_i(q) | \Psi_i(q, r(t)) > = < \Psi_i(q) | U(t, t_0) | \Psi_i(q) > .$$

With the Magnus form of the time-evolution
operator $U(t, t_0)$ [19], we obtain

$$a_{it} = < \Psi_i | \exp \left\{ \frac{-i}{\hbar} \int_{-\infty}^{t_1} \tilde{H}_{\text{int}}(t_1) dt_1 \cdot \frac{1}{2\hbar^2} \int_{-\infty}^{t_1} dt_2 \int_{-\infty}^{t_1} dt_3 \left\{ \tilde{H}_{\text{int}}(t_1), \tilde{H}_{\text{int}}(t_2) \right\} \right\} \Psi_i > . \tag{7}$$
The particle-nucleus interaction \( H_{\text{int}}(\mathbf{q}, \mathbf{r}(t)) \) is defined assuming vibrational excitation of the target nucleus. So, for small deformation of the nuclear shape defined by

\[
R(\theta, \varphi) = R_{0,e}(1 + \Sigma_{n} z_{2n} Y_{2n}(\theta, \varphi))
\]

\[
z_{2n} = \frac{\beta_{2}}{\sqrt{2 \lambda + 1}} \left( b_{2n} + (-)^{n} b_{2n}^{-} \right)
\]

(8)

the particle-nucleus interaction energy may be represented by

\[
V(| \mathbf{r} - \overline{R}(\theta, \varphi) |) = V_{0}(\mathbf{r} - \overline{R}_{0,e}) + 
+ \sum_{n=1}^{\infty} \frac{(-\delta R)^{n}}{n!} R_{0,e}^{n} \int dV \bigg|_{R=R_{0,e}} \quad (9)
\]

\[
(\delta R)^{n} = \sum_{l_{m}} S_{lm}^{(n)} (i^{l} Y_{lm}(\theta, \varphi))^{*}
\]

\[
S_{lm}^{(n)} = i^{l} \sum_{l_{m}} \sum_{l_{m}'} \left( \frac{\hat{l} \cdot \hat{l}'}{4 \pi} \right)^{1/2} \times
\]

\[
\times \left( \frac{l'}{\mu} \frac{l'}{\mu'} \frac{l}{l-m} \right) \left( \begin{array}{l} \frac{l}{0} \frac{l}{0} \frac{l}{0} \end{array} \right) S_{l_{m}l_{m}'}^{(n-1)} z_{2n}^{*}
\]

with initial relation

\[
S_{l_{m}}^{(1)} = z_{2n}^{*}.
\]

The expression (10) is

\[
V^{eb}(| \mathbf{r} - R(\theta, \varphi) |) = \frac{2 z^{2} e^{2}}{R_{e}} \left( 3 - \frac{R_{e}^{2}}{r^{2}} \right) \theta(R_{e} - r) + \frac{z^{2} e^{2}}{r} \theta(r - R_{e})
\]

\[
V^{eb}(r) = \frac{3}{2} \frac{2 z^{2} e^{2}}{l + 1} I_{lm} \theta(R_{e} - r) + \frac{R_{l}^{2}}{r \rho_{i+1} \rho_{l+1}} O_{lm} \theta(r - R_{e})
\]

\[
I_{lm} = - \sum_{n=1}^{\infty} \frac{n!}{(l + m - 3)!} S_{lm}^{(n)}
\]

\[
O_{lm} = \sum_{n=1}^{l+2} \frac{(l + 2)!}{n! (l + 3 - n)!} S_{lm}^{(n)}
\]

\[
I_{00} = O_{00} = 0.
\]

The nuclear potential

\[
V^{\text{Nuc}}(| \mathbf{r} - \overline{R}(\theta, \varphi) |) = V_{0}^{\text{Nuc}}(| \mathbf{r} - \overline{R}_{0} |) + \sum_{lm} V_{lm}^{\text{Nuc}}(r) (i^{l} Y_{lm})^{*}
\]

\[
V_{lm}^{\text{Nuc}} = \sum_{n=1}^{\infty} S_{lm}^{(n)} \frac{(-)^{n}}{n!} R_{0}^{n} \int dU \bigg|_{R=R_{0}}.
\]

\[
R_{0,e} \text{ defines the equilibrium radius of the nuclear mass (R_{e}) or charge (R_{e}) distribution.}
\]

The scalar term \( V_{0}(| \mathbf{r} - \overline{R}_{0,e} |) \) we identify with the spherical potential used to describe the elastic scattering and the other terms in (9) we may identify with the inelastic interaction.

So, the explicit form of the Coulomb and nuclear interaction potentials are:

The Coulomb potential

\[
V^{eb}(| \mathbf{r} - R(\theta, \varphi) |) = \frac{2 z^{2} e^{2}}{R_{e}} \left( 3 - \frac{R_{e}^{2}}{r^{2}} \right) \theta(R_{e} - r) + \frac{z^{2} e^{2}}{r} \theta(r - R_{e})
\]

\[
= \frac{4 \pi z^{2} e^{2}}{2} \sum_{l=1}^{\infty} \frac{1}{2 \lambda + 1} Y_{2n}(\tilde{r}) \times
\]

\[
\times \int \rho(r) r^{2} r_{<}^{l-1} Y_{2n}(\tilde{r}) d\tilde{r}
\]

\[
r_{<}, r_{>} \text{ being the smaller and larger of } r, r'.
\]

With the charge density function

\[
\rho(\tilde{r}) = \frac{3}{4 \pi R_{e}^{3}} \theta(\overline{R}_{e}(\theta', \varphi') - \tilde{r}) \text{ with } \theta(r) = 1 \quad r > 0 \quad \theta(r) = 0 \quad r < 0 .
\]

The expression (10) is

\[
V^{eb} = V_{0}^{eb} \sum_{lm} V_{lm}^{eb}(r) (i^{l} Y_{lm}(\theta, \varphi))^{*}
\]

with

\[
V_{0}^{eb} = \frac{2 z^{2} e^{2}}{R_{e}} \left( 3 - \frac{R_{e}^{2}}{r^{2}} \right) \theta(R_{e} - r) + \frac{z^{2} e^{2}}{r} \theta(r - R_{e})
\]

\[
V_{lm}^{eb} = \frac{2 z^{2} e^{2}}{2 l + 1} \left( \frac{R_{l}^{2}}{R_{e}^{2}} \right) I_{lm} \theta(R_{e} - r) + \frac{R_{l}^{2}}{r \rho_{l+1} \rho_{l+1}} O_{lm} \theta(r - R_{e})
\]

\[
I_{lm} = - \sum_{n=1}^{\infty} \frac{(l + m - 3)!}{n! (l - 2)!} S_{lm}^{(n)}
\]

\[
O_{lm} = \sum_{n=1}^{l+2} \frac{(l + 2)!}{n! (l + 3 - n)!} S_{lm}^{(n)}
\]

\[
I_{00} = O_{00} = 0.
\]
If we assume small deformation of order $A$ for the nuclear surface, the residual Coulomb and nuclear particle-nucleus interactions may be limited in their developments [12], [15] to the first order terms in $\alpha_{np}$ and the general form of $\tilde{H}_{int}(\Omega)$ is

$$
\tilde{H}_{int}(\Omega, t) = \tilde{H}_{c}(\Omega) + \tilde{H}_{n}(\Omega) = \frac{\beta J}{\sqrt{(2 \lambda + 1)}} g_\lambda f_s(r(t)) \sum_{\mu} Y_{\mu}(\Omega(t)) \{ (-)^\mu b_{-\mu, \mu}^* \exp(i \omega_\lambda t) + b_{\mu, \mu} \exp(- i \omega_\lambda t) \}
$$

with

$$ g_\lambda f_s(r(t)) = -\frac{1}{r} \frac{dU}{dr} \bigg|_{r=R} .
$$

$r = r(t), \Omega = \Omega(t)$ are defined by the classical trajectory.

When this assumption is satisfied, the commutators $[\tilde{H}^{\pm}(t), \tilde{H}^{\pm}(t')]$ are C-numbers and the Magnus expansion of $U(t, t_0)$ is limited to the second order. So, we obtain

$$
a_{\lambda f} = \langle f | \exp(- i \Omega^+) \exp(- i \Omega^-) \times \exp(- i/2(\Omega^+ + \Omega^-)) | i >
$$

with

$$
\Omega^\pm = \frac{1}{\hbar} \int_{-\infty}^{+\infty} \tilde{H}_{int}(t) dt.
$$

In this multiple excitation semi-classical theory, the explicit form of the one phonon vibrational state excitation cross-section shall be [see formulas (4), (16)-(18)].

$$
\frac{d\sigma_1}{d\Omega} = \frac{\eta^2}{4 k^2} \sin^{-4} \frac{\theta}{2} \left( \sum_{\mu=1}^{2N} | a_{\mu 2} |^2 \right)
$$

or

$$
\frac{d\sigma_2}{d\Omega} = \frac{\eta^2}{4 k^2} \sin^{-4} \frac{\theta}{2} \chi^2(\theta, \xi) \exp(- \chi^2(\theta, \xi))
$$

with

$$
\chi = \frac{\eta \Delta E}{2 E}
$$

and where

$$
\chi^2(\theta, \xi) = P^{(1)}_{\lambda 2}
$$

$$
P^{(1)}_{\lambda 2} = \sum_{\mu} | b_{\mu 2} |^2 = \frac{\beta J}{2 \lambda + 1} \left( \frac{3 \eta}{2 \lambda + 1} \right)^2 \left( \frac{R_e}{a} \right)^{2 \lambda} \sum_{\mu} Y_{\mu} \left( \frac{\pi}{2}, 0 \right) \left( \frac{R_e}{a} \right)^{2 \lambda} \times \left| I_{\mu}(\theta, \xi) - \frac{V_0 R_0 a^2}{\sigma} \frac{2 \lambda + 1}{6} \frac{k}{\eta E_0 R_e^2} N_{\mu}(\theta, \xi) \right|^2
$$

with the Coulomb classical orbital integral [16]

$$
I_{\mu}(\theta, \xi) = \int_{-\infty}^{+\infty} dw \exp(i \xi(w + \epsilon \sinh w)) \left( \cosh w + \epsilon + i \sqrt{(\epsilon^2 - 1) \sinh w} \right)^\mu
$$

is the probability defined by first order time-dependent perturbation theory

$$
P^{(1)}_{\lambda 2} = \sum_{\mu} | b_{\mu 2} |^2
$$

$$
b_{\mu 2} = \frac{1}{i \hbar} \int_{-\infty}^{+\infty} \exp(i \omega_\lambda t) < \lambda | V^{(1), \epsilon'}_{\lambda}(t) + V^{(1), N}_{\lambda}(t) \lambda | 0 > dt .
$$

$V^{(1), \epsilon'}_{\lambda}$ and $V^{(1), N}_{\lambda}$ defined by the first order terms in $\alpha_{np}$ of the developments (12)-(14) are respectively the Coulomb and nuclear residual interaction potential. In the present calculation, only the external Coulomb interaction potential has to be used and a Saxon-Woods shape is selected for the nuclear potential. Explicitly we have

$$
V^{(1), \epsilon'}_{\lambda}(t) = \frac{4 \pi \kappa e}{2 \lambda + 1} r(t)^{-\lambda-1} \sum_{\mu} Y_{\mu}(\Omega(t)) ME_{\mu}
$$

with the multipole electric moment

$$
ME_{\mu} = \frac{3}{4 \pi} Z_2 e R_e^2 \alpha_{\mu}
$$

or

$$
V^{(1), N}_{\lambda}(t) = -\frac{V_0 R_0 a^2}{\sigma} \frac{2 \lambda + 1}{6} \frac{k}{\eta E_0 R_e^2} N_{\mu}(\theta, \xi)
$$

For convenience, we use the focal coordinate system and the parametric description of the Rutherford trajectory defined in [16]. With $a = \eta/k$ and $\varepsilon = \sin^{-1} \theta/2$, the explicit form of $P^{(1)}_{\lambda 2}$ is:

$$
P^{(1)}_{\lambda 2} = \sum_{\mu} | b_{\mu 2} |^2 = \frac{\beta J}{2 \lambda + 1} \left( \frac{3 \eta}{2 \lambda + 1} \right)^2 \left( \frac{R_e}{a} \right)^{2 \lambda} \sum_{\mu} Y_{\mu} \left( \frac{\pi}{2}, 0 \right) \left( \frac{R_e}{a} \right)^{2 \lambda} \times \left| I_{\mu}(\theta, \xi) - \frac{V_0 R_0 a^2}{\sigma} \frac{2 \lambda + 1}{6} \frac{k}{\eta E_0 R_e^2} N_{\mu}(\theta, \xi) \right|^2
$$

with the Coulomb classical orbital integral [16]

$$
I_{\mu}(\theta, \xi) = \int_{-\infty}^{+\infty} dw \exp(i \xi(w + \epsilon \sinh w)) \left( \cosh w + \epsilon + i \sqrt{(\epsilon^2 - 1) \sinh w} \right)^\mu
$$
and the nuclear classical orbital integral
\[ N_{sd}(\theta, \zeta) = \int_{-\infty}^{+\infty} dw \exp(i\zeta(w + \varepsilon \sinh w)) \times \]
\[ \times \left( 1 + \exp\left( a(e \cosh w + 1) - R_0 \right) \right)^{2} \]
\[ \frac{\exp\left( a(e \cosh w + 1) - R_0 \right)}{\sigma} \]
\[ \frac{(e \cosh w + 1)^{\lambda + \mu - 1}}{\sigma} \]
\[ \lambda + \mu \text{ even}. \]

3. Results and discussion. — This theory has been applied to the inelastic scattering of heavy ions \( \text{N}^{14} \) by \( \text{Sn}^{122-124} \) at incident energies of 44.5, 48.5 and 52.5 MeV around the Coulomb barrier. Excitation of levels \( 2^+ \) of one phonon \( \lambda = 2 \) and \( 3^- \) of one phonon \( \lambda = 3 \) have been studied. The characteristic parameters for \( \text{Sn}^{122} \) are given in Table II.

<table>
<thead>
<tr>
<th>( E_0 ) MeV</th>
<th>( \eta = \sqrt{\eta_I \eta_t} )</th>
<th>( k = \sqrt{k_I k_t} ) (fm(^{-1}))</th>
<th>( \zeta = \eta_I - \eta_t )</th>
<th>( a = \eta/k ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.5</td>
<td>31.13</td>
<td>4.862</td>
<td>0.444</td>
<td>6.403</td>
</tr>
<tr>
<td>48.5</td>
<td>29.80</td>
<td>5.079</td>
<td>0.389</td>
<td>5.868</td>
</tr>
<tr>
<td>52.5</td>
<td>28.63</td>
<td>5.287</td>
<td>0.345</td>
<td>5.415</td>
</tr>
</tbody>
</table>

The differential excitation cross sections are calculated for different values of the nuclear parameters \( V_0 \) and \( \sigma \) and for \( R = r(A_p^{1/3} + A_e^{1/3}) \) with \( r = r_0 = 1.5 \) fm and \( r = r_e = 1.3 \) fm respectively for the mass and charge spherical distribution of the nucleus.

In the « Coulomb + Nuclear (C + N) theory », the differential cross-section is thus defined as the superposition of three terms
\[ d\sigma^{\text{tot}} = d\sigma^{\text{eb}} + d\sigma^{\text{Nu}} + d\sigma^{\text{int}} \]
where \( d\sigma^{\text{eb}} \) is the Coulomb excitation differential cross-section, \( d\sigma^{\text{Nu}} \) is the nuclear excitation differential cross-section, depending on \( V_0^2 \) and \( d\sigma^{\text{int}} \) is the negative interference term of the repulsive Coulomb interaction and the attractive nuclear one, depending on \( V_0 \).

The negative interference term \( (V_0) \) is the first correction of the \( (C + N)^* \) theory on the \( (C)^* \) one and the nuclear term \( (V_0^2) \) prevails over the interference term, only for large scattering angles.

For increased incident energies, the effect of the nuclear interaction is present in the excitation differential cross-section in a larger angular region and its maximum amplitude goes to larger angles or, which is the same, deeper penetration of the classical orbits into the nuclear interaction region. (See Fig. 1 and 2.)

For increased \( V_0 \) values, the negative discrepancy between the \( (C + N) \) and the \( (C) \) differential cross-sections occurs at smaller angles. As the positive
Angular distributions for the octupole transition in Sn$_{122}$ bombarded with N$^{14}$ ions at $E_0 = 44.5, 48.5$ and 52.5 MeV. The results are presented for two values of the parameter $Q : a = 0.56$ fm and $a = 0.40$ fm.

The nuclear term prevails on the negative interference term for smaller angles, a big minimum followed by faster ascent of the differential cross-section is observed at smaller angles. (See Fig. 3-4-5.)

For increased $\sigma$ values, the discrepancy observed between the (C + N) and (C) differential cross-sections depends strongly on the incident energy. At lower energies (44.5, 48.5 MeV) the classical orbits penetrate a nuclear region limited at (1) or (2) in figure 6, for...
which the nuclear interaction is smaller for smaller $\sigma$ values. See figure 4. At 52.5 MeV energy, the classical 
orbits penetrate to the region (3) in figure 6 and so, 
for scattering angles larger than $\theta_0$, the nuclear inter-
action for small $\sigma$ values is larger. See figures 5 and 7.

FIG. 6. — Nuclear potential variation for two values of the 
nuclear parameter $\sigma$.

FIG. 7. — Variation with the nuclear parameter $\sigma$ of the nuclear 
contribution to the angular distribution for the octupole transi-
tion in Sn$^{122}$ bombarded with N$^{14}$ ions at $E_0 = 52.5$ MeV.

In this paper, we only present the results of octupole excitation. One may indeed see that the nuclear 
correction introduced in Coulomb excitation 
differential cross-sections is smaller in quadrupole than 
in octupole excitation. This appears clearly by 
comparing the interaction potentials defined as the 
superposition of the nuclear surface interaction and 
the Coulomb one for the $2^+$ and $3^-$ excitation (curves $2^+$ and $3^-$ on Fig. 8) with the corresponding 
Coulomb residual interaction only (curves $\lambda = 2$ 
and $\lambda = 3$ on Fig. 8).

FIG. 8. — The potential interaction defined as the superposition 
of the Coulomb and nuclear interaction potentials for $\lambda = 2$ 
and $\lambda = 3$ excitation ($2^+$ and $3^-$ (C + N)) is compared to 
the Coulomb interaction only ($2^+$ and $3^-$ (C)).

Similar qualitative results are obtained in Sn$^{124}$.
The quantitative determinations of reduced transition 
probabilities are, in the present theory, based on an 
assumed proportionality between the inelastic 
scattering cross-sections and the BE parameters derived 
from Coulomb excitation. The two processes do not, 
however, necessarily measure the same matrix elements. 
The « apparent » reduced transition probabilities $\text{BE}_3$ 
of Sn$^{122-124}$ defined at $180^\circ$ by the ratio of (C + N) 
and (C) differential cross-sections are compared with 
the experimental values of Lemberg [2]. A good fit is 
obtained for some admissible values of the nuclear 
and Coulomb parameters : $V_0 = 6$ MeV, $\sigma = 0.56$ fm, 
$r_0 = 1.5$ fm and $r_e = 1.3$ fm. See Table I. So, it appears 
that a (C) theoretical analysis of such experiments 
defines a $\beta_3$ deformation value increasing with 
incident energy.

4. Conclusion. — On the basis of these calculations 
we can draw the following conclusions :

— The nuclear interaction between incident and 
target nuclei may be significant even below the
Coulomb barrier $E_B$. It seems to be necessary to take into account the interference effects between Coulomb and nuclear interactions if we hope to fit the increasing BE$_2$ values observed when the incident energy passes through the Coulomb barrier.

The excitation probabilities are rather sensitive to the nuclear surface potential parameters (diffuseness and depth of the nuclear potential); so these interference effects might serve as a tool to measure these parameters.

The interference effects between Coulomb and nuclear interactions and the contribution of pure nuclear interaction fluctuate sensitively with the scattering angle.

Recently, theoretical results on multiple excitation near the Coulomb barrier have been presented by Lukyanov et al. [20]. Their excitations cross sections with transitions to rotational $2^+$ and $4^+$ states in an even nucleus (Nd$^{150}$) are calculated at one scattering angle only ($\theta = 150^\circ$). Their calculations are limited by the validity of the sudden approximation and of the $g$ eff. ($\theta$) approximation as defined in reference [21]. Similar qualitative conclusions have been drawn. Here, our main objective was to develop a theory available for heavy ion scattering and which could give some fit to the « anomalous BE$_2$ » values obtained in some experiments.

To develop some more complete theories about the present subject it would be necessary to have some more experimental results on BE$_2$ values and on differential cross-sections of such scattering at the Coulomb barrier energy. So, it would be very interesting to have some results like those of Lemberg in the N$^{14}$-Sn$^{116-122-124}$ at different scattering angles or those presented by Wang et al. [5] in the scattering of C$^{12}$ at 125.6 MeV and O$^{16}$ at 166.4 MeV on Pb$^{208}$ for $30^\circ < \theta_{cm} < 50^\circ$. In concluding, we must also note that the Coulomb-nuclear interference effects in the excitation cross-sections are more important for larger $\lambda$-transfer. So it would be very interesting to study some examples of large $\lambda(= 4, 5)$ excitations in scattering process at the Coulomb barrier energy. In fact, a more realistic theory of scattering between complex nuclei requires treatment exactly in the same way of Coulomb and nuclear interaction potentials.

So, a coupled channel calculation or the first order DWBA approximation performed with a optical model nuclear potential in addition to the Coulomb model nuclear potential in addition to the Coulomb potential in addition to the Coulomb model nuclear potential in addition to the Coulomb model nuclear potential in addition to the Coulomb potential in addition to the Coulomb model nuclear potential in addition to the Coulomb potential.