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A METHOD FOR SYNTHESIS OF DESIGNS OF MULTILAYER STRUCTURES WITH PRESCRIBED FREQUENCY DEPENDENT PROPERTIES

By G. TRICOLES,

Résumé. — Ce procédé de synthèse est basé sur un coefficient de réflexion approché dans le cas d'ondes planes. Les calculs montrent que les coefficients de réflexion exact et approché diffèrent peu lorsqu'ils sont faibles. Les maximums des coefficients de réflexion se produisent à peu près pour la même longueur d'onde, mais leurs valeurs sont très différentes. On a déduit des schémas de couches multiples à partir d'une variation donnée du facteur de transmission en fonction de la fréquence. On a pu vérifier par des calculs exacts que le schéma donne approximativement les propriétés prévues. Le processus synthétique n'est pas systématique parce que le nombre de couches est arbitraire. En pratique, ce nombre devrait être le plus petit possible. De plus, la résolution simultanée des équations quadratiques ne donne pas en général parfaitement l'équation (9). On donne des interprétations des propriétés des couches multiples. L'augmentation du nombre des couches ou du chemin optique modifie le nombre des composantes de Fourier utilisables. L'épaisseur des couches est en relation avec la largeur de bande et, d'après les équations (5), (10) et (11) lorsque \( f = f_m/2 \), chaque couche est une lame quart d'onde. Un autre travail a montré certains rapports entre cette méthode approchée et le calcul d'impédance.

Abstract. — This synthesis method is based on an approximate reflection coefficient for plane waves. Computations show that the exact and approximate reflection coefficients differ slightly for small values. Maxima of the reflection coefficients occur at about the same wavelength, but values differ considerably. Multilayer designs were derived starting with a specification of transmission as a function of frequency. Exact calculations verified that the design does approximate specified performance. The synthesis procedure is not systematic because the number of layers is arbitrary. In practice, this number would be minimized. Also, solving the simultaneous quadratics, eq. (9) is not straightforward in general.

Interpretations are provided of multilayer performance. Increasing numbers of layers or optical paths in a layer varies the number of Fourier components available. Layer thickness is related to bandwidth, and from eqs. (5), (10, and (11) when \( f = f_m/2 \), each layer is a quarter wave plate. Other work has shown some of the relationship of this approximate method to the impedance theory of iterative structures [5, 6].

1. Introduction. — Stratified dielectric media are used as filters in optics and as enclosures for microwave antennas. In both cases, analysis is made using plane wave theory with wavelength, angle of incidence, wave polarization, and number of layers being variables. Analysis can be based on the summation, boundary value, or the impedance methods [1]. Thus the problem of analyzing multilayer dielectrics seems routine, especially when computers are available. Fabrication problems seem less routine; however, these will not be discussed here.

In this paper an approach is given to synthesis of designs of multilayers. This procedure differs from an analytical approach in that a design is not sought by repeated analysis of likely structures. Rather, starting with desired transmission properties as a function of frequency, a set of dielectric constants and thicknesses can be obtained. The procedure is based on an approximate formalism. By considering single reflections at each interface a trigonometric polynomial is obtained for the transmission coefficient. By expressing desired transmission properties as a Fourier polynomial, and equating the two polynomials, a multilayer configuration can be deduced.

There has been recent interest in synthesis of filter designs [2, 3, 4]. The present method differs from Pohlack's in the manner of specifying desired properties. In the present paper all layers need not be of equal optical path. The multilayer problem is related to the problem of spacing dielectric supports in coaxial cables [5].

Attention is restricted to multilayer dielectrics in which each region is homogeneous isotropic, linear, and non-absorbing.

2. Approximate polynomial. — A polynomial can be derived which approximates the reflection coefficient of a multilayer dielectric sheet. Using this polynomial, the configuration (dielectric constants and thicknesses) can be derived of a multilayer dielectric which approximates a specification of power reflection as a function of frequency.

For a single linear, homogeneous, and isotropic dielectric sheet immersed in another such dielectric of infinite extent, the amplitude reflection coefficient for plane waves is

\[
R_{eq} = \frac{r_{eq} + r_{12} e^{i\theta_{12}}}{1 + r_{eq} r_{12} e^{i\theta_{12}}} \tag{1}
\]
where \( r_{01} \) and \( r_{12} \) are Fresnel reflection coefficients, 
\( \varphi_1 \) is \( \pi a_1 \lambda_1^{-1} (\varepsilon' \sin^2 \theta)^{1/2} \), 
\( \varepsilon' \) is the relative dielectric constant, 
\( \theta \) is the angle of incidence, 
\( \lambda_0 \) is the free space wavelength, and \( d \) is the thickness of the sheet.

Polarization is introduced as either parallel or perpendicular separately. For a multilayer dielectric a similar formula can be derived. For a sheet of \( N \) layers, as sketched in figure 1, the reflection coefficient is

\[
R_{0,N+1} = \frac{r_{01} + R_{1,N+1} e^{i\varphi_1}}{1 + r_{01} R_{1,N+1} e^{i\varphi_1}} \tag{2}
\]

**Fig. 1.** An \( N \) layer multilayer structure.

where \( R_{1,N+1} \) is the reflection coefficient for the \( N - 1 \) layer sheet consisting of media 2 through \( N \) bounded by semi-infinite media of parameters identical with those of media 1 and \( N+1 \). Thus the fraction of intensity reflected, that is, the power reflection coefficient, is on omitting subscripts the asterisk denoting the complexe conjugate.

Now consider \( R_{02} \) defined by eq. (1). An approximation to \( R_{02} \) is

\[
R_{02}^* = r_{01} + r_{12} e^{i\varphi_1} \tag{3}
\]

Zeros of \( R_{02}^* \) and \( R_{02} \) coincide, for the denominator in eq. (1) is greater than zero.

For identical media bounding the dielectric sheet, zeros of \( R_{02}^* \) occur for

\[
d(\varepsilon - \sin^2 \theta)^{1/2} = n\lambda_0 \lambda_1, \quad n = 0, 1, 2 \ldots \tag{4}
\]

A similar polynomial can be written for a multilayer sheet:

\[
R_{0,N+1}^* = r_{01} + r_{12} E_1 + r_{23} E_1 E_2 + \cdots + r_{N,N+1} \prod_{j=1}^{N} E_j \tag{5}
\]

where

\[
E_n = e^{i\varphi_n} \quad \text{and} \quad \varphi' = 4\pi a_n (\varepsilon_n - \sin \theta)^{1/2} / \lambda_0.
\]

Here \( a_n \) is an integer, which specifies the ratio of the electrical thickness, or optical path, of a layer relative to that of a certain minimum path. Thus the optical paths in all layers need not be identical.

Physically, \( R_{0,N+1}^* \) can be interpreted as a sum of waves of unit amplitude reflected once at each interface. \( R_{0,N+1}^* \) contains less than the first order reflection at each interface as consideration of the single layer case \( (N = 2) \) shows.

Hereafter are considered only symmetric structures consisting of several dielectrics on one side of a center layer and an identical set on the other side. This configuration is illustrated in figure 2. For such a structure

\[
R_{0,N+1}^* = r_{01} + r_{12} E_1 + \cdots + r_{N-1} E_i + \prod_{j=1}^{N} E_j \tag{6}
\]

For this case the reflection coefficient approximated as

\[
|R|^2 = A_0 + A_1 \cos \varphi' + A_2 \cos^2 \varphi' + \cdots + A_M \cos^M \varphi', \quad \tag{7}
\]

\( M \) is an integer determined by the number of layers and the \( a_n \). The \( A_n \) depend on the number of layers and on the \( a_n \). They are combinations (not linear) of Fresnel coefficients.

3. Approximation of specified transmission properties. — Suppose \( |R|^2 \) as defined in eq. (2) is desired as a specified function of frequency for a multilayer structure of the type in figure 2. Now expand \( |R|^2 \) as a Fourier polynomial, so that

\[
|R|^2 = \sum K \cos K \varphi'. \tag{8}
\]

Equate eqs. (7) and (8). Then

\[
B_n = A_n (r_{01}, r_{12}, \ldots r_{N,N+1}), \quad n = 0, 1, 2 \ldots M. \tag{9}
\]

Equations (9) are to be solved for the \( r_{ij} \), if possible.

The number of these equations depend on the electrical thicknesses of the various layers and the number of layers in the structure being considered. Since the \( r_{ij} \) depend on the angle of incidence, a solution must be effected for a fixed angle of incidence. From the \( r_{ij} \), the refractive indices or dielectric constants of the layers can be found.

In this procedure the number of assumed layers is arbitrary but fixed. This number determines the number of spectral components available in eq. (7) since it determines the number of eqs. (9) that have \( A_n \) unequal to zero. Equations (9) impose conditions on the \( r_{ij} \); however, it may not be possible to satisfy all these conditions in general.
Nevertheless, since this method is approximate, it may be sufficient to satisfy only some of the conditions, probably those involving only the larger $B_n$. Also, the range of $n$ over which the $B_n$ differ appreciably from zero may be consistent with a reasonable number of layers for certain classes of functions which specify $|R|^2$.

Thicknesses of the various layers can be found after the $r_L$ are known. With the form of eq. (8), the desired transmission, or reflection, properties, expressed through $|R|^2$ are periodic in $\varphi$ and hence in frequency. Layer thickness can be found from the factor changing the scale of the orthogonality interval from 0 to $2\pi$ for the variable $\varphi$ to 0 to $f_m$ for frequency where $f_m$ is the value of the frequency at the right end of the period. Now

$$\varphi' = 4\pi d_n (\varepsilon_n' - \sin^2 \theta)^{1/2} f c^{-1}$$

(10)

where $c$ is the speed of light. For $\varphi' = 2\pi$, $f = f_m$, so

$$2d_n = c (\varepsilon_n' - \sin^2 \theta)^{-1/2} f_m^{-1} = \lambda_m (\varepsilon_n' - \sin^2 \theta)^{-1/2}.$$ (11)

Thus $d_n$ is proportional to the wavelength corresponding to the frequency $f_m$.

Since eqs. (9) are simultaneous quadratic equations, their approximate solution, say by graphical means, may be all that is practical; however, these consistency requirements on the $r_L$ may not be too important, so far as practical design of a multilayer is concerned. It seems that in expressing requirements on $|R|^2$, functions should be used which have values of $B_n$ that are similar to the $A_n$ obtained from realizable multilayer structures. Some reasonable classes of functions are described in the next section.

4. Selection of functions for expressing transmission properties. — Consider eq. (7). By assuming values of indices and thicknesses for multilayers with a fixed number of layers, $|R|^2$ can be evaluated. From such a procedure, repeated for different numbers of layers, classes of function can be inferred which are reasonable for expressing $|R|^2$, which is defined in eq. (8). It is particularly convenient to consider structures such that only a single absolute value of Fresnel coefficient enters. There are two classes of this kind of structure. First, that in which are found only two values of the refractive index, one high and one low. For simplicity, one can neglect the difference between the lower index and the medium bounding the multilayer. Second, there is that class in which the index of any layer on one side of the center layer is the geometric mean of the indices of the layers bounding that layer. Both these configurations are well known in optics, in microwaves, and in radio.

Computations of $|R|^2$, defined in eq. (7), were made for several cases. These are described next.

First are those for a five layer structure with indices increasing geometrically toward the center and all layers of equal optical paths (or electrical thickness). Index profile and $|R|^2$ are shown in figure 3. For another five layer structure but now with alternately high and low indices, $|R|^2$ is shown in figure 4, for all layers of equal path. In the alternating index case but with the center layer twice as long in optical path as the other layers, $|R|^2$ is shown in figure 5. Increasing the number of layers can them be interpreted as increasing the number of spectral components in the trigonometric polynomial of eq. (7).
fig. 5. — Refractive index profile and approximate reflection coefficient $|R_2|^2$ of a five layer structure with alternately high and low indices. Center layer optical path twice that in any other layer.

Some variation is necessary in $|R|^2$ in order that physically realizable structures be found. A simple example illustrates this point. From a single interface only one term appears in $|R_{01}|^2$, it is a constant $r_{01}^2$, and would satisfy $|R|^2 = \text{const.}$

5. An example. — Transmission, hence reflection was specified through the following periodic function which in one period is defined, with $f$ being frequency as,

\[
|R|^2 = \begin{cases} 
0 & \text{for } 0 \leq f < 10^6 \text{ cps}, \\
1 & \text{for } 10^6 \text{ cps} < f < 5 \times 10^6 \text{ cps}, \\
0 & \text{for } 5 \times 10^6 \text{ cps} < f < 25 \times 10^6 \text{ cps}, \\
1 & \text{for } 25 \times 10^6 \text{ cps} < f < 29 \times 10^6 \text{ cps}, \\
0 & \text{for } 29 \times 10^6 \text{ cps} < f < 30 \times 10^6 \text{ cps}.
\end{cases}
\]

Recall that $R^2 = \Sigma B_k \cos K\varphi'$. The $B_k$ are shown in figure 8. A configuration was tried, for which indices increased according to the geometric mean rule toward the center. Three sets of optical paths for the layers were tried. These were in the ratios 11111, 11211 and 11311. For each of these sets of ratios of path, a set of $A_n$ results from eqs. (6) and (7). These $A_n$ are also plotted in figure 8. It would seem that the set of thicknesses should be used as a design for which the $A_n$ best fit the $B_n$. From figure 8 it seems the $A_n$ for the thickness set 11111 best fit the $B_n$ about equally and better than do the $A_n$ for the thickness set 1111.

Refractive indices can be determined because all absolute values of $r$ in eq. (6) are equal for the configuration attempted. By equating the $A_n$ in terms of $r^2$ and the corresponding $B_n$ for the trial

layer configuration of alternating indices. These are shown in figure 7. Note that $|R|^2$ and $|R_{01}|^2$ differ slightly near their zeros but differ appreciably for large $|R|^2$ and $|R_{01}|^2$. 

fig. 6. — Trial functions for expressing reflection coefficient requirements.

fig. 7. — Exact and approximate reflection coefficient against normalized frequency.

fig. 8. — Expansion coefficients for synthesis problem. The $B_n$ express the expansion of $|R|^2$, the desired performance. The $A_n$ represents the approximate reflection coefficient $|R_{01}|^2$ for an assumed five layer configuration. The three sets of $A_n$ are for optical path lengths in different ratios as shown.
function above, a set of values of \( r^2 \) can be found. Again this is for each choice of thickness ratios. That is, a different set is found for each choice of a set of layer thickness. Since \( r \) is to have a single absolute value, the best choice of thicknesses should be one such that a graph of \( r^2 \) is a straight horizontal line. From figure 9 it seems that thicknesses or optical paths for the various layers in the ratios 11211 and 11311 satisfy this condition better than does the choice 11111. From figure 9, mean values of \( r^2 \) were found for the three sets of thicknesses. Thus the \( E_n \) were determined. Thicknesses were found using eq. (11).

From the mean value of \( r^2 \), an approximate value of \( r \) leads to the dielectric constants (or refractive indices). This mean value of \( r \) will differ depending on the thickness ratios, hence a different set of indices will be found. For the three cases above parameter values are listed in Table 1.

![Fig. 9. — Calculated values of square of Fresnel interface coefficients. The best set of optical path lengths was chosen as that which led to a curve of \( r^2 \) most closely approximating a straight horizontal line.](image)

![Fig. 10. — Results of exact calculation for synthesis problem and desired transmission performance. The desired performance, i.e., design objective is indicated by the function with abrupt jumps. Exact calculations made on configurations obtained from the synthesis method described in the text are indicated by open or closed circles. Parameters used in calculations are given in Table 1. Thickness ratio refers to optical path lengths as a multiple of a minimum optical path. For all layers equally thick optically (11111), calculations made with the parameters of set \( A \) in Table 1 are indicated by the open circles; for set \( B \), the dots apply.](image)

<table>
<thead>
<tr>
<th>Layer Number</th>
<th>SET A ( 11211 )</th>
<th>SET B ( 11311 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.63</td>
<td>2.41</td>
</tr>
<tr>
<td>2</td>
<td>7.05</td>
<td>5.8</td>
</tr>
<tr>
<td>3</td>
<td>18.5</td>
<td>14.0</td>
</tr>
<tr>
<td>4</td>
<td>7.05</td>
<td>5.8</td>
</tr>
<tr>
<td>5</td>
<td>2.63</td>
<td>2.41</td>
</tr>
</tbody>
</table>

Exact computations of transmission are shown in figure 10, where the desired properties are shown also. Computations were made using a set of parameters for thickness ratios 11211 and another set for thickness ratios 11311. For the thickness ratios related as 11111, computations were made using both sets of dielectric constants for the 11211 and 11311 cases, with thickness adjusted accordingly. It is seen in figure 10 that desired transmission is approximated. In addition the parameters that were expected to give closer approximation do so.

The values of dielectric constant found in the center layer seem too high to be practical. More recent work with alternately high and low indices has yielded successful solutions of similar synthesis problems using the present method.

6. Another example. — Consider as a synthesis problem the transmission performance described in figure 11 by the abruptly varying function. Again assume a five-layer configuration but now with indices alternately high and low. By applying the procedure described in Section 5, dielectric constants and thicknesses were found. Dielectric constants were 2.5 and 1.0. The latter number represents an idealization, but is not unrealistic because certain microwave structures with dielec-
tric constants of 1.1 or lower are quite common. Slight differences such as these can be compensated by adjusting thickness according to eq. (11). Thicknesses found were 0.320 cm and 0.503 cm.

Calculations of transmission according to the exact theory are compared with the design objective in figure 11.

Fig. 11. — Results of exact calculation for second synthesis problem and desired transmission property. The abrupt curve is the desired property.

7. Comparison with measurement. — In figure 12 are shown results of measurements, of exact calculations, and of approximate calculations for a third example for normal incidence. Parameters were derived as in the preceding sections and are shown also in figure 12.

Fig. 12. — Results of measurement, exact calculation, and approximate calculation for a third example.

Discussion

M. VASICEK. — I hope this lecture hangs together with the lecture presented by Knittl to-day afternoon. The first papers about this synthesis were published by Pohlack about 1952.

M. WELFORD. — Many years ago Schröder calculated a polynomial approximation to the reciprocal of the transmission and he also found that this agreed very well with the exact computation.

Réponse : The utility of approximate polynomials seems greater than one might expect at first for such calculations.

REFERENCES