Dense Disparity Map Estimation Respecting Image Discontinuities : A PDE and Scale-Space Based Approach

Luis J. Alvarez, Rachid Deriche, Javier Sánchez, Joachim Weickert

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Dense Disparity Map Estimation Respecting Image Discontinuities: A PDE and Scale-Space Based Approach

Luis Alvarez — Rachid Deriche — Javier Sánchez — Joachim Weickert

N° 3874

January 2000

THÈME 3

Rapport de recherche

ISSN 0249-6399 ISRN INRIA/RR--3874--FR+ENG

Dense Disparity Map Estimation Respecting Image Discontinuities: A PDE and Scale-Space Based Approach

Luis Alvarez * , Rachid Deriche † , Javier Sánchez ‡ , Joachim Weickert §

Thème 3 — Interaction homme-machine,
images, données, connaissances
Projets Robotvis

Rapport de recherche n° 3874 — January 2000 — 31 pages

Abstract: We present an energy based approach to estimate a dense disparity map between two images while preserving its discontinuities resulting from image boundaries. We first derive a simplified expression for the disparity that allows us to easily estimate it from a stereo pair of images using an energy minimization approach. We assume that the epipolar geometry is known, and we include this information in the energy model. Discontinuities are preserved by means of a regularization term based on the Nagel–Enkelmann operator. We investigate the associated Euler–Lagrange equation of the energy functional, and we approach the solution of the underlying partial differential equation (PDE) using a gradient descent method. In order to reduce the risk to be trapped within some irrelevant local minima during the iterations, we use a focusing strategy based on a linear scale-space. We prove the existence and uniqueness of the underlying parabolic partial differential equation. Experimental results on both synthetic and real images are presented to illustrate the capabilities of this PDE and scale-space based method.

Key-words: Stereoscopic Vision, Disparity Map, Anisotropic Diffusion, Scale-Space, Partial Differential Equations, Regularization, Finite Difference Methods

* Departamento de Informática y Sistemas - Universidad de Las Palmas, Campus de Tafira - SP-35017 Las Palmas, Spain - E-Mail: lalvarez@dis.ulpgc.es - WWW:http://serdis.dis.ulpgc.es/∼*lalvarez*

† INRIA Sophia-Antipolis - 2004, Route des Lucioles - F-06902 Sophia-Antipolis, France - E-Mail: Rachid.Deriche@sophia.inria.fr - WWW: http://www-sop.inria.fr/robotvis/personnel/der-eng.html

‡ Departamento de Informática y Sistemas - Universidad de Las Palmas, Campus de Tafira - SP-35017 Las Palmas, Spain - E-Mail: jsanchez@dis.ulpgc.es - WWW:http://serdis.dis.ulpgc.es/∼*jsanchez*

§ Computer Vision, Graphics, and Pattern Recognition Group - Department of Mathematics and Computer Science - University of Mannheim - D-68131 Mannheim, Germany - E-mail: Joachim.Weickert@ti.uni-mannheim.de - WWW:http://www.ti.uni-mannheim.de/∼bmg/weickert

Estimation Dense de Cartes de Profondeur avec Préservation des Discontinuités Images: Une approche Multi-Echelle à Base d'EDP

Résumé : Ce rapport traite du problème de l'estimation dense de cartes de profondeur à partir d'une paire d'images obtenues à l'aide d'un système stéréoscopique faiblement calibré. L'accent est principalement mis sur une estimation en accord avec les discontinuités en niveaux de gris présentes dans les images et sur l'analyse mathématique de l'EDP associée à la résolution de ce problème. Nous dérivons en premier une grandeur, reliée à la disparité, qui nous permet de formuler le problème de son estimation dans un cadre variationnel de manière simple et efficace. On suppose que la géométrie épipolaire est connue, et on intègre cette information dans le critère variationnel, avec une contrainte sur la grandeur à estimer afin ses discontinuités correspondent au mieux à ceux observés sur les images. Ceci est mis en oeuvre au travers d'un terme de régularisation basé sur l'opérateur de Nagel-Enkelmann, bien adapté à ce type de contrainte. L'équation d'Euler-Lagrange associée est dérivée; l'existence et l'unicité de la solution de l'EDP parabolique associée est prouvée et sa solution approchée par une technique de descente de gradient. Une approche multi-échelle linéaire est enfin mise en oeuvre afin d'augmenter les performances en convergence de la méthode proposée. Plusieur résultats expérimentaux, obtenus sur des images synthétiques et réelles, illustrent les différentes potentialités de cette nouvelle approche qui permet d'intégrer dans un même formalisme le problème de l'estimation de cartes de profondeur et celui de la prise en compte des discontinuités en niveau de gris dans les images.

Mots-clés : Vision stéréoscopique, Carte de disparité, Diffusion Anisotrope, Multi-Echelle, Équations aux Dérivées Partielles, Régularisation, Méthodes aux Différences Finies

1 Introduction

We present a variational approach to recover a dense disparity map from a set of two *weakly calibrated* stereoscopic images. To solve this problem, we first make full use of the knowledge of the so-called *fundamental matrix* to derive the equations that relate corresponding pixels in the two views, and then combine regularization and scale-space tools to estimate iteratively and hierarchically the disparity map. The solution obtained at a coarse spatial scale is used to restrict searching at finer scales. We minimize an energy term that takes into account the epipolar line constraint as well as the edge information constraint through an appropriate regularization term.

Over the years, numerous algorithms for stereo vision have been proposed, which use different strategies.

- **Feature based:** Those algorithms establish correspondences between some selected features extracted from the images [22], such as edge pixels [41, 43], line segments [5, 31, 32] or curves [11, 39, 47]. Their main advantage is to yield accurate information and to manipulate reasonably small amounts of data, thus gaining in time and space complexity. Their main drawback is the sparseness of the recovered depth information. This class of methods has been widely used some years ago, when it was not possible to retrieve a dense and accurate reconstruction within a reasonable amount of time.

- **Area based:** In these approaches, dense depth maps are provided by correlating the grey levels of image patches in the views being considered, assuming that they present some similarity [14, 17, 21, 40]. These methods are well adapted for relatively textured areas; however, they generally assume that the observed scene is locally fronto-parallel, which causes problems for slanted surfaces and in particular near the occluding contours of the objects. Lastly, the matching process does not take into account the edge information which is actually a very important information that should be used in order to get reliable and accurate dense maps.

- **Phase based:** Another class of methods is based on the Fourier phase information, which can be considered as a sort of gradient-based optical flow method, with the time-derivative approximated by the difference between the left and right Fourier-phase images [7, 18, 19, 26, 29, 58]. Hierarchical methods are also used here in order not to get trapped in some local minima.

- **Energy based:** A last kind of approach which does not suffer any of the shortcomings presented above, consists of solving the correspondence problem in a minimization and regularization formulation [6, 8, 24, 45, 46, 51, 59]. An iterative solution of the discrete version of the associated Euler–Lagrange equation is then used in order to estimate depth. For instance, in [46], the authors propose a method to directly compute the depth map $Z(x, y)$ as the minimum of the following energy:

$$S(Z) = \iint \left(I_l(x,y) - I_r(f_1(x,y,Z(x,y))\right)^2 \, dx\, dy + \nu \iint \Phi(|\nabla Z|)\, dx\, dy$$

where $f_1(.,.,.)$ depends on the camera intrinsic and extrinsic parameters derived from an off-line calibration step. The minimum of the above energy is calculated using the associated Euler–Lagrange equation.

The method we present in this paper follows the last strategy and can be considered in some sense as a generalization of the work presented in [46] to the *weakly calibrated* case, with the following important originalities:

- As it has been already said, the method presented in this article considers a *weakly calibrated* stereoscopic system, i.e the stereoscopic system is not calibrated, as in [46], and only the knowledge of the so-called *fundamental matrix* is known. The intrinsic and extrinsic parameters of the camera are not known.
- Consequently, one computes a sort of *projective* depth directly related to the well-known *disparity* measure from the grey-level images intensities, while in [46], the authors directly estimate the depth information. We do not have to deal with any intermediate process such as rectification [4, 44, 48, 60]. The *projective depth* information is directly issued as a function $m \rightarrow D(m)$ of the image point.
- This method addresses the problem of accurately determining the dense disparity map while smoothing and regularizing this disparity map along the contours of the grey level image and inhibiting smoothing across the *image* discontinuities. In [46], the regularization process was performed in order to inhibit smoothing the depth map across the estimated *depth* discontinuities. On the other hand, in $S(Z)$ the preservation of discontinuities in the depth Z is obtained by the *nonlinear* diffusion operator associated to the function Φ, while in our method, the preservation of discontinuities in the disparity map is obtained using an anisotropic *linear* operator which allows to develop discontinuities in the disparity map across the edges of the image I_l. This is achieved by considering a Nagel–Enkelmann regularization term that has already proven to be very useful in optical flow estimation [2, 36].
- Another important item is related to the way we proceed in order to avoid converging to irrelevant minima. A focusing strategy embedding the method in a linear scale-space is used, as it has already been successfully applied in optical flow estimation [2, 3]. This also relates our work to stereo methods using linear scale-space such as [27, 30, 37, 38].
- Finally, in order to show that our method leads to a mathematically correct concept, we prove the existence and uniqueness of the solution of the parabolic equation which governs our method.

The paper is organized as follows: In Section 2, we present the formalism of the matching process and discuss all relevant features of our model. In Section 3, we prove the existence and uniqueness of the solution of the underlying parabolic partial differential equation. In Section 4, we discuss numerical aspects of the algorithms, and in Section 5 we present