Nodal statistics-based structural pattern detection for graph collections characterization
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Motivations
- Human consciousness states can be differentiated by nodal organization in brain functional connectivity networks [1]
  - structural pattern definition
  - extend node role discovery to graph collections [2, 3]
  - graph collections comparison based on structural pattern

Nodal statistics-based structural pattern on single graph

Def. Nodal statistics based equivalence relation. We consider undirected unweighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and refer to a nodal statistics $s : \mathcal{V} \rightarrow s(\mathcal{V})$ any function of the adjacency matrix. The equivalence relation $\sim_s$ associated with a nodal statistics $s$, on the nodes set $\mathcal{V}$ of a graph is:

$$v \sim_s u \iff s(u) = s(v).$$

The equivalence relation associated with any collection of statistics $\mathcal{S} = \{s_i\}_{i=1}^{n}$, is defined as:

$$a \sim_{\mathcal{S}} b \iff a \sim_{s_i} b, \quad \text{for all } i = 1, \ldots, n. \quad (1)$$

Def. Structural Pattern. Its induced partition $P$ on $\mathcal{V}$,

$$P_{\mathcal{S}} = \frac{\mathcal{V}}{\sim_{\mathcal{S}}} = \{[a], \forall a \in \mathcal{V}\},$$

defines the structural pattern of $\mathcal{G}$ associated with the statistics collection $\mathcal{S}$.

Def. Node role. The class of equivalence $[a] = \{b \in \mathcal{V} | a \sim_{\mathcal{S}} b\} \iff s(a) = s(b)$ corresponds to node role.

Structural patterns for graph collections characterization

Def. Correspondence structural pattern score. Let $\mathcal{G}, \mathcal{G}'$ be two graphs having same vertices $\mathcal{V}$ and let $\mathcal{S}$ be a statistics collection whose associated partitions are $P_{\mathcal{S}}, P'_{\mathcal{S}}$ on $\mathcal{G}, \mathcal{G}'$ respectively. Given bijective mapping from $P_{\mathcal{S}}, P'_{\mathcal{S}}$ to an initial segment of the natural numbers as enumerations, let $c(\mathcal{v}_i), c'(\mathcal{v}_i)$ be the enumeration of the classes of $\mathcal{v}_i$, the correspondence structural pattern score between $\mathcal{G}, \mathcal{G}'$ is defined as:

$$C(\mathcal{G}, \mathcal{G}') = \max_{\Gamma} \frac{1}{|\mathcal{V}|} \sum_{i=1}^{|\mathcal{V}|} \lambda(\tau(c(\mathcal{v}_i)) = c'(\mathcal{v}_i)) \quad (3)$$

where $\pi$ is the set of all coupling between the elements in $P_{\mathcal{S}}$ and the elements in $P'_{\mathcal{S}}$ and $\lambda$ is the indicator function.

Def. Nodal-percentage of participation. Given a graphs collection $\mathcal{G} = \{\mathcal{G}_i = (\mathcal{V}_i, \mathcal{E}_i)\}_{i=1}^{N}$ and a statistics collection $\mathcal{S}$ we count the percentage of participation of each node of $\mathcal{V}$ in non-trivial classes:

$$\forall v \in \mathcal{V}, \quad \text{PP}_{\mathcal{S}}(v) = \frac{1}{|\mathcal{G}|} \sum_{i=1}^{|\mathcal{G}|} \mathbb{I}(v \in \mathcal{G}_i \cap \#\mathcal{V}_i \neq 0)$$

Nodal-percentage of participation captures brain homotopy

The symmetry in real data and brain models reveals the expected hemisphere similarity in the participation of analogue regions.

References

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