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1 Self-similar solutions of shallow water equations with porosity

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17 ABSTRACT

18 Simulated free surface transients in periodic urban layouts have been reported to be self-similar in the
19 space-time domain when averaged on the scale of the building period. Such self-similarity is
20 incompatible with the head loss model formulae used in most porosity-based shallow water models.
21 Verifying it experimentally is thus of salient importance. New dam-break flow laboratory experiments
22 are reported, where two different configurations of idealised periodic buildings layouts are explored. A
23 space-time analysis of the experimental water level fields validates the self-similar character of the
24 flow. Simulating the experiment using the two-dimensional shallow water model also yields self-
25 similar period-averaged flow solutions. Then, the Single Porosity (SP), Integral Porosity (IP) and Dual
26 Integral Porosity (DIP) models are applied. Although all three models behave in a similar fashion when
27 the storage and connectivity porosities are close to each other, the DIP model is the one that upscales
28 best the refined 2D solution.

29 Keywords: Urban floods, scale model, porosity model, flux, source term

30 1 Introduction

31 Shallow water models with porosity have arisen over the past two decades as an alternative to
32 classical two-dimensional shallow water models for the modelling of flows over complex
33 topography and in complex geometries (Bates, 2000; Defina, 2000). Urban flood modelling has
34 become a typical application field of such models (Chen et al., 2012; Guinot and Soares-Frazão,
35 2006; Hervouët et al., 2000; Özgen et al., 2016b; Sanders et al., 2008; Schubert and Sanders,
36 2012; Soares-Frazão et al., 2008; Viero and Valipour, 2017). Many variations of porosity
37 models have been presented in the literature. The Single Porosity (SP) model (Bates, 2000;
38 Defina, 2000; Guinot and Soares-Frazão, 2006; Hervouët et al., 2000) uses a single porosity to
39 describe the flow storage and connectivity properties of the urban medium. The Integral
40 Porosity (IP) model (Sanders et al. 2008) uses a storage and a connectivity porosity to
41 incorporate the effects of urban anisotropy in the governing equations (Kim et al., 2015).
42 Anisotropic conservation laws may also be defined to account for anisotropy in SP models
43 (Viero and Valipour, 2017; Viero 2019) via a tensor formulation (Ferrari and Viero, 2020) or
44 through a binary porosity indicator (Varra et al., 2020). A depth-dependent version is presented

45 in Özgen et al. (2016a). The Multiple Porosity model (MP) uses a partition of the urban domain
46 into several flow regions, with different degrees of connectivity and anisotropy (Guinot, 2012).
47 The approach bears similarities with the multilayer BCR/CRF approach proposed in Chen et
48 al. (2012). The Dual Integral Porosity (DIP) model (Guinot et al., 2017) has been proposed as
49 a variation of the IP model, with improved flux and source term formulae. A model with three
50 porosity parameters has been proposed recently (Bruwier et al., 2017). Higher-order numerical
51 schemes are proposed to further increase the accuracy of the results especially in cases with
52 topographies (Ferrari et al., 2020).

53 Two key issues arise in the derivation and application of porosity-based shallow
54 water models. The first is the flux model, the second is the source term model. The flux
55 model has a direct influence on the wave propagation properties of the flow solutions. Very
56 different wave propagation speeds are obtained for the SP, IP and DIP models (Guinot et al.,
57 2017). This suggests that experiments featuring wave propagation properties may be
58 instrumental in discriminating the various porosity models. Many models have been proposed
59 for the source term arising from building drag, from drag coefficient-based formulae (Sanders
60 et al., 2008) to drag tensors (Guinot et al., 2017) and generalized tensor formulations
61 (Velickovic et al., 2017). All these models take the form of Equations Of State (EOS)
62 involving the flow variables (the water depth, the flow velocity or unit discharge vector) that
63 can be validated against steady flow experiments (see Velickovic et al., 2017). However,
64 benchmarking the various available formulations with refined, transient flow simulations
65 (Guinot, 2017; Guinot et al., 2017) indicates that an additional momentum dissipation
66 mechanism is at work in the presence of positive transients. This momentum dissipation
67 mechanism makes the wave propagation speeds different depending on whether the flow is
68 falling or rising (Guinot et al., 2017). This effect had been foreseen in Guinot (2012) to arise
69 from a particular configuration of the MP model. It cannot be modelled by a momentum
70 source term in the form of an EOS because it acts directly on the wave propagation speeds of
71 the transients. In the presence of sharp transients propagating along the preferential directions
72 of the urban layout, this momentum dissipation term exerts a significant influence on the
73 simulated flow behaviour (Guinot, 2017). However, to date, no experimental evidence of such
74 a mechanism has been provided. While successful applications of porosity models against
75 experimental data sets have been presented in the literature (Kim et al., 2015; Özgen et al.,
76 2016b), the number of building blocks involved is often too small to allow for a conclusive
77 interpretation of the results.

78 The present paper serves three objectives. The first is to present a data set for dam-
79 break scale model experiments in idealized urban layouts. This data set is to be used for
80 model validation and/or benchmarking. The modelled urban layout counts up to 20 building
81 periods in the longitudinal direction, which, to our best knowledge, is the largest number of
82 block periods used in transient, scale model experiments. The second objective is to
83 benchmark the SP, IP and DIP models against these experiments and to assess the validity of
84 the abovementioned transient momentum dissipation mechanism. The third objective is to
85 check whether refined 2D shallow water models are accurate enough to reproduce the dam-
86 break experiments reported here and may therefore be used to produce reference solutions for
87 the benchmarking of porosity models.

88 Section 2 presents the models used in the benchmarking phase and the theoretical
89 implications of the source term formulation on solution properties. Section 3 presents the
90 experimental setup and results. Section 4 is devoted to model benchmarking. Conclusions are
91 given in Section 5.

92 2 Porosity models and solution self-similarity

93 2.1 Porosity models

94 The present subsection details the various formulations explored in the present work. Three
 95 different porosity models are considered: the Single Porosity (SP) model, the Integral Porosity
 96 (IP) model, and the Dual Integral Porosity (DIP) model. The analysis is restricted to one-
 97 dimensional flow configurations for the sake of consistency with the experiments reported
 98 hereafter. The governing equations for the three models can be written as

$$99 \quad \partial_t \mathbf{u} + (\mathbf{I} - \mathbf{M}) \partial_x \mathbf{f}(\mathbf{u}) = \mathbf{s} \quad (1a)$$

$$100 \quad \mathbf{u} = [\phi_\Omega h, \phi_\Omega hu]^T, \quad \mathbf{s} = [0, s_x]^T \quad (1b)$$

$$101 \quad \mathbf{f} = \begin{cases} \phi_\Omega [hu, hu^2 + gh^2/2]^T & \text{(SP)} \\ \phi_T [hu, hu^2 + gh^2/2]^T & \text{(IP)} \\ \left[\phi_\Omega hu, \frac{\phi_\Omega^2}{\phi_T} hu^2 + \phi_T gh^2/2 \right]^T & \text{(DIP)} \end{cases} \quad (1c)$$

$$102 \quad s_x = g\phi_\Omega h(S_0 - S_f) - C_D h|u|u \quad (1d)$$

$$103 \quad \mathbf{M} = \mu \begin{bmatrix} 0 & 0 \\ 0 & \mu \end{bmatrix}, \quad \mu = \begin{cases} 0 & \text{if } \partial_t h \leq 0 \\ 1 & \text{if } \partial_t h > 0 \end{cases} \quad (1e)$$

104 where \mathbf{u} , \mathbf{f} and \mathbf{s} are respectively the conserved variable, flux and source term vectors, \mathbf{I} and
 105 \mathbf{M} are respectively the identity and momentum dissipation matrices, C_D is the building drag
 106 coefficient per unit depth, g is the gravitational acceleration, h is the water depth, S_0 and S_f are
 107 respectively the bottom slope and bottom friction slope, u is the x -flow velocity, ϕ_Ω and ϕ_T
 108 are respectively the storage and connectivity porosities, and μ is a dimensionless coefficient
 109 between 0 and 1 accounting for momentum dissipation. The shallow water equations are
 110 obtained as a particular case of the SP, IP and DIP models by setting $\phi_\Omega = \phi_T = 1$ and

111 $C_D = \mu = 0$ in equations (1a-c). The SP model is a particular case of both the IP and DIP

112 models with $\phi_\Omega = \phi_T$. The main difference between the three models lies in how the

113 connectivity porosity ϕ_T acts on the fluxes. Originally introduced by Sanders et al. (2008),

114 the connectivity porosity is the fraction of the frontal area available for mass and momentum

115 transfer. While it is not relevant to the SP model, ϕ_T exerts a salient influence on the wave

116 propagation speeds of the IP and DIP models. From a physical point of view, it should be

117 taken smaller than ϕ_Ω in the IP model, so as to account for the effects of building obstruction

118 on the flow. Using $\phi_\Omega < \phi_T$ in the IP model is known to increase the propagation speed of the

119 waves artificially. In the DIP model, the configuration $\phi_\Omega < \phi_T$ is not permissible because it

120 yields complex-valued wave speeds (Guinot et al., 2017), with the consequence that

121 hyperbolicity is lost and initial- and boundary-value problems become ill-posed.

122 Two types of source term models have been proposed so far for porosity models: turbulent

123 head loss model proportional to the square or another power of the flow velocity (Guinot and

124 Soares-Frazão, 2006; Özgen et al., 2015; Sanders et al., 2008; Velickovic et al., 2017), and

125 transient momentum dissipation models, active only when transients involving positive waves

126 arise (Guinot, 2012; Guinot et al., 2017). In one-dimensional flow configurations, as explored

127 in the present work, turbulent head loss models can be written as in equation (1d). This type

128 of model, however, has been shown to be insufficient to reproduce the behaviour of transients

129 involving positive waves (Guinot et al., 2017; Guinot, 2017). For this reason, the transient

130 momentum dissipation model (1a, e) has been proposed in Guinot et al. (2017). This model
 131 was validated against systematic refined 2D simulations of urban dam-break problems in
 132 (Guinot, 2017). A salient feature of this model is that it leads to self-similar solutions when
 133 applied to the solution of Riemann problems, a feature that was identified in (Guinot, 2012)
 134 and confirmed in (Guinot, 2017). The next subsection is devoted to the analysis of self-similar
 135 solutions.

136 2.2 Source term and solution self-similarity

137 The objective of the present section is to derive the conditions under which the solutions of
 138 the one-dimensional dam-break problem are self-similar when computed using porosity
 139 models in the presence of source terms. A solution \mathbf{u} of the governing equations (1a-e) is said
 140 to be self-similar in (x, t) if it verifies the following condition:

$$141 \quad \mathbf{u}(x, t) = \mathbf{v}(x/t) \Leftrightarrow \mathbf{u}(kx, kt) = \mathbf{u}(x, t) \quad \forall k \quad (2)$$

142 It is inferred from equation (2) that \mathbf{u} is constant along any straight line originating from the
 143 origin $(0, 0)$ in the (x, t) plane (Figure 1).

144 Consider first equation (1a) without the momentum dissipation matrix \mathbf{M}

$$145 \quad \partial_t \mathbf{u} + \partial_x \mathbf{f}(\mathbf{u}) = \mathbf{s} \quad (3)$$

146 The exact expression for the flux function \mathbf{f} (SP, IP or DIP in equation (1c)) needs not be
 147 known at this stage. For a periodic one-dimensional building layout, the flux \mathbf{f} is a function of
 148 \mathbf{u} alone because the porosities can be taken uniform, by setting the averaging domains equal
 149 to one spatial period. Solutions of the one-dimensional, initial value dam-break problem fulfil
 150 equation (3) with the following initial conditions:

$$151 \quad \mathbf{u}(x, 0) = \begin{cases} \mathbf{u}_L & \text{for } x < 0 \\ \mathbf{u}_R & \text{for } x > 0 \end{cases} \quad (4)$$

152 Solution self-similarity is investigated as in Lax (1957). Introducing the variable
 153 change $(X, T) = (kx, kt)$ where k is an arbitrary constant, using the chain rule, dividing by k
 154 leads to

$$155 \quad \partial_T \mathbf{u} + \partial_X \mathbf{f}(\mathbf{u}) = \frac{1}{k} \mathbf{s} \quad (5a)$$

$$156 \quad \mathbf{u}(X, 0) = \begin{cases} \mathbf{u}_L & \text{for } X < 0 \\ \mathbf{u}_R & \text{for } X > 0 \end{cases} = \mathbf{u}(x, 0) \quad (5b)$$

157 In the case $\mathbf{s} = 0$, equations (5a-b) are equivalent to equations (3, 4). Consequently,
 158 $\mathbf{u}(X, T) = \mathbf{u}(x, t)$ and the self-similarity property (2) is valid.

159 When the source term \mathbf{s} is non-zero, the solutions of the IVP (5a-b) are not
 160 necessarily self-similar. If \mathbf{s} obeys an equation of state, that is, $\mathbf{s} = \mathbf{s}(\mathbf{u})$, equation (5a) is not
 161 invariant with respect to the scaling factor k . In contrast, self-similarity is preserved if the
 162 source term takes the form proposed in Guinot et al. (2017):

$$163 \quad \mathbf{s} = \mathbf{M}(\mathbf{u}) \partial_x \mathbf{f}(\mathbf{u}) \quad (6)$$

164 where \mathbf{M} is a matrix, the coefficients of which are functions of \mathbf{u} . Self-similarity is proved by
 165 introducing again the variable change $(X, T) = (kx, kt)$ into Equation (6). The chain rule

166 $\partial_x = \partial_X X \quad \partial_x = k \partial_X$ yields the following property for \mathbf{s}

$$167 \quad \mathbf{s} = \mathbf{M}(\mathbf{u}) k \partial_X \mathbf{f}(\mathbf{u}) = k \mathbf{M}(\mathbf{u}) \partial_X \mathbf{f}(\mathbf{u}) \quad (7)$$

168 Substituting equation (7) into (5a) gives

$$169 \quad \partial_T \mathbf{u} + \partial_x \mathbf{f}(\mathbf{u}) = \mathbf{M}(\mathbf{u}) \partial_x \mathbf{f}(\mathbf{u}) = \mathbf{s} \quad (8)$$

170 Equation (8) is identical to Equation (3). The form (6) thus leaves equation (5a) invariant with
 171 respect to the scaling factor k and the self-similarity property (2) is verified. A transient
 172 source term in the form (6) is proposed in the DIP model (Guinot et al., 2017) to account for
 173 the dissipation of momentum originating from positive transients propagating into building
 174 block layouts. Comparisons of DIP model outputs to pore-scale averaged refined shallow
 175 water simulations show that this source term is active only for rising water levels and that
 176 only this form of source term allows the wave propagation properties of transients to be
 177 preserved and accurate solutions to be reproduced (Guinot et al., 2017). Note that this source
 178 term alone is not sufficient to account for all energy losses because it is zero under steady
 179 state conditions, while significant steady state head losses have been reported when the flow
 180 is not aligned with the main street directions (Velickovic et al., 2017). However, it is
 181 predominant in the case of sharp transients propagating along the main street directions.

182 The self-similar behaviour of the solutions of the Riemann problem, if verified in
 183 reality, may serve as a powerful tool for discriminating between alternative momentum source
 184 term closures in shallow water models with porosity.

185 3 Experiments

186 3.1 Experimental set-up

187 Dam-break flow experiments were conducted at the Hydraulics Laboratory of the Institute of
 188 Mechanics, Materials and Civil Engineering (Université catholique de Louvain, Belgium).
 189 The flume is 29.5 m long and 1 m wide. The bed is horizontal and its Manning friction
 190 coefficient is estimated to be $n = 0.011 \text{ s m}^{-1/3}$ from previous experiments (e.g. Soares-Frazão
 191 and Zech 2008, Velickovic et al., 2017). Two periodic block configurations were studied. In
 192 Configuration 1 (Figure 2), the blocks are placed in such a way that 3/4 of the flume section is
 193 obstructed and each block occupies a fraction 6/16 of the plan view area, therefore 62.5 % of
 194 the plan view area is available to water storage, and 25 % of the model width is available to
 195 the flow. This yields $(\varphi_\Omega, \varphi_T) = (0.625, 0.25)$. In Configuration 2 (Figure 3), the blocks
 196 occupy 30% of the total plan view area, thus making 70 % of the total area available for water
 197 storage, and half of the cross-section is periodically obstructed by the blocks. Consequently,
 198 $(\varphi_\Omega, \varphi_T) = (0.7, 0.5)$. The blocks are made of marine plywood presenting a very smooth
 199 surface, so that no additional friction effects are expected. The water was initially at rest in
 200 the upstream part of the flume, with an initial depth $h_0 = 0.350 \text{ m}$. In the downstream part, a
 201 thin layer of water was present, with a depth ranging from 5 mm to 1.5 cm depending on the
 202 replicate. Since several experimental replicates were performed, the downstream depth,
 203 stemming from the residual water from the previous replicates, was not exactly the same for
 204 all replicates. The upstream and downstream sections of the flume were separated by a gate
 205 located 9.73 m from the upstream end of the flume. The gate was lifted very quickly (within
 206 less than 0.5 s) to simulate the breaking of a dam. The water level at each gauging point was
 207 measured using ultrasonic probes (Baumer) placed at a fixed distance above the flume. These
 208 probes record the distance to the water surface with an accuracy of 0.1 mm and a maximum
 209 temporal resolution of 0.05 s, depending on the number of simultaneous measurements. In
 210 addition to the probes located in the flume, a probe was placed in front of the gate in order to
 211 identify the starting time of the experiment ($t = 0$), thus allowing the signals collected from
 212 the other probes to be synchronized.

213 The water levels were recorded every 0.2 s using four probes, at 114 and 97 gauging points
 214 for Configurations 1 and 2 respectively, resulting in more than 300 experimental runs. The
 215 time span involved in these plots is [0 s, 30 s], with one block-averaged value every second.
 216 The gauging points are located along the main flow path and inside the cavities between two
 217 successive blocks (see Figure 2 for Configuration 1, Figure 3 for Configuration 2). The
 218 probes are labelled in the following way: PU x_y for a probe located in the upstream part at the
 219 coordinates $(-x, y)$ and PD x_y for a probe located in the downstream part at the coordinates
 220 (x, y) . For example, probe PD1_125 is located in the downstream part, at $(x, y) = (1, 0.125)$
 221 where the coordinates are expressed in m. Sample locations of the probes are given in Table 1
 222 for Configuration 1 and Table 2 for Configuration 2.

223 The repeatability of the experiments was checked, so that measurements obtained
 224 from different runs could be combined. Indeed, more than 300 experimental runs in total were
 225 needed to cover the whole measurement range, so it is mandatory to check for the
 226 repeatability of the experiments in order to combine the results of the different runs into a
 227 single data set. The repeatability at probe PD2_125 is illustrated in Figure 4 for three different
 228 runs.

229 3.2 Results

230 The purpose is to check the self-similar character of the large-scale water depth field. To do
 231 so, it is necessary to devise a procedure to compute the block-averaged, experimental water
 232 depth fields. The block-averaged water depths are computed using all the experiment
 233 replicates:

$$234 \quad h_{\text{av}}(x_i, t) = \frac{1}{R} \sum_{j=1}^R \sum_{k=1}^N w_k h_{i,k,j}(x, t), \quad \sum_{k=1}^N w_k = 1 \quad (9)$$

235 where $h_{i,k,j}$ is the water depth measured by the k th probe in the i th block period during the j th
 236 replicate, N and R are respectively the number of probes per block period and the number of
 237 experiment replicates, and w_k is the weight of the k th probe within a given block period. The
 238 averaging domain is centred around the lateral street, as shown in Figure 5. Figure 5 also
 239 shows the numbering scheme for the probes in Configuration 1. The sensitivity of the
 240 averaging results to the distribution of the weights w_k is assessed by using two strongly
 241 contrasted weight distributions, labelled A and B. The weight distributions for these two
 242 weighting approaches are given in Table 3. Approach A gives equal weight to all probes,
 243 while Approach B gives a 50% weight to the average of the probes within the lateral street,
 244 and 1/4 to each of the probes at the boundary of the averaging domain. Figure 6 shows the
 245 behaviour of h_{av} obtained for the two weighting approaches. The two curves are very close to
 246 each other, with differences much smaller than the measurement precision of the probes.
 247 Since no significant difference is observed between the two approaches, the equal weight
 248 approach is retained for Configuration 2.

249 The block-averaged water depth is plotted as a function of x/t for all block periods
 250 in Figure 7. Note that the time interval used for these plots is [0 s, 11 s] for Configuration 1
 251 and [0 s, 9 s] for Configuration 2. This is because after these times, the dambreak waves are
 252 observed to reach the downstream or upstream end of the flume. The flow pattern at later
 253 times switches from an initial value problem to a boundary value problem and studying
 254 solution self-similarity becomes meaningless. Although the plots in Figure 7 seem to indicate
 255 that the experimental profiles gather along a single S-shaped curve in the $(x/t, h_{\text{av}})$ plane, a
 256 certain amount of scattering is observed in the experimental data. In the case of a perfectly

257 self-similar solution, all experimental points should fall exactly onto a single curve. The
 258 question then arises whether the present scattering can be explained by experimental
 259 uncertainty.

260 This question is answered by incorporating the error boxes in the experimental plots.
 261 This is done in Figure 8 for Configuration 1 and Figure 9 for Configuration 2. The reader is
 262 referred to the electronic version of the paper, where the various series are plotted using
 263 different colours to allow for a better identification of the series. The experimental uncertainty
 264 for probe positioning was $\Delta x = \pm 1$ cm .

265 The time resolution of the data logger is less than 1 s. But the uncertainty stemming
 266 from the estimation of the starting time ($t = 0$), identified using the probe placed in front of
 267 the gate, is estimated as ± 2 s.

268 The uncertainty $\Delta h = \pm 1$ cm in the water depth is inferred from the min-max
 269 difference between the individual time series recorded by the probes. Let $h_{av}(x, t, j)$ be the
 270 average water depth obtained at location x_i at time t for the j th replicate. The uncertainty in
 271 the water depth is obtained by taking the difference between the maximum and minimum
 272 water depths obtained from the various replicates at the same time and location

$$273 \quad \Delta h = \max_{x_i, t} \left(\max_j (h_{av}(x_i, t, j)) - \min_j (h_{av}(x_i, t, j)) \right) \quad (10)$$

274 This uncertainty is significantly larger than the measurement precision of the probes. It only
 275 illustrates the impossibility of obtaining perfectly replicable experiments. This is due to many
 276 factors. To start with, small time difference in the water depth time series from one replicate
 277 to the other may result in large depth differences because sharp fronts are dealt with.

278 Moreover, oscillations with an amplitude up to 1.5 cm were observed in the upstream part of
 279 the experimental device during the filling phase, which makes it very difficult to guarantee
 280 that the upstream water level was exactly the same for all replicates. Lastly, a small leakage
 281 was observed across the gate during the filling phase before the replicates. Consequently, the
 282 downstream part of the channel was never completely dry between successive replicates. Also
 283 note that a number of outliers had to be removed from the experimental data set (in some
 284 occasions, initially dry beds were recorded by the probes as having an initial depth of -1.5
 285 cm). As shown in Figure 8, for Configuration 1 it is possible to draw a curve intersecting all
 286 the experimental error boxes, except for a few block periods. The curve is a spline that is
 287 fitted by minimizing the following objective function.

$$288 \quad J = \sum_{i,j,k} e_{i,j,k} \quad (11a)$$

$$289 \quad e_{i,j,k} = \max \left(0, h_{i,j,k} - \frac{\Delta h}{2} - h_{\text{fit}}(x_i, t), h_{\text{fit}}(x_i, t) - h_{i,j,k} - \frac{\Delta h}{2} \right) \quad (11b)$$

290 Where $h_{\text{fit}}(x_i, t)$ is the fitted function. The fitting error $e_{i,j,k}$ is zero if the fitted h lies within
 291 the experimental error box, indicating that the difference between the fitted value and the
 292 experimental one can be explained by the experimental imprecision. If the fitted function
 293 value $h_{\text{fit}}(x_i, t)$ lies outside the experimental error box, it means that the experimental
 294 imprecision cannot explain fully the difference between the experimental values and the fitted
 295 ones. In this case, the error is taken as the distance between the fitted value and the nearest
 296 bound of the experimental error box. This approach was used in the past to maximize the
 297 plausibility of a model, that is, to determine a unique model setup that best explains the
 298 measurements in the light of the experimental error (see Majdalani et al. (2018) for detailed
 299 considerations on the approach). The series for $x = -1$ m and $x = 4$ m are clearly outliers.
 300 For Configuration 2 (Figure 9), the error boxes for the series $x = -2.50$ m , $x = 1.25$ m and
 301 $x = 2$ m are not intersected by the fitted curve either. These few series excepted, the fitted
 302 curves are intercepted by the error boxes of all experimental series, which tends to validate
 303 the self-similar behaviour.

304 That the measurements at a given location depart from the expected self-similar
 305 behaviour may be explained by several factors. A first possible reason is the local invalidity
 306 of the hydrostatic pressure distribution assumption. The self-similar character of the solution
 307 is a direct consequence of the hyperbolic nature of the shallow water model. In the case of
 308 non-negligible vertical accelerations, the shallow water model becomes invalid and corrective
 309 terms, usually dispersive, must be incorporated in the governing equations. The solution of
 310 the Riemann problem for second- and higher-order partial differential equations is not self-
 311 similar in (x, t) . This may explain for instance the behaviour of the experimental points
 312 $x = -1$ m in Configuration 1 (Figure 8), because the first block period is located very close to
 313 the gate, in a region where the free surface is steep and vertical accelerations are not
 314 negligible. A second possible reason is a systematic error during probe recording and/or
 315 calibration. A third reason could be a consistent bias introduced by a slightly altered
 316 geometry. It is striking indeed to notice that, for a given block period, the records are
 317 consistently above or below the fitted curve. This seems to indicate a consistent behaviour of
 318 the hydrodynamics of the block period under consideration. That a given block average
 319 departs significantly from the rest of the series might be due to an altered geometry compared
 320 to the ideal layout (e.g. a block slightly shifted or with dimensions slightly different compared
 321 to specifications, etc.). The outlying series in Figures 8 and 9 can be attributed to a
 322 combination of all these reasons.

323 4 Model results and discussion

324 4.1 Results

325 The experiments are simulated using four models: the two-dimensional shallow water model,
 326 the Single Porosity (SP) model, the Integral Porosity (IP) model, and the Dual Integral
 327 Porosity (DIP) model. The 2D shallow water model uses a 1 cm×1 cm square grid, which
 328 results in more than 140,000 elements. The 2D shallow water equations are solved using the
 329 finite volume-based, MUSCL-EVR technique (Soares-Frazão and Guinot, 2007), with a
 330 maximum CFL of unity (see Soares-Frazão and Guinot, (2007) for the derivation of the
 331 stability criterion). The porosity models are one-dimensional. The governing equations for the
 332 three porosity models are not solved numerically but semi-analytically. The first generalized
 333 Riemann invariant (Lax, 1957) is integrated across the rarefaction wave that connects the left
 334 state to the intermediate region of constant state:

$$335 \quad \mathbf{du} // \mathbf{K}^{(1)} \quad \text{across} \quad \frac{dx}{dt} = \lambda_1 \quad (12)$$

336 Where $\mathbf{K}^{(1)}$ and λ_1 are respectively the first eigenvector and eigenvalue of the Jacobian
 337 matrix of \mathbf{f} with respect to \mathbf{u} . Owing to the simple structure of the Jacobian matrix, the matrix
 338 of eigenvectors in shallow water models is given by

$$339 \quad \mathbf{K} = \begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix} \quad (13)$$

340 Consequently, the first generalized Riemann invariant across the p th wave is given by

$$341 \quad du_2 = \lambda_1 du_1 \quad \text{across} \quad \frac{dx}{dt} = \lambda_1 \quad (14)$$

342 where u_k ($k=1,2$) is the k th component of \mathbf{u} . The formulae for the wave propagation speeds
 343 in the SP, IP and DIP models are given in (Guinot et al., 2017). While Equation (14) can be
 344 integrated analytically for the SP and IP models, its integration for the DIP model is
 345 performed using an RK2 procedure with a depth increment $\delta h = 1$ cm .

346 All simulations are run using the initial condition $(h_L, h_R) = (0.35 \text{ m}, 0 \text{ m})$. The fine
 347 grid solution computed every 0.1 second by the two-dimensional shallow water model is
 348 averaged over the block periods in order to allow for a comparison with the experimental
 349 results.

350 The results of the refined 2D shallow water model are compared to the experimental
 351 results in Figure 10. It is noticed that the refined 2D solution also tends to follow an S-shaped
 352 curve. Only do a few points strongly depart from the main trend. Inspecting the period-
 353 averaged results shows that these points belong to the block periods in the immediate
 354 neighbourhood to the initial discontinuity. This tends to confirm the finding by Guinot (2017)
 355 that more than one spatial period may be needed to observe the self-similarity of the refined
 356 solution. While the block-averaged refined 2D solution quite closely follows the experimental
 357 data in Configuration 1 (Figure 10, top), it fails to replicate the curvature and the sudden
 358 steepening of the data cloud near $x/t = 0$ in Configuration 2 (Figure 10, bottom).

359 The results of the SP, IP and DIP models are plotted together with the experimental
 360 data in Figure 11. For Configuration 1, the combination of porosities $(\phi_T, \phi_\Omega) = (0.25, 0.625)$
 361 yields strongly contrasted model responses. The SP model consistently overestimates the
 362 propagation speed of the transient. The IP model fails to reproduce the change in the
 363 curvature of the free surface in the upstream part of the flume (negative x/t values) but
 364 reconstructs successfully the downstream part. This, however, is regarded as a mere
 365 coincidence in that the refined 2D model (of which the porosity model is supposed to be an
 366 approximation) is not as good as the IP model in this part of the profile. Of the three porosity
 367 models, the DIP model is the only one that combines a successful reproduction of the
 368 curvature in the upstream part of the flume with a satisfactory approximation of the
 369 experimental behaviour in the downstream part. For Configuration 2, the contrast between the
 370 two porosities $(\phi_T, \phi_\Omega) = (0.5, 0.7)$ is much milder and all three models behave in a very
 371 similar fashion. All three fail to reproduce the steepening of the profile near $x/t = 0$, just as
 372 the refined 2D model does. Nevertheless, the DIP model is the most successful of the three in
 373 upscaling the refined 2D shallow water solution in both configurations.

374 4.2 Discussion

375 Besides setting up an experimental database, an objective of the present work was to check
 376 the relevance of incorporating a transient momentum dissipation term in the governing
 377 equations of porosity models. The three model outputs plotted in Figure 11 seems to indicate
 378 that such a momentum dissipation model is of limited interest, at least for the flow
 379 configurations reported here. The reason is that the transient momentum dissipation model
 380 operates only when the free surface is rising (thus only for positive x/t) and its main effect
 381 is to slow down the propagation of the waves. In Configuration 1, it may allow the
 382 downstream part of the SP profile to be corrected to fit better the experimental results, but it
 383 would have no action on the upstream part of the profile. Correcting the propagation speeds
 384 of the IP and DIP profiles for $x/t > 0$ would only contribute to increase the discrepancy
 385 between these profiles and the experimental points, so introducing dissipation in these two
 386 models is not advisable. In configuration 2, all three models exhibit similar behaviours and fit
 387 the experimental data rather well in the region $x/t > 0$. Consequently, slowing down the
 388 propagation speeds in this region is of limited interest. The main discrepancy with the
 389 experimental results of Configuration 2 is observed in the upstream part of the flume, a region
 390 over which the free surface is falling, thus cancelling the transient momentum dissipation
 391 term.

392 A salient feature of the modelling is that all models, including the refined 2D shallow
 393 water model, fail to reproduce the steep part of the experimental profiles near $x/t = 0$. This
 394 may be attributed to strong vertical accelerations occurring at the gate at the early times of the
 395 experiment. Such acceleration may modify locally the pressure field compared to the
 396 theoretically hydrostatic pressure distribution. This may result in modified wave propagation
 397 speeds and altered free surface profiles compared to those given by the solution of the shallow
 398 water equations.

399 As shown in Figures 10 and 11, the water depth profile is much better reproduced by
 400 all models for Configuration 1 than for Configuration 2. This can be explained by a number of
 401 factors. First, the water depth profile is much steeper near $x/t = 0$ in Configuration 2 than in
 402 Configuration 1. Consequently, vertical accelerations are much stronger in Configuration 2
 403 than in Configuration 1, thus making the hydrostatic pressure distribution assumption more
 404 questionable in the second configuration than in the first, especially in the neighbourhood of
 405 the gate. Second, the ratio ϕ_T/ϕ_Ω is larger in Configuration 2 than in Configuration 1.
 406 Therefore, the positive and negative waves propagate faster along the main channel in the
 407 second configuration. This entails stronger reflection phenomena against the walls of the
 408 building blocks and within the lateral streets, with the consequence that difference between
 409 the water depths and velocities in the main street and in the lateral ones is larger in
 410 Configuration 2. Almost all porosity models (except for the MP model) being based on the
 411 assumption of a uniform distribution of the water levels and the flow velocity within an
 412 averaging block, the more transient the configuration, the more inaccurate this underlying
 413 assumption.

414 5 Conclusions

415 An experimental data set for laboratory scale dam-break experiments in idealized urban
 416 layouts is presented. The experiments are carried out using a scale model of periodic, aligned
 417 building blocks. Two geometric configurations, corresponding to two different storage and
 418 connectivity porosity combinations, are used: $(\phi_\Omega, \phi_T) = (0.625, 0.25)$ and
 419 $(\phi_\Omega, \phi_T) = (0.7, 0.5)$. The water levels are recorded as functions of time at 6 points in each
 420 block period. The experimental, block-averaged values are shown to obey a self-similar
 421 behaviour in the (x, t) space.

422 The experiment is reproduced satisfactorily using a two-dimensional refined shallow
 423 water model, except in the close neighbourhood of the initial discontinuity. The block-
 424 averaged water depths computed by the refined 2D model are also shown to follow a self-
 425 similar behaviour. A theoretical analysis shows that the self-similar behaviour can be
 426 preserved by a pore scale-averaged model only if the momentum source term does not obey
 427 an equation of state but is a function of the momentum flux divergence. This is the case of the
 428 transient momentum dissipation term introduced in the DIP model. Reproducing the
 429 experiments using the SP, IP and DIP models shows that (i) the DIP fluxes outperform the SP
 430 and IP fluxes when the contrast between the connectivity and storage porosities is large ($\phi_\Omega / \phi_T = 2.5$),
 431 (ii) all three models exhibit very similar behaviours for a ratio $\phi_\Omega / \phi_T = 1.4$,
 432 (iii) all models, including the refined 2D shallow water model, fail in reproducing the steep
 433 part of the water depth profiles near $x/t = 0$.

434 Lastly, while satisfactory in theory because it preserves solution self-similarity, the
 435 transient momentum dissipation term proposed in the DIP model (Guinot et al., 2017) is
 436 found useless to reproduce the present experiments. Many reasons may be given for this, such
 437 too small a number of block periods to allow for an accurate upscaling. Further research

438 should be directed to enriching the experimental database by exploring a wider range of flow
439 conditions and geometries.

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446 **Supplemental data**

447 The detailed experimental data can be obtained upon request to the corresponding author.

448 **Notation**

449 CD = drag coefficient per unit depth (m^{-1})

450 \mathbf{f} = flux vector

451 h = water depth (m)

452 k = scaling factor (-)

453 \mathbf{M} = dissipation matrix

454 \mathbf{s} = source term vector

455 S_0 = bottom slope

456 S_f = friction slope

457 T, t = time coordinates (s)

458 \mathbf{u} = conserved variable vector

459 u, v = x - and y -flow velocities ($m\ s^{-1}$)

460 X, x = longitudinal space coordinates (m)

461 y = transverse space coordinate (m)

462 μ = momentum dissipation coefficient (-)

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- 530

531

532 **Table 1. Sample probe coordinates for Configuration 1. x-period: 1 m.**

| Identification | x (m) | y (m) |
|----------------|--------|-------|
| PU85_125 | -8.500 | 0.125 |
| PU8_125 | -8.000 | 0.125 |
| PU8_300 | -8.000 | 0.300 |
| PU8_455 | -8.000 | 0.455 |
| PU8_600 | -8.000 | 0.600 |
| PU8_800 | -8.000 | 0.800 |

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534 **Table 2. Sample probe coordinates for Configuration 2. x-period: 1.25 m.**

| Identification | x (m) | y (m) |
|----------------|--------|-------|
| PU875_125 | -8.750 | 0.150 |
| PU875_375 | -8.750 | 0.375 |
| PU875_575 | -8.750 | 0.575 |
| PU875_800 | -8.750 | 0.800 |
| PU85_250 | -8.500 | 0.250 |
| PU8125_250 | -8.125 | 0.250 |
| PU775_250 | -7.750 | 0.250 |

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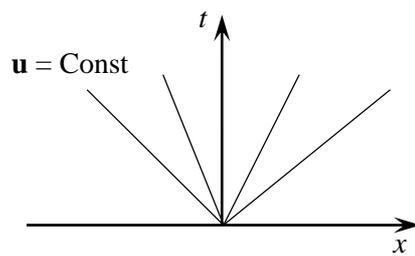
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538 **Table 3. Probe weights used in the sensitivity analysis of the averaging procedure (Configuration 1).**

539

| k | wk Approach A | wk Approach B |
|---|---------------|---------------|
| 1 | 1/7 | 1/4 |
| 2 | 1/7 | 1/10 |
| 3 | 1/7 | 1/10 |
| 4 | 1/7 | 1/10 |
| 5 | 1/7 | 1/10 |
| 6 | 1/7 | 1/10 |
| 7 | 1/7 | 1/4 |

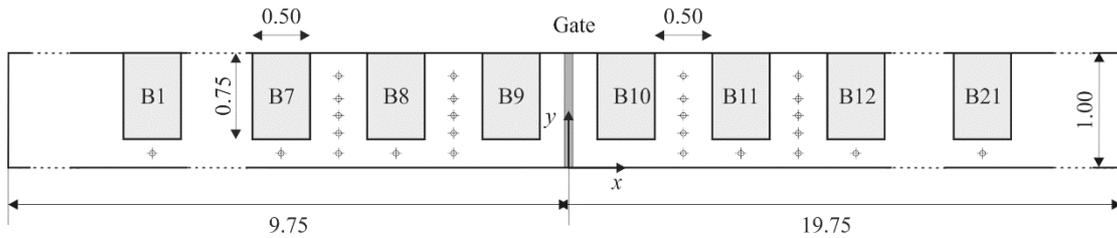
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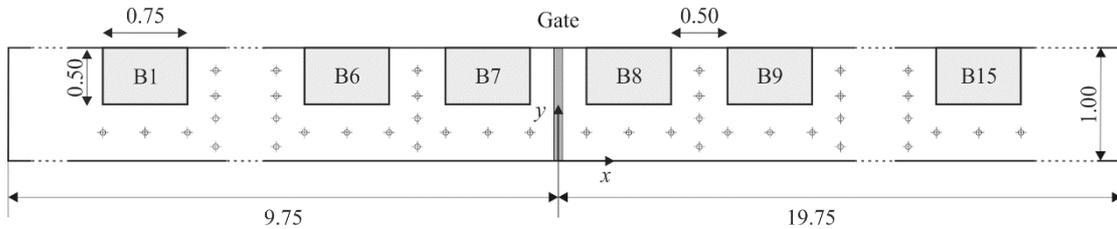
542 **Figure 1. Self-similarity of the flow solution in the (x, t) space. The lines $u = \text{Const}$ are straight**543 **lines.**

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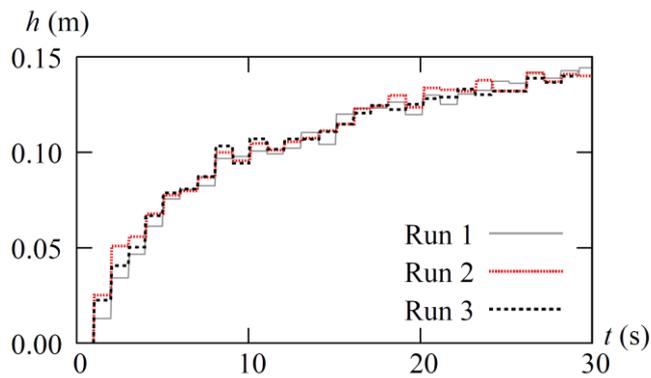
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Figure 2. Definition sketch and dimensions (m) for Configuration 1



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Figure 3. Definition sketch and dimensions (m) for Configuration 2



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Figure 4. Repeatability of the experiment for Configuration 1, probe PD2_125

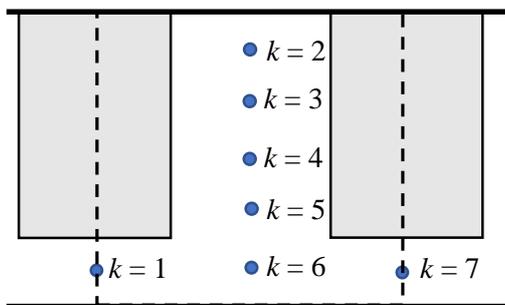


Figure 5. Configuration 1. Probe numbering within a block period. Dashed lines: boundaries of the block period.

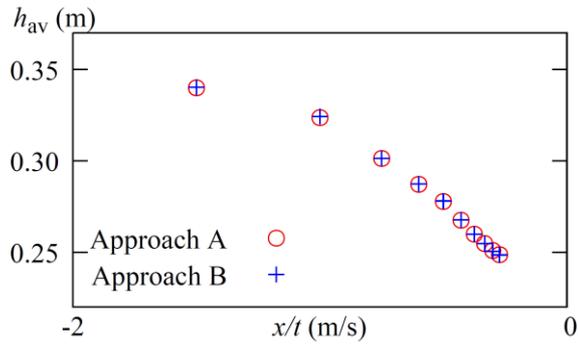


Figure 6. Sensitivity analysis of the averaging approach. h_{av} computed using Approaches A and B (Configuration 1).

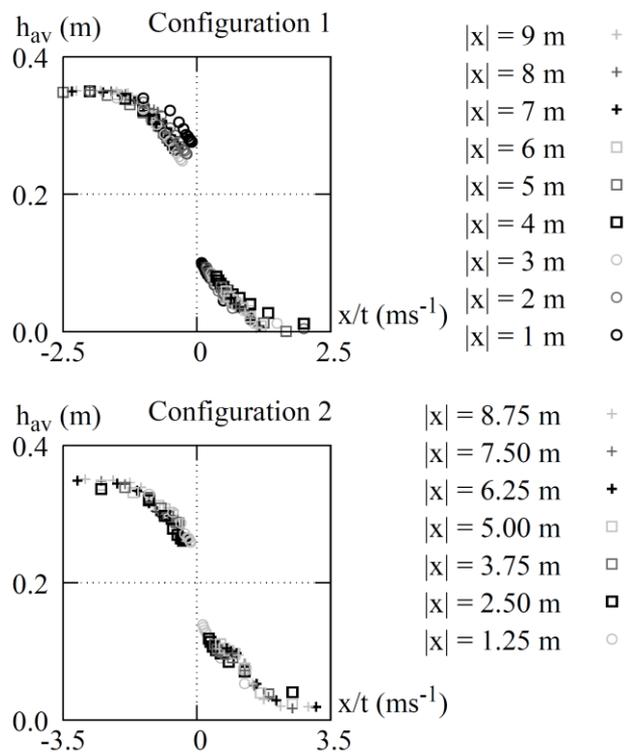
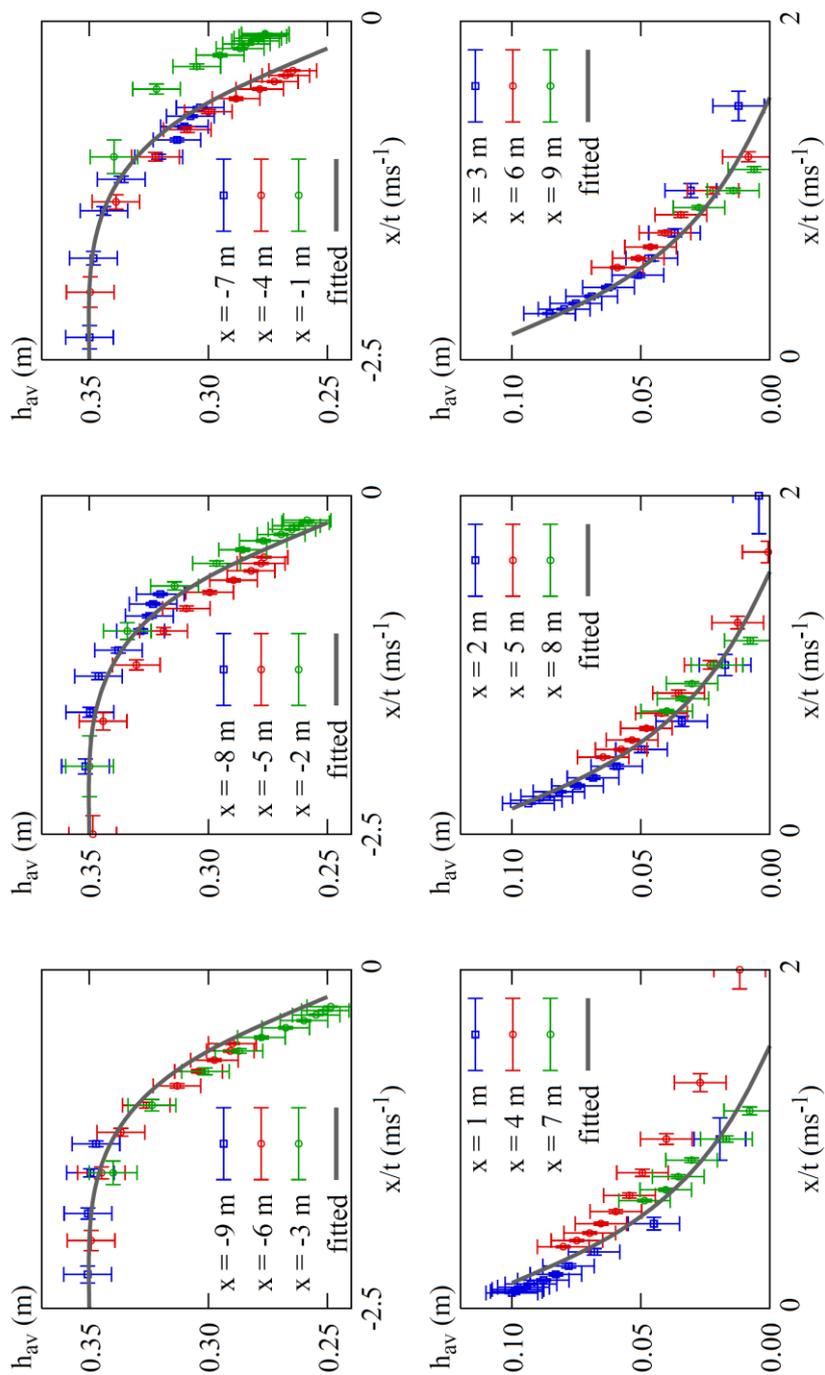


Figure 7. Block-averaged water depth as a function of x/t .

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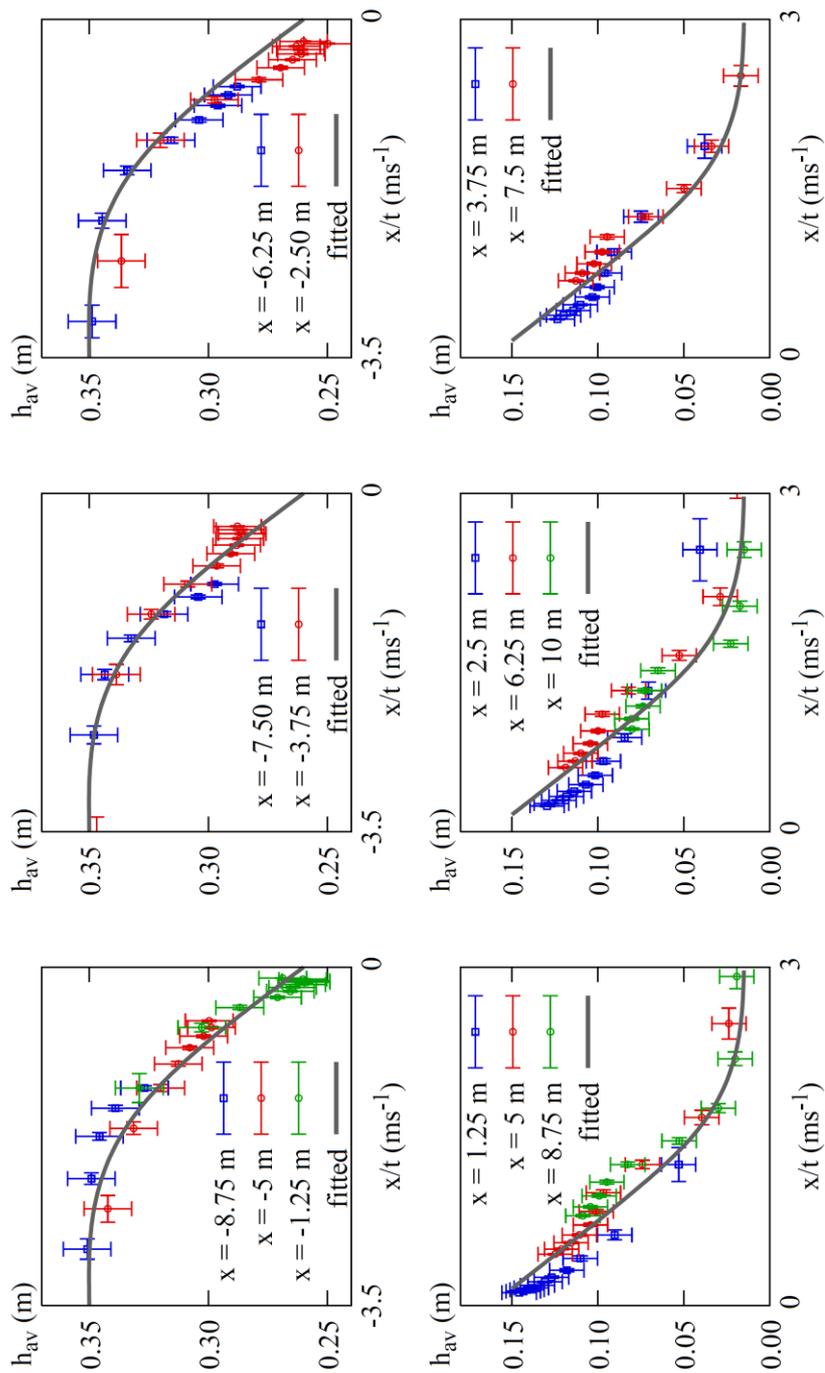
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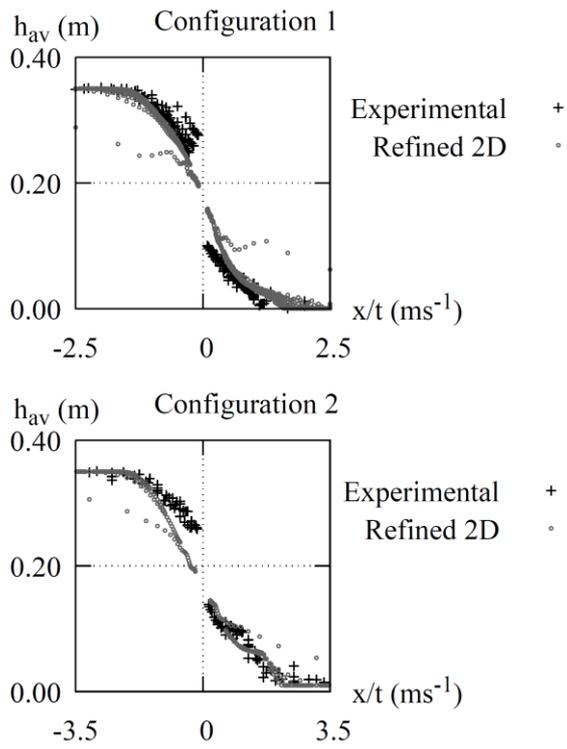
Figure 8. Configuration 1. Experimental uncertainty and fitted curve for solution self-similarity assessment.



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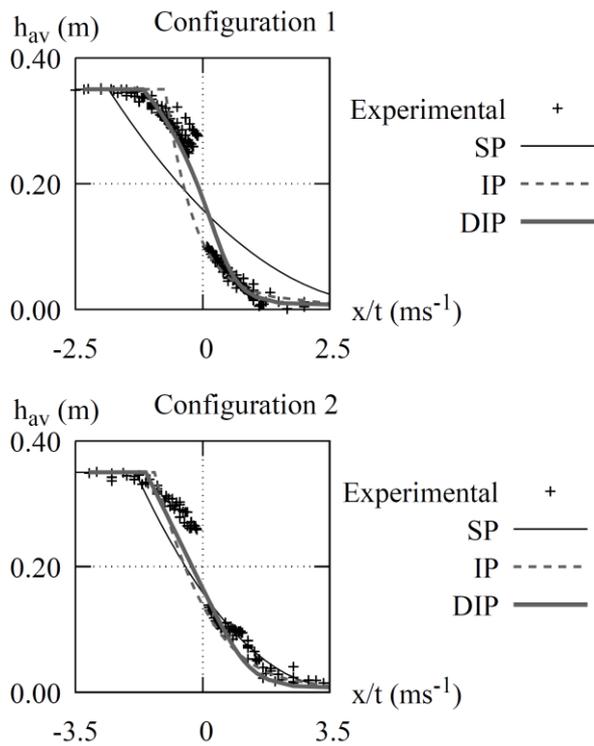
Figure 9. Configuration 2. Experimental uncertainty and fitted curve for solution self-similarity assessment.

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Figure 10. Comparison of the block-averaged refined 2D shallow water solution and experimental results.



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Figure 11. Comparison of the SP, IP and DIP porosity model solutions and experimental results.