Introduction to the papers of TWG 15 Teaching mathematics with technology and resources

Alison Clark-Wilson, Ornella Robutti, Melih Turgut, Gülay Bozkurt, Daniel Thurm

Introduction to the papers of TWG 15

Teaching mathematics with technology and resources

Alison Clark-Wilson[1], Ornella Robutti[2], Melih Turgut[3], Gülay Bozkurt[4] and Daniel Thurm[5]

[1]University College London, London, United Kingdom; a.clark-wilson@ucl.ac.uk

[2]University of Turin, Italy; ornella.robutti@unito.it

[3]Norwegian University of Science and Technology, Norway; melih.turgut@ntnu.no

[4]Izmir Demokrasi University, Turkey; gulay.bozkurt@idu.edu.tr

[5]University of Siegen, Germany; daniel.thurm@uni-siegen.de

Keywords: Technology uses in education, mathematics instruction, teaching mathematics, teacher competences, teacher behavior, teacher professional development

Introduction

The two technology groups at CERME (TWG15 and TWG16) adopt a broad view of technology resources in mathematics education to include tangible devices (hardware), and the associated applications (software), within the context of learning and teaching. TWG15 addresses the related issues that mostly concern teachers and teacher education. Previous discourse at ERME conferences embraced a wide variety of research topics, theoretical and methodological approaches. Most recently, this has focused on: teachers' uses of students' (digital) productions; sorting and organising digital content such as: simulations, applets and Open Educational Resources (OERs); the teaching of computational thinking in, and through, mathematics; teachers' choices and beliefs concerning technology use; and the ever-challenging process to develop useful theories and new pedagogies. The group is keen to learn more about actual uses of technology in classrooms (and beyond) to understand both the prevailing classroom practices and the implications of this on policy, practice and theory. The Covid-19 pandemic offered such a context and two papers (Barlovits, Kolokytha, Ludwig & Fessakis and Ramirez) and one CERME12 award-winning poster (Bini) reported such research from Chile, Germany and Italy respectively.

The group concluded the following perspectives and suggestions for future research. Continued effort is needed to enable wider understanding of the theories and methods applicable to this domain. More specifically, a greater awareness of the epistemologies that underpin these theories and methods, particularly when adopting more than one frame and/or attempting to network them in some way. The prevalence of quantitative studies led the group to conclude that increased attention is needed to the sampling of research participants and their associated characteristics, which can greatly impact the research claims that can be made when reporting findings. In addition, the roles and activities of participants within research studies is also an area of growing interest for the field. Finally, the nature of innovation within the field of

technology in mathematics education results in many future research opportunities that have both pedagogical and mathematical implications, and make this a vibrant area for study.

Organisation of TWG15 and its sub-themes

TWG15 accepted 22 papers and 6 posters for presentation at the conference, which reported research from 20 countries (Argentina, Austria, Canada, Chile, China, Denmark, England, France, Germany, Ghana, Greece, Ireland, Israel, Italy, Norway, Portugal, Spain, Sweden, Turkey & United States). Each paper was allocated to one of six sub-themes and all authors of papers recorded 5-minute presentations of their papers, which were made available for asynchronous viewing by all CERME participants. In each TWG session, the allocated video presentations were broadcast prior to the TWG discussions. Each Sub-theme chair structured the whole group and breakout group discussions around key ideas that emerged from the related papers. In the text that follows, we summarize these ideas and suggest implications for our field of research. We conclude with 'hot topics' for the TWG15 community, which emerged from our final collaborative discussions.

Sub-theme 1: Theoretical frames for investigating the teaching of mathematics with technologies (Chair: Melih Turgut)

Tools and technologies for teaching and learning mathematics continually change. Accordingly, it naturally takes time for teachers to adapt the use of technologies in classroom practice, a process that is known to be challenging. Alongside, pedagogies also evolve and emerging theoretical/conceptual perspectives are needed to provide insights on how to integrate both new technologies alongside new functionalities of well-adopted tools. Many theoretical and conceptual frameworks have been developed to help understand the inevitable gaps between teachers' existing content and technological knowledge, and the processes through which new technologies transform classroom practice. The TWG15 papers provided a broad spectrum of emerging theoretical perspectives for designing and understanding teaching practice. We underline two main applications of theories: (i) as more deductive research frames, such as Ruthven's *Structuring Features of Classroom Practices* (SFCP) (Ruthven, 2009) (Simsek, Bretscher, Clark-Wilson & Hoyles), Gueudet and Trouche's (2009) *Documentational Approach to Didactics* (Basturk-Sahin & Tapan-Broutin), and Koehler and Mishra's *TPACK* (2009) and TPACK developments (i.e. Bray & Oldham; Lyublinskaya & Du; Meier & Oliveira); and (ii) as multiple frameworks that are merged to offer new theoretical/conceptual lenses, such as: Verillon and Rabardel's (1995) *Instrumental Genesis* and Robert and Rogalski's (2002) *Double Instrumental Approach* (Haspekian & Fluckiger), and *TPACK* and Valsiner's (1997) *Zone Theory* (Lindenbauer, Lavicza & Weinhandl).

Barlovits et al. propose a set of "design requirements" for the development of mobile environments for teaching distance mathematics, where they combine a community of inquiry lens, with e-pedagogy and mobile learning models. Gonscherowski and Rott conduct an interview study to explore pre-service and in-service teachers' argumentation and justification regarding digital tool use in mathematics education. Gonscherowski and Rott consider a combined analysis tool; teaching phases and classification of the use of digital tools (Clark-Wilson, Robutti & Thomas, 2019) and they further elaborate different levels of *decision-*

making competencies about appropriation of the digital technologies. Haspekian and Fluckiger combine the five components of the Double Approach (Robert & Rogalski, 2002) with notions of instrumental distance and didactic landmarks to analyse teachers' instrumental geneses when integrating programming in their practices. Their combined analysis addresses teachers' difficulties, which mainly concern '... the changes that ICT introduce[s] at cognitive and meditative levels'.

Sub-theme 2: Methodologies and methods for investigating the teaching of mathematics with technologies (Chair: Alison Clark-Wilson)

The TWG15 papers and posters predominantly report empirical research that spans a diversity of methodologies. Each study is underpinned by the respective researchers' epistemologies, which frame their thinking on what it is possible to know and conclude from their research findings. Consequently, this TWG session encouraged participants to think critically about if, and how, the application of research methods in different studies did (or did not) shed light on the chosen phenomena, and in accordance with the selected theoretical frame.

The majority of studies adopted qualitative research designs that sought to reveal aspects of the different phenomena at hand: (i.e., Engelhardt & Roth; Meier & Oliveira; Vilchez & Lemmo; Speer & Eichler; Tortoriello & Veronesi). Common qualitative methods included questionnaires, surveys, observations, interviews (individual, focus group), and document scrutiny/analysis. In some cases, data is analysed to present findings quantitatively. For example, in the study by Lyublinskaya and Du, a novel data visualization method reveals interesting characteristics of pre-service teachers' development of TPACK over time.

The second most common method used in the studies (i.e. Gavor, Clark-Wilson & Hoyles; Kristinsdóttir, Hreinsdóttir & Lavicza; and Lindenbauer, Lavicza & Weinhandl) is the increasingly used *design (or design-based) research approach* (Bakker, 2018). At the heart of such an approach is the aim to research innovations (i.e. design new materials or approaches) for which existing theories and methods might not be appropriate. Instead, more 'humble' theories inform iterative cycles or research that involve different methods to enable the theoretical, methodological and practical knowledge to emerge.

Two mixed method studies were presented. Fan and colleagues propose a mixed method approach to answer the question: How do mathematics secondary school teachers in China use digital resources in their teaching? They design a questionnaire for teachers about their uses of digital resources, before, during and after lessons, according to the three dimensions (content, function and infrastructure). The results are analysed quantitatively, alongside a sample of follow-up interviews, which are analysed qualitatively. The quantitative results show that more than one half of the teachers use digital resources often or always, and especially during lessons. However, they less frequently use resources after lessons, and the variety of resources used before, during and after is still limited. Interviews are used to ask the reasons for the use of specific resources. Segal and Biton's study, conducted in Israel, research teachers' perceptions of their work in the Whatsapp environment, also using questionnaires and interviews, and additionally Whatsapp messages and observations of four teaching groups.

Results of this study are at two levels: for students, in feeling free to make mistakes without fear, and for teachers, who gained in their professional technological knowledge.

The three studies by Thurm, Geraniou and Jankvist, Müller and Wachte, and Sharkia and Kohen all adopted quantitative designs.

Finally, TWG15 discussed how well we paid attention to, and made explicit in our research, our methods for sampling the participants of our studies. The prevalence for 'opportunity sampling' of pre- and in-service teachers, was acknowledged as a concern when set alongside claims for wider application or generalisation of research findings.

Sub-theme 3: Teachers' different roles in diverse educational settings (Chair: Ornella Robutti)

There is a recent increase in research concerning the role of teachers within in educational settings and processes. From being recognized as a "dimension" in studies on mathematics teachers (Robutti et al., 2016), to becoming one of the four fundamental themes of ICMI Study 25 *Tools and resources used/designed for teacher collaboration and resulting from teacher collaboration* (Robutti et al., in press). TWG15 interpreted teachers' roles within the different institutions as: prospective teachers (e.g., undergraduates in mathematics courses with a clear aim in preparing future teachers); pre-service teachers (e.g., participants of university-led teacher education program); in-service teachers (e.g., within their professional work context, or in formal projects); teacher educators (e.g., participating in formal or informal teacher education settings). The papers presented below relate to teachers in the aforementioned roles, focusing on the different types of involvement for teachers as participants in the research.

One role is teachers as *designers* of tasks using technology (Guerrero-Ortiz & Camacho-Machín; Speer & Eichler; Fahlgren, Szabo, & Vinerean): teachers are protagonists and have an active role in the design of tasks. Another role for teachers is that which is presented by Lyublinskaya and Du. In their study mathematics pre-service teachers are *learners* of technologies to be used in teaching mathematics. Their learning trajectories are analysed using the TPACK frame within the institutional context of a non-discipline-specific online educational technology summer course. The findings show that the pre-service teachers grew in their discipline-specific TPACK, under the influence of personal and contextual factors. The study demonstrates an innovative use of TPACK, combined with digital timelining analysis, to describe teachers' growth in professionalism as dynamic processes.

Jacinto and Carreira show another kind of teachers' involvement in the research: as *problem solvers*, with the use of technology. Contextualising the teacher's approach to technology in the general lens of *humans-with-media*, the study questions the role of technology within the mathematics teachers' problem-solving processes. The study reports the gain in the teacher's competences according to the MPST model, facilitated through the different micro-cycles in the overall process.

A community of inquiry comprising teachers as *participants* constitutes another kind of involvement: active participation is a distinctive feature of the community, intended not only in presence but also at distance. The ASYMPTOTE project (Barlovits et al.) is presented as an

evolution and adaptation of the MathCityMap system in the COVID-19 pandemic era. The particular setting of the technology, which uses the idea of decision trees, enables teachers to design differentiated tasks for students in a flexible way, directing students to different learning paths, according to their performances.

Sub-theme 4: Innovations in technology for teaching mathematics (Chair: Daniel Thurm)

Technologies in mathematics education are constantly evolving. “New” innovative technologies emerge and become over time more established technologies. For example, dynamic geometry systems, computer-algebra systems, or function plotters - technologies which were considered “new” in the 1990 – have by now a strong research base. However, even for those technologies there is still much to be understood with respect to teaching and learning which, is evidenced for example by the papers of Fahlgren, Vinerean, and Szabopaper (task design for dynamic geometry environments) or Jacinto and Carreira (problem solving using spreadsheets). At the same time some TWG15 papers focused on more emerging technologies, e.g., videogames (Vilchez & Lemmo), WhatsApp-messenger (Segal & Biton), videos in a flipped classroom setting (Sharkia & Kohen) and multitouch environments (Bakos).

Vilchez and Lemmo presented research on a game-based approach, concluding that the selected videogame has potential to support the teacher to scaffold students’ relational thinking during whole-class discussions. In particular, the authors hypothesize that observing students play the videogame might implicitly help teachers to orchestrate classroom discussions.

Another emerging technology not so widely investigated from the perspective of the teacher is WhatsApp. Segal and Biton’s research focuses on opportunities for learning and teaching that can be created using WhatsApp as a social network. While teachers report positively on many aspects of using WhatsApp some teachers felt limited to generate collaboration between students, as they would in a regular class. From the researcher’s point of view, decentralized learning made it difficult for the teachers to capture the learning processes of the students.

Sharkia and Kohen investigate how online videos in a flipped classroom setting can effectively utilize an Inquiry-Based Learning (IBL) approach. For this they analyze seven filmed lectures from an advanced mathematics course. The discussions during the TWG15 session suggested extending the investigation to include the students’ views and experiences, since IBL is fundamentally a student-centered practice.

Bakos focuses on teaching and learning with the app TouchTimes, which is a multi-touch iPad application. By using the notion of double instrumental genesis, Bakos examines how teachers experience TouchTimes as learners, alongside their subsequent transitions to adopting it as a didactical instrument. The paper highlights that the notion of instrumental distance can be a promising way to examine the impact of integrating emerging digital technology on mathematics teachers’ practices.

The TWG15 papers evidence how the constant emergence of technological innovations brings some recurring challenges for the educational research community. For example, new technologies might require new pedagogies, for which existing theories may neither be

sufficient, nor sensitive enough, to detect new phenomena. Conversely, there is the challenge to relate research on emerging technologies to research on established technologies and the "old" theories (Jankvist & Misfeldt, 2021). The TWG discussed how the research community can respond to the accelerated emergence of "new" technologies and in what way "new" technologies demand and/or inspire "new" pedagogies. It was suggested that looking back at the characteristics of technologies which remain constant might help to us to better connect our research to existing knowledge and practices. For example, focusing on dynamic mathematical representations could be a common theme across different technologies. Finally, it was highlighted that use of (and research on) digital technologies is often inherently linked to a particular mathematical concept or topic. Hence, for researchers and teachers alike, the challenge to grapple with both the mathematics and the tool is very real, as clearly described in the paper authored by Bakos.

Sub-theme 5: Pedagogical approaches and mathematical content (Chair: Gülay Bozkurt)

As indicated in the previous theme, technology use demands different pedagogical approaches, which are also likely to be affected by the aspects of mathematics that is being taught. Hence, in this theme, we focused on elaborating the foci of *pedagogical approaches* and *mathematical content* in our studies. The studies discussed in the TWG15 indicated a great variety in both their pedagogical framing and the particular aspect(s) of the mathematics curriculum such as: developing problem solving skills in algebra (Jacinto & Carreira); integrating computer science in mathematics education (Haspekian & Fluckiger); encouraging modelling activity in proportions and areas (Guerrero & Camacho); and improving distance education (Barlovits, Kolokytha, Ludwig & Fesakis). Within the TWG15 discussions on this theme, we particularly focused on perspectives pointed out in the three papers by: Abu Raya and Olsher; Bretscher; and Simsek, Bretscher, Clark-Wilson and Hoyles.

The paper of Abu Raya and Olsher explores the potential of a technological environment (STEP) on formative assessment as a process of teachers' pedagogical approach. They particularly examine the effect of accessible learning analytics on teachers' formative assessment practices, by providing them with means to respond to student submissions. The FaSMEd framework is adopted as an analytical lens through which to focus on teachers' interactions with the STEP environment's learning analytics. The researchers explore the impact of this on both class discussion, and the content for teaching functions. Findings indicate that teachers use all the technology's functionalities, enabling them to advance learners' understanding through a class discussion that deployed the five key strategies of formative assessment interactions.

Bretscher examines one teacher's knowledge about a particular aspect of geometry, circle theorems, focusing on his transition between dynamic and static technologies through which he would support students' related conceptual understanding. The researcher uses the TPACK framework to explore such knowledge with a particular focus on comparisons between angle definition and measurement within GeoGebra, and a paper-and-pencil environment. The author's analysis of a clinical interview with the teacher, reveals an example of this teacher's dilemma on defining angles in GeoGebra indicating his technological content knowledge

(TCK). Bretscher concludes “mathematical knowledge for teaching using technology is always situated, since the technological context in which it is being applied is central to its meaning”.

Simsek, Bretscher, Clark-Wilson and Hoyles apply Ruthven’s (2009) SFCP framework into teachers’ domain specific practices. They select the construct of curriculum script in their aim to characterize teachers' knowledge regarding key aspects of their practice. Focusing on three different teachers’ classroom practices and interview accounts in the context of English secondary schools, this study highlights differences in both the quantity of teachers’ anticipated or identified misconceptions about geometric similarity in their curriculum scripts, and the ways they use the dynamic mathematical technology in addressing such misconceptions.

Emerging “hot topics” relevant to the TWG15 community

During the final session, the TWG15 divided into self-identified breakout groups of early, mid and late career researchers/practitioners for the purpose of reflecting on the implications of the TWG15 session inputs (and associated discussions) for our field. Each group generated and discussed a long list of personal ‘hot topics’, prior to agreeing those that are summarized below.

Early career researchers want to get to know and deepen knowledge on theories that were introduced within the different papers’ frameworks and the group discussions, e.g., on TPACK and (Double) Instrumental Genesis as major theories; or specific theories like FASMEd as possible components of theoretical frameworks. Also, this group is keen to learn more about how these frameworks are operationalized within the respective research methodologies. The group also want to explore more the social, political and economic aspects of the use of technology within mathematics teaching, from both teachers’ and students’ perspectives (i.e. the availability of technology, the learning opportunities offered, etc.).

Mid-career researchers identified a methodological hot topic, which concerns the design of research instruments that capture mathematical and digital competences for teaching and/or the associated knowledge and beliefs/orientations. This seems important both for conceptualizing what the competences are, and for supporting teachers’ professional learning. Associated with this, how to develop methods to help them understand whether such knowledge is associated with improved pupil outcomes? On a theoretical level, the selection of theoretical framework(s), how frameworks are put to use (operationalized) within studies and, in the case when multiple frameworks are used, how they are synthesized (e.g., examining the resulting data from different perspectives) is also an ongoing hot topic. Finally, the group acknowledged the need for research designs that include strong connection, communication and collaboration with pre-service and in-service teachers to enable both a deeper understanding of teacher characteristics and practices, and to be able to develop more impactful teacher education and development programmes/initiatives.

Late career researchers identified the hot topic of how best to identify and connect theories, frameworks and methodologies. The group poses three questions, “What are the considerations that guide us as researchers in choosing a theoretical framework?”; “Are the existing theoretical frameworks suitable for the purpose of analyzing the research data?”; and “How to merge different theoretical frameworks for the analysis and characterization of findings, including mergers between theories on the integration of technologies in teaching and research and

theories from other fields?". For example, Segal and Biton's study, which explores teachers' perceptions of the contribution of teaching and learning in the WhatsApp environment, implies that theoretical frameworks from the field of social sciences might be profitable. A second hot topic concerns the need for methodologies that take account of the evolution of teachers' practices over time, a perspective that demands a mixed (quantitative/qualitative) study design. Finally, the group was most concerned about the need for studies that conclude findings that can be applied directly to teachers practices and/or support the design of teacher education/preparation programmes.

References

Bakker, A. (Ed.) (2018). *Design Research in Education: A Practical Guide for Early Career Researchers*. Routledge.

Clark-Wilson, A., Robutti, O., & Thomas, M. (2020). Teaching with digital technology. *ZDM Mathematics Education, 52*(7), 1223–1242. https://doi.org/10.1007/s11858-020-01196-0

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218. https://doi.org/10.1007/s10649-008-9159-8

Jankvist, U.T., & Misfeldt, M. (2021). Old Frameworks—New Technologies. *Canadian Journal of Science, Mathematics and Technology Education, 21*(2), 441–455.

Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., ... & Joubert, M. (2016). ICME international survey on teachers working and learning through collaboration: *ZDM*, *48*(5), 651-690. https://doi.org/10.1007/s11858-016-0797-5

Robutti, O., Trouche, L., Cusi, A., Psycharis, G., Kumar, R. & Pynes, D. (forthcoming). *Tools and resources used/designed for teacher collaboration and resulting from teacher collaboration*. ICMI Study 25 Volume, Lisbon, February 2020. New ICMI Study Series, Springer.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Education & Didactique*, *3*(1), 131–159. https://doi.org/10.4000/educationdidactique.434

Koehler, M.J., & Mishra, P. (2009). What is technological pedagogical content knowledge (TPACK)? *Contemporary Issues in Technology and Teacher Education*, *9*(1), 60–70.

Robert, A., & Rogalski, J. (2002). Le système complexe et cohérent des pratiques des enseignants de mathématiques: Une double approche. *Canadian Journal of Science, Mathematics and Technology Education, 2*(4), 505–528. https://doi.org/10.1080/14926150209556538

Valsiner, J. (1997). *Culture and the development of children's action: A theory of human development* (2nd ed.). John Wiley & Sons.

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology of Education*, *10*(1), 77–101. https://doi.org/10.1007/BF03172796