Introduction to the work of TWG 2: Arithmetic and number systems

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Introduction

Learning and teaching arithmetic and number systems through activities in kindergarten and school is a relevant and broad field in mathematics education. In kindergarten, children start to develop a number concept of natural numbers with counting and subitizing. Primary school aims at evolving a deep conceptual understanding of numbers and basic arithmetic operations. The transition from natural numbers to rational numbers is a key challenge for mathematics education in the first years of secondary school. Only if students have a deep understanding of natural numbers, will they be able to develop conceptual understanding and procedural skills in rational numbers. Even if a great variety of topics comes with learning and teaching arithmetic and number systems there are common goals, such as developing conceptual understanding and building number- and structure-sense as foundations for flexibility in mental and written arithmetic operations. Research in arithmetic and number systems does not only focus on typical content, but also on models for teaching and learning, approaches for heterogeneous and inclusive classrooms, analogue and digital tools to support understanding, and, not to forget, teachers' competencies and cultural practices.

Working Group 2 was formed in 2011 and has developed as a forum for discussing theoretical and empirical research on the teaching and learning of arithmetic and number systems. Over the last decade, our work has aimed to acquire and enhance knowledge about students' understanding and meaningful learning in this content area regarding different ages and achievement levels. The scope of the TWG comprises kindergarten to 12th grade and emphasizes research-based specifications of domain specific frameworks, concepts and goals, analysis of learning processes and learning outcomes in different classroom cultures, as well as innovative teaching and diagnostic approaches that attend to both procedural and conceptual knowledge.

This year, we faced the same variety of topics as in previous conferences, but even more diversity in theoretical approaches and measurements. This was a great opportunity to negotiate our understanding of terms and clarify our theoretical and methodological approaches. The group intensively discussed 15 papers and 6 posters in the plenary whole group as well as in small groups.

Discussed papers and posters

The presented and discussed papers and posters can be pooled in four thematic groups: Number sense, understanding of basic ideas, strategies in mental arithmetic, teaching approaches and methodological approaches.

Number sense

Nathalie Bisaillon explores the role of groupitizing (the ability to recognize structured quantities without counting) in building internal representations of quantities to enhance students number sense. Three activities are presented to highlight the links between groupitizing skills, the construction of dynamic and imagistic mental representations and the comprehension of numeration. It is hypothesized that the use of these activities with 7–8-year-olds would enhance their number sense.

Brumm Leonie and Elisabeth Rathgeb-Schnierer present their initial study on comparing strategies in numerosity estimation in grade 3 students. Their interest focused on using two and three-dimensional tasks, and the relationship between strategies and task characteristics. The aim of the study is to reveal effective strategies which lead to high estimation accuracy according to the type of task. (poster)

Zübeyde Er and Perihan Dinç Artut investigate the strategies of gifted students in grade 5 in number sense problems. Twenty-one students undertook twenty-five questions in a number sense test where both opinions and answers were analysed. The results of this test revealed that the majority of strategies used are rule-based strategies not number sense based.

Elvira Fernandez-Ahumada, Natividad Adamuz-Povedano, Enrique Martinez-Jimenez and Jesús Montejo-Gámez present a review of instruments on assessment of number sense and mature number sense. The authors highlighted the existence of 10 instruments, of which the majority focused on the assessment of early number sense, mainly for ages between 3 and 8 years, while there are far fewer instruments for the assessment of the so-called mature number sense. (poster)

Astrid Junker investigates counting strategies of one first-grade student with a high proficiency in foundational number-sense (FoNS). A task-based, semi-structured counting interview was conducted with the aim to qualitatively analyze counting strategies. The results suggest that a student with proficiency in number-sense does not necessarily exhibit flexible counting strategies in the number range 10-20.

Pernille Bødtker Sunde and Judy Sayers investigate perspectives of six Danish first-grade teachers on teaching and learning number and addition. Data was gathered by a semi-structured interview with open-ended questions that allow to reveal teachers' emphasis in teaching on number. Data was analyzed based on the FoNS framework. The results show that teachers hardly mention estimation, quantity discrimination and number pattern when they report on their teaching practice in first grade.

Understanding of basic ideas

Marei Fetzer and Kerstin Tiedemann developed a theoretical approach in how to introduce multiplication to support children's understanding of this operation. The theoretical approach makes the distinction between basic ideas (Grundvorstellungen), strategies and representations of multiplication. The authors illustrate their approach through a German textbook example.

Aurelien Ovide, Lalina Coulange and Grégory Train examine the potentials of a specific approach for teaching and learning fractions. Data was collected throughout teaching experiments, one designed by Brousseau for 5th graders and others designed by the author for 8th graders. The results

highlight the potential of commensuration meaning to enable students to access a broad conceptualization of multiplicative comparison relationships between fractions.

Strategies in mental arithmetic and flexibility

Claudia Corriveau, Doris Jeannotte and Sandrine Michot investigate how the way the manipulatives are used on a fraction task influence the students' reasoning. The methodological and analytical work is based on the concepts of affordance and didactic variables. The analysis shows that introducing familiar manipulatives in new tasks forces the students to develop new ways of doing and thinking.

Timo Flückiger and Elisabeth Rathgeb-Schnierer present the development and piloting of a standardized, semi-structured interview guide designed to capture second and third graders abilities in flexible mental calculations. This interview guide will allow for more generalizable conclusions on the cognitive elements sustaining the solution process based on students' explanations and justifications of solution methods. (poster)

Ioannis Papadopoulos and Michail Karakostas investigate students' strategy choices when performing mental and written fraction arithmetic. They find that the written algorithm was the preferred strategy in both mental and written calculations. The students' arguments for their choice of the algorithm was accuracy, speed and easiness, although the algorithm was in fact the most time-consuming strategy for all items.

Maria Pericleous investigates how a number line can be used as a vehicle for mathematical understanding. The author uses concept cartoons as a tool to support the role of the number line to foster and develop conceptual understanding of simple calculation strategies. Nineteen participants, ages 7-8 years old, engaged in whole class discussion drawing on their perceptions of constructing and reading open number lines.

Anders Månsson addresses the inter-coder reliability of three researchers when categorizing mental computation strategies of prospective elementary teachers (PETs). PETs' mental computation strategies were captured questionnaires (15 two-digit addition problems) and categorized by strategies described in literature. Based on the high consensus in categorization, the author concludes that the questionnaire and the categories are appropriate and reliable to investigate PETs' strategies.

Steven Van Vaerenbergh, Irene Polo-Blanco, Lara González-de Cos and Juncal Goñi-Cerveradeveloped investigate the strategies used by students with an autism spectrum disorder diagnosis when solving Cartesian product problems. An exploratory and descriptive investigation was conducted with 26 students (6-12 years). Results show a low success in solving problems by the participants, but a variety of correct strategies were found, predominantly operation strategies.

Cristina Zorrilla, Pedro Ivars, Ceneida Fernández and Salvador Llinares present a study in progress that aims to examine characteristics of the transition from natural to rational numbers when grade 6 students (11-12 years old) solve multiplicative structure problems. Data was collected throughout a teaching experiment (with three phases), a pre-test, an instruction, and a post-test. Preliminary results show different levels of success according to the numerical set used and the type of problem solved.

Teaching approaches

Silke Friedrich and Elisabeth Rathgeb-Schnierer investigate whether students work on open-ended tasks according to their learning and performance levels. This initial study compares 160 grade 3 students' attitudes and achievement levels in mathematics, compared to the complexity of the equations individuals created. Results suggest that those who scored highly in achievement tests invent particularly complicated equations. (poster)

Laura Korten presents the development of a diagnosis-guided support programme for primary school students with mathematical learning difficulties. The programme will be conducted by university students, which will stimulate, develop and refine their own fostering and supporting competences. Initial results suggest university students need specific criteria to enable them to support children's strategies more effectively. (poster)

Sze Man Yeung and Taro Fujita introduce and provides some examples of productive practices in the context of a study that aims to investigate how learning environments of productive practices can be embedded into the daily lessons as a part of the curriculum for basic skills and higher-order skills training. Number pyramids in a grade 2 classroom will be used, and students' mathematical thinking processes while doing productive practices will be analyzed. (poster)

Methodological approaches and tools

Mayu Akoi and Carl Winsløw pursue the aim of elaborating a large-scale model of an arithmetic curriculum. Based on the Anthropological Theory of Didactic they created an epistemological reference model for the entire domain of arithmetic in a Japanese primary school from grades 1 to 6. The model allows observed lessons to be evaluated in the context of the entire curriculum as well as the comparison of different curricula.

Einat Heyd-Metzuyanim, Avital Elbaum-Cohen and Michal Tabach introduce a tool for analyzing the arithmetic discourse of students. This tool allows to map students' participation in the discourse on a continuum between ritual and explorative. Therefore, the individual performance of a student is assessed based on eight characteristics (e.g., objectification, flexibility or focus on process or procedure) that enable to construct a ritual/explorative ratio.

Anna Lisa Simon, Benjamin Rott and Maike Schindler introduce the use of eye-tracking analysis to explore students' strategy use when naming and locating numbers on a marked number line. When measuring response time, they found that the use of reference points, e.g. gazing at the nearest reference point, was more efficient than strategies based on counting procedures, such as counting from the beginning of the number line.

Summary

The discussed theoretical and empirical projects show a huge variety, but also common goals for current and future research: We aim to investigae students' development of conceptual knowledge, number sense, flexibility and adaptive expertise in arithmetic. Additionally, we intend to develop and evaluate teaching approaches that lead to a deep understanding of arithmetic and number systems.