JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 104, NO. B3, PAGES 4861-4875, MARCH 10, 1999

Atmospheric torque on the Earth and comparison with atmospheric angular momentum variations

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Abstract. The purpose of this paper is to compute atmospheric torques on the Earth, including the oceans, with an emphasis on the equatorial components. This dynamic approach is an alternative method to the classical budget-based angular momentum method for viewing atmospheric effects on Earth's orientation in space. The expression of the total torque interaction between the atmosphere and the Earth is derived from the angular momentum balance equation. Such a torque is composed of three parts due to pressure, gravitation, and friction. Each of these torque components is evaluated numerically by a semi-analytical approach involving spherical harmonic approximations, and their orders of magnitude are intercompared. For the equatorial components the pressure and gravitational torques have far larger amplitudes than that of the friction torque; these two major torques have the same order of magnitude but opposite signs, and the value of the sum of the torques is shown to be close to the equatorial components of the atmospheric angular momentum time derivative s, as would be expected in a consistent model-based analysis system. The correlation between the two time series is shown to be very good at low frequency and decrease slowly with increasing frequency . The correlation is still significant (≥ 0.7) up to 0.5 cycle per day, but the correlation coefficient reduces to 0.5 at the diurnal frequency band, indicating the difficulty of calculating rapidly changing model-based torques within an atmospheric analysis system.

1. Introduction

The atmospheric effects on Earth's rotation are generally investigated using the atmospheric angular momentum (AAM) approach. It assumes an angular momentum balance between the Earth and the atmosphere: to any change of AAM there corresponds a variation, with opposite sign, of the angular momentum of the solid Earth and hence a perturbation in its rotation vector [*Munk and MacDonald*, 1960; *Barnes et al.*, 1983]. These AAM values are derived from meteorological data according to a procedure introduced by *Munk and MacDonald* [1960] and further described by *Barnes et al.* [1983], for instance.

However, an alternative method to the momentum approach exists, consisting of considering the mechanical system Earth and the atmospheric effect as an external torque acting on that system. This torque approach has not been investigated as often as has the angular momentum method because of the difficulty of calculating all its components comprehensively, owing in part to its dependence on atmospheric models.

In Newtonian physics the variation of the total angular momentum of a physical system is only due to external torques acting on that system. Equivalently, when the atmosphere alone is considered, the time derivative of AAM is equal to the total torque on the atmosphere. This torque results from various interactions with the solid Earth, with the ocean, and from other external phenomena, such as the lunisolar tides acting on the atmosphere.

To express the torque from the solid Earth on the atmosphere, we develop analytically the angular momentum conservation equation for a moving fluid in contact with a solid body. The pressure torque portion of this expression has been evaluated by *Dehant et al.* [1996] (see also *Bizouard* [1996]), using a spherical harmonic development of the topography (which can eventually be extended to the bathymetry to account for ocean bottom pressure torques) and using diurnal S_1 pressure tide given by *Haurwitz and Cowley* [1973]. *Gegout et al.* [1998] updated this computation by using new data for the S_1 pressure. In the present paper we extend the semi-analytical approach of Dehant et al. to other torques, and we evaluate them numerically from a time series of variables from an atmospheric analysis system.

Carrying out torque computations to obtain atmospheric effects on the Earth's rotation is of particular interest for the following reasons:

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Paper number 1998JB900063.
0148-0227/99/1998JB900063$09.00

1. Unlike the AAM approach, the torque approach allows us to understand the Earth-atmosphere interaction processes that affect the angular momentum exchange between Earth and atmosphere, and it allows us to evaluate their relative contributions.

2. The AAM approach assumes an isolated Earth-atmosphere system, while the torque approach does not. This may be important because ocean-atmosphere interaction and the effects of the external tidal forces can be important on the time-scales under consideration here, such as the diurnal.

3. The AAM approach implicitly supposes that the ocean either behaves fully as an inverted barometer (IB) or a non-IB; the IB relation is typically valid at periods longer than a few days, but it is not verified for shorter periods when ocean dynamics are important. The torque approach will be very easily extended to account for variable ocean-atmosphere relationships.

4. It is worthwhile to compare the two approaches in order to confirm their reliability and obtain their accuracy, or to understand the limitation of the numerical procedures.

The torque approach was developed in the past essentially to compute the atmospheric effect on the rotation rate [*Swinbank*, 1985 ; *Salstein and Rosen*, 1994], and the impact of atmospheric torque on the long periodic polar motion has been studied by *Wahr* [1982]. More recently, *Dehant et al.* [1996], as already mentioned, computed the pressure torque caused by the 24-hour barometric tide S_1 and its seasonal modulations in order to determine the corresponding effects on nutations.

Section 2 gives a report of a procedure used to compute the atmospheric torque on the Earth more generally than it has been done up to now. In section 2.1 a general analytical expression of the total interaction torque as a function of surface variables has been derived from the angular momentum balance equation and the Navier-Stokes equation applied to the atmosphere. Then this expression has been carefully analyzed, reduced, and physically interpreted (sections 2.2 – 2.4). Section 3 of the paper is devoted to the numerical computation in the time domain of the atmospheric torque from surface pressure and wind data, according to the analytical expressions derived. A very important issue that we address is whether the time series based on the torque approach, partially from an atmospheric model, is compatible with the time series based on the AAM approach (based on analyses) spanning the same period (and implicitly for an IB-ocean; that is, in this case there is no pressure coupling on the bathymetry). Therefore we have also reconstructed the atmospheric torque from the AAM time series (section 3.3). Finally, in section 4 we compare the two sets of results. If a significant agreement would be shown, this would be an important confirmation of the reliability of both AAM and torque approaches. We discuss this issue in terms of correlation and scale factor at different timescales.

2. Torques Acting on the Atmosphere and Relation With the Atmospheric Angular Momentum

2.1. Derivative of the Angular Momentum and Torques Acting on a Fluid Moving Around a Rigid Body

The interaction torque between the Earth and the atmosphere is contained in the total torque acting on the atmosphere, $\vec{\Gamma}_A$, provided by the angular momentum balance

$$\vec{\Gamma}_A = \frac{D\vec{h}_A}{Dt}, \qquad (1)$$

where D/Dt is the time derivative in a celestial frame and $\vec{h}_A$ is the atmospheric angular momentum given in the celestial frame by

$$\vec{h}_A = \int_{\mathrm{Atm}} \vec{r} \wedge \vec{v}\, dM, \qquad (2)$$

where $\vec{r}$ is the position vector and $\vec{v}$ is the speed of the mass element dM. Let $\vec{v}_r$ be the velocity of the mass element dM in the terrestrial frame, and let $\vec{\Omega}$ be the Earth's rotation vector; then $\vec{v} = \vec{v}_r + \vec{\Omega} \wedge \vec{r}$. Assuming that the mass of the atmosphere is constant (for instance, the budget of the water content is not taken into account), we have

$$\frac{D\vec{h}_A}{Dt} = \int_{\mathrm{Atm}} \frac{D}{Dt}\left[\vec{r} \wedge \vec{v}\right]\, dM; \qquad (3)$$

that is,

$$\frac{D\vec{h}_A}{Dt} = \int_{\mathrm{Atm}} \vec{r} \wedge \left[\frac{D\,\vec{v}_r}{Dt} + \frac{D\,\vec{\Omega}}{Dt} \wedge \vec{r} + \vec{\Omega} \wedge \vec{v}_r + \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{r})\right] dM\,. \qquad (4)$$

Let d/dt be the time derivative in the terrestrial frame. Then we have

$$\frac{D\vec{v}_r}{Dt} = \frac{d\vec{v}_r}{dt} + \vec{\Omega} \wedge \vec{v}_r\,, \qquad \frac{D\vec{\Omega}}{Dt} = \frac{d\vec{\Omega}}{dt}\,. \qquad (5)$$

Thus, if ρ is the density of the atmosphere and dV is the volume of the mass dM, we get

$$\frac{D\vec{h}_A}{Dt} = \int_{V_A} \vec{r} \wedge \left[\frac{d\,\vec{v}_r}{dt} + \frac{d\,\vec{\Omega}}{dt} \wedge \vec{r} + 2\vec{\Omega} \wedge \vec{v}_r + \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{r})\right] \rho\, dV\,, \qquad (6)$$

where V_A is the volume of the atmosphere.

The atmosphere is accelerated by means of several external forces, with its decelerations related to flow, which drags the mass: angular momentum is changed by local velocity variations or by mass displacement toward a point where the velocity field takes another value. These two kinds of variation are summarized by the equation

$$\frac{d\vec{v}_r}{dt} = \frac{\partial \vec{v}_r}{\partial t} + (\vec{v}_r \cdot \vec{\nabla})\vec{v}_r, \tag{7}$$

where $(\vec{v}_r \cdot \vec{\nabla})\vec{v}_r$ is the acceleration induced by advection and $\partial \vec{v}_r/\partial t$ is the acceleration relative to the rotating frame ($\vec{\nabla}$ expresses the gradient operator and the center dot is the scalar product). Hence

$$\frac{D\vec{h}_A}{Dt} = \int_{V_A} \rho\, \vec{r} \wedge \left(\frac{\partial \vec{v}_r}{\partial t} + 2\vec{\Omega} \wedge \vec{v}_r + \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{r}) + \frac{d\vec{\Omega}}{dt} \wedge \vec{r} + \left(\vec{v}_r \cdot \vec{\nabla}\right) \vec{v}_r \right) dV. \tag{8}$$

The first term is due to the relative acceleration; the second, third, and fourth terms are due to the inertial effects (Coriolis, centrifugal, and frame acceleration, respectively); and the fifth term is due to advection. The volume integral of the advection term is zero, as is shown in Appendix A, and the angular speed of the frame is supposed to be constant. The corresponding terms can thus be removed from (8). The acceleration $\partial \vec{v}_r/\partial t$ of the fluid particle is given by the Navier-Stokes equation in a rotating reference frame:

$$\frac{\partial \vec{v}_r}{\partial t} = \rho\vec{F} - \vec{\nabla}\left(p + \eta \vec{\nabla} \cdot \vec{v}_r\right) + \eta \Delta \vec{v}_r - 2\vec{\Omega} \wedge \vec{v}_r - \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{r}), \tag{9}$$

where η is a viscous parameter, p is the atmospheric pressure, and $\vec{F}$ is the external force ($\Delta = \vec{\nabla} \cdot \vec{\nabla}$ expresses the Laplacian). As a first approximation, the mass of air is conservative; then the divergence of the flow vanishes:

$$\vec{\nabla} \cdot (\rho \vec{v}_r) = 0; \tag{10}$$

that is,

$$\rho\, \vec{\nabla} \cdot \vec{v}_r + \vec{v}_r \cdot \vec{\nabla}\rho = 0\,. \tag{11}$$

As the velocity of the flow is essentially horizontal and the gradient of the density is essentially vertical, the second term in (11) is essentially zero. The velocity divergence can thus be neglected. Substituting (9) into (8) and taking into account the earlier remarks, we get

$$\frac{D\vec{h}_A}{Dt} = \int_{V_A} \vec{r} \wedge \left(\rho\vec{F} - \vec{\nabla}p + \eta\Delta\vec{v}_r\right) dV\,. \tag{12}$$

Using (C6) of Appendix C and the formula of the rotational (equation C3), it can be shown that we have, for any scalar function f,

$$\int_{V_A} \vec{r} \wedge \vec{\nabla} f \, dV = \int_{S_A} (\vec{r} \wedge \hat{n})\, f dS, \tag{13}$$

where $\hat{n}$ is the outer normal unit vector to the bottom surface of the atmosphere, S_A ($\hat{n}$ is directed toward the center of the Earth). Thus (12) is equivalent to

$$\frac{D\vec{h}_A}{Dt} = \int_{V_A} \vec{r} \wedge \left(\rho\vec{F} + \eta\Delta\vec{v}_r\right) dV + \int_{S_A} \vec{r} \wedge \hat{n}\,(-p)\, dS\,, \tag{14}$$

The first volume integral provides the body torque due, on the one hand, to the forces external to the atmosphere, among them the Earth gravity, and, on the other hand, a term linked to the viscosity. The second expression shows one component of the total interaction torque expressed in terms of a surface integral, that is, the pressure torque (with the opposite sign for the torque on the Earth). The latter will be identified as the friction torque. The viscous term in (14) is

$$\int_{V_A} \eta \vec{r} \wedge \Delta \vec{v}_r \, dV\,. \tag{15}$$

Working in Cartesian coordinates, it can be shown after some algebra that

$$\vec{r} \wedge \Delta \vec{v}_r = \vec{\nabla} \cdot \vec{\nabla}\,(\vec{r} \wedge \vec{v}_r) - 2\vec{\nabla} \wedge \vec{v}_r\,. \tag{16}$$

When (16) is substituted into (15) and the Green-Ostrogradski theorem (Appendix C, equation (C1)) is applied to the first term, and the rotational formula (Appendix C, equation (C3)) is applied to the second term, assuming that the viscous parameter η is uniform, one then gets

$$\int_{V_A} \eta \vec{r} \wedge \Delta \vec{v}_r\, dV = \int_{S_A} \eta \vec{\nabla}(\vec{r} \wedge \vec{v}_r) \cdot \vec{n}\, dS + \int_{S_A} 2\eta \vec{v}_r \wedge \hat{n}\, dS. \tag{17}$$

Developing $\vec{\nabla}\,(\vec{r} \wedge \vec{v}_r)$ in Cartesian coordinates, we can show that

$$\vec{\nabla}(\vec{r} \wedge \vec{v}_r) \cdot \hat{n} = \vec{r} \wedge (\hat{n} \cdot \vec{\nabla})\vec{v}_r - \vec{v}_r \wedge \hat{n}\,. \tag{18}$$

Hence we have

$$\int_{V_A} \eta \vec{r} \wedge \Delta \vec{v}_r\, dV = \int_{S_A} \eta \vec{r} \wedge (\hat{n} \cdot \vec{\nabla})\vec{v}_r\, dS + \int_{S_A} \eta \vec{v}_r \wedge \hat{n}\, dS. \tag{19}$$

It turns out, as is shown in Appendix B, that the first term is exactly the opposite of the torque exerted by a viscous fluid acting on a rigid body. The numerical computation of the second term proved that it is negligible.

2.2. Torques Related to the Earth-Atmosphere Interaction

In section 2.1 we have deduced the torques from the angular momentum budget equation of a fluid around a solid body, in the general case. The analytical expressions found for these torques are now applied in the case of the Earth-atmosphere system. We have three torques: (1) the torque produced by external forces, mostly gravitational,

$$\int_{V_A} \vec{r} \wedge \rho \vec{F}\, dV; \tag{20}$$

(2) the normal pressure torque,

$$\int_{S_A} [\vec{r} \wedge \hat{n}(-p)]\, dS; \tag{21}$$

and (3) the tangential friction torque,

$$\int_{S_A} \vec{r} \wedge \left(\eta \hat{n} \cdot \vec{\nabla} \vec{v}_r\right)\, dS, \tag{22}$$

where $\hat{n}$ is the unit normal to the surface directed toward the center of the body.

In the particular case of the rotating Earth in contact with the moving atmosphere, for computing the Earth-atmosphere coupling, the external force is reduced to the Earth's gravitation. Indeed, the lunisolar gravitational attraction on the atmosphere as well as the other forces acting on it (such as the magnetic coupling with solar wind, the ocean-atmosphere interaction, etc.) are not due to the solid Earth and are not considered here. Moreover, they are associated with indirect effects on the atmospheric pressure and density. These effects on the Earth are thus incorporated in the pressure torque and the gravitational torque. The same conclusion can be drawn for the solar heating effects on the atmosphere.

For studying the effect on the Earth's rotation we would have to deal with the torque that the atmosphere exerts on the Earth, that is, the opposite of the total interaction torque considered here (because of the law of the action and reaction).

2.3. Expressions of the Interaction Torques Between Earth and Atmosphere Using a Spherical Harmonic Expansion

In order to be able to obtain numerical evaluation of the different torques appearing in the interaction between the Earth and the atmosphere, we have decided in this paper to use spherical harmonic expansion. This allows us to avoid numerical instability due to the time derivative of a high-frequency function.

2.3.1. Pressure torque. In the work of *Dehant et al.* [1996] a numerical computation of the pressure torque was based on spherical harmonic expansions of the surface pressure and the shape of the Earth's surface. The surface pressure is expressed as

$$p = P_0 \left[1 + \sum_{n=1}^{n_{\max}} \sum_{m=0}^{n} (p_{nm} \cos m\lambda + \tilde{p}_{nm} \sin m\lambda) P_{nm}(\cos\theta)\right], \tag{23}$$

where P_0 is the mean atmospheric pressure, λ is the longitude, θ is the colatitude, and $P_{nm}(\cos\theta)$ are the associated Legendre polynomials.

The shape of the Earth is expressed in a similar way as

$$r = r_0 \left[1 + \sum_{n=1}^{n_{\max}} \sum_{m=0}^{n} (u_{nm} \cos m\lambda + \tilde{u}_{nm} \sin m\lambda) P_{nm}(\cos\theta)\right], \tag{24}$$

where r_0 is the mean radius of the Earth ($r_0 = 6,371,000$ m). It should be noted that the largest departure from the mean sphere is associated with the ellipsoidal shape and is given by the coefficient $u_{20} = -2/3f$, where $f \approx 1/300$ is the geometrical flattening of the ellipsoid.

The pressure torque acting on the atmosphere is given by

$$\vec{\Gamma}_p = \int_{S_A} \vec{r} \wedge (-p)\hat{n} dS\,, \tag{25}$$

because $\vec{r} \wedge \hat{n} = \vec{r} \wedge \vec{\nabla} r$, by the defininton of $\hat{n}$. Substituting r by this expression in (24) gives the pressure torque in spherical coordinates:

$$\vec{\Gamma}_p = -\int_{S_A} \vec{r} \wedge p r_0 \vec{\nabla} \left[\sum_{n=1}^{n_{\max}} \sum_{m=0}^{n} (u_{nm} \cos m\lambda + \tilde{u}_{nm} \sin m\lambda) P_{nm}(\cos\theta)\right] dS. \tag{26}$$

This expression has been developed by *Dehant et al.* [1996] as a function of the Earth shape and pressure. Here we consider the pressure effect on the atmosphere, the opposite of the expressions of *Dehant et al.*; that is,

$$\begin{aligned}
\Gamma_p^x &= -\sum_{n=1}^{n_{\max}} \sum_{m=0}^{n} D \Big[(1+\delta_{m0})(n-m)(n+m+1) (u_{n,m}\, \tilde{p}_{n,m+1} - \tilde{u}_{n,m}\, p_{n,m+1}) \\
&\quad + (1-\delta_{m0}+\delta_{m1})(u_{n,m}\, \tilde{p}_{n,m-1} - \tilde{u}_{n,m}\, p_{n,m-1})\Big] \\
\Gamma_p^y &= -\sum_{n=1}^{n_{\max}} \sum_{m=0}^{n} D \Big[-(1+\delta_{m0})(n-m)(n+m+1) (\tilde{u}_{n,m}\, \tilde{p}_{n,m+1} + u_{n,m}\, p_{n,m+1}) \\
&\quad + (1-\delta_{m0}+\delta_{m1})(u_{n,m}\, p_{n,m-1} + \tilde{u}_{n,m}\, \tilde{p}_{n,m-1})\Big]
\end{aligned} \tag{27}$$

$$\Gamma_p^z = \sum_{n=1}^{n_{\max}} \sum_{m=0}^{n} 2D\,(\tilde{u}_{n,m}\, p_{n,m} - u_{n,m}\, \tilde{p}_{n,m}),$$

with

$$D = \frac{(n+m)!}{(2n+1)(n-m)!}\pi r_0^3.$$

Values Γ_p^x, Γ_p^y, and Γ_p^z are the three components of the torque in the classical Tisserand reference frame. Over the oceans the pressure torque exerted on the atmosphere should be computed by taking the geoid as surface. However, for practical purposes and as is done in atmospheric models, the ocean surface has been approximated by the reference ellipsoid, which is the (2,0) spherical harmonic of the Earth topography. In fact, as the differences between the geoidal surface and the ellipsoid amount to only several hundreds of meters, we have shown that this approximation produces a maximum error amouting to 0.5% of the ocean pressure torque. It should be pointed out that the oceanic response to the atmospheric forcing is dynamic and depends on the frequency of the excitation (see *Munk and MacDonald* [1960] and *Ponte et al.* [1991]). This response can not be computed from the atmospheric forcing in a simple way. In this paper we will compute the torque acting on the atmosphere due to solid Earth and the ocean, or the (equivalent) torque acting on the solid Earth and the ocean due to the atmosphere in the simplified case of the IB ocean. This case corresponds to what is done for the AAM computation. If we are concerned with the AAM changes, we are concerned with the torques acting on the atmosphere. In particular, there is the pressure torque on the atmosphere boundaries (the pressure torque is the only torque that is different in the IB ocean and non-IB ocean cases).

The lower atmospheric boundary includes the topography of the continent and the surface of the ocean. There is no direct action of the atmospheric pressure on the bathymetry. So the angular momentum change of the atmosphere does not directly take into account the bathymetry because the ocean surface is unrelated to the bathymetry, an hypothesis nearly equivalent to the IB. The interaction between the solid Earth and the ocean as well as the effects on the Earth's rotation of the oceanic response to atmospheric forcing will be treated later . Similarly, the importance of the ocean dynamics will be evaluated.

2.3.2. Gravitational torque. We shall show that the gravitational torque can be reduced to a surface integral involving atmospheric surface pressure. The gravitational force due to the Earth acting on an element dV_A of the atmosphere is given by

$$d\vec{F} = \rho_A \vec{\nabla}\Phi \, dV_A, \tag{28}$$

where ρ_A is the density of dV_A and Φ is the Earth's gravitational potential given by the spherical harmonic decomposition

$$\Phi = \frac{GM}{r}\left[1 + \sum_{n=2}^{n_{\max}} \left(\frac{r_0}{r}\right)^n \sum_{m=0}^{n} P_n^m(\cos\theta)\,(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\right], \tag{29}$$

where G is the gravitation constant ($G = 6.67259\ 10^{-11}$ ($\mathrm{m^3 kg^{-1} s^{-2}}$)) and M is the Earth's mass ($M = 5.9737\ 10^{24}$(kg)). Thus the gravitational torque on the atmosphere induced by the geopotential is

$$\vec{\Gamma}_G = \int_{V_A} \vec{r} \wedge \rho_A \vec{\nabla}\Phi \, dV_A \,. \tag{30}$$

Let $\hat{n}_{\mathrm{grav}}$ be the normal unit vector to any equipotential of the geopotential (directed toward the center of the Earth): $\hat{n}_{\mathrm{grav}} = \vec{\nabla}\Phi/\|\vec{\nabla}\Phi\|$. Let us assume that the surface pressure p is only due to the weight of the atmosphere above the column:

$$\int_{\text{bottom of atm.}}^{\text{top of atm.}} \rho_A \|\vec{\nabla}\Phi\| dr \approx p \,. \tag{31}$$

Hence it can be shown that (30) is equivalent to

$$\vec{\Gamma}_G = \int_{S_A} \vec{r} \wedge p\hat{n}_{\mathrm{grav}} \, dS, \tag{32}$$

neglecting the variation of $\vec{r}$ and $\hat{n}_{\mathrm{grav}}$ with the height (thin layer approximation). Substituting the expression $\hat{n}_{\mathrm{grav}} = 1/g\vec{\nabla}\Phi$, with the spherical harmonics of Φ, one gets

$$\vec{\Gamma}_G = \int_{S_A} \vec{r} \wedge p r_0 \vec{\nabla}\left[\sum_{n=2}^{n_{\max}} \sum_{m=0}^{n} (C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\ P_{nm}(\cos\theta)\right] dS. \tag{33}$$

This expression is totally analogous to the expression of the pressure torque (equation 26), if we substitute the topographic coefficients by the geopotential coefficients divided by g and take the opposite sign. Substituting the expression of p and evaluating the torque as previously, one obtains developments similar to (27) where the spherical harmonic coefficients of the gravitational geopotential C_{nm} and S_{nm} replace those of the Earth's topography, u_{nm} and $\tilde{u}_{nm}$, respectively. The final expression of the gravitational torque in the three directions can thus be obtained from the expression of the pressure torque (equation 27):

$$
\begin{aligned}
\Gamma_G^x &= \sum_{n=2}^{n_{\max}} \sum_{m=0}^{n} D \Big[(1+\delta_{m0})(n-m)(n+m+1) \\
& \quad (C_{n,m}\, \tilde{p}_{n,m+1} - S_{n,m}\, p_{n,m+1}) \\
&+ \quad (1-\delta_{m0}+\delta_{m1}) \\
& \quad (C_{n,m}\, \tilde{p}_{n,m-1} - S_{n,m}\, p_{n,m-1}) \Big] \\
\Gamma_G^y &= \sum_{n=2}^{n_{\max}} \sum_{m=2}^{n} D \Big[-(1+\delta_{m0})\,(n-m) \\
& \quad (n+m+1)\,(S_{n,m}\, \tilde{p}_{n,m+1} \\
&+ \quad C_{n,m}\, p_{n,m+1}) + (1-\delta_{m0}+\delta_{m1}) \\
& \quad (C_{n,m}\, p_{n,m-1} + S_{n,m}\, \tilde{p}_{n,m-1}) \Big] \\
\Gamma_G^z &= - \sum_{n=2}^{n_{\max}} \sum_{m=0}^{n} 2D\,(S_{n,m}\, p_{n,m} - C_{n,m}\, \tilde{p}_{n,m}).
\end{aligned}
\tag{34}
$$

Restricting the topography and the geopotential to the degree 2 and order 0, it can be easily seen that the pressure and gravitational coefficients are given by

$$
\vec{\Gamma}_p^{20} = \frac{12\pi}{5} r_0^3 \begin{pmatrix} u_{20}\, \tilde{p}_{21} \\ -u_{20}\, p_{21} \\ 0 \end{pmatrix}
$$

$$
\vec{\Gamma}_G^{20} = -\frac{12\pi}{5} r_0^3 \begin{pmatrix} C_{20}\, \tilde{p}_{21} \\ -C_{20}\, p_{21} \\ 0 \end{pmatrix}, \tag{35}
$$

with $C_{20} = -J_2 = -1.08 \times 10^{-3}$ and $u_{20} = -2/3f$, where f is the geometrical ellipticity of the Earth. As the ratio $C_{20}/u_{20} \approx 3/5$, we may conclude that were the topography and potential restricted to degree 2 and order 0, the gravitational torque compensates about three fifths of the pressure torque.

2.3.3. Friction torque. The tangential friction torque on the atmosphere is from (22):

$$
\vec{\Gamma}_f = \int_{S_A} \vec{\eta} r \wedge \left(\hat{n} \cdot \vec{\nabla} \vec{v}_r \right) dS ,
$$

which depends on the density and the wind speed. The wind stress $-\eta \hat{n} \cdot \vec{\nabla} \vec{v}$ is available in the form of two vector fields (f_θ, f_λ), which give the friction force on the ground in the colatitude direction ($\hat{\theta}$ is the unit vector in this direction) and the longitude direction ($\hat{\lambda}$ is unit vector in this directions). So we have

$$
\vec{\Gamma}_f = -\int_{S_A} \left(\vec{r} \wedge \hat{\theta} \right) f_\theta \, dS_A - \int_{S_A} \left(\vec{r} \wedge \hat{\lambda} \right) f_\lambda \, dS_A \,. \tag{36}
$$

It can easily be seen here that the topography does not play an important role. We shall thus consider a constant radius vector (r_0, is the mean radius of the Earth), and finally,

$$
\vec{\Gamma}_f = r_0 \int_{S_A} \begin{pmatrix} -f_\theta \sin\lambda - f_\lambda \cos\theta \cos\lambda \\ f_\theta \cos\lambda - f_\lambda \cos\theta \sin\lambda \\ f_\lambda \sin\theta \end{pmatrix} dS_A \,. \tag{37}
$$

As the shape of the Earth does not appear in (37), one needs no spherical harmonic development for the wind stress to compute this torque.

3. Numerical Computation of the Interaction Torques and the Atmospheric Angular Momentum in the Time Domain

3.1. Data Used and Their Preparation

3.1.1. Atmospheric data. The atmospheric values are derived from the analysis-forecast system of the U.S. National Aeronautics and Space Administration NASA Goddard Earth Observing System - version 1 (GEOS-1) data assimilation system [*Schubert et al.*, 1993]. This system was designed to assimilate the variety of the space-based meteorological observations expected from NASA's planned missions, together with other data, such as the ground-based radiosonde network. This analysis system has been run in a retrospective mode for the period March 1980 to November 1993, and it produced a set of standard meteorological fields, such as winds and pressures, as well as other fields, such as stresses, which are produced with the help of an underlying meteorological model. The spatial resolution is on a 2° (in latitude) x 2.5° (in longitude) grid. The temporal resolution of the archived upper air fields, such as the winds, is every 6 hours, and that of the surface variables, such as pressure and wind stress, is every 3 hours. Finally, we perform every 3 hours a spherical harmonic decomposition of the surface pressure up to degree 70, as required by the expressions (27) and (34) of the pressure and gravitational torques, respectively.

3.1.2. Earth shape. The spherical harmonic development of the topography has been derived from the NASA GEOS-1 model, with the same spatial resolution as that of the pressure. The grid is that used in the atmospheric model. We assume that the oceanic surface follows the ellipsoid of reference (see section 2.3.1).

3.1.3. Geopotential. The geopotential field has been derived from the JGM3 model [*Tapley et al.*, 1996]. This model gives spherical harmonic coefficients of the potential up to degree 70.

3.2. Interaction Torques in Time Domain

The pressure torque, the gravitational torque, and the friction torque have been computed numerically from the NASA GEOS-1 system yielding a value every 3 hours from 1980 to 1992, according to the expressions (27), (34), and (37), respectively. The sum of these torques represents the Earth-atmosphere interaction, which in theory may also be obtained from the time derivative of the atmospheric angular momentum if no other torque acting on the atmosphere is considered and if the atmospheric model and data assimilation system are perfectly consistent.

To obtain AAM, wind and surface pressure values from GEOS-1 atmospheric analyses are used; such analyses are the product of observations assimilated with modeled parameters. Note that in the atmospheric models the nonspherical terms of the geopotential are implicitly taken into account because the atmospheric analyses are given on a grid for which the vertical coordinates have the geopotential height (in meters) as a dynamic variable at levels of constant pressure, and the Earth's surface intersects various levels of geopotential height. Thus the values of the model parameters at constant pressure or at constant geopotential vary in the horizontal direction.

For presentation clarity we have separated the high-frequency components and the low-frequency components. To that aim, we have applied a low-frequency band-pass filter (half cutting period of 10 days) and sampled these torques with a 10-day step. The residuals provide the high-frequency part of these torques. We have then applied a discrete Fourier transform separately to the high-frequency and the low-frequency time series.

The equatorial components of these torques in the terrestrial frame are plotted in Figure 1 (low-frequency part) and Figure 2 (high-frequency part). It can be seen that the pressure torque and the gravitational torque have the same order of magnitude whereas the frictional torque is 2 orders of magnitude smaller. The pressure torque is the largest, but it is partly counterbalanced by the gravitational torque for about one third of its value, as was foreseen in section 2.3.2. These results are not true for the axial component because the friction torque is larger than the gravitational one.

The contributions to the pressure and gravitational torque of each degree of the spherical harmonic development of the Earth shape and of the geopotential are given in Figure 3, plotted on a logarithmic scale, for the equatorial components. The large value at the second-degree harmonic confirms that the major contribution

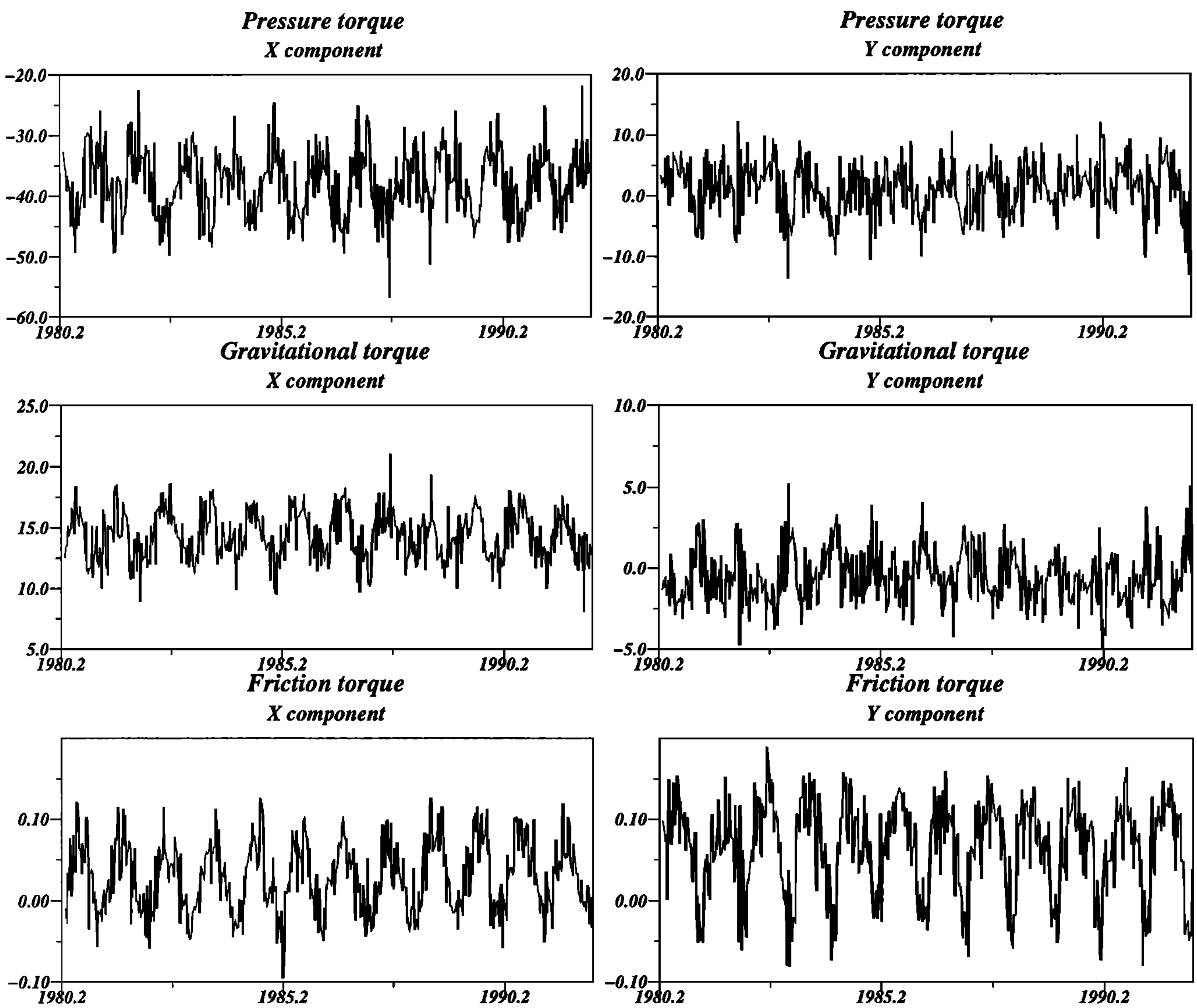

Figure 1. Low-frequency part of the torques in the time domain (units are 10^{20} Nm).

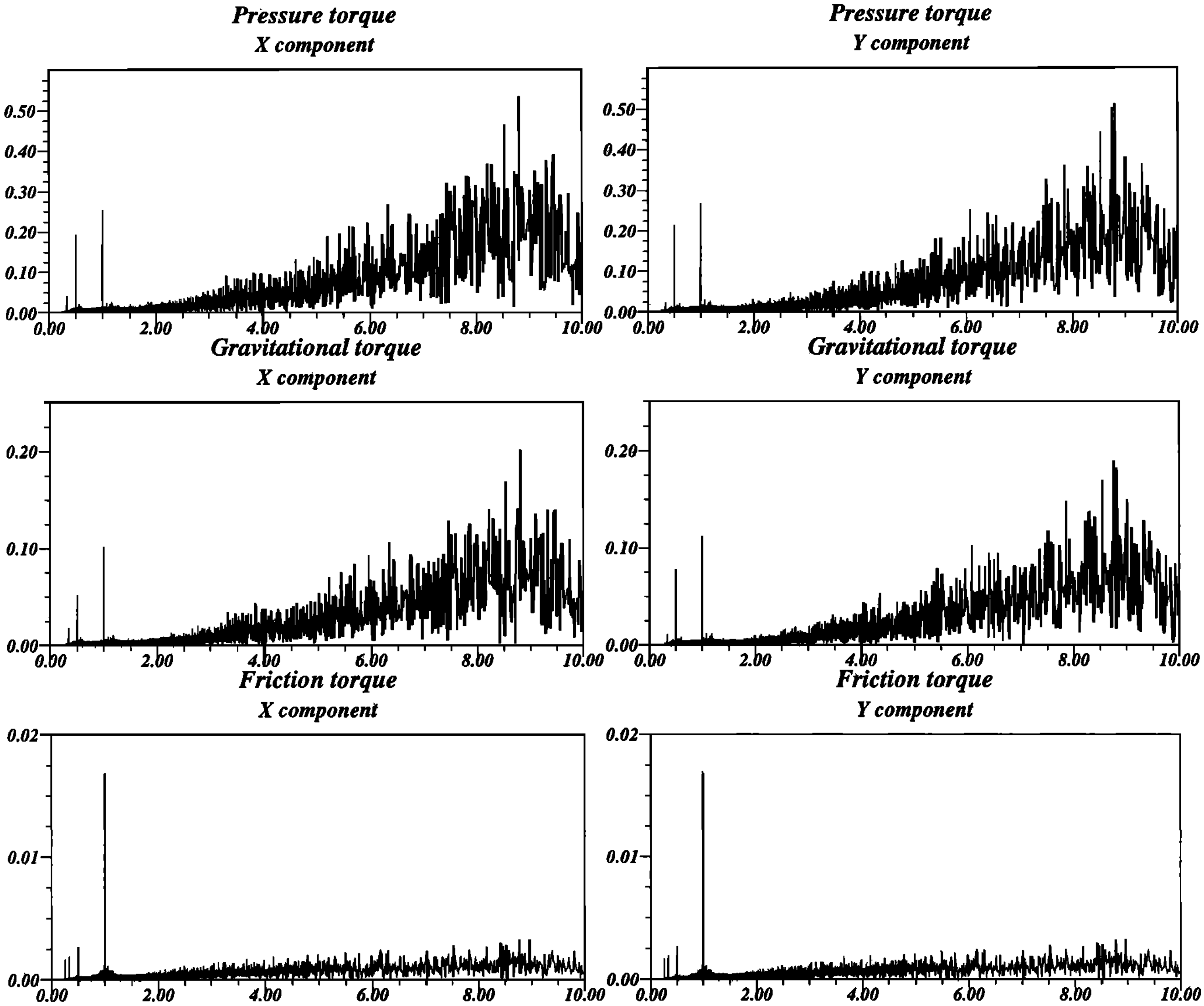

Figure 2. High frequency part of the torques in the frequency domain in cycle per day (units are $10^{20} Nm$).

is due to the Earth flattening (spherical harmonic of degree 2 and order 0).

The axial component of the interaction torque has also been calculated, but the pressure and gravitational torques of this component are independent of the Earth flattening. The friction as well as the coupling with higher topographic features determine the axial torque. We can thus expect a large sensitivity of the axial torque to the high degrees and orders of the spherical harmonics of the grid. Because of the relatively low resolution of the grid, our results are fairly sensitive, as confirmed by the comparison with the AAM derivative in section 4.

3.3. Atmospheric Angular Momentum in Time Domain

Atmospheric angular momentum (AAM) has been computed according to the methods outlined by *Salstein et al.* [1993] from the following expressions given by *Barnes et al.* [1983]:

$$\vec{h}_A = \vec{h}_A^M + \vec{h}_A^w, \tag{38}$$

where $\vec{h}_A^M$ is the matter term expressed by

$$\vec{h}_A^M = \frac{a^4}{g}\Omega \int_{S_A} p \sin^2\theta \begin{pmatrix} \cos\theta\cos\lambda \\ \cos\theta\sin\lambda \\ \sin\theta \end{pmatrix} d\theta \, d\lambda, \tag{39}$$

and $\vec{h}_A^w$ is the wind (motion) term expressed by

$$\vec{h}_A^w = \frac{a^3}{g} \int_{V_A} \sin\theta \begin{pmatrix} v_\theta \sin\lambda + v_\lambda \cos\theta \cos\lambda \\ v_\theta \cos\lambda - v_\lambda \cos\theta \sin\lambda \\ -v_\lambda \sin\theta \end{pmatrix} dp \, d\theta \, d\lambda, \tag{40}$$

where v_θ is the zonal wind speed along the colatitude direction (colatitude increasing and constant longitude)

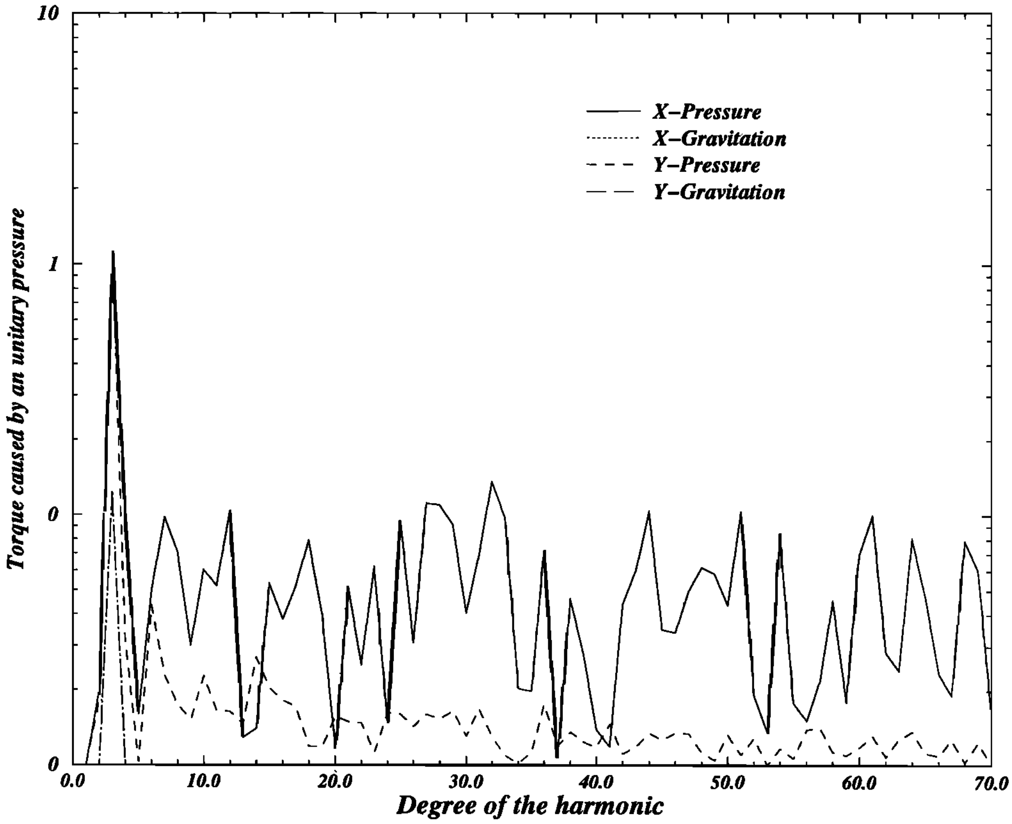

Figure 3. Contribution to the pressure and gravitational torques of each degree of the spherical harmonic decomposition of the Earth's shape. Unit of the torques is 10^{20} Nm.

and v_λ is the wind along the longitude direction (longitude increasing and constant colatitude). The expression of the matter term is obtained from the atmospheric pressure assuming that the surface pressure is the weight of the air masses integrated over the above column.

4. Comparison of the Total Earth Atmosphere Interation Torque and AAM

4.1. Method of Comparison

The relation between the AAM and the torque exerted on the atmosphere is given by the angular momentum balance equation. From the components h^i_A of the atmospheric angular momentum in the terrestrial frame, we can obtain the corresponding torque acting on the atmosphere and express it in the celestial reference frame according to *Munk and MacDonald* [1960]:

$$\begin{aligned} \Gamma^1_A &= \dot{h}^1_A - \Omega h^2_A \\ \Gamma^2_A &= \dot{h}^2_A + \Omega h^1_A \\ \Gamma^3_A &= \dot{h}^3_A . \end{aligned} \qquad (41)$$

The equations allow us to validate our computations given the approximation that we applied in computing the Earth-atmosphere interaction torque. We can check that there are no additional torques on the atmosphere.

A time series of the torque acting on the atmosphere can be obtained from (41) from the AAM time series. The comparison is difficult because of the sensitivity of forming time derivatives of AAM. Performing such calculations increases the high- frequency noise. In order to avoid such noise, we have used a semi-analytical derivative algorithm. First we calculate a Fourier series of the AAM time series using

$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty}(A_n \cos\frac{n\pi t}{T} + B_n \sin\frac{n\pi t}{T}) , \qquad (42)$$

and then we take the time derivative on each frequency component of the Fourier series analytically:

$$\frac{df(t)}{dt} = \sum_{n=1}^{\infty}\frac{n\pi}{T}(-A_n \sin\frac{n\pi t}{T} + B_n \cos\frac{n\pi t}{T}) . \qquad (43)$$

The precision of such computations is estimated to be better than 10^{16}Nm.

4.2. Comparison in the Time Domain

In Figure 4 and Figure 5 we have plotted the equatorial components of the total interaction torque and the total torque on the atmosphere derived as the time derivative of the AAM time series. These two time series appear to be very coherent. Indeed, the correlation

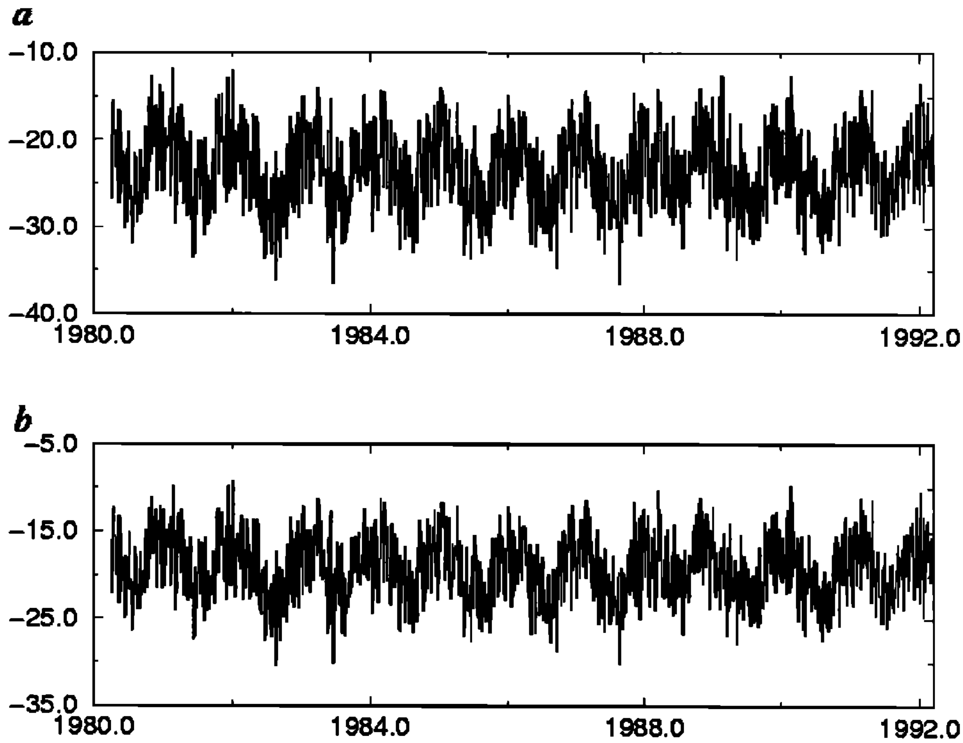

Figure 4. Equatorial components of the (a) total interaction torque and (b) atmospheric angular momentum (AAM) time derivative in the terrestrial frame for the x component. Unit of the torques is 10^{20} Nm.

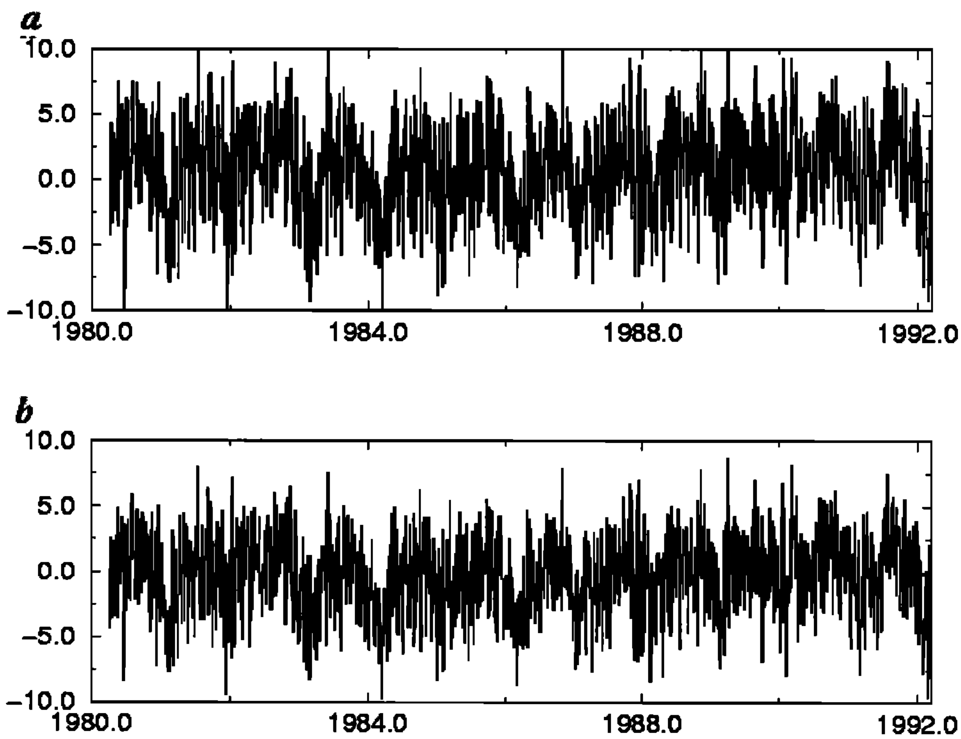

Figure 5. Equatorial components of the (a) total interaction torque and (b) AAM time derivative in the terrestrial frame for the y component. Unit of the torques is 10^{20} Nm.

coefficient r is 0.970 for the x component and 0.965 for the y component.

The difference between the AAM derivative and the total torque series for the x and y components, plotted in Figure 6, appears to contains both a high-frequency signal and an annual wave.

For the axial direction the correlation between the interaction torque and the AAM derivative (not shown) is not significant. This lack of agreement here is likely to be due to the inaccuracy of computing the axial interaction torque with our spherical harmonic approach, and it could be improved using a more precise grid system. As was noted in section 3.2, the grid resolution may indeed be too low to compute correctly the axial components of the pressure and gravitational torques.

4.3. Comparison in the Spectral Domain

As is suggested by the comparison in the time domain, it is worthwhile to estimate the correlation coefficients between the equatorial torque and momentum derivative series for each frequency band. To do so, each time series is filtered by a band-pass filter, and the residual parts of each series are compared. For this comparison the bandwidth of the filter is taken rather large. For each band we have then computed the correlation coefficients between the two time series of residuals, and we fit their regression coefficient a of the total interaction torque against the AAM derivative.

In the context of this comparison we also compute the correlation between series as a function of the frequency band, related to the coherency function, a method that is similar to correlation within frequency bands.

From the regression and correlation results shown in Figure 7, several conclusions can be drawn:

1. There is a scale factor between the AAM time derivative and the Earth-atmosphere interaction torque that we computed. This regression coefficient has the value of 0.8 for the low-frequency part of the signal (up to 0.45 cycle per solar day). A possible explanation for the scale factor is the sensitivity of the pressure torque to the topography. Using another topographic grid, we have indeed obtained a very different pressure torque. In particular, we have experienced the fact that by using a different topographic grid, the scaling factor can be increased up to 5, maybe more.

2. The correlation coefficients are very close to 1 at low frequency (larger than 0.9 until a frequency of 0.45 cycle per solar day).

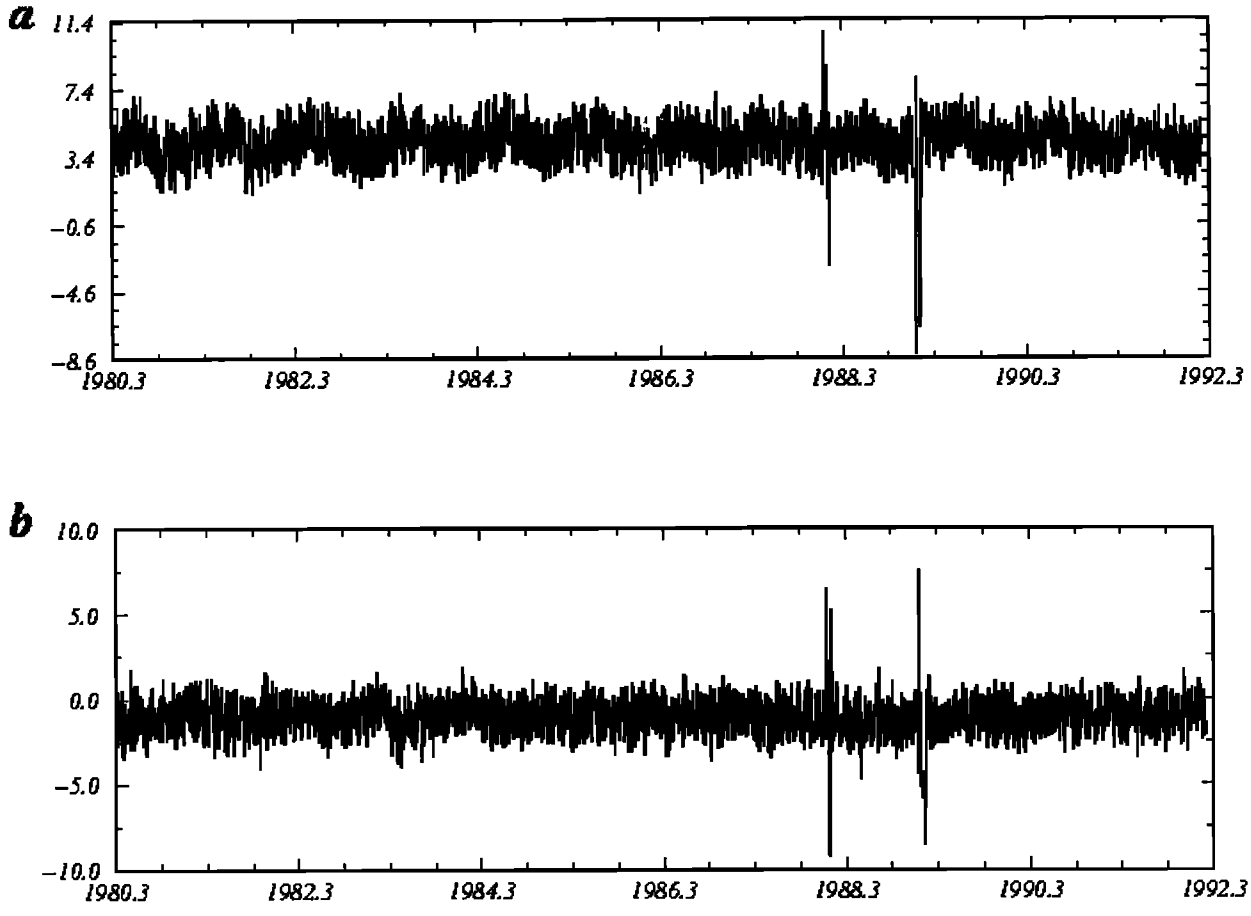

Figure 6. Differences between the total interaction torque and the AAM time derivative for the (a) x component and (b) y component. Unit of the torques is 10^{20} Nm.

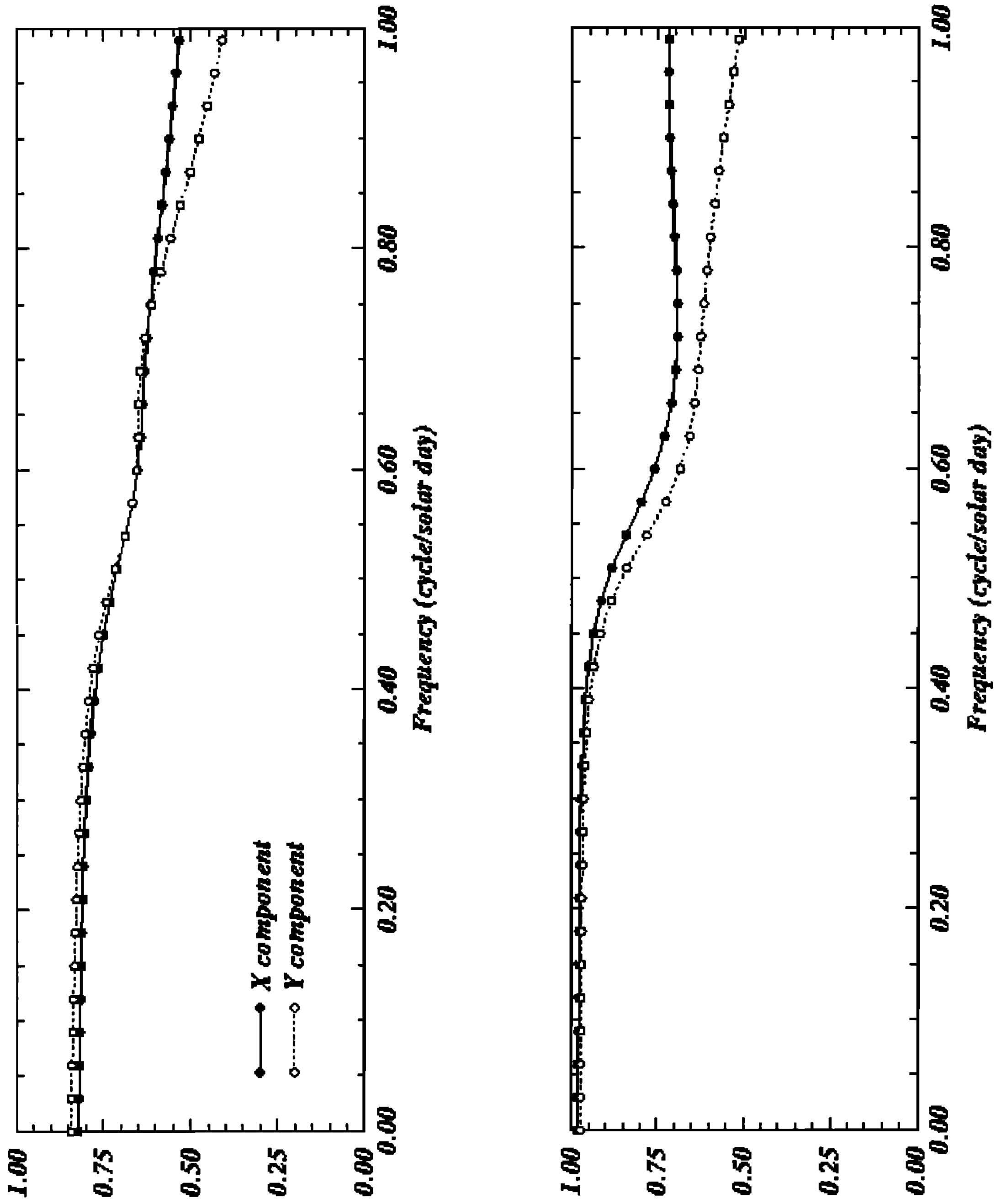

Figure 7. (a) Regression coefficient and (b) correlation coefficient, between the AAM time derivative and the total interaction torque in frequency bands from 0 cycle per solar day to 1 cycle per solar day.

3. The correlation and the regression coefficients slowly decrease with increasing frequency.

4. Except for the quasi-diurnal frequency, the x and y components have a very similar behavior in both time series.

5. The x components of the torque and the AAM time derivative are significantly correlated (correlation coefficient of 0.71) at the 1 cycle per day frequency. The y components are not. This point is the major difference between the two curves.

5. Conclusion

Firstly the analytical expression of the total interaction torque between the atmosphere and the Earth including the oceans has been deduced from the angular momentum balance applied to the atmosphere. Three torques were examined: the pressure torque, the gravitational torque, and the friction torque. The following approximations have been made: (1) the flow of air is conservative; that is,the total atmospheric mass is conserved; (2) the viscous parameter η is uniform; (3) the atmosphere height is very small with respect to the Earth radius, which is the thin layer approximation; (4) the radial wind speed (vertical motion) is much smaller than the tangential speed; and (5) the surface pressure is the weight of the air mass integrated over the air column. All of these approximations are assumed at all the timescales considered.

Secondly, data from the NASA Goddard Earth Observing System data assimilation system were used to obtain a numerical evaluation of the interaction Earth-atmosphere torques as well as for the atmospheric angular momentum every 3 hours from March 1980 to March 1992. We have shown that the equatorial friction torque is negligible, that the equatorial pressure torque is counterbalanced for about one third by the equatorial gravitational torque, and that the remaining time series present similar variations.

Thirdly, we have then compared the time series of the total interaction torque with the time series of the time derivative of the AAM. On this comparison we compared and concluded that the total time derivative of the atmospheric angular momentum derived from the analysis data can be explained by the Earth-atmosphere interaction with no external torque on the atmosphere. The equatorial components of these two vectors are globally consistent since their correlation coefficient is about 96%. However, a disagreement appears at the highest frequencies, higher than 0.5 cycle per day, which we discuss here. A possible explanation for a discrepancy at high frequencies is that the time derivative of the AAM differs from the total interaction torque, because some extraterrestrial torque acts on the atmosphere. A second explanation is that at high frequencies the pressure torque on the mountains could not be well taken into account because of the low spatial resolution of the grid system used (more than 200 km). While such an explanation appears plausible for the axial component of the total interaction torque, it may not be valid for the equatorial component, which is almost completely determined by the equatorial flattening part. Only a higher spatial resolution data set would provide a definite answer to that question.

Lastly, we address the fact that the angular momentum is derived from the analyzed fields of the GEOS-1 data assimilation system (DAS), whereas the torques are derived essentially from model forecast of the global circulation model within the data assimilation system. The two approaches are strongly related but may have inconsistency if the model forecasts are not matched by atmospheric data that are ingested into the model.

In our paper we have explained that the AAM approach is equivalent to considering the ocean as an inverted barometer. The good correspondence between the Earth-atmosphere interaction torque series and the AAM time derivative series in the time domain corroborates this assertion.

The torque approach will further give us the possibility of considering a more complex dynamical ocean (IB is not verified at periods lower than a few days), which is particularly important for the computation of the atmospheric effect on nutations. We also plan to further consider the ocean response to the atmospheric forcing in these computations and the Earth's response to atmosphere, ocean tides, and atmospheric forcing on the ocean.

Appendix A: Demonstration That Advection Does Not Produce Any Torque

We shall prove that

$$\mathcal{A} = \int_{V_A} \rho\vec{r} \wedge (\vec{v}_r \cdot \vec{\nabla}\vec{v})\, dV = 0.$$

From (C5) of Appendix C we have

$$\vec{r} \wedge (\vec{v}_r \cdot \vec{\nabla}) = \vec{\nabla}(\vec{r} \wedge \vec{v}_r) \cdot \vec{v}_r\ . \tag{A1}$$

Hence

$$\mathcal{A} = \int_{V_A} \rho\vec{\nabla}(\vec{r} \wedge \vec{v}_r) \cdot \vec{v}_r\, dV.$$

However, from (C4) (see Appendix C),

$$\nabla(\vec{r} \wedge \vec{v}_r)_i \cdot \rho\vec{v}_r = \nabla \cdot [(\vec{r} \wedge \vec{v}_r)_i\, \rho\vec{v}_r] - (\vec{r} \wedge \vec{v}_r)_i\, \vec{\nabla} \cdot (\rho\vec{v}_r).$$

As the flow is conservative, its divergence is zero, and we thus get

$$\mathcal{A} = \int_{V_A} \begin{pmatrix} \nabla \cdot [(\vec{r} \wedge \vec{v}_r)_x\, \rho\vec{v}_r] \\ \nabla \cdot [(\vec{r} \wedge \vec{v}_r)_y\, \rho\vec{v}_r] \\ \nabla \cdot [(\vec{r} \wedge \vec{v}_r)_z\, \rho\vec{v}_r] \end{pmatrix} dV.$$

Then applying the Green-Ostrogradski theorem (see Appendix C, equation (C1)), we get

$$\mathcal{A} = \int_{S_A} \rho(\vec{v}_r \cdot \hat{n})(\vec{r} \wedge \vec{v}_r)\, dS\,. \tag{A2}$$

As the surface velocity is parallel to the surface, the scalar product with the unit outer normal is equal to zero. Hence $\mathcal{A}$ is proven to be zero.

Appendix B: Expression of the Friction Torque Exerted by the Atmosphere on the Earth

Let us prove that the expression

$$\int_{S_A} \eta \vec{r} \wedge \left(\hat{n} \cdot \vec{\nabla}\right) \vec{v}_r \, dS \tag{B1}$$

is the expression of the friction torque on the atmosphere. To this aim, we shall express the tangential force that the atmosphere exerts on a surface element of the Earth. Let (τ_1, τ_2, n') be the local Cartesian coordinate system of which the axes are directed along two orthogonal vectors in the tangential plane to the surface element, and the outer is normal to the surface, that is, $\hat{\tau}_1$, $\hat{\tau}_2$, and $\hat{n}'$, respectively. The stress tensor of the atmosphere is expressed in any Cartesian coordinate system (x_1, x_2, x_3) by

$$\sigma_{ij} = -p\delta_{ij} + \eta \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right), \tag{B2}$$

where v_i are the components of the velocity of the fluid $\vec{v}_r$.

The tangential force exerted on the surface element is thus

$$\vec{T} = (\sigma_{\tau_1 n'} \hat{\tau}_1 + \sigma_{\tau_2 n'} \hat{\tau}_2) dS \tag{B3}$$

with

$$\sigma_{\tau_1 n'} = \eta(\frac{\partial v_{\tau_1}}{\partial n'} + \frac{\partial v_{n'}}{\partial \tau_1}) \tag{B4}$$

$$\sigma_{\tau_2 n'} = \eta(\frac{\partial v_{\tau_2}}{\partial n'} + \frac{\partial v_{n'}}{\partial \tau_2}). \tag{B5}$$

However, at the surface the component of the speed that is normal to the surface remains constant, equal to zero, and thus has derivatives with respect to τ_1 and τ_2 coordinates that are equal to zero. One gets finally

$$\vec{T} = \eta(\frac{\partial v_{\tau_1}}{\partial n'} \hat{\tau}_1 + \frac{\partial v_{\tau_2}}{\partial n'} \hat{\tau}_2) dS = \eta(\hat{n}' \cdot \vec{\nabla} \vec{v}_r) dS. \tag{B6}$$

The friction torque exerted by the Earth on the atmosphere comes from the opposite force and finally is equal to

$$\int_{S_A} \eta \vec{r} \wedge \left(-\hat{n}' \cdot \vec{\nabla}\right) \vec{v}_r \, dS = \int_{S_A} \eta \vec{r} \wedge \left(\hat{n} \cdot \vec{\nabla}\right) \vec{v}_r \, dS. \tag{B7}$$

Appendix C: Vectorial Analysis Formulae Used in This Paper

Let V be a volume, S be the surface enclosing this volume, $\hat{n}$ be the outer normal unit vector to this surface, $\vec{A}$ be a vector field, f be a scalar field, and $\vec{u}$ be an arbitrary vector; we have the following theorems.

Transformation of Volume Integrals Into Surface Integrals

Green-Ostrogradski or divergence theorem

$$\int_V \vec{\nabla} \cdot \vec{A}\, dV = \int_S \vec{A} \cdot \hat{n}\, dS \tag{C1}$$

Gradient formula

$$\int_V \vec{\nabla} f\, dV = \int_S f \hat{n}\, dS \tag{C2}$$

Rotational formula

$$\int_V \vec{\nabla} \wedge \vec{A}\, dV = -\int_S \vec{A} \wedge \hat{n}\, dS \tag{C3}$$

Miscellaneous Formulae

$$\vec{\nabla} \cdot (f\vec{A}) = f\, \vec{\nabla} \cdot \vec{A} + \vec{\nabla} f \cdot \vec{A} \tag{C4}$$

$$\vec{\nabla}(\vec{r} \wedge \vec{v}_r) \cdot \vec{u} = \vec{r} \wedge (\vec{u} \cdot \vec{\nabla})\vec{v}_r - \vec{v}_r \wedge \vec{u} \tag{C5}$$

$$\vec{\nabla} \wedge (f\vec{A}) = f\, \vec{\nabla} \wedge \vec{A} + \vec{\nabla} f \wedge \vec{A} \tag{C6}$$

Acknowledgments. O. de Viron would like to thank A. Quinet for his help in understanding the atmospheric processes, J.-L. Colot for having introduced him to Computer Algebra, P. Vautrin for providing him with data analysis programs, P. Defraigne for her numerous and interesting discussions, P. Nelson for his help in the data processing, and R. Gross for the constructive scientific discussions about the AAM approach. O. de Viron acknowledges also the support from the Symbolic Computing Group (CSO) of the Université Libre de Bruxelles. C. Bizouard would like to thank the financial support of the Royal Observatory of Belgium for short-term visits in this institute. He ended this study at the Space Research Center of the Polish Academy of Sciences, where he was financed by a fellowship in the framework of bilateral French-Polish agreement. O. de Viron is financially supported by the Belgium state program of the Service du Premier Ministre, Service fédéraux des affaires scientifiques, techniques, et culturelles. Support for D. Salstein is from NASA Earth Science Enterprise. Computations were performed at the NASA Goddard Computer Center. The authors would like to thank the two anonymous reviewers and the Associated Editor B. Chao for their interesting and constructive remarks and comments.

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(Received February 4, 1998; revised October 8, 1998; accepted October 16, 1998.)