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Logical forms of chronicles

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Abstract

A chronicle is a temporal model introduced by Dousson *et al.* for situation recognition. In short, a chronicle consists of a set of events and a set of real-valued temporal constraints on the delays between pairs of events. This work investigates the relationship between chronicles and classical temporal-model formalisms, namely TPTL and MTL. More specifically, we answer the following question: *is it possible to find an equivalent formula in such formalisms for any chronicle?* This question arises from the observation that a single chronicle captures complex temporal behaviours, without imposing a particular order of the events in time.

For our purpose, we introduce the subclass of *linear chronicles*, which set the order of occurrence of the events to be recognized in a temporal sequence. Our first result is that any chronicle can be expressed as a disjunction of linear chronicles. Our second result is that any linear chronicle has an equivalent TPTL formula. Using existing expressiveness results between TPTL and MTL, we show that some chronicles have no equivalent in MTL. This confirms that the model of chronicle has interesting properties for situation recognition.

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1 Introduction

The problem of representing complex behaviours has a lot of applications to monitor dynamic systems based on their functioning traces. Detecting complex behaviours in these traces is useful to identify a faulty state of a system or to label traces with higher-level events.

One possible application of this problem is the improvement of health-care systems. A major issue is to evaluate the incidence of a disease or a treatment in a real-world population. This can be done through the analysis of Electronic Health Records (EHR). EHR are health-administrative databases that gather all delivered cares at hospital. This gives a longitudinal view – or a temporal trace – of patients' treatments and their responses. The difficulty with EHR data is that they do not necessarily hold the desired medical information (such as the patient's medical status). This requires to infer the medical status of a patient from observable information.

A practical solution is to define a proxy of a status by a complex situation to match with patient's care pathways. Such proxy is called a *computable-phenotype* or a *phenotype* [8]. The more expressive the phenotype language, the more accurate the evaluation of incidence.

Then, we advocate for the primary importance of the temporal dimension to accurately represent phenotypes. Indeed, systems such as GLARE [27] emphasize management of temporal knowledge to formalize clinical guidelines, including comprehensive treatment of temporal constraints. But, contrary to clinical guidelines that uses complex reasoning on few care pathways (*e.g.*, guideline compliance [5]), our objective is to find the occurrences of a specified complex situation in a large set of patients (*i.e.*, millions of patients). This requires computational efficiency. To sum up, we need a temporal query language to specify

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phenotypes, *i.e.* complex situations, and that can efficiently be matched in large collections of temporal sequences.

Behind this problem lies the classical tradeoff of computer science: the expressiveness of temporal queries *vs.* their computational efficiency. Researchers aim to find computational models that achieve the best compromise between these two opposite objectives. This compromise also depends on the context of usage, and it is not unique. Thus it has been addressed in a wide range of research fields: knowledge reasoning, temporal logic, model checking, temporal databases, complex-event processing, ... Representing complex situations is studied from the origin of logic-based artificial intelligence to represent and reason about temporal facts. Indeed, the representation of actions and formalisms/logics to reason with them are very central to AI. Many knowledge-reasoning formalisms have been proposed: temporal logic of action [19], situation calculus [20], event calculus [21], ... They are very expressive but knowledge-reasoning tools are not efficient enough for being practically applied on massive data.

As the problem of specifying situations is of particular interested for monitoring dynamic or reactive systems, dedicated formalisms attracted a lot of interest at the crossroad of model checking and temporal logic. We can mention Linear Temporal Logic (LTL) [24] when dealing with discrete time, or Metric Temporal Logic (MTL) [18] for real-valued time. Their success comes from their expressiveness and their clear semantics. Many temporal systems are based on these logics. This is especially the case of temporal databases, which extend the principles of relational databases to timed records. A query language such as TSQL2 [4] combines relational operators and LTL operators. DatalogMTL [29] combines datalog language and MTL operators and has been used to query log data [7]. Finally, temporal models have also been developed in the field of complex-event processing and stream reasoning to address the specific questions of recognition efficiency. Kervac and Piel [17] survey such techniques including ETALIS language [2] or chronicles [13]. These tools provide expressive temporal models suitable to efficiently recognize temporal patterns in real-valued timed sequences.

In the present article, we focus on the notion of chronicle. A chronicle is a temporal model introduced by Dousson *et al.* [13] for situation recognition. In short, a chronicle consists of a set of events and a set of real-valued temporal constraints on the delays between pairs of events. It describes situations that can be recognized within a temporal sequence, *i.e.*, a sequence of timestamped events (with no durations). Chronicles are close to, but not equivalent to, temporal constraint networks [12]. They have the following interesting characteristics:

- they are user-friendly. Their graphical representation makes them attractive for a wide range of applications where temporal patterns have to be analyzed by domain experts;
- they are used with computational efficiency in a wide range of tasks: planning, diagnosis, system modelling, and also data mining. In 1999, Dousson *et al.* [14] proposed an algorithm to discover chronicles from a set of temporal sequences. In this latter context, chronicles form a very expressive class of models, and many works have been proposed in the field of pattern mining to extract frequent or discriminant chronicles [10, 11]. Chronicle recognition algorithms are also computationally efficient [13].
- they are *a priori* expressive. Despite their apparent simplicity, a single chronicle model captures a wide range of practical temporal situations. For instance, they do not imposes a strict order of appearance on the events. A chronicle involving a and b event types can match sequence that contains a and b whatever their order of appearance.

These characteristics make chronicles a first-class citizen to represent complex situations to match in temporal sequences.

A natural question then arises from the remark about the expressiveness of chronicles: what is the relationship between chronicles and classical temporal-model formalisms? The intuition that chronicles are expressive is based on practical uses but, to the best of our knowledge, their expressiveness has not been compared to that of alternative temporal models. A situation can also be seen as a property of a reactive system. Recognizing a situation is similar to matching the property of a reactive system on a single trace [28]. Temporal logics such as Linear Temporal Logic (LTL) [24], Metric Temporal Logic (MTL) [18] or Timed Propositional Temporal Logic (TPTL) [1], have been widely studied to specify temporal properties of reactive systems, and more specifically for the pattern-recognition task. Contrary to LTL or CTL (Computational Tree Logic [9]), which deal with sequences of events in time, MTL and TPTL deal with events having real-valued timestamps. Surprisingly, there is no existing result stating the relationship between temporal logics and chronicles. It is worth noting that chronicles are already equipped with efficient recognition algorithms and that the purpose of the comparisons is to evaluate the expressive power of chronicles but not to gain efficiency with the use TPTL or MTL tools.

This article compares the expressiveness of chronicles with two temporal logics, $\mathsf{TPTL}_\Diamond$ [1, 6] and MTL. The main results are that chronicles can be expressed with $\mathsf{TPTL}_\Diamond$ formulas (in the pointwise semantics [6]), but in general they cannot be expressed with MTL formulas. To obtain this result, we first introduce a notion of *linear chronicles*, and show that any chronicle is equivalent to a finite conjunction of linear chronicles. We then propose a transformation of linear chronicles into $\mathsf{TPTL}_\Diamond$. The impossibility to express chronicles in MTL is then obtained by adapting a result from Bouyer *et al.* [6].

2 Chronicles

In this section, we introduce the basic notions and notations of chronicles. We start by introducing the definitions of temporal sequences and chronicles, and give their semantics through the definition of an *occurrence* of a chronicle in a temporal sequence. Then, we introduce the subclass of *linear* chronicles. This subclass highlights the specificity of the chronicle model, which allows to leave the order of occurrence of some events unspecified. A first result is that any chronicle has an equivalent disjunctive set of linear chronicle. This result is our first step further toward a translation into $\mathsf{TPTL}_\Diamond$.

2.1 Syntax and semantics

Given a finite alphabet Σ of event types, we first introduce the notion of timed sequence over Σ, and then we define chronicles. We let $\leq_\Sigma$ denote a total order on the elements of Σ. In the following, event types are capital letters and $\leq_\Sigma$ is the alphabetic order. For $m \in \mathbb{N}$, we write $[m]$ for the set $\{i \in \mathbb{N} \mid 1 \leq i \leq m\}$.

▶ **Definition 1** (Timed sequence). *A* ***timed sequence*** *of length n over a finite alphabet Σ is a finite sequence $\rho = \langle(\sigma_1, \tau_1), \ldots, (\sigma_n, \tau_n)\rangle$ in $(\Sigma \times \mathbb{R}_{\geq 0})^n$ where for all $1 \leq i < n$, it holds $\tau_i \leq \tau_{i+1}$.*

Using the notations of e.g. Ouaknine *et al.* [22] or Bouyer *et al* [6], a timed sequence $\langle(\sigma_1, \tau_1), \ldots, (\sigma_n, \tau_n)\rangle$ corresponds to the timed word $\langle \sigma = \sigma_1 \cdots \sigma_n, \tau = \tau_1 \cdots \tau_n \rangle$. We may identify timed sequences and their corresponding timed words in the sequel, as long as no ambiguity arises.

Let us now define the notion of chronicle introduced by Ghallab [15]. Our definition of chronicle is borrowed from Besnard and Guyet [3].

▶ **Definition 2** (Chronicle). *A* ***chronicle*** *is a pair* $(\mathcal{E}, \mathcal{T})$ *where*

- $\mathcal{E}$ *is a* ***multiset*** *over* Σ, i.e. $\mathcal{E}$ *is of the form* $\{\!\{c_1, \ldots, c_m\}\!\}$ *(in which repetitions are allowed) such that* $c_i \in \Sigma$ *for* $i = 1, \ldots, m$ *and* $c_1 \leq_\Sigma \cdots \leq_\Sigma c_m$. *We impose the latter condition for technical reasons explained at Remark 6;*
- $\mathcal{T}$ *is a set of* ***temporal constraints****,* i.e. *expressions of the form* $(c, o_c)[t^-, t^+](c', o_{c'})$ *such that*
  1. $c, c' \in \mathcal{E}$ *and*
  2. $t^-, t^+ \in \mathbb{Q} \cup \{-\infty, +\infty\}$ *and*
  3. $o_c, o_{c'} \in [m]$ *and* $o_c < o_{c'}$ *and*
  4. $c_{o_c} = c$ *and* $c_{o_{c'}} = c'$.

The size of a chronicle $(\mathcal{E}, \mathcal{T})$ *is the size* m *of its multiset* $\mathcal{E}$.

▶ **Example 3.** Let $\Sigma = \{A, B, C\}$; The pair

$$\left(\{\!\{A, B, B, C\}\!\}, \left\{ \begin{array}{ll} (A,1)[-3.5,2](B,2), & (A,1)[-2,2.3](C,4), \\ (B,2)[-1,5](C,4), & (B,2)[0.1,2](B,3), \\ (B,3)[-1,5](C,4) & \end{array} \right\} \right)$$

is a chronicle. It involves two occurrences of event type B, and one occurrence of event types A and C. It has no direct temporal constraints between $(A,1)$ and $(B,3)$.

Such a chronicle can be represented as a directed graph, with one vertex per event in the multiset, and edges labelled with the temporal constraints. Figure 1 represents the chronicle above.

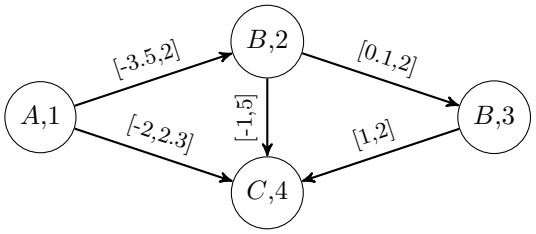

Figure 1 Graphical representation of the chronicle in Example 3.

We now define the semantics of chronicles, via the notion of occurrence of a chronicle in a timed sequence:

▶ **Definition 4** (Occurrence of a chronicle). *Let* $\boldsymbol{s} = \langle(\sigma_1, \tau_1), (\sigma_2, \tau_2), \ldots, (\sigma_n, \tau_n)\rangle$ *be a timed sequence of length* n, *and* $\mathscr{C} = (\mathcal{E} = \{\!\{c_1, \ldots, c_m\}\!\}, \mathcal{T})$ *be a chronicle of size* m. *Chronicle* $\mathscr{C}$ *is said to occur in* $\boldsymbol{s}$ *if, and only if, there exists an injective function* $\varepsilon \colon [m] \to [n]$, *hereafter called* embedding, *such that:*

1. *for all* $1 \leq i < m$, $\tau_{\varepsilon(i)} < \tau_{\varepsilon(i+1)}$ *whenever* $c_i = c_{i+1}$,[1]
2. *for all* $1 \leq i \leq m$, $\sigma_{\varepsilon(i)} = c_i$,
3. *for all* $1 \leq i, j \leq m$, $\tau_{\varepsilon(j)} - \tau_{\varepsilon(i)} \in [t^-, t^+]$ *whenever* $(c_i, i)[t^-, t^+](c_j, j) \in \mathcal{T}$.

[1] This condition is not always required for occurrences of chronicles, but it appears for instance in [3]. As we explain later, our results also holds when this condition is lifted. Similarly, in some settings it might be convenient to remove the injectiveness condition of the embedding functions, and again this would be easy to deal with in our translations.

Then, $\tilde{\boldsymbol{s}} = \{(\sigma_{\varepsilon(1)}, \tau_{\varepsilon(1)}), \ldots, (\sigma_{\varepsilon(m)}, \tau_{\varepsilon(m)})\}$ *is an* occurrence *of* $\mathscr{C}$ *in* $\boldsymbol{s}$.

Chronicle $\mathscr{C}$ *is said to match the sequence* $\boldsymbol{s}$*, denoted* $\mathscr{C} \Subset \boldsymbol{s}$*, if, and only if, there is at least one occurrence of* $\mathscr{C}$ *in* $\boldsymbol{s}$.

The temporal constraints of a chronicle are conjunctive: an embedding of a chronicle has to satisfy all of them. As a consequence, a chronicle with two temporal constraints $(c, o_c)[t^-, t^+](c', o_{c'})$ and $(c, o_c)[u^-, u^+](c', o_{c'})$ relating the same pair of events is equivalent (w.r.t. occurrences) to a single temporal constraint with the interval $[t^-, t^+] \cap [u^-, u^+]$. It follows that in any chronicle, $\mathcal{T}$ can be assumed to contain at most one temporal constraint per pair of events. For any two indices i and j in $[m]$ with $i < j$, we write $t^-_{i,j}$ and $t^+_{i,j}$ for the rational values such that $(c_i, i)[t^-_{i,j}, t^+_{i,j}](c_j, j)$ is the (unique) temporal constraint between (c_i, i) and (c_j, j).

On a similar note, chronicles do not allow temporal constraints on pairs of events (c_i, i) and (c_j, j) when $i \geq j$; such constraints are trivial when $i = j$, while when $i > j$, the constraint $(c_i, i)[t^-, t^+](c_j, j)$ is equivalent (w.r.t. occurrences) to the constraint $(c_j, j)[-t^+, -t^-](c_i, i)$, which in turn can be intersected with the other constraints relating (c_i, i) and (c_j, j).

A chronicle that occurs in no sequences in said *inconsistent*. This is in particular the case of chronicles with unsatisfiable temporal constraints. For instance, the chronicle $(\{\!\{A, B, C\}\!\}, \{(A, 1)[1, 2](B, 2), (B, 2)[3, 4](C, 3), (A, 1)[-2, -1](C, 3)\})$ is inconsistent. Indeed, because of the first two temporal constraints, C must occur after A (B after A and C after B), but the third constraint enforces A to occur before C.

In this article, we do not bother about the minimality or the satisfiability of the temporal constraints. Dechter *et al.* [12] proposed reasoning techniques about temporal constraints to narrow the intervals of temporal constraints (w.r.t. some equivalence) or identify insatisfiable temporal constraints. Such techniques may apply for chronicles also, but they are not required for the results presented in this article.

The problem we address in this paper is whether chronicles can be represented by equivalent temporal logic formulas. In such a case, theoretical results and algorithms could be used to better understand chronicles, and possibly improve existing chronicle-matching algorithms [16].

▶ **Example 5.** Consider again the chronicle depicted at Fig. 1. This chronicle can be seen to occur in the following timed sequences:

- $\langle (A, 1.8), \boldsymbol{(A, 3.5)}, \boldsymbol{(B, 3.9)}, \boldsymbol{(B, 4.1)}, (C, 4.2), \boldsymbol{(C, 5.7)} \rangle$
- $\langle \boldsymbol{(B, 0.2)}, \boldsymbol{(B, 0.9)}, \boldsymbol{(C, 2.5)}, (B, 3.2), \boldsymbol{(A, 3.7)}, (A, 4.7) \rangle$

Events in bold are the events that form the embedding of the chronicle. The second temporal sequence illustrates that the chronicle can have occurrences with different orders of event types. This is possible thanks temporal constraints allowing negative delays.

▶ Remark 6. Notice that in a timed sequence, several events may occur at the same time, and such "simultaneous" events can appear in an occurrence of a chronicle. However, because the embeddings ε are required to be injective, a single event in a timed sequence cannot be used to match different copies of the same event in a chronicle. For instance, chronicle $(\{\!\{A, A\}\!\}, (A, 1)[-2, 2](A, 2))$ cannot occur in a timed sequence containing a single event A.

3 Disjunction of linear chronicles

We have seen that a chronicle expresses only conjunctive temporal constraints. This is obviously limiting to represent complex temporal behaviours for which there are possible

alternative situations to represent. A natural solution is to use several chronicles, one for each situation and to define the situation recognition as a disjunctive matching of the chronicles.

▶ **Definition 7** (Occurrence of (disjunctive) collection of chronicles). *Let $\mathcal{C} = \{(\mathcal{E}_1, \mathcal{T}_1), \ldots, (\mathcal{E}_h, \mathcal{T}_h)\}$ be a collection of h chronicles and $\boldsymbol{s}$ be a timed sequence. A subsequence $\tilde{\boldsymbol{s}}$ of $\boldsymbol{s}$ is an **occurrence** of $\mathcal{C}$ if, and only if, there exists a chronicle $(\mathcal{E}, \mathcal{T}) \in \mathcal{C}$ such that $\tilde{\boldsymbol{s}}$ is an occurrence of $(\mathcal{E}, \mathcal{T})$. A collection $\mathcal{C}$ of chronicles matches a sequence $\boldsymbol{s}$, denoted $\mathcal{C} \Subset \boldsymbol{s}$, whenever there is at least one occurrence of $\mathcal{C}$ in $\boldsymbol{s}$.*

We now consider a variation of chronicles called *linear chronicles* (LC for short). A linear chronicle is a chronicle equipped with a permutation of the events in its multiset, prescribing the order of occurrence of those events. This is detailed more formally in the following definition.

▶ **Definition 8** (Linear chronicle). *A linear chronicle is a triple $\mathscr{L} = (\{\!\{c_1, \ldots, c_m\}\!\}, \mathcal{T}, \pi)$, where $(\{\!\{c_1, \ldots, c_m\}\!\}, \mathcal{T})$ is a chronicle and π is a permutation of $[m]$.*

A linear chronicle $\mathscr{L} = (\{\!\{c_1, \ldots, c_m\}\!\}, \mathcal{T}, \pi)$ occurs in a timed sequence $\boldsymbol{s} = \langle(\sigma_1, \tau_1), (\sigma_2, \tau_2), \ldots, (\sigma_n, \tau_n)\rangle$ whenever there exists an embedding $\varepsilon\colon [m] \to [n]$ witnessing that the chronicle $(\{\!\{c_1, \ldots, c_m\}\!\}, \mathcal{T})$ occurs in $\boldsymbol{s}$, and such that $\varepsilon \circ \pi$ is increasing.

Intuitively, a linear chronicle is a chronicle for which the order of the events is fixed (via π): in any occurrence, event $c_{\pi(i)}$ always occurs before event $c_{\pi(j)}$ when $i < j$.

▶ Remark 9. Condition 1 in Def. 4 states that for any two identical events c_i and c_j in a chronicle with $i < j$, the time at which c_i is matched must be strictly earlier than the time at which c_j is matched. In particular, for any embedding ε, we must have $\varepsilon(i) < \varepsilon(j)$. Assume that $\pi^{-1}(j) < \pi^{-1}(i)$; since $\varepsilon \circ \pi$ is increasing, we would have $\varepsilon(\pi(\pi^{-1}(j))) < \varepsilon(\pi(\pi^{-1}(i)))$, i.e., $\varepsilon(j) < \varepsilon(i)$. Then no embeddings would exist, and the linear chronicle is inconsistent. As a consequence, for a linear chronicle to be consistent, if two identical events c_i and c_j are such that $i < j$, we must have $\pi^{-1}(i) < \pi^{-1}(j)$; in other terms, π has to preserve the order of identical events.

The occurrence of a disjunctive collection of linear chronicles is defined in the very same way as for disjunctive collections of plain chronicles. Two collections of (possibly linear) chronicles are said *equivalent* if any occurrence of one of them is also an occurrence of the other one. As a special case, this defines an equivalence relation between single chronicles and collections of linear chronicles. We use this notion in the following proposition:

▶ **Proposition 10.** *For any chronicle $\mathscr{C} = (\mathcal{E}, \mathcal{T})$, there exists an equivalent disjunctive collection of linear chronicles.*

Proof. Write $\mathcal{E} = \{\!\{c_1, \ldots, c_m\}\!\}$. For any occurrence $\boldsymbol{s}$ of $\mathscr{C}$, with embedding ε, there is a permutation π of $[m]$ such that $\varepsilon(\pi(j)) < \varepsilon(\pi(j+1))$. We can thus write $\mathscr{C}$ as the disjunction, over all permutations π of $[m]$, of the linear chronicles obtained from $\mathscr{C}$ by adding the ordering π.

That the resulting disjunction of linear chronicles is equivalent to the original chronicle is straightforward: on the one hand, all linear chronicles are obtained from the original one by imposing an order of the events, so that any occurrence of any of the linear chronicles is an occurrence of the original chronicle; on the other hand, as already argued above, any occurrence of the original chronicle satisfies a specific order defined by some permutation π, so that it is an occurrence of one of the linear chronicles. ◀

▶ Remark 11. The number of linear chronicles in the disjunction obtained in our proof can be bounded by $m!$ (the factorial of m). Chronicles without any temporal constraints are easily seen to achieve this bound. By Remark 9, when several copies of the same event appear in the multiset, their order has to be preserved by π, which would reduce the number of permutations to consider.

As can be expected, linear chronicles can be turned in a special form where the timing constraints reflect the order of events:

▶ **Proposition 12.** *For any linear chronicle $\mathscr{L} = (\mathcal{E}, \mathcal{T}, \pi)$, there exists an equivalent linear chronicle $\mathscr{L}' = (\mathcal{E}, \mathcal{T}', \pi)$ such that each interval occurring in $\mathcal{T}'$ is either a subset of $\mathbb{R}_-$ or a subset of $\mathbb{R}_+$.*

Proof. Consider a temporal constraint $(c_{\pi(i)}, \pi(i))[t^-, t^+](c_{\pi(j)}, \pi(j))$ in $\mathcal{T}$. First assume that $i < j$ (the other case is symmetric). For any sequence $\boldsymbol{s} = \langle(\sigma_1, \tau_1), \ldots, (\sigma_n, \tau_n)\rangle$ matched by $\mathscr{L}$, for any embedding $\varepsilon\colon [m] \to [n]$, we must have $\varepsilon(\pi(i)) < \varepsilon(\pi(j))$ (by definition of a matching for a linear chronicle), i.e., $\varepsilon(\pi(j)) - \varepsilon(\pi(i)) \in [0, +\infty)$. Under this requirement, the temporal constraint which imposes that $\tau_{\varepsilon(\pi(j))} - \tau_{\varepsilon(\pi(i))} \in [t^-, t^+]$ is then equivalent to the temporal constraint $\tau_{\varepsilon(\pi(j))} - \tau_{\varepsilon(\pi(i))} \in [0, t^+]$, so that the temporal constraint $(c_{\pi(i)}, \pi(i))[t^-, t^+](c_{\pi(j)}, \pi(j))$ in $\mathcal{T}$ can be replaced by the temporal constraint $(c_{\pi(i)}, \pi(i))[0, t^+](c_{\pi(j)}, \pi(j))$, while preserving the same set of occurrences. ◀

▶ **Example 13** (Equivalent collection of linear chronicles)**.** Figure 2 represents chronicle $\mathscr{C} = (\{\!\{A, B, C\}\!\}, \{(A, 1)[-2, 1](C, 3), (B, 2)[3, 4](C, 3)\})$ and an equivalent collection of three linear chronicles.

The timing constraint $(B, 2)[3, 4](C, 3)$ imposes that B must occur before C, but A can occur either before B, or between B and C, or after C.

We can verify that the temporal constraints of $\mathscr{C}$ forbids the other orders. Then, we can derive one linear from each order and from the temporal constraints of $\mathscr{C}$. This leads to the three linear chronicles depicted in Figure 2.

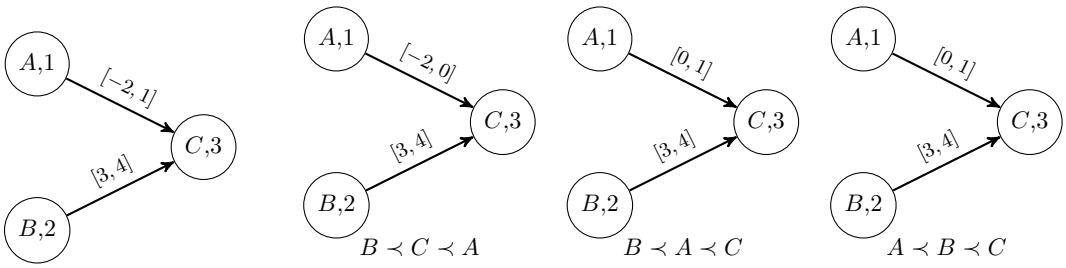

Figure 2 On the left: a chronicle $\mathscr{C}$. On the right: collection of three linear chronicles equivalent to $\mathscr{C}$. The order of event at the bottom of each linear chronicle illustrates their π; Thus, the two rightmost linear chronicles are distinct.

▶ Remark 14. As claimed above, Prop. 10 extends to the setting where we remove Condition 1 in Def. 4. Indeed, this condition imposes the order of occurrence of identical events in a matching; our translation into a disjunction of linear chronicles can thus easily be adapted by dropping all permutations that do not satisfy this condition.

Similarly, Prop. 10 extends to the setting where embeddings are allowed not to be injective: for this it suffices to transform chronicles into disjunctions of (still exponentially many) linear chronicles in which some of the identical events are merged.

4 Chronicles and TPTL$_\Diamond$

The objective of this section is to establish a first result between chronicle and a temporal logic, namely the TPTL$_\Diamond$. We start by recalling the syntax and the semantics of TPTL and TPTL$_\Diamond$, then we propose the construction of a TPTL$_\Diamond$ formula equivalent to any given linear chronicle. With the result of the previous section, the main result is that an equivalent TPTL$_\Diamond$ formula can be constructed for any chronicle.

4.1 Timed Propositional Temporal Logic (TPTL)

The Timed Propositional Temporal Logic (TPTL) is a timed extension of LTL. It uses clock variables to explicitly represent timing constraints in formulae. Below, we define the syntax and semantics of TPTL borrowed from Bouyer et al. [6]. Formulae of TPTL are built from letters in Σ, Boolean connectives, *Until* operators ($\mathcal{U}$), clock resets and clock constraints:

$$\text{TPTL} \ni \varphi ::= \sigma \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \varphi_1 \mathcal{U} \varphi_2 \mid x.\varphi \mid x \sim c$$

where σ is a letter in Σ, x is a clock variable drawn from a finite set X, $c \in \mathbb{Q}$ is a rational number, and $\sim \in \{\leq, <, =, >, \geq\}$.

We are interested in the pointwise semantics, which interprets TPTL over timed sequences. More precisely, models are (finite) sequences $\rho = (\sigma_i, \tau_i)_{i\in[n]}$. The satisfaction of a formula at a position i of such a sequence depends on the values of the clock variables that appear in the formula. Writing $v\colon X \rightharpoonup \mathbb{R}^+$ for a partial valuation of the clock variables, the satisfaction relation can then be inductively defined as follows:

$(\rho, i, v) \models \sigma$	if, and only if,	$\sigma = \sigma_i$
$(\rho, i, v) \models \varphi_1 \wedge \varphi_2$	if, and only if,	$(\rho, i, v) \models \varphi_1$ and $(\rho, i, v) \models \varphi_2$
$(\rho, i, v) \models \neg\varphi$	if, and only if,	it is not the case that $(\rho, i, v) \models \varphi$
$(\rho, i, v) \models \varphi_1 \mathcal{U} \varphi_2$	if, and only if,	there exists $j > i$ such that $(\rho, j, v) \models \varphi_2$ and for all $i < k < j$, $(\rho, k, v) \models \varphi_1$
$(\rho, i, v) \models x.\varphi$	if, and only if,	$(\rho, i, v[x \mapsto \tau_i]) \models \varphi$
$(\rho, i, v) \models x \sim c$	if, and only if,	$v(x)$ is defined, and $\tau_i - v(x) \sim c$

As can be observed in this definition, formulas of the form $x.\varphi$ (called *clock resets*) have the effect of storing the current time τ_i in clock x; at any later time τ_j, the value $\tau_j - v(x)$ corresponds to the amount of time that elapsed since the last reset of clock x: this justifies the semantics of formulas of the form $x \sim c$ (*clock constraints*).

▶ **Example 15.** Formula $x.[\alpha \mathcal{U} (\beta \wedge x \leq 10)]$ states that event β has to occur within 10 time units, and that only event α is allowed to occur in the meantime. Similarly, $x.[(\alpha \wedge x \leq 10) \mathcal{U} \beta]$ also states that eventually β must occur and that only events of type α can occur in the meantime, but additionally all of these events α must occur within the first 10 time units.

TPTL comes with several classical shorthands: conjunctions $\varphi_1 \vee \varphi_2$ are obtained as $\neg(\neg\varphi_1 \wedge \neg\varphi_2)$; $\top$ stands for $\sigma \vee \neg\sigma$ (for some fixed $\sigma \in \Sigma$); $\Diamond\varphi$ stands for $\top \mathcal{U} \varphi$ (and means that φ eventually holds in a strict future), and $\Box\varphi$ stands for $\neg\Diamond\neg\varphi$ (and means that φ always holds in the strict future).

TPTL$_\Diamond$ denotes the fragment of TPTL that uses only the $\Diamond$ modality (and limitations on the use of negation):

$$\text{TPTL}_\Diamond \ni \varphi ::= \sigma \mid \neg p \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \Diamond\varphi \mid x.\varphi \mid x \sim c.$$

▶ **Example 16.** The $\mathsf{TPTL}_\Diamond$ formula $x.\Diamond(\alpha \wedge \Diamond(\beta \wedge x \leq 10))$ states that (at least) one occurrence of α and one occurrence of β have to occur in that order within the next 10 time units.

4.2 $\mathsf{TPTL}_\Diamond$ formulae for linear chronicles

Contrary to TPTL operators, temporal constraints in chronicles do not impose an order on the occurrence of events. But linear chronicles do impose such an order; relying on Prop. 10, we present an encoding of chronicles in $\mathsf{TPTL}_\Diamond$.

Let $\mathscr{L} = (\mathcal{E} = \{\!\{c_1, \ldots, c_m\}\!\}, \mathcal{T}, \pi)$ be a linear chronicle of size m. We characterize $\mathscr{L}$ by a $\mathsf{TPTL}_\Diamond$ formula $\varphi_{\mathscr{L}}$ over $\mathcal{E}$ (seen as a finite set) and using a set $X = \{x_i \mid 1 \leq i \leq m-1\}$ of $m-1$ clocks; formula $\varphi_{\mathscr{L}}$ is obtained through the inductive definition of a collection of $\mathsf{TPTL}_\Diamond$ formulae $(\varphi^i_{\mathscr{L}})_{1\leq i\leq m}$, such that $\varphi_{\mathscr{L}} = \bar{\Diamond}\varphi^1_{\mathscr{L}}$, where $\bar{\Diamond}\phi$ stands for $\Diamond\phi \vee \phi$ (and means that ϕ holds now or at some point in the future).

The collection of formulae $(\varphi^i_{\mathscr{L}})_{i=1\ldots m}$ is defined as follows: if $m = 1$, then $\varphi^1_{\mathscr{L}} = c_{\pi(1)}$; otherwise,

$$\varphi^1_{\mathscr{L}} = \big(c_{\pi(1)} \wedge x_{\pi(1)}.\Diamond\varphi^2_{\mathscr{L}}\big) \tag{1}$$

and for all $2 \leq i \leq m-1$,

$$\varphi^i_{\mathscr{L}} = c_{\pi(i)} \wedge \mathscr{T}_i(\mathscr{L}) \wedge x_{\pi(i)}.\Diamond\varphi^{i+1}_{\mathscr{L}} \tag{2}$$

and finally

$$\varphi^m_{\mathscr{L}} = c_{\pi(m)} \wedge \mathscr{T}_m(\mathscr{L}) \tag{3}$$

where

$$\mathscr{T}_i(\mathscr{L}) = \bigwedge_{\substack{(c_{\pi(k)},\pi(k))[l,u](c_{\pi(i)},\pi(i))\in\mathcal{T}\\ \pi(k)<\pi(i)}} \big(l \leq x_{\pi(k)} \leq u\big) \wedge \big((\pi(i) > 1 \wedge c_{\pi(i)} = c_{\pi(i)-1}) \rightarrow x_{\pi(i)-1} > 0\big) \tag{4}$$

Formula $\varphi^m_{\mathscr{L}}$ has size linear in m. Notice that, in the last conjunct of Eq. (4), the condition to the left of the implication is *static* (it only depends on the chronicle $\mathscr{L}$), and its role is simply to decide whether the condition on $x_{\pi(i)-1}$ has to be imposed.

We prove that this construction correctly encodes chronicles:

▶ **Proposition 17.** *For any linear chronicle $\mathscr{L}$ and any sequence ρ, it holds $\mathscr{L} \Subset \rho$ if, and only if, $\rho \models \varphi_{\mathscr{L}}$.*

Proof. Let $\mathscr{L} = (\{\!\{c_1, \ldots, c_m\}\!\}, \mathcal{T}, \pi)$ be a linear chronicle of size m, and $\rho = \langle(\sigma_1, \tau_1), (\sigma_2, \tau_2), \ldots, (\sigma_n, \tau_n)\rangle$ be a sequence. If $m = 1$, the chronicle has no timing constraints, and the result is straightforward. We now assume that $m > 1$.

We begin with the direct implication, assuming that $\mathscr{L} \Subset \rho$. By Def. 8, this means that there exists $\varepsilon\colon [m] \to [n]$ such that:

1. $\tau_{\varepsilon(i)} < \tau_{\varepsilon(i+1)}$ whenever $c_i = c_{i+1}$ for all $1 \leq i < m$,
2. $\sigma_{\varepsilon(i)} = c_i$ for all $1 \leq i \leq m$,
3. $\tau_{\varepsilon(j)} - \tau_{\varepsilon(i)} \in [l, u]$, whenever $(c_i, i)[l, u](c_j, j) \in \mathcal{T}$ for all $i < j$.

For all $1 \leq i \leq m$, we define $v_i\colon \{x_{\pi(j)} \mid 1 \leq j < i\} \to \mathbb{R}_+$ by $v_i(x_{\pi(j)}) = \tau_{\varepsilon(j)}$. By Property 2 of ε, we have that $\rho, \varepsilon(\pi(i)), v_i \models c_{\pi(i)}$ for all $i \in [m]$. By Property 3, we have

that $\tau_{\varepsilon(\pi(i))} - v_i(x_{\pi(k)}) = \tau_{\varepsilon(\pi(i))} - \tau_{\varepsilon(\pi(k))} \in [l,u]$ for any $(c_{\pi(k)},\pi(k))[l,u](c_{\pi(i)},\pi(i)) \in \mathcal{T}$ with $i,k \in [m]$ such that $\pi(k) < \pi(i)$. Moreover, by Property 1, if $c_{\pi(i)-1} = c_{\pi(i)}$ (and $\pi(i) > 1$), then $\tau_{\varepsilon(\pi(i)-1)} < \tau_{\varepsilon(\pi(i))}$. It follows that $\tau(\varepsilon(\pi(i))) - v_i(x_{\pi(i)-1}) > 0$, which means that $\rho, \varepsilon(\pi(i)), v_i \models x_{\pi(i)-1} > 0$. In the end, we have shown that $\rho, \varepsilon(\pi(i)), v_i \models \mathscr{T}_i(\mathscr{L})$ for all $i \in [m]$.

We now prove by downward induction that $\rho, \varepsilon(\pi(i)), v_i \models \varphi^i_{\mathscr{L}}$. By the two properties above, we have $\rho, \varepsilon(\pi(m)), v_m \models c_{\pi(m)} \wedge \mathscr{T}_m(\mathscr{C})$, which proves our base case. Assuming that $\rho, \varepsilon(\pi(i+1)), v_{i+1} \models \varphi^{i+1}_{\mathscr{L}}$ for some $i \in [m-1]$, we prove that $\rho, \varepsilon(\pi(i)), v_i \models \varphi^i_{\mathscr{L}}$. Again, from the above two remarks, we have $\rho, \varepsilon(\pi(i)), v_i \models c_{\pi(i)} \wedge \mathscr{T}_i(\mathscr{L})$. It remains to prove that $\rho, \varepsilon(\pi(i)), v_i \models x_{\pi(i)}.\ \Diamond\varphi^{i+1}_{\mathscr{L}}$. This directly follows from the following facts: $\rho, \varepsilon(\pi(i+1)), v_{i+1} \models \varphi^{i+1}_{\mathscr{L}}$, and $\varepsilon(\pi(i)) < \varepsilon(\pi(i+1))$ (since $\varepsilon \circ \pi$ is increasing). By induction, we get $\rho, \varepsilon(\pi(1)), v_1 \models \varphi^1_{\mathscr{L}}$, which entails $\rho \models \varphi_{\mathscr{L}}$.

We now assume that $\rho \models \varphi_{\mathscr{L}}$. We construct an embedding ε of $\mathscr{L}$ in ρ inductively: formally, at each step j, we define $\varepsilon(\pi(j))$ so that

1. $\varepsilon(\pi(j))$ satisfies the conditions of Def. 4;
2. letting $w_j(x_{\pi(k)}) = \tau_{\varepsilon(\pi(k))}$ for all $k \in [j]$, we have $\rho, \varepsilon(\pi(j)), w_j \models \Diamond\varphi^{j+1}_{\mathscr{L}}$.

We initialize the induction as follows: since $\rho \models \bar{\Diamond}\varphi^1_{\mathscr{L}}$, there exists $\varepsilon(\pi(1))$ such that $\rho, \varepsilon(\pi(1)), \emptyset \models \varphi^1_{\mathscr{L}}$. By definition of $\varphi^1_{\mathscr{L}}$, this entails that

1. $\rho, \varepsilon(\pi(1)), \emptyset \models c_{\pi(1)}$, hence $\sigma_{\varepsilon(\pi(1))} = c_{\pi(1)}$; the other two conditions for being an embedding hold vacuously;
2. $\rho, \varepsilon(\pi(1)), \emptyset \models x_{\pi(1)}.\Diamond\varphi^2_{\mathscr{L}}$, which entails $\rho, \varepsilon(\pi(1)), w_1 \models \Diamond\varphi^2_{\mathscr{L}}$, where w_1 is the partial embedding defined only for $x_{\pi(1)}$ with $w_1(x_{\pi(1)}) = \tau_{\varepsilon(\pi(1))}$.

Now, assume that the result holds up to some step $\pi(j)$ for some $j < m$; we extend it to $\pi(j+1)$. Since $\rho, \varepsilon(\pi(j)), w_j \models \Diamond\varphi^{j+1}_{\mathscr{L}}$, there exists $\varepsilon(\pi(j+1)) > \varepsilon(\pi(j))$ for which $\rho, \varepsilon(\pi(j+1)), w_j \models \varphi^{j+1}_{\mathscr{L}}$. By definition of $\varphi^{j+1}_{\mathscr{L}}$, we get:

1. $\rho, \varepsilon(\pi(j+1)), w_j \models c_{\pi(j+1)}$, so that $\sigma_{\varepsilon(\pi(j+1))} = c_{\pi(j+1)}$. Additionally, $\rho, \varepsilon(\pi(j+1)), w_j \models \mathscr{T}_{j+1}(\mathscr{L})$, so that for each $(c_{\pi(k)},\pi(k))[l,u](c_{\pi(j+1)},\pi(j+1)) \in \mathcal{T}$, we have $l \leq \tau_{\varepsilon(\pi(j+1))} - \tau_{\varepsilon(\pi(k))} \leq u$. Finally, if $\pi(j+1) > 1$ and $c_{\pi(j+1)} = c_{\pi(j+1)-1}$, then $\rho, \varepsilon(\pi(j+1)), w_j \models x_{\pi(j+1)-1} > 0$, which means $\tau_{\varepsilon(\pi(j+1))} - \tau_{\varepsilon(\pi(j+1)-1)} > 0$, i.e., $\tau_{\varepsilon(\pi(j+1)-1)} < \tau_{\varepsilon(\pi(j+1))}$.
2. $\rho, \varepsilon(\pi(j+1)), w_j \models x_{\pi_{j+1}}\Diamond\varphi^{j+2}_{\mathscr{L}}$ (unless $j+1 = m$), which entails $\rho, \varepsilon(\pi(j+1)), w_{j+1} \models \Diamond\varphi^{j+2}_{\mathscr{L}}$.

In the end, we have built an embedding ε witnessing the fact that $\mathscr{L} \Subset \rho$. ◀

Our main result follows:

▶ **Theorem 18.** *Any chronicle $\mathscr{C}$ admits an equivalent* TPTL$_\Diamond$ *formula $\varphi_{\mathscr{C}}$.*

▶ Remark 19. Again, notice that our construction easily extends to the case where Condition 1 in Def. 4 is removed: it suffices to drop the last part of the definition of $\mathscr{T}_i(\mathscr{L})$ (Eq. (4)).

▶ Remark 20. In our definition of timed words and TPTL, we have taken the approach of seeing events as letters: if several events can take place at the same date, they are encoded as several consecutive letters in the timed word, all having the same timestamp; for instance, $w = (A, 3.2)(B, 3.2)(C, 4.1)$ corresponds to two events A and B occurring at the same date, and C occurring later.

Another approach would consist in seeing events as atomic propositions: in that setting, each timestamp would be unique, and would be associated with a (non-empty) set of events. The timed word w above would then correspond to $w' = (\{A, B\}, 3.2)(\{C\}, 4.1)$. Notice that this would not allow to have two occurrences of the same event at the same time; because of

Condition 1 in Def. 4, removing multiple simultaneous copies of the same event preserves occurrences of chronicles.

Conceptually, our translation would still work, but we would have to allow the operator $\Diamond$ to also consider the present position; in other terms, we would have to replace each occurrence of $\Diamond$ in $\varphi^m_{\mathscr{L}}$ with $\bar{\Diamond}$. Strictly speaking, this would involve an exponential blow-up of the $\mathsf{TPTL}_\Diamond$ formula, since $\bar{\Diamond}\phi$ rewrites as $\phi \vee \Diamond\phi$, which requires duplicating formula ϕ.

▶ **Example 21** ($\mathsf{TPTL}_\Diamond$ formula for linear chronicles)**.** This example illustrates the construction of a $\mathsf{TPTL}_\Diamond$ formula for the linear chronicle depicted at Fig. 3; the order $B \prec A \prec C_3 \prec C_4$ for the events corresponds to the permutation π defined by $\pi(1) = 2$, $\pi(2) = 1$, $\pi(3) = 3$, $\pi(4) = 4$. We denote this linear chronicle by $\mathscr{L}$.

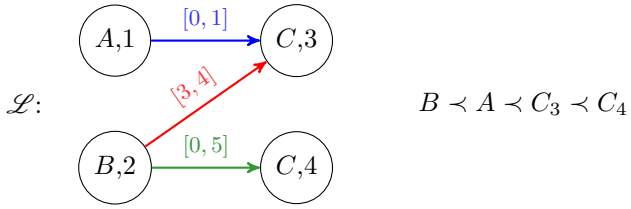

Figure 3 An example of a chronicle

To construct a $\mathsf{TPTL}_\Diamond$ formula corresponding to $\mathscr{L}$, each event except the last one (according to π) is associated to a clock (x_1 for event $(A, 1)$, x_2 for event $(B, 2)$, x_3 for event $(C, 3)$).

Let us then define the temporal constraints according to Eq. (4):

$$\begin{aligned}
\mathscr{T}_1 &= \emptyset \\
\mathscr{T}_2 &= \emptyset \\
\mathscr{T}_3 &= x_1 \leq 1 \wedge x_1 \geq 0 \wedge x_2 \leq 4 \wedge x_2 \geq 3 \\
&= x_1 \leq 1 \wedge x_2 \leq 4 \wedge x_2 \geq 3 \\
\mathscr{T}_4 &= x_2 \leq 5 \wedge x_2 \geq 0 \wedge x_3 > 0 \\
&= x_2 \leq 5 \wedge x_3 > 0
\end{aligned}$$

The colors in formula match the color in Fig. 3. Formula $\mathscr{T}_2$ is empty because A is the first event in the chronicle multiset. Formula $\mathscr{T}_1$ is also empty, but for a different reason: there are no temporal constraints between A and B. It is also worth noticing that the constraints $x_3 > 0$ has been added to the temporal constraints of $\mathscr{T}_4$ because the implicit order between chronicle events having the same event type. The inequality is strict because of the injectiveness condition (see Remark 6).

Then, we construct the formula by induction following the order defined by π. It adds temporal constraints on the clocks marking the occurrence of previous events (relatively to time instants of the current event occurrence).

Thus, we obtain the following $\mathsf{TPTL}_\Diamond$ formula equivalent to chronicle $\mathscr{L}$:

$$\varphi_{\mathscr{L}} \quad = \quad \bar{\Diamond}(B \wedge x_2.\Diamond(A \wedge \mathscr{T}_2 \wedge x_1.\Diamond(C \wedge \mathscr{T}_3 \wedge x_3.\Diamond(C \wedge \mathscr{T}_4))))$$

The brown part of the formula is φ^4, which is the final case in the inductive construction of the formula. The black part is the initial case of the formula, which starts by the B event (the first event, according to π). The $\bar{\Diamond}$ operator catches the case of a sequence starting with a B. The remaing parts (orange for φ^3 and purple for φ^2) are the regular induction cases.

This formula can be rewritten after simplification and use of classical operators as follows:

$$\varphi_{\mathscr{L}} = \Diamond(B \wedge x_2.\Diamond(A \wedge x_1.\Diamond(C \wedge x_1 \leq 1 \wedge x_2 \leq 4 \wedge x_2 \geq 3 \wedge x_3.\Diamond(C \wedge x_2 \leq 5 \wedge x_3 > 0))))$$
$$\vee (B \wedge x_2.\Diamond(A \wedge x_1.\Diamond(C \wedge x_1 \leq 1 \wedge x_2 \leq 4 \wedge x_2 \geq 3 \wedge x_3.\Diamond(C \wedge x_2 \leq 5 \wedge x_3 > 0))))$$

▶ Remark 22. Notice that while each single linear chronicle is translated into a linear-size $\mathsf{TPTL}_\Diamond$ formula, the translation of a plain (non-linear) chronicle into a $\mathsf{TPTL}_\Diamond$ formula generally involves an exponential blow-up. This is not a concern for this paper, but proving that this blow-up cannot be avoided is an interesting direction for future work.

5 Metric temporal logic and chronicles

Metric Temporal Logic (MTL) is another timed extension of LTL that could be suited to encode chronicles. We first define its syntax and semantics, before dealing with the problem of encoding chronicles.

Given a finite alphabet Σ of atomic events, formulae of MTL are built up from Σ by Boolean connectives and time-constrained versions of the temporal operator *Until* as follows:

$$\mathsf{MTL} \ni \varphi ::= \sigma \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \mathcal{U}_I \varphi_2$$

where σ ranges over Σ, and $I = [l, u]$ is a temporal interval with l and u in $\mathbb{Q}$.

As for TPTL, in the pointwise semantics, MTL is evaluated at a position $i \in \mathbb{N}$ along a timed word $\rho = (\sigma_i, \tau_i)_{i \in [n]}$ as follows:

$(\rho, i) \models \sigma$	if, and only if,	$\sigma = \sigma_i$
$(\rho, i) \models \varphi_1 \wedge \varphi_2$	if, and only if,	$(\rho, i) \models \varphi_1$ and $(\rho, i) \models \varphi_2$
$(\rho, i) \models \neg\varphi$	if, and only if,	it is not the case that $(\rho, i) \models \varphi$
$(\rho, i) \models \varphi_1 \mathcal{U}_I \varphi_2$	if, and only if,	$\exists j \geq i$ s.t. $(\rho, j) \models \varphi_2$ and $\tau_j - \tau_i \in I$ and $\forall i < k < j$, $(\rho, k) \models \varphi_1$

As previously, we use the classical shorthands such has $\Diamond_I \varphi$, which stands for $\top \mathcal{U}_I \varphi$, and $\Box_I \varphi$, which stands for $\neg\Diamond_I \neg\varphi$.

It is not hard to observe that any MTL formula can be expressed in TPTL: formula $\phi \mathcal{U}_I \psi$ can be expressed as $x.(\phi \mathcal{U} (\psi \wedge x \in I))$. It was shown in Bouyer et al. [6] that TPTL is strictly more expressive than MTL in the general case. An example of a TPTL formula that has no equivalent MTL formula in the pointwise semantics is:

$$\phi = x.\Diamond(b \wedge \Diamond(c \wedge x \leq 2)) \tag{5}$$

Base on this counterexample of the equivalence between MTL and TPTL, we exhibit a chronicle that has no equivalent in MTL. Consider the following chronicle:

$$\mathscr{C} = (\{\!\{A, B, C\}\!\}, \{(A,1)[0,+\infty](B,2), (B,2)[0,+\infty](C,3), (A,1)[0,2](C,3)\}) .$$

The first two constraints impose that A, B and C must appear in that order (or possibly with the same timestamp). The last temporal constraint states that C must occur within two time units after A.

Using our transformation, the TPTL formula representing chronicle $\mathscr{C}$ is a disjunction of

$$\varphi^0_{\mathscr{C}} = \bar{\Diamond}(a \wedge x.\Diamond(b \wedge x \geq 0 \wedge y.\Diamond(c \wedge x \leq 2 \wedge y \geq 0))),$$

and of formulas of the form

$$\bar{\Diamond}(b \wedge y.\Diamond(a \wedge y \leq 0 \wedge x.\Diamond(c \wedge x \leq 2))$$

(one for each non-trivial permutation). Formula $\varphi^0_{\mathscr{C}}$ simplifies as

$$\bar{\Diamond}(a \wedge x.\Diamond(b \wedge \Diamond(c \wedge x \leq 2))),$$

which contains formula ϕ of Eq. (5) as a subformula.

Of course, the fact that ϕ occurs as a subformula of $\varphi^0_{\mathscr{C}}$ does not imply that $\mathscr{C}$ cannot be characterized in MTL. In order to formally prove this fact, we rely on the proof of [6] that some TPTL formulas have no MTL equivalent: consider the timed sequences depicted on Fig. 4: first notice that, for any integer p, chronicle $\mathscr{C}$ occurs in $\mathcal{A}_p$, while it does not occur in $\mathcal{B}_p$. On the other hand, Lemma 8 in [6] states that no MTL formula involving constants that are integer multiple of $\frac{1}{p}$ can distinguish between $\mathcal{A}_p$ and $\mathcal{B}_p$.

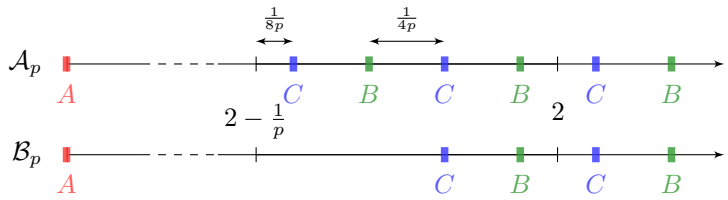

Figure 4 Two timed sequences not distinguishable by MTL with constants that are multiple of $1/p$

Now, if there were an MTL formula $\psi_{\mathscr{C}}$ characterizing exactly chronicle $\mathscr{C}$, then for some integer p_0, this formula would involve constants that are integer multiple of p_0, and this formula would take the same value on $\mathcal{A}_{p_0}$ and $\mathcal{B}_{p_0}$, contradicting the fact that it exactly corresponds to $\mathscr{C}$.

It follows:

▶ **Theorem 23.** *There exist chronicles (with only three events) that admit no equivalent* MTL *formula.*

It is interesting to notice that the counterexample is a very simple chronicle: it only has three events and one useful temporal constraint. In addition, the temporal constraints do not straddle zero. They are all included in $\mathbb{R}_+$. Our initial intuition was that such temporal constraints would be a problem for translating chronicle in MTL. But the problem does not come from them. Intuitively, MTL formula does not need memory to be recognized but the recognition of a chronicle requires to store the position of all the occurrences of multiset events to check the temporal constraints. If chronicles of size 2 have equivalent formulae in MTL, the counterexample above illustrates a chronicle of size 3 – requiring to store the position of two events – a non-equivalent formula in MTL.

6 Conclusion

This work started from our need to specify complex situations to recognize disease or treatment from patient care pathways. In this medical context, specifying temporal arrangements of events is of particular interest to have accurate specifications. Then, our problem is to find a formalism that is both expressive and efficient to recognize complex situations in large datasets of patients.

In this article we investigated the temporal model of chronicle and compared its expressiveness to two temporal logics TPTL and MTL. Chronicle is a temporal model that emerged in the field of complex event processing. It can be used to efficiently monitor a stream of events. Despite its seeming simplicity, the temporal constraints of a chronicle allows to specify situations without presupposition on the events order. Then, such temporal models were intuitively qualified as *highly* expressive but, to the best of our knowledge, no formal comparison with other temporal formalisms were made. Then, our objective was to better qualify the expressiveness of chronicles through their comparison against MTL.

In this article we have shown that any chronicle as an equivalent formula in $\mathsf{TPTL}_{\Diamond}$, but some chronicles have no equivalent formula in MTL. This confirms that chronicles have an interesting expressiveness.

This first result opens interesting perspectives: the formulation of TPTL equivalent to chronicle can be a cornerstone for new results with other temporal models, and more especially other models from the field of event processing. It also raises the question of the equivalence with TPTL: would it be possible to find an equivalent collection of chronicles for a TPTL formula? If it is possible, chronicles may be a new approach for recognition of TPTL formulae.

The negative result we had with MTL also opens new questions. We identify a plausible reason for non-equivalence that guides us toward other logics that would be also interesting to compare chronicles. Our first target is MTL with past operators [25]. We conjecture there is no equivalence with chronicles.

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