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To get an overview of how many and which studies in the literature specifically address the implementation of innovations from mathematics education research, we have conducted a systematic literature review. We report on the subset of 19 studies from the review, dealing with the implementation of instructional sequences aiming to enhance students' learning of mathematical concepts or competencies. The research question is: Which mathematical concepts and/or competencies are in play, and what characterizes the implementation of the instructional sequences? Results show that spatial reasoning, patterns, and structure gained the most interest, addressed in 6 studies. The other studies are relatively evenly spread over the concepts: algebra, arithmetic, calculus, number theory and proof, and the competencies: proportional reasoning and problem-solving. Seven studies, with long-term goals, describe a design for scaling the implementation.

Keywords: Implementation research, mathematical concepts, mathematical competencies, goals.

Introduction

That implementation research (IR) in mathematics education research (MER) has gained momentum during the past few years is beyond any doubt. Since 2017, a thematic working group (TWG 23) at CERME has been dedicated to the topic. In 2021, a new journal—*Implementation and Replication Studies in Mathematics Education* (IRME)—was launched by the well-established Dutch publishing house, Brill. Finally, also in 2021, a special issue of ZDM was dedicated to the topic of implementation research in mathematics education. Ongoing discussions in relation to IR in MER concern, for example, the use of theoretical constructs from outside the field of MER (e.g. health science, economics, etc.) versus those available inside of MER; what we should take implementability to mean in relation to IR in MER; to what extent IR should mainly address large scale studies; etc. (Jankvist et al., 2021). Yet, it seems to us that to engage in these discussions on a more enlightened basis, a natural starting point is to get an overview of both how many and which studies in the MER literature specifically address "implementation". We have taken on this task by conducting a systematic literature review.

Taking on this task involved a few delicate considerations on our behalf, since most of the reported research studies in our field, at least from an inclusive standpoint, may be considered as studies addressing some kind of implementation. Two criteria were decided upon and enforced in order to avoid a too large number of papers to consider. Firstly, we limited the review to include papers that clearly stated dealing with some kind of implementation. Secondly, we limited the review to only consider studies published in the top twenty quality-ranked MER journals following the recent journal categorization by Williams and Leatham (2017). The literature search was carried out on February 4-5, 2021, and initially resulted in 1,093 papers, which through a screening process (see the following section) were reduced to 98 papers. In this paper, we focus on a smaller subset, consisting of 19 papers, dealing with the implementation *of instructional sequences to enhance students' learning* on specific mathematical concepts, or competencies. The research question is: *Which mathematical concepts and/or competencies are in play, and what characterizes the implementation of the instructional sequences?*

Review methodology

We conducted the literature searches in ERIC (EBSCO) searching for manuscripts with implement* in the title and/or abstract, journal by journal of our top 20 samples (Williams & Leatham, 2017). The advantage of doing the entire search in one database is that it is easy to collect the results in one folder. To ensure that no article had been overlooked, we repeated the search implement* in the title and/or abstract on each journal's website. We found 1,093 peer-reviewed articles fitting the inclusion criteria. We used the software *Covidence* to manage our literature review.

Each paper was screened by two reviewers. The screening was made in two steps. First, we screened the title and abstract. In cases where we were hesitant, e.g., because the abstract did not provide sufficient information, we chose to forward the paper to full-text screening. In the full-text screening, papers were included if they followed Century and Cassata's (2016) definition of IR:

> ... as systematic inquiry regarding innovations enacted in controlled settings or in ordinary practice, the factors that influence innovation enactment, and the relationships between innovations, influential factors, and outcomes. (Century & Cassata, 2016, p. 170)

As evident from this quote, another central term in IR is that of *innovation*. Innovation refers to the practical implementation of ideas resulting from research that involve a change (e.g., in behavior or practice) for the individuals enacting them. (Century & Cassata, 2016).

Of a total of 139 papers, 97 remained after the full-text screening (see table 1). These were categorized in terms of, Curriculum reform (31); Curriculum materials (22); Professional development projects (25); and finally, Mathematical concepts, competencies, and instructional sequences, (19). The latter is the focus of this paper. We define an instructional sequence as one, or more, cohesive series of lessons that address a concept (e.g., fractions) or a competency (e.g., problem posing and solving). There are no clear cuts between the categories. An instructional sequence may stem from a new curriculum material, that is implemented through a PD project, due to curriculum reform. Decisive for how the categorization is done is the focus of the paper.

The data extraction from these papers included general information on the author(s), title, purpose statement(s), country where the study was conducted, research question(s), methods, target group, and results. The specific information about the implementations included what kind of innovation from mathematics education the study concerned, specific or general goals in the short- or long term, phase of implementation studied, stakeholders responsible for the implementation, and identified factors of influence for the outcomes of the implementation.

Table 1: Process for inclusion of studies

	Inclusion Criteria	**Exclusion Criteria**	**Excluded**
Title and abstract screening 1093 studies imported	Implement* in abstract or title	Innovations that do not stem from results from ME Studies that do not involve a change in behavior or practice Papers not belonging to the sample of 20 journals	955
Full-text screening 139 studies imported 97 remained for data extraction	The studies should fit the definition of implementation research and one or more criteria for doing implementation research (Century & Cassata, 2016)	Does not fit the definition of implementation research. The innovation is not from ME. Not in the sample of 20 journals. Full text is not available. Duplicate	21 8 5 4 3

IR theoretical constructs applied

While most MER studies aim at investigating and improving student learning, implementation studies often consider a dimension of scaling (Coburn, 2003). Scaling, in turn, is entirely dependent on the timing of scaffolding (Helenius, n.d.) between three major groups of *stakeholders*, namely agents from *practice*, *research*, and *policy* (Krainer, 2021). Agents from *practice* include all teachers who carry out teaching but also principals and other staff who are responsible for the teaching that is carried out in an educational context. Agents from *research* refer to both actively involved researchers and the overall production of reported research findings from the research community that informs implementation projects. Agents from *policy* refer to all agents with the power to spread innovations over school districts, make decisions on the direction and budgeting for educational efforts, as well as the administrative superstructure required to implement political decisions.

Our interpretation of *goals* draws on Krainer's (2021) description of four different kinds of goals for IR: 1) Concrete and short-term goals; 2) Concrete and long-term goals; 3) General and long-term goals, and 4) General and short-term goals. While the *general-concrete* dichotomization is introduced by Krainer, we find it more linguistically natural to instead speak in terms of the opposites, *general* versus *specific*. Therefore, when we operationalize Krainer's goals in our analysis we use *specific* instead of concrete. For the dimensions short-term and long-term, we conceptualize the longevity aspect in terms of scale-up possibilities for the innovation (Coburn, 2003).

Specific and short-term goals: In this category we put papers addressing a specific limited goal to enhance teaching and learning. The innovation concerns a mathematical concept or competency. When we classify something as belonging to short-term goals, we consider the intended life-cycle of the innovation. As a consequence, implementations without a scaling plan that are studied with a longitudinal research methodology will be classified as short-term. Our definition of scaling follows Coburn's (2003) notions of depth, sustainability, spread, and shift in reform ownership. *Depth* refers to change in classroom practice that goes beyond a shift in teaching resources and the introduction of specific activities. Coburn argues that scaling includes a shift in teachers' beliefs, norms for communication, and pedagogical practices. *Sustainability* concerns the scaffolding tools that are left to maintain the vitality of the innovation after the support of the reform leaders are withdrawn from the organization. When Coburn considers *spread,* she, in addition to scaling to other schools and classrooms, also includes spread within the organization. Finally, Coburn adds the dimension of a *shift in reform ownership* to the notion of scale. When reform is launched, the ideas and activities are owned by the creators of the reform. According to Coburn, the authority to scale the implementation needs to shift to the districts, schools, and teachers. Only then can scaling in depth, sustainability, and spread be maintained. If none of these scaling dimensions are discussed, or implied, the paper is considered to be a short-term implementation. *Specific and long-term goals,* on the other hand, discuss at least some dimension of scaling.

General and long-term goals refer to innovations that intend to change the practice of mathematics teaching in general, as opposed to a focus on changing the teaching of a specific concept, subject, or competency. For example, as a result of alarms from international tests, politicians may plan for increasing the mathematics teachers' general content knowledge at scale and/or state-wide curriculum reforms, to be implemented with long-term goals. At the other end of the spectrum, we find locally introduced projects within organizations that aim to fundamentally change teaching locally. Thus, with *general and short-term goals,* we refer to non-content or non-competency-specific innovations. For example, limited periods where a new model of the organization of classroom teaching in mathematics is tried without an existing plan for scaling.

Results

We summarize the answer to our question: *Which mathematical areas of concepts and/or competencies are in play, and what characterizes the implementation of the instructional sequences?* in terms of specific- or general-, and short-term or long-term goals in table 2 below.

Table 2: Results

Concepts/competencies	Author(s)	Target population(s)	Characteristics of goals
Algebra (2)	(Adiredja et al., 2020) (Tsai & Chang, 2009)	Undergraduate Grade 8	Specific and short-term Specific and short-term
Arithmetic (3)	(Savard & Polotskaia, 2017) (Tyminski et al., 2014) (Polotskaia & Savard, 2018)	Grades 1-4 Pre-service teachers Grade 1-2	Specific and short-term Specific and short-term Specific and short-term
Calculus (1)	(Carter et al., 2016)	Undergraduate	Specific and long-term
Fractions (2)	(Osana & Royea, 2011) (Thanheiser et al., 2016)	Pre-service teachers Pre-service teachers	Specific and short-term Specific and short-term
Number Theory (1)	(Strømskag, 2017)	Pre-service teachers	Specific and long-term
Problem-solving (1)	(Leung, 2013)	Elementary in-service teachers	Specific and short-term
Proofs (1)	(Stylianides & Stylianides, 2009)	Pre-service teachers	Specific and short-term
Proportional reasoning (2)	(Howe et al., 2011) (Wright, 2014)	Grade 7 Grade 8	Specific and long-term Specific and short-term
Spatial reasoning and patterns (6)	(Papic et al., 2011) (Mulligan et al., 2018) (Mulligan, Oslington, et al., 2020) (Mulligan, Woolcott, et al., 2020) (Patahuddin et al., 2020) (Pollitt et al., 2020)	Pre-school Grades 3-5 Pre-school Grades 3-4 Inservice teachers Pre-school in-service teachers	Specific and long-term Specific and long-term Specific and long-term Specific and long-term Specific and short-term Specific and short-term

Concluding discussion

The first thing that we notice from table 2 is that the targeted concepts and competencies are well spread, although with one exception, namely spatial reasoning. The reason that spatial reasoning and patterns have gained greater interest than other concepts and competencies is that the same research group, working on a large-scale implementation, authored four of the six studies on spatial reasoning and patterns (Mulligan et al., 2018; Mulligan, Oslington, et al., 2020; Mulligan, Woolcott, et al.,

2020; Papic et al., 2011). Second, concerning goals, we notice that all studies are specific; a result driven by the delimitation of the category's focus on some concept or some competence. Third, only seven studies address implementation where scaling is a part of the project. Of these seven studies, four belong to the large-scale project, mentioned above, about patterns and structure in Australia (Mulligan et al., 2018; Mulligan, Oslington, et al., 2020; Mulligan, Woolcott, et al., 2020; Papic et al., 2011). The Mulligan et al. studies describe (briefly) plans for *depth*, *sustainability*, *spread,* and *shift in reform ownership*. The intervention program in Papic et al. (2011) reported on *spread* through replication and adaptions to other preschools. One paper belongs to a large-scale project on proportional reasoning in the UK (Howe et al., 2011), where *sustainability* is supported by free web-based modules with lesson plans. One paper is a theoretical paper drawing a picture of how an instructional design for teaching number-theory could be *spread*, within the organization, and also to other schools and classrooms, without the drawback of considering limitations in the organization (Strømskag, 2017). Finally, one paper has a plan for *spread* within the organization but no plan for scaling to other schools and classrooms (Carter et al., 2016).

Reviewing the 20 top-ranked journals ended up in only 19 papers addressing specific instructional sequences on concepts and/or competencies. How can that be? We hypothesize that the discourse surrounding implementation research is somewhat new in MER. The papers in this sample are from the year 2009 to a peak in 2020, with five papers. Further, given that we limited the categorization addressed here to specific instructional sequences on concepts and/or competencies, all goals in this subset are classified as specific. Taken together, that can constitute a problem if we want to understand how large-scale projects work. This is because stakeholder groups of policymakers, in particular those on a school-district level, seldom operate with just specific goals. On the contrary, curriculum reforms are often justified by general goals. We believe that the picture will change when we deepen our study to the other identified categories, i.e., curriculum reform, curriculum materials, and PD projects. Regarding short-term and long-term goals, only papers belonging to large-scale projects, where stakeholders from all levels are involved, explicitly discuss scaling. Depending on the goals, different requirements are placed on the involvement of different stakeholders. Stakeholders with the power to make general decisions are necessary for scaling in school districts. Projects aiming at long-term implementation thus need a plan—and a theory—for how to involve stakeholders at different levels at the 'right' points in time.

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