Towards a dual integrated modeling approach to the teaching and learning of mathematics: Challenges and tensions in implementation

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This paper elaborates on the theoretical conceptualization of a so-called dual integrated modeling approach to the teaching and learning of modeling and the teaching and learning of mathematics through modeling. Departing from a sociocultural perspective on teaching, the models and modeling perspective, and using the previously developed theoretical constructs of the Teaching Triad and the Expanded Mediated Triangle, we discuss modeling in terms of (i) micro modeling in which modeling is used as a vehicle to teach and learning other curricula content; and (ii) macro modeling in which modeling is a self-standing mathematics curricula goal. We illustrate this dual integrated approach by analyzing and discussing a lower secondary teachers' implementation of a modeling activity focusing on a simple statistical investigation and measures of central tendency in statistics.

Keywords: Dual integrated modeling approach, expanded mediational triangle, measures of central tendency, models and modeling perspective, teaching triad.

Introduction

There are different goals and rationales for implementing modeling tasks in the teaching and learning of mathematics (Blum & Niss, 1991; Kaiser & Sriraman, 2006). One way to broadly characterize these is to draw on Julie and Mudaly's (2007) discussion of modeling as either *a self-standing mathematical content* or as *a vehicle for learning other more specific (mathematics) ((curricula)) objectives* (see also Niss & Blum (2020)). By enabling the option to put 'mathematics' and 'curricula' in brackets in the last sentence, "*(mathematics)*" and "*((curricula))*", the latter category then includes socio-critical and discursive perspectives on modeling (cf. Barbosa, 2006) as well as ethnomodeling (Orey & Rosa, 2021). There are plenty of examples of research studying modeling as understood more or less exclusively in one or the other of these two categories. In the case of modeling as a self-standing mathematical content much research focusing on the development of students' modeling competencies qualifies, see Cevikbas et al. (2021) for examples. Regarding modeling as a vehicle for learning other more specific (mathematics) ((curricula)) content, Barbosa (2006) and Orey and Rosa (2021) illustrate this strand of research, as do much of the research carried out based on the realistic mathematics education (RME) programme (cf. Gravemeijer, 1999). However, theoretical and empirical research on integrated modeling approaches, combining both rationales and goals outlined above and given close to similar emphasis on both, is to our knowledge spares. Even in the so-called *educational modeling perspectives* (cf. Kaiser & Sriraman, 2006), which potentially have pedagogical as well as subject-related goals, are commonly divided into two strands focusing either on *didactical modeling* or *conceptual modeling*.

In this theoretical paper, we seek to start building and elaborating on a theoretical foundation for a dual integrated modeling approach based on the models and modeling perspective (MMP) (Lesh & Doerr, 2003). We use the MMP to at the macro level of teaching focus on modeling as a self-standing mathematical content in its own right (*macro modeling*), as well as at the micro level of teaching using modeling as a vehicle for teaching more specific mathematical content matter (*micro modeling*). In terms of Blum and Niss' (1991) six basic approaches to include applications and modeling in mathematics teaching and learning, our conceptualization of this dual integrated modeling approach is akin to what they term "*[T]he interdisciplinary integrated approach*" (p. 61, italics in original).

As part of our initial work and ongoing thinking, we in this paper outline our current theoretical conceptualization of this dual integrated modeling approach. We also report on a first attempt to apply this framework as an analytical lens, to discern its adequacy to capture aspects and challenges of teaching that arises when the dual approach is implemented in the classroom. The question guiding our theoretical exploration in this paper is: *What teaching challenges come to the fore in the initial phase of when a teacher tries to adapt a dual integrated modeling approach in his/her teaching?*

To further theorize, we use a sociocultural perspective on teaching to try and capture and describe teaching challenges on both the micro- and macro level of the teaching and learning of, and through, modeling, when a lower secondary teacher implements a modeling activity involving a simple statistics investigation focusing on measures of central tendency in statistics.

Theoretical considerations

We now elaborate on the notion of the dual integrated modeling approach adapted in the paper, and how we conceptualize modeling at both a micro- and macro level (*micro*- and *macro modeling*) using the Teaching Triad (Jaworski, 1994) and the Expanded Mediational Triangle (Engeström, 1998).

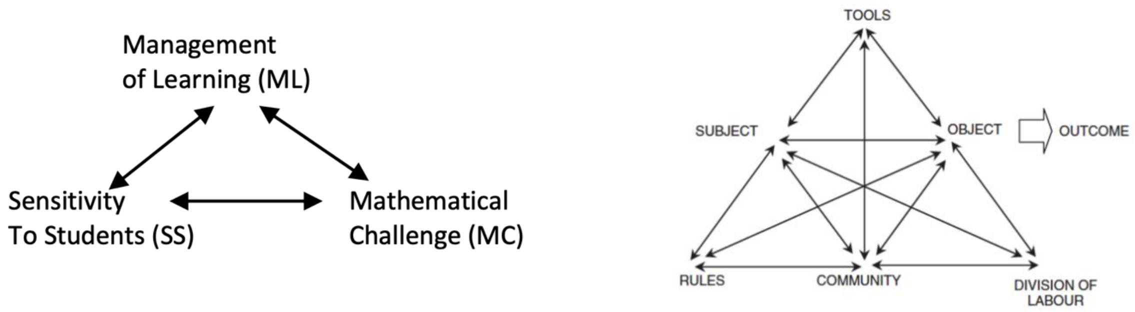
The models and modeling perspective

The perspective on modeling adapted in this paper is the *models and modeling perspective* (MMP), in which a model is defined as a general system consisting of elements, relationships, rules and operations that can be used to describe, predict, make sense of, or explain some other system. A mathematical model focuses on the structural characteristics of the system at hand (Lesh & Doerr, 2003). Learning from a MMP is understood as developing useful and generalized models consisting of a set of concepts and procedures. The concepts are used to describe or explain the mathematical objects and aspects in the context relevant to the phenomenon studied using or re-using the procedures to engage in or create goal-directed constructions, manipulations, or predictions (Lesh & Harel, 2003). In the MMP three different types of structurally related activities organized in so-called *model development sequences* can be used to purposefully support students' learning towards a given learning goal: *model eliciting activities* (MEAs) which aim to elicit the students' ideas they bring to the activity; *model exploration activities* (MXAs) that focus on the underling mathematical structure elicited by students; and *model application activities* (MAAs) were students apply their models in similar or new contexts (Lesh et al., 2003). In all three types of activities students iteratively engage in expressing, testing, revising, and developing their models (Lesh & Doerr, 2003; Lesh et al., 2003).

The Teaching Triad and the Expanded Mediational Triangle

Following Jaworski and Potari (2009) and Jaworski et al. (2017) we in addition stress the sociocultural aspects of teaching as a mediating process connecting the content of mathematics, students, and teachers. In particular, we elaborate on the notions of *micro*- and *macro modeling* from both an institutional perspective and the nature of more local interactions between teachers and students engaged in modeling activities.

The Teaching Triad (TT) captures three interdependent and interlinked dimensions of teaching (see Figure 1a): *Management of Learning (ML)* aims at describing the organization of the learning environment by the teacher such as the planning and orchestrating of tasks and activities, use of resources, and forms of working in the classroom; *Sensitivity to Students (SS)* focuses on students' affective-, cognitive- and social needs and the ways in which these are considered by the teacher in interactions; and *Mathematical Challenge (MC)* on what mathematical content, thinking and activities are offered to students in the learning environment including for example posed questions, sets and sequences of tasks, and metacognitive demands and processing.



(a) (b)

Figure 1: (a) The Teaching Triad (Jaworski, 1994); and
(b) The Expanded Mediational Triangle (Engeström, 1998)

The Expanded Mediational Triangle (EMT; see Figure 1b) by (Engeström, 1998) models the structure of human activity built around Leont'ev's three levels of human activity where the concepts *activity*, *action* and *operation* are central and dialectical theoretical constructs. An *activity* is described as a complex evolving structure of mediated and collective human agency connected to specific *motive* that both distinguishes activities from each other and gives the activity directionality. Activities are constituted by *actions* directed toward specific and conscious *goals* that realize and sustain the activity. Actions in turns are constituted of *operations* that realize the actions in the activity and are carried out under the premises determined by the *conditions* of the activity and the environment in which the activity takes place (Leont'ev, 1979). In the EMT, the *subject* transforms the *object* into an *outcome* using *tools* and artifacts. However, the mediation facilitating this transformation is both supported and conditioned by the culture and *community* carrying out the activity. Here, the *rules* account for norms, conventions and regulations within a community or activity, whereas the *division of labour* highlights both the actual division of labour and responsibility for achieving the goal/motive of the actions/activity and status- and power relations within the community (Engeström, 1998).

The dual integrated modeling approach: micro- and macro modeling in terms of TT and EMT

In terms of EMT, we consider the activity in the dual integrated modeling approach to be the teacher teaching mathematics with the overall motive of the activity for the students to learn mathematics within the boundaries of the teacher's and students' school setting. The actions that constitute the activity are the actions of the teacher as s/he based on her/his set goals interact and engage with the students to support their learning. One of the goals in the dual integrated modeling approach is for the teacher to draw on and implement the ideas and principles of the models and modeling perspective, and indeed to for the students to learn modeling, meaning for the teacher to make the students cognizant about the modeling processes that inherently comes with the way of working based on the models and modeling perspective and with model development sequences (macro modeling). In realizing the actions, the teacher carries out various operations conditioned by the conditions manifested in the activity and the institutional and cultural setting in which the activity is carried out. In other words, and analogous to Jaworski et al. (2017), we conceptualize the teacher as the *subject* in the activity with the *objective* to teach and support students to learn and make sense of mathematical content (such as modeling as a self-standing mathematical content – macro modeling). The teacher does this using various *tools* in mediating the mathematics – including pedagogical and didactical strategies and theories – and in particular the elements of the models and modeling perspective and micro modeling (modeling as a vehicle). However, the mediation of mathematics is in addition supported as well as conditioned by the cultural norms of the schooling (*community*), the school's norms and regulations (*rules*), and status- and power relations between actors in the classroom (*division of labour*).

In the dual integrated modeling approach this framing using the EMT is complemented by the TT framework to really focus in on the acts of teaching and how the teachers' actions and managing of the learning are related to the mathematical content and the students' learning – and in particular the role and function of all aspects related to micro modeling (the use of modeling to teach other content) and how these relate and support developing macro modeling as a mathematical content to be learnt.

An example of the use of the dual integrated modeling approach

The example we will now discuss and theorize on comes from a grade 7 classroom of 22 students of which the majority was girls (86%). The class was considered to be somewhat loud and challenging to keep on track in order to not lose focus on the topic at hand and the work needed to be done (providing clues about governing behavioral- and interactional norms in the class (community) as well as the power- and status relations in the classroom (division of labour)). The teacher had taught the class for almost a year at the time when the modeling activity, The Paper Helicopter Activity, was implemented.

The teacher participated in a project aimed at developing the teaching of statistics using the MMP, and he was familiar with, and had some prior experiences with, the fundamental philosophy and ideas of this perspective. In the context of this paper, the adoption of the MMP as the framework guiding and supporting the design of the learning opportunities for the students, induced principles for how to implement micro modeling in terms of model development sequences. With respect to the content matter of statistics being in focus in the project, simple statistical investigations and measures of central tendency in statistics, macro modeling in the context of this paper is (somewhat artificially) equated with the whole statistical (model) investigating process.

In addition, one of the goals the teacher expressed for participating in the project was to be able to work more independently from the textbook used in the school (breaking an established rule) and for his students to be comfortable in not relying on the textbook all the time as the sole source for mathematical knowledge and truth, and hence wishing to change the epistemic status of the textbook (a community norm) as well as its authoritative status (shifting the division of labour).

Within the project focusing on the teaching of statistics, the teacher and the two authors together developed a sequence spanning 12 lessons of on average 60 minutes covering the statistical content prescribed in the curricula; mainly descriptive statistics, simple statistical investigations, and measures of central tendency (mean, median and mode). The allocated amount of time and predefined mathematical subject-matter content establish natural conditions on what actions and operations were feasible to implement and enact to support the students mathematical learning. To document the teaching, the classroom was videotaped using two cameras: one following the teacher's movement and actions in the classroom which in addition picked up all his interactions and conversations (using a small portable microphone); and a fixed camera in the back of the classroom facing front to capture the dynamics of the classroom. The teacher also recorder short pre- and post-lesson audio-memos to document the teachers thinking regarding the goals and plans made before the lesson, and then the teacher's reflections on what happened in the classroom in relation to these goals and plans during the lesson. All audio-memos were collected, as were all the written work done by the students. In the account provided here we only draw on the video and audio data. Next, we use data from this project in a first attempt to apply and evaluate the outlined dual integrated modeling approach as a lens to identify and teaching challenges that surfaced during an implementation of a modeling activity involving a simple statistical investigation and focusing on measures of central tendency in statistics.

Identifying of teaching challenges applying the dual integrated modeling approach as a lens

As part of the sequence of the 12 lessons on statistics the teacher introduced an adapted version of the paper helicopter activity originating from the work by Box (1992) who used it to teach experimental design to engineering students. The activity has also recently been used as a modeling activity involving conjecturing, experimenting, and evaluating 10-11-year-old students' ideas about statistical distributions (Kawakami, 2017). In the activity, the students were presented with a scenario to evaluate three competing designs of airdrop devices by conducting a small statical investigation of miniature replicas of the designs in terms of paper helicopters (see Figure 2). The aim for the students were to decide which helicopter (a) had the longest flying-time; and (b) which was the more accurate in terms of coming closets to the airdrop target – and also considering if difference in the loaded weight might influence the decisions. The class worked on the activity on and off during four consecutive lessons (lessons 6-9 in the sequence), whereby we will focus on the first two lessons.

Figure 2. Helicopter

In the first lesson, the teachers used 20 minutes to introduce the activity using a by the researchers pre-prepared PowerPoint presentation (a new tool) aimed at raising the students' interest and provide a meaningful as well as motivating framing for the activity. This had the effect that the class fell dead silent and focused all their attention on the teacher's introduction. This behavior was atypical for the class in question, indicating that the use of this type of presentations might be an effective strategy in managing the learning (ML) to tackle this class' students' attitudes toward mathematics and what it means to do mathematics in a, for the students, intriguing and inspiring way (SS), and hence make them better prepared to engage in the mathematical challenges (MC) to come. The introduction ended with the teacher leading a MEA-inspired whole class discussion around the questions *What features of the paper helicopter affects its (a) precision (how far from the intended target it lands)? (b) travel time (time spent in the air after being released?* and *How do you think these different features affect the precision and travel time of the helicopter?* The features suggested by the students to affect the precision were wind, distance to fall, air pressure and gravitational force. Features suggested to affect the travel time were the size of the rotor blades, the weight of the helicopter, the body-shape of the helicopter, and the falling velocity. Note that all the features related to precision are about external factors that might affect the behavior of the helicopter rather than actual features *of* the helicopter.

The second lesson (70 minutes) began with the teacher recapitulating and discussing the students' suggested ideas of what factors might influence the paper helicopter. The teacher explained that the class were to evaluate three competing paper helicopter designs by collecting and analyzing data to determine which design is the best. In addition, the teacher also qualified the explicit goal of the activity to in addition investigate (i) if heavier load decreases the travel time? (ii) if heavier load increases the precision? (iii) if larger rotor blades increase the travel time? and (iv) if larger rotor blades increase the precision? After having established these goals, the teacher turned to illustrate an airdrop with a prepared cut out paper helicopter, explicitly showing how to fold the rotors of the helicopter to form a suitable angle relative to its body, how to hold it to minimize interference on its trajectory, and from what height to drop the helicopter (the edges of the ceiling lamps). In doing this, the teacher also multiple times explicitly stressed the need for the students to be consistent in their experiment procedure and data collection. Initially, this way of managing the learning (ML) was interpreted as being over-sensitive to the students' needs (SS) just to get them to understand the mathematical challenge at hand (MC). However, this later turned out to be an important clarification act by the teacher to eliminate misconceptions, as the students' following discussions revealed that some of the student had the impression that the task dealt with real maneuvering helicopters.

Most of the time in the second lesson were spent on collecting and analyzing the data in groups. The students took measurement series of 10 values using either 0, 1 or 2 paperclips as helicopter wight. At this point the students were not totally free to explore the data and think about how their data set could be used to answer the posed questions. Rather, they were implicitly prompted by the semester overall planning listing the content for the week as "mean, median and mode" as well as explicitly by the teacher's statements like "I wonder what the mean, median and mode of that data will tell us about which helicopter is best?!". Here, a tension between two conflicting goals came to the fore: the teacher's goal for the students to engage in realistic statistical investigation and to freely explore the data set (macro modeling) on the one hand, and to use the activity as a context for the students to learn more about (and apply) mean, median and mode (micro modeling). This tension was then accentuated when the nature of the students' collected data not in all cases was suitable for determining the mode of the data.

In summary, the first attempt to apply the dual integrated approach as a lens on a teaching sequence provided information on teaching challenges related to establishing student's autonomy for learning both micro and macro modeling simultaneously. The teachers' role, highlighted by our theoretical analysis, of directing students on what to learn and at the same time leave enough space and time for students' own experience in both micro and macro modeling is at the core of the teaching challenges.

Discussion and conclusions

This paper provides the first steps of theorizing toward a theoretical conceptualization of a so-called *dual integrated modeling approach* to the teaching and learning of (i) modeling; and (ii) mathematics through modeling. In this initial work, a potentially fruitful interplay between what is discerned from the TT- and the EMT-related framing of the ongoing teaching has come to the fore. On the one hand, the EMT provides a background helping to contextualize and deeper understand the relationships and dynamics revealed in the TT-based analysis of students' learning as mediated by the teacher's actions in managing of the mathematical challenges (MC) while being sensitive to students' various needs (SS) given the organized and planned management of learning (ML). An example of how the EMT elevates the effect of the teacher's actions, is the effect the use of the introductory PowerPoint had on focusing the students and making them more susceptible and ready to engage with the mathematical content. On the other hand, challenges for the teacher in obtaining the learning goal revealed by the TT-based analysis highlight conflicts and tensions between the micro level of teaching as constrained by institutional and cultural aspects manifested at the macro level provided by the EMT. As such, the TT-based analysis might be helpful in pointing at more systemic oriented changes needed in the EMT to improve the teachers' possible actions to increase the students' opportunities to learn mathematics in the classroom. This could for example be introducing a new tool such as a pedagogic strategy or assessment tool that facilitate the teacher in transforming his/her objectives for the students to learn mathematics toward the desired outcome, or to pin-point where measures need to be taken, such as replace dysfunctional power relationships (division of labour) or classroom behavioral norms (rules).

The development of the dual integrated modeling approach is just in its infancy, and all notions and concepts still need to be further concretized and elaborated. However, our analysis shows that the dual integrated approach as a lens provides some useful information for developing teaching practice, and we hope that what we have presented in this paper can spark an interesting discussion and result in productive suggestions for how to continue develop this line of thinking and theorizing.

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