# Collective and individual mathematical progress: Layering explanations

Tommy Dreyfus, Naneh Apkarian, Michal Tabach, Chris Rasmussen

# Collective and individual mathematical progress: Layering explanations

Tommy Dreyfus[1], Naneh Apkarian[2], Michal Tabach[3] and Chris Rasmussen[4]

[1]Tel Aviv University, Israel; tommyd@tauex.tau.ac.il

[2]Arizona State University, USA; napkaria@asu.edu

[3]Tel Aviv University, Israel; tabachm@tauex.tau.ac.il

[4]San Diego State University, USA; crasmussen@sdsu.edu

*Keywords: Individual, collective, mathematical progress, methodology, Sierpiński triangle.*

## Introduction and background

We report on the latest development of our efforts to coordinate analyses of individual and collective mathematical progress. We build on and extend a series of theoretical-methodological analyses aimed at networking Abstraction in Context (AiC, Dreyfus et al. 2015), and Documenting Collective Activity (DCA, Rasmussen & Stephan 2008). AiC is commonly used for the analysis of knowledge construction by individuals or small groups of students, and DCA, which accompanies the emergent perspective, is commonly used for analyzing the mathematical progress of the whole class or a small group of students (Hershkowitz et al. 2014; Rasmussen et al. 2015; Tabach et al. 2014; Tabach et al. 2020). Our research goal is to further develop a methodological approach for characterizing the interplay of mathematical progress across individuals, small groups, and the whole class. We refer to this approach as "collective and individual mathematical progress: Layering explanations" (CIMPLE, pronounced as "simple"). "Layering explanations" pertains to the use of both theories on the entire data set, and to transparently layering analysis upon analysis, unlike our previous efforts that leveraged AiC on small group work (SGW), and DCA on whole class discussions (WCD). The significance of this ongoing work lies in the identification of nuanced ways in which students' knowledge progresses in inquiry-oriented classrooms.

The context for this study was a semester-long intact graduate level mathematics course on chaos and fractals at a State University in the USA. Ten of the eleven students were pursuing a master's in mathematics education. The students worked in four stable groups: A (Carmen, Jen and Joy); B (Kevin, Elise and Mia); C (Soo, Kay and Shani); and D (Curtis and Sam). All names are pseudonyms. Groups A and B were video-recorded during SGW; the class was video-recorded during WCDs.

## Analysis and results

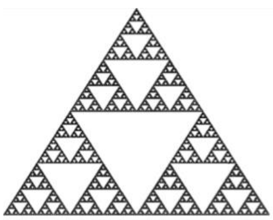

**Figure 1: The ST**

In Lesson 9, students carried out the first few iterations of a recursive geometric process (given a triangle, connect its midpoints and remove – or color white – the middle triangle); if continued infinitely, this process produces the Sierpiński triangle (ST). Students were asked to imagine the ST and discuss what they could say about the area and the perimeter of the ST. After 3 WCDs and 3 SGWs, the instructor (who had listened in on Group A) convened the class and asked Carmen and Joy to report on their opposite views of the perimeter. According to Carmen, as one keeps zooming in, the entire triangle is "going to be white, so there's no area, so there's nothing to … put a fence around; so there'd be no perimeter". According to Joy, "if you zoom in…, there is more and more to fence; Until…" Joy was unsure whether the perimeter increases to a finite value or to infinity. Although in the preceding SGW and WCD episodes the students had made little progress constructing knowledge about the features or the length of the perimeter, Elise immediately reacted to Carmen and Joy by Connecting Area to Perimeter (CAP): "what you are coloring in *is* perimeter, to some extent". She was followed by Kevin, and later Curtis: "The perimeter of the white is also the perimeter of the black" (PW=PB); and in between Carmen: "The fence is guarding both properties". Our analysis shows that in these 2 minutes, several students constructed the CAP knowledge element (according to AiC) and PW=PB started functioning-as-if-shared (FAIS) in the class (according to DCA). In other words, the students constructed new (to them) knowledge within a WCD, and this new knowledge immediately began to FAIS in the class. This constitutes substantial mathematical progress, achieved in a pattern that is very different from the standard trajectory.

Earlier in the same lesson, the students made mathematical progress in two further patterns. In one, "Area goes to 0" passed from FAIS in Group A to FAIS in the whole class without any reaction or even question. In the other one, the everyday metaphor of "zooming in" which had earlier appeared in a movie about the fractal nature of the coast of Britain was appropriated by the students as a tool to deal with the infinite nature of the ST (as used by Joy and Carmen above).

There are certainly additional patterns of mathematical progress. We conclude that the standard trajectory is only one of many possible ones for mathematical progress in inquiry-oriented classrooms. The coordination of AiC and DCA is an efficient methodology to identify such patterns.

## Acknowledgment

Research partially supported by the Israel Science Foundation under grant 438/15.

## References

Dreyfus, T., Hershkowitz, R., & Schwarz, B. (2015). The nested epistemic actions model for abstraction in context. In Bikner-Ahsbahs, Knipping & Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (pp. 185-217). Springer.

Hershkowitz, R., Tabach, M., Rasmussen, C., & Dreyfus, T. (2014). Knowledge shifts in a probability classroom. *ZDM Mathematics Education, 46*(3), 363–387. https://doi.org/10.1007/s11858-014-0576-0

Rasmussen, C., & Stephan, M. (2008). A methodology for documenting collective activity. In Kelly, Lesh & Baek (Eds.), *Handbook of design research methods in education* (pp.195–215). Routledge.

Rasmussen, C., Wawro, M., & Zandieh, M. (2015). Examining individual and collective level mathematical progress. *Educational Studies in Mathematics, 88*(2), 259-281. https://doi.org/10.1007/s10649-014-9583-x

Tabach, M., Hershkowitz, R., Rasmussen, C., & Dreyfus, T. (2014). Knowledge shifts in the classroom—A case study. *Journal of Mathematical Behavior, 33*, 192–208. https://doi.org/10.1016/j.jmathb.2013.12.001

Tabach, M., Rasmussen, C., Dreyfus, T., & Apkarian, N. (2020). Towards an Argumentative Grammar for Networking. *Educational Studies in Mathematics, 103*, 139-155. https://doi.org/10.1007/s10649-020-09934-7