Supporting teachers' early statistical development and PCK through lesson study: an informal inferential reasoning experience with kindergarten through fourth grade teachers

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Developing statistical thinking at the early school levels is a challenge that in-service primary school teachers must face. One of the actions that would achieve this goal is aimed at generating teaching proposals from the teachers themselves in their schools, from contexts that give meaning to the analysis of data in situations of uncertainty from an informal perspective. This report presents how collaboratively, a group of preschool and primary school teachers and researchers participated in a Lesson Study Group on the Informal Statistical Inference (ISI) approach for nine weeks. This allowed them to move in a cycle of goal setting, planning, implementation and evaluation of a lesson plan, acquiring knowledge of the content and pedagogical knowledge of ISI reflecting on their professional learning and the teaching of ISI.

Keywords: Early statistics, primary education, informal statistical inference, lesson study, pedagogical content knowledge.

Introduction

Statistical thinking makes it possible to more accurately analyze the world and overcome the fallibility of intuition. The use of statistics allows decisions to be made about phenomena in which uncertainty is present and facilitates the establishment of inferences in situations that we try to foresee according to the behavior of the data. It is necessary to incorporate at the school level experiences that allow students to face critically various information sources that provide data, such as those related to epidemiological phenomena, scientific findings, electoral results, weather forecasts, economic models, among others. In turn, they are required to be able to recognize variation and understand the randomness present in many daily activities. Given this scenario, teachers could use informal statistical inference (ISI) as a theoretical and pedagogical approach that contributes to developing in their students' ways of reasoning in situations of uncertainty, understanding ISI as "a generalized conclusion expressed with uncertainty and evidenced by, yet extending beyond, available data" (Ben-Zvi, et al., 2015, p. 293).

On the other hand, preservice teachers and in-service teachers, require a pedagogical content knowledge of the informal inferential reasoning (IIR) around central concepts of ISI, so that they understand key ideas in statistics, anticipate the difficulties and errors of students and can build tasks that provide opportunities for the development of inferential reasoning (Leavy, 2010). In this sense, the present study addresses early statistical education with preschool and primary educators and investigates their pedagogical content knowledge (PCK) by planning a learning sequence on ISI in early childhood education designed collaboratively within a lesson study group (LSG), and by researching their lessons in the classroom.

Lesson study for the transformation of teaching

To initiate a Lesson Study cycle, a team of 3 or more teachers is needed to form a LSG, which meets periodically to establish a lesson plan. Through Lesson Study cycles, understood as a process of instructional improvement, teachers can transform traditional ways of teaching mathematics into an instance of teacher professional development. Research has shown that it is a way for teachers to develop their work collaboratively and understand how their students learn. Lesson Study (LS) focuses attention on teamwork and shared responsibility around a lesson plan, its implementation, and its improvement. Thus, one or more teachers prepare the lesson, selecting the materials needed to achieve the objective stipulated in the lesson plan. Subsequently, one of the teachers involved in the planning implements the lesson, which in some cases is observed by other teachers or researchers. Once the lesson is completed, the teachers and observers meet in a session in which the implemented lesson is reviewed and analyzed, which will allow for the improvement of the lesson plan and its subsequent dissemination. According to Isoda et al. (2021), LS allows the development of teachers' Content Knowledge and Pedagogical Knowledge, promotes their ability to observe students' productions in class, motivates the improvement of their teaching proposals and allows the establishment of collaborative relationships among colleagues.

Teachers' knowledge and Informal Statistical Inference

Since the 1980s, Shulman (1987) stated that teaching begins when the teacher reflects on what needs to be learned and how students will be taught. In these reflective processes, beliefs, implicit theories, and other forms of thinking interact with context variables to shape actions that take place in the classroom. The PCK construct seeks answers that contribute to specifying the professional knowledge needed in teaching to intensify student learning (Shulman, 1987).

The framework presented in Table 1 (Chick et al., 2006), accounts for the components of PCK that are evident in teaching and the way pedagogical and content knowledge are intertwined. This framework considers three components: Clearly PCK (involves aspects that are more of a mixture of content and pedagogy); Content knowledge in a pedagogical context (involves aspects drawn more directly from the content); and Pedagogical knowledge in a content context (includes knowledge drawn more directly from pedagogy).

Table 1: Framework for analyzing Pedagogical Content Knowledge (Chick et al., 2006)

PCK Category	
Clearly PCK	Teaching Strategies
	Student Thinking
	Student Thinking - Misconceptions
	Cognitive Demands of Task
	Appropriate and Detailed Representations of Concepts
	Explanations
	Knowledge of Examples
	Knowledge of Resources
	Curriculum Knowledge
	Purpose of Content Knowledge
Content knowledge in a pedagogical context	Profound Understanding of Fundamental Mathematics (PUFM)
	Deconstructing Content to Key Components
	Mathematical Structure and Connections

PCK Category	
	Procedural Knowledge
	Methods of Solution
Pedagogical knowledge in a content context	Goals for Learning
	Getting and Maintaining Student Focus
	Classroom Techniques

The research in which this report is framed, involves characterizing the progression of the IIR from kindergarten through fourth grade attending, in turn, to the development of the PCK of the teachers when designing and implements a lesson plan in an LSG. In this sense, it is considered that the PCK accounts for the knowledge of teachers to understand the concepts, problems or emerging difficulties adapted according to the interests and skills of the students (Estrella et al., 2015).

This framework is combined with the informal statistical inference components characterized by Makar and Rubin (2009) for the generation of teaching proposals that seek to promote IIR in students at the school level. Various frameworks have been proposed when characterizing the ISI and the reasoning that it supports, whose common components correspond to using data as evidence requiring to assess and consider the available data to establish arguments associated with a question or problem, privileging certain evidence that they provide over personal experiences or opinions; generalizing beyond the data accounts for the ability to communicate conclusions derived from particular data, generating inferences that apply to a broader set; and expressing uncertainty implies manifesting the uncertain in generalization, being aware that statements are not certainties, but consider a margin of error.

In this way, one of the purposes of this study focuses on how the work in a LSG that seeks to promote ISI can contribute to the development of the PCK of the teachers linked to the experience. Although the work with the teachers included stages of planning, execution and improvement of a lesson plan, this report focuses on the following question: How does a teacher's PCK manifest itself when planning a lesson plan on ISI at the preschool to fourth grade levels?

Methodology

To answer this question, -and based upon consensus among the authors-, a qualitative, descriptive-interpretative approach is adopted, analyzing the dialogues transcribed from nine video recordings: seven working sessions within the LSG and two lessons of implementation of the lesson plans of each teacher, according to the categories adapted from the PCK components of Chick and collaborators (2006), integrating the key components of the IIR.

Participants

The LSG consisted of six teachers (three primary school teachers, one preschool educator, two special educators) and three researchers with expertise in statistics education, two of them with LS research experience. Regarding their years of professional experience, three of the teachers in this school had more than 15 years of teaching experience, two of them had between 10 and 15 years of experience and one had less than 10 years of teaching experience. The teachers work in a municipal educational establishment located in the Metropolitan Region of Chile and carried out the execution of the lesson plans with a total of 70 students distributed in grades K-4.

The LS cycle for generating a lesson plan that promotes IIR.

The LSG met weekly for 2 hours during 9 weeks. The activities of the LS cycle in which the teachers had to go through were delineated by the team of researchers based on four work axes: A. Authentic experiences of informal inferential reasoning; B. Planning a learning sequence; C. Implementation, revision and adjustment of the lesson plan that promotes IIR; and D. Analysis and reflection of the LS process.

In Table 2, although the activities are presented sequentially, some actions are carried out cyclically throughout the process, for example implementation of the lesson plan, review and adjustment of the lesson, identify student needs, reformulate objectives, propose new possible teaching interventions, among others.

Table 2: Overview of the activities according to the LSG work

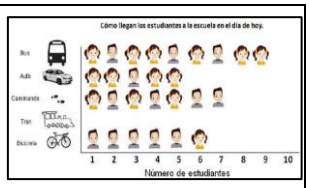
Week	Activity
1–2	**Authentic experiences of informal inferential reasoning: Exploring content knowledge in a pedagogical context** •Task 1. How will Tom get to school tomorrow? •Task 2. Which car would he choose to win the race? What elements characterize IIR: Comparing responses of LSG teachers and some other teachers in the country, in light of the components: Using data as evidence, expressing uncertainty, and generalizing beyond data.
3–4	**What do we expect from the IIR of K-4 students? - Planning a learning sequence** Identify student needs, formulate objectives, set a task to promote IIR, anticipate student difficulties and responses, outline possible teaching interventions, and determine aspects to assess.
5	**Implementation of the lesson plan** The planned lesson plan is implemented, some teachers and researchers observe and record, in videos and/or field notes, what happens.
6	**Review and adjustment of the lesson plan according to post-session analysis** The experiences of the teachers in the implementation are shared, the LSG reviews and adjusts the lesson plan considering some suggestions and recommendations that arise in the post-session analysis.
7	**Implementation of the adjusted version of the lesson plan** The lesson plan is implemented in light of the proposed adjustments and the LSG's recommendations.
8 - 9	**Analysis and reflection of the LS process** Video records and field notes are reviewed to identify strengths and aspects of the intended lesson plan, and group reflection on all activities of the process describing the LS experience, professional learning and challenges, socioemotional aspects and some discoveries.

ISI tasks proposed to the teachers in the first LS stage

In the first LS stage, the teachers had two experiences that triggered their RII (see Table 3). The first situation is from the inferential domain and is based on one of the items reported by Watson and Callingham (2003). The second situation belongs to the probability domain and is related to a randomized experiment with dice. After its realization, the in-service teachers were able to know the elements that have been characterized at the research and pedagogical level of the ISI approach, with the purpose of integrating these elements in the lesson plan that would be formulated, implemented and evaluated in the following stages.

Table 3: ISI tasks proposed to the educators and teachers in the first stage of the LS

Situation	Concepts	ISI Ability	Resources

Reading a data representation with a missing data to make a prediction in a scenario of uncertainty.	Variable, categories of the variable; data; majority; prediction.	Predicting the variable category in which the missing data correspond beyond the data provided in the representation.	
Conducting an experiment of throwing two dice simultaneously, advancing on game board according to the sum of the results of the dice faces.	Random variable; sample; population; random experiment; variability; sample size; prediction.	Predicting the greatest chance of winning by considering various samples.	

The lesson plan and the core task proposed within the LSG

The lesson plan was jointly designed based on previous LSG discussions on IIR and was intended to be implemented transversally with K-4 students. This plan considers a central statistical task of a playful type involving the randomized coin-tossing experiment, called "the frog race".

The game, whose instructions are illustrated in Figure 1, favors the manipulation of the material and the recording of data on a game board. Before starting the game, each player must choose the frog that he/she thinks can reach the goal before the other two. The game ends when one of the frogs has reached the goal. The game can be repeated as many times as desired, although three or more completed games are recommended.

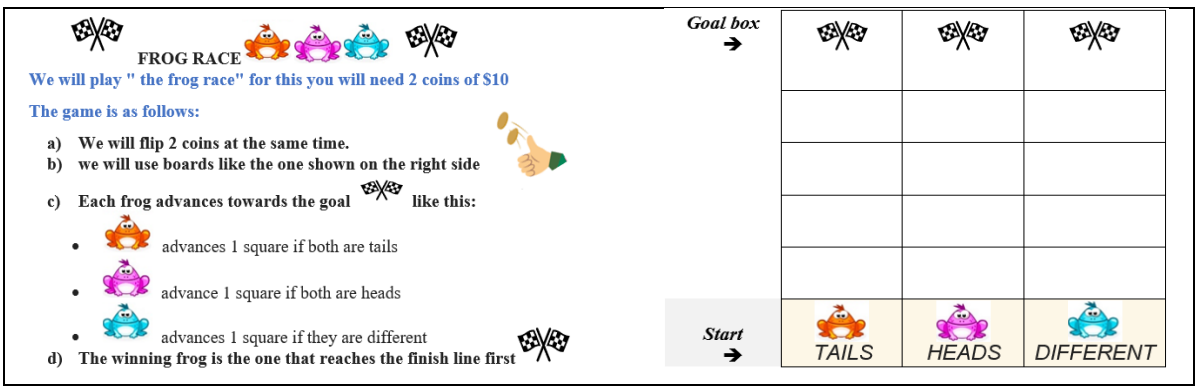


Figure 1: Board and game instructions “Frog race” (Estrella et al., 2022)

After completing several games with the data recorded on the boards, students are encouraged to establish generalizations beyond the data, using arguments based on the samples obtained (their own and those of their classmates) and to express uncertainty about which of the frogs is more likely to win; the guiding question was "If you had to give someone advice, which frog would you advise them to choose? Why?". Each teacher gave two lessons on ISI in two groups of the same course.

Results

To account for teachers' PCK when generating, implementing, and improving a lesson plan that promotes IIR, some illustrative episodes were selected from the categories proposed by Chick et al. (2006). Given the limitation of this paper, we chose to show only the lesson planning stage and the categories used to analyze the teachers' interventions (each teacher is labeled with T1 to T5, and the researchers as R1 and R2).

Conformation of a lesson plan

As an instance prior to the planning of the lesson plan, the teachers faced IIR eliciting tasks, as an authentic experience that not only allowed them to activate their content knowledge on ISI, but also enabled them in a subsequent exercise, to analyze their own and other colleagues' answers, considering the IIR components brought into play. At the same time, they learned in parallel some aspects of the ISI approach as an alternative that makes possible the teaching of contents associated with uncertainty prior to the use of formal inference techniques. These actions were key in the formulation of a central task of the lesson plan.

Some manifestations of teachers' PCK according to the categories (1) Clearly PCK, (2) Content knowledge in a pedagogical context and (3) Pedagogical knowledge in a content context, when participating in an LSG are illustrated in Table 4 below.

Table 4: PCK manifestations of two teachers according to PCK categories

Stage of the LS cycle	**PCK Category**		
Planning	1	2	3
T1: The central problem of the class... considering K to 4th grade, What scenario... evidence... express uncertainty...	X	X	X
T2: It would have to be something like the car race thinking that something super meaningful and known to the kids...			X
T1: You can think of the generalization in the fact that if they see that a car is winning... When they are doing the game ...If a partner asks them	X	X	
T1: A visual question to see who is winning... visually they realize who is winning... and when the children realize that they are winning... they will immediately say "yes", I experience this daily with my students. They want to get to the winning situation, yes or yes!	X		X

In this episode, T1 points out in his intervention key aspects to promote in his students, identifying components of the IIR that are fundamental for students to understand and establish ISI, in particular the use of data as evidence and the establishment of expressions with uncertainty. T2 proposes to the LSG to involve playfulness as a fundamental feature of the task, so that students can become emotionally attached to its development. Then T1 points out to his colleagues that employing a visual representation could allow students to reason through identifying patterns in the data, and analyze the results more favorably. Finally, T1 discusses strategies for engaging students.

Table 5: PCK manifestations of five teachers according to PCK categories

Stage of the LS cycle	**PCK Category**		
Planning	1	2	3
R1: Regarding the car race, in kindergarten I think there would be difficulty with the sum of the dice, ... there would be too much prominence of the teacher, 2 dice are going to be slower (e.g. 6 and 6).	X		
T1: Maybe the children could roll [roll the dice] and one [teacher] could count.			X
T2: But T1, that has to be done [live and direct with the students] how do you plan to do it with Kindergarten in this [Pandemic] scenario?			X
T1: A of course .... I was thinking about that... how do you apply it? It has to be something we can apply now [Pandemic] right?			X
T3: With R2 we thought of a race, but with colored frogs, and that, instead of rolling dice, you would roll a coin 2 times or 2 coins at the same time. The frogs would advance, for example, the orange one if the two coins go "tails", the pink one if the two coins are "heads"., and the blue one if different [head and tail].		X	

R1: Given the level of the children it could be 2 coins. I want to know which frog you would vote for.	X		
T4: I'm leaning...I have no prior training ...but I think there's a better chance of 1 or 1 [heads and seal]. R1: and why? T4: I don't know... Thinking it's harder for [both] 2 heads and 2 tails to fall out. R2: Do you have 2 coins? let's toss them 10 times!		X	
T5: I already did it, I got 2-times [both] head, 1-time tails [both], and 7-times tail and head T4: I won!		X	
T1: Playing the game I feel it's super practical, because the parents could pull or the children, and helping them to advance. We [teachers] wouldn't have to do it, my kids could do it.			X
T4: It could be playing as a family and completing a register...	X		

In the episode (see Table 5), R1 invites to anticipate certain difficulties of the students in the operation of a randomized experiment of throwing two dice simultaneously given the limited numerical scope of the kindergarten through fourth grade students. Some LSG members propose alternatives to reduce its complexity, in this case using two coins (three non-equiprobable events) instead of dice (six equiprobable events; or 36 non-equiprobable events). On the other hand, teachers discuss some pedagogical strategies that can be adjusted regarding their management in an online learning environment given the pandemic. In turn, R1's intervention provokes teachers to bring into play their content knowledge in the pedagogical context of task planning vis-à-vis possible predictions, the operation of the randomized experiment, and possible outcomes.

Discussion

It is recognized that teachers face challenges in teaching statistics, as many lack experiences in school or in their initial teacher training, particularly on aspects specific to early statistics (Estrella et al., 2020). Although research regarding teachers' PCK about ISI and ways of preparing to teach ISI is scarce (de Vetten et al., 2017), it is also recognized that teacher professional development experiences conducive to promoting PCK on ISI and associated reasoning can be generated. In this perspective, the manifestation of K-4 teachers' PCK was analyzed when planning, executing and improving the lesson plan on ISI.

Authentic informal inferential reasoning experiences (Table 3) helped to foster in teachers, an awareness of the uncertainty inherent in the outcomes presented in the situations, which led to proposing a central task for K-4 students to develop their IIR early; in turn, allowed teachers to analyze their own IIR, and to delimit future actions regarding the planning, management and evaluation of a learning sequence that could develop the IIR of their students. The planned learning sequence considered the materials, motivation, online education context, and students' prior knowledge and experiences, delineating possible teaching interventions, in the core task promoting IIR. During the implementation, review, and adjustment of the lesson plan, teachers were able to reflect on the achievement of the lesson objectives, providing feedback leading to the fine-tuning of the lesson. Finally, the instance of reflection of the LS process allowed teachers to make explicit the professional challenges, the perceived socioemotional aspects and some discoveries.

Although this report shows part of the comprehensive analysis of teachers' PCK in only one of the stages of the LSG cycle, the categories employed can be considered relevant to analyze in detail the manifestation of aspects of PCK in relation to IIR. The generation of statistical learning sequences created with the expertise of the teachers themselves together with LSG collaborating researchers in

an LS that lasted several weeks, hold promise for the promotion of statistical thinking at the early school levels. It is projected that the spirit of improvement of the LS methodology will permeate both the learning sequence constructed and the experience and teaching of this group of teachers.

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