Concepts introduced in Norwegian textbooks before, during and after the New Math period

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For a nuanced discussion on the legacy of New Math, detailed knowledge of the content of mathematics before, during and after New Math is needed. In this article, we mirror a study of Icelandic textbooks, studying at what ages certain New Math-connected concepts were introduced in 14 Norwegian textbook series for age 7-15 published in the 1950s, 60s, 70s and 80s. As was the case with Iceland, set-theoretical concepts appeared with New Math and were mostly gone by the 80s, while axioms of the number field and the study of number were to a large extent present throughout the period studied. These were introduced at an earlier age in the 70s. We find that some New Math-related concepts were introduced in textbooks before New Math entered on a full scale, unlike Iceland. Like Iceland, many other concepts were introduced in the 70s and stayed in textbooks.

Keywords: New Math, textbooks in mathematics, mathematical concept

Introduction

New Math was an international movement that began in the mid-twentieth century. It has been strongly associated with the set concept. Since the set concept lost its place in textbooks quite soon, it is tempting to believe that the long-term impact of New Math was small. As Schubring (2021) points out, many believe that New Math affected all countries similarly. For informed discussions on this period within the history of mathematics education, more nuanced pictures of the legacy of New Math in each country are important. In this article, we present such a picture for Norway, mirroring a similar study for Iceland by Bjarnadóttir (2017).

In 1966, an international meeting for experts on mathematics teaching at the primary level was held in the UNESCO Institute for Education in Hamburg. In the report from this meeting, a list of basic topics was proposed that should be included in primary school. These included concepts from set theory (Williams, 1967, p. 47). However, set theory was just one of several topics in mathematics that were introduced to students either as a new topic or for younger students. According to Kilpatrick (2012), internationally many of the ideas of New Math are still part of school mathematics.

In this paper, our research questions are: Which concepts related to New Math were introduced in the textbook series for compulsory school from age 7 in Norway and to what degree did they survive into the 80s?

The starting point for New Math

The Royaumont seminar in 1959 was “the starting point for the coordinated international efforts to reform mathematics teaching based on the conception of ‘modern mathematics’” (Schubring, 2014,

p. 89). Teachers, school leaders and researchers from many countries, including Norway, Sweden and Denmark attended this seminar (OEEC, 1961). The starting point for New Math was a desire to teach mathematics that corresponded to modern society with its tremendous acceleration in technical, economic and social development (Christiansen, 1967). According to Christiansen, the curriculum in mathematics was previously narrow and the primary focus was to practice formal skills, while mathematical concepts and the interaction between these were often less emphasized.

The conclusions from the Royaumont seminar stressed that one must move away from memorizing facts and procedures to experimenting, discovering and making mathematics with physical objects. "This experimentation must lead to the abstraction of the quality of a set called number" (Schubring, 2014, p. 93). In the beginning one should focus on the ideas and later the mathematical concepts sets, subsets, mapping etc.

At the Royaumont seminar, it was also outlined how one could reform mathematics teaching in school. Regional cooperation was recommended. The authorities in Norway, Sweden, Denmark and Finland were positive about Nordic co-operation and in 1960 the Nordic Committee for the Modernization of Mathematics Education (NKMM) had its first meeting (NKMM, 1967). In 1967 a report was published on the work of this committee, providing a description of the current situation in the four countries, followed by goals for the schools' mathematics teaching. Several experimental texts for textbooks at different grade levels were developed and experimental teaching was carried out. The report also includes a proposal for a concrete curriculum for first to twelfth grade.

For the first three school years, two experimental texts were developed by NKMM, one Swedish and one Danish-Finnish, written by Bundgaard and Kyttä. "The basic ideas are the same in the two series, but they differ in many details" (Håstad, 1967, p. 99). For grades four to six, NKMM had trouble finding a suitable writing team and therefore they decided to translate the experimental texts published in the United States in the large project School Mathematics Study Group (NKMM, 1967, p. 108). For grades seven to nine, NKMM developed experimental texts in geometry and algebra.

Iceland did not participate in the Nordic collaboration, nevertheless the result of the collaboration influenced Iceland through, for example, the choice of a Danish-Finnish textbook series. The Bundgaard-Kyttä textbooks series was translated to Icelandic (Bjarnadóttir, 2017), together with textbooks for age 10-12 written by Agnete Bundgaard alone. We will call the combined series the Bundgaard series. Bjarnadóttir (2017) has studied how students in Iceland were introduced to certain mathematical concepts, often related to New Math, and at what age they were introduced to these concepts. In her study, the content of this textbook series is compared with the content of other series in use in Iceland before, during and after the introduction of New Math. In Iceland, there was just one textbook series in mathematics from 1939 to 1966 (Bjarnadóttir, 2017). The results of her study are that the topics that were new in the Bundgaard series were "the use of set theoretical concepts and the notation for building up the number concept and understanding of operations through repeated reference to the axioms of the number field" (p. 58). Negative numbers were not mentioned in it. Bjarnadóttir concludes that this textbook series went far in meeting the

requirement that "mathematicians" had for mathematics in primary school. Here, she is most likely referring to those mathematicians who participated in the Royaumont seminar in 1959. The topics that survived in textbooks after the New Math period were primes and divisibility, mental arithmetic and approximation and estimation. The topics that did not survive were replaced by an introduction to statistics, probability, the use of variables and solving simple equations. These were new topics when the Bundgaard series was replaced by an Icelandic textbook series in the 70s.

Bjarnadóttir's study shows that there were more than set theory concepts that were introduced to the students with New Math. Some of these were also introduced for students in the textbooks in the period after New Math, not necessarily for the students at the same age.

Norway participated in the cooperation and was therefore naturally affected by this participation. But unlike Iceland, in Norway there were many publishers of textbooks for primary schools. After a temporary curriculum in mathematics with New Math was introduced in 1971, several textbook series were published with content adapted to this new curriculum. Soon, mathematicians as well as parents of school children protested. In the final curriculum in mathematics, which was approved in 1976, New Math content was greatly reduced (Gjone, 1985, p. VII:31; Solvang & Mellin-Olsen, 1980, p. 1:18).

"The residue of the new math era may be difficult to see in today's school mathematics, but it is there" (Kilpatrick, 1997, p. 956). Kilpatrick mentions inequality as a subject that came in with New Math in USA and has remained in school mathematics.

Method

As outlined above, Bjarnadóttir (2017) studied the content of textbooks that were published in Iceland before, during and after the introduction of New Math. Based on Bjarnadóttir's study, we have done a similar study of when students encounter the various concepts for the first time in Norwegian textbooks.

Bjarnadóttir first developed tables with mathematical concepts based on literature on New Math, then she analysed how and when students were introduced to these concepts in Icelandic textbooks. Bjarnadóttir chose the following categories *set theoretical concepts and notation; structure of the number field;* and *study of numbers.* We have chosen to use Bjarnadóttir's categories to be able to compare the Icelandic and the Norwegian situation.

The two authors of this paper have each studied each textbook series separately and noted where the students are first introduced to the various concepts. When in disagreement, we have studied the textbooks together and come to an agreement on where the students meet the concept for the first time. We have noted both the first implicit treatment of a concept and the first explicit one. For reasons of space, in the tables in this article, we only include the first explicit introduction. If the concept is only introduced implicitly, we have marked this by putting parentheses around the age. We exemplify what we mean by an implicit introduction of a concept through the commutative law for addition. When several examples and tasks in a row have the same numbers (as for example 2+3 and 3+2), without the commutative law being mentioned, we conclude that there is an implicit introduction to this concept.

After preparing tables that show the age at which students are introduced to a topic for the first time, we have analysed these tables to compare different periods, which also makes it possible to compare the result from Norway with the situation in Iceland.

In this study, we analyse the content of textbooks that were published before, during and after the period of New Math, that is the period 1950 to 1980s. Early in the 1970s, several textbooks with New Math were published. We have chosen to include all known series from this period. In the period before and after New Math, we have chosen series from Aschehoug and Cappelen, as these published textbook series for the whole period. They have partly the same authors as the books that were published in the 1970s. We also conjecture that they were used by many schools in Norway, although there exists no statistics to determine this. We have also included Tanum, during and after the introduction of New Math (see Table 1), to include one more textbook series for the 80s. None of the Norwegian textbook series published in the 1970s were directly linked to NKMM's experimental texts.

Table 1: Textbooks included in our analysis. Textbooks from 1970s are marked for clarity

	Publisher	**Year**	**Age**	**Title**	**Author**
A1	Aschehoug	1953-	7-13	Nå regner vi	Aam, Johannesen, Slaatto, Paulsen
A2	Aschehoug	1964-	7-15	A. regneverk for folkeskolen	Paulsen, Slaatto
A2+	Aschehoug	1971-	7-9	A. regneverk, mengder og tall	Paulsen
A3	Aschehoug	1980-	7-12	Vår matematikk	Harboe
A4	Aschehoug	1987-	7-15	Regnereisen	Venheim, Breiteig m. fl.
C1	Cappelen	1962-	7-15	C. regneverk: matematikk	Bue
C2	Cappelen	1971-	7-15	C. matematikkverk	Bue, Gjerdrum
C3	Cappelen	1980-	7-12	Jeg regner	Gjerdrum, Bue
D1	Dreyer	1972-	7-12	Grunnskolens matematikk	Eicholz, Arneberg
E1	Eli	1972-	7-15	Ny regning	Dentrup, Kjeldberg, Kjeldberg
G1	Gyldendal	1972-	7-12	Matematikk for grunnskolen	Myrmo, AAs, Grymer, Ridar
N1	NKI	1973-	7-12	Tal og teikn	Rudjord, Bjørklund, m.fl.
T1	Tanum	1970-	7-12	Min matematikk	Viken (red.)
T2	Tanum	1980-	7-12	Min matematikk	Viken

Dreyer (D1) was based on a US textbook series, adapted to Norwegian conditions. Eli (E1) was based on a collaboration between Norwegian and Danish teachers. Gyldendal (G1) refers in the teacher manual to NKMM's report but does not claim to build directly on it. Tanum (T1) was based on a Swedish textbook series and refers also to NKMM. The other textbook series (A2+, C2 and N1) were modernizations of previous Norwegian textbook series. A2+ was written as a supplement to an existing textbook series to introduce students to set theory. This supplement was only for

students aged 7 to 9, older students used A2 textbooks. In our analysis, we have first analysed A2 (under the heading A2), then the combined series of A2 and A2+ (under the heading A2+).

When comparing our results with the findings of Bjarnadóttir (2017), we need to keep in mind that the two textbook series before New Math in her analysis were used in the periods 1922-37 and 1927-80. The textbooks we have studied were published much later. In addition, the Icelandic textbook series before New Math were for students from the age of 10 because children were expected to learn some mathematics at home before starting school at age 10.

Results

We will follow the structure of Bjarnadóttir (2017), looking first (briefly) at concepts from set theory, then axioms of the number field and finally what Bjarnadóttir calls "topics of number".

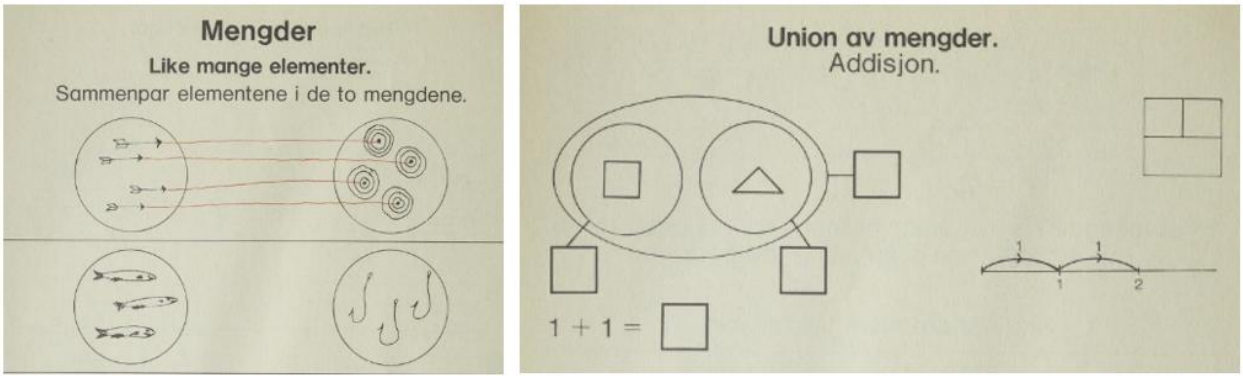

Figure 1: A section from A2+ from 7-year-olds. To the left with introduction to Sets (p. 10) and to the right with introduction to Union (p. 30)

Our analysis shows, not unexpectedly, that set theory entered the textbooks in the 1970s for the students aged 7 (see Figure 1 for the introduction in a textbook series) but were not included in the textbooks that were published after the New Math movement had passed. This is similar to what Bjarnadóttir found in Iceland.

For axioms of the number field (Table 2), the picture is different. Most of these were present in the 60s or even in the 50s, but in the 70s series, they were presented to younger students. In the 80s, it is more complicated – some of the axioms are left out, some are presented later, while some are even presented earlier.

The commutative law of addition is introduced implicitly for 7-year-olds in all time periods, from the 50s to the 80s. However, there is variation in when students are introduced to this law explicitly, from age 8 to age 13. It also varies when students are introduced to the commutative law of multiplication. If the law is first introduced implicitly, then it happens when students are 7 or 8 years old. It is often explicitly introduced when students are 9 years old. The same applies to the distributive law, but it tends to be introduced explicitly in the 70s (under New Math). The associative laws of addition and multiplication are introduced explicitly at the secondary level (from 13 years) in the 60s and at the primary level (under 13 years) in the 70s. The associative laws appear implicitly or not at all in the textbooks of the 1980s.

The additive identity is implicitly included for 7-year-olds throughout the period, and explicitly included later. Only in the 80s is it mentioned explicitly to 7-year-olds. The fact that addition/subtraction and multiplication/division are inverse operations is included in almost all textbook series, either explicitly or implicitly. But it varies when students are introduced to these connections. It seems that students were introduced to these connections at a younger age in the 70s.

In the 50s, the role of 0 in multiplication is introduced to 9-year-olds. In the 60s, this has been moved to when the students are 13-14 years old, and in the 70s and 80s it is again in most of the textbook series for 8-9-year-olds. The role of 0 in division is not mentioned in the 50s and 80s. In the 60s, it is mentioned in the same textbook as 0 in multiplication, and in the 70s it is mentioned at a higher grade level if at all.

Table 2: The age-groups of students for which axioms of the number field are presented

Topic\Textbook	**50s**	**60s**		**70s**							**80s**			
	A1	**A2**	**C1**	**A2+**	**E1**	**C2**	**D1**	**G1**	**N1**	**T1**	**A3**	**A4**	**C3**	**T2**
Commutative law														
of addition	(7)	13	9	13	9	8	8	(7)	10	(7)	(7)	14	(7)	(7)
of multiplication	13	14	9	9	9	8	9	(8)	9	(8)	11	9	9	9
Distributive law	(9)	14	13	9	9	(9)	9	(9)	9	(10)		14	(8)	(8)
Associative law														
of addition		13	13	8	9	8	7	9	10	7			(8)	
of multiplication		14	13	9	9	9	9	10	10	(9)				(9)
Identity -additive	(7)	(7)	14	(7)	13	(7)	10		12	(7)	7	7	(7)	(8)
-multiplicative	(10)	14	14	9	13		8	8	12	(8)		9		(8)
Inverse -additive		13	14	13	12			(12)	(10)			13		
-multiplicative			13		15			12	(12)	12		15		
0 in multiplication	9	14	13	9		8	8	8	9		9	9	8	
0 in division		14	13	14	(15)		9		12					
Negative numbers		12	12	12	11	11	12	12	10	11	12	12	11	10
Inverse operations														
add./subt.	13		9	13		(9)	9	9	8			7	(8)	(10)
mult./div.	10	14	13	9	(9)	10	9	(9)	8	10		11	(9)	10

Within study of number (Table 3), most topics were present already in the 60s. However, many of these were introduced far earlier in the 70s. Again, the picture is mixed as we enter the 80s.

While the number line is not included in the series from the 50s, it is included in most later series from the age 7. In some textbooks, especially in the 80s, it appears a little later. The connection between even numbers and odd numbers was introduced earlier with the New Math: for 7-8-year-olds students in the 70s as compared to 12-14-year-olds in the 60s. We see that the early introduction lasts into the 80s.

It varies how old students are when they are introduced explicitly to primes, factorization and divisibility (from 8 years to 14 years), but these concepts are often introduced simultaneously.

Students are introduced to bases other than ten in some of the textbooks in the 60s and 70s, but not in any of the textbooks in the 80s. Modular systems is never a topic in the textbooks studied.

In the 60s and 70s, students were introduced to equations at the age 7-9, while in the 80s they were introduced to equations at the age 10-12. This is different from the variable concept. It appeared in the textbooks for 11-12-year-olds in the 1960s, for 7-9-year-olds in the 1970s and again for 11-12-year-olds in the 1980s (except in one textbook series where students were introduced to this concept when they were 9 years of age).

Statistics is not included in the textbook series from the 50s, later, students are introduced to statistics between the ages of 8 and 11. In only two of the textbook series do we find probability, one of them in the 70s and one in the 80s.

Table 3: The age-groups of students for which topics on numbers are presented

Topic\Textbook	**50s**	**60s**		**70s**							**80s**			
	A1	**A2**	**C1**	**A2+**	**E1**	**C2**	**D1**	**G1**	**N1**	**T1**	**A3**	**A4**	**C3**	**T2**
Number line		7	9	7	7	7	7	7	7	8	11	9	7	9
Number relations: Even & odd		14	12	7	8	8	8	7	7	(8)	8	7	8	8
primes	11	14	12	14	12	12	9	11	10	12		10		11
Factorization	11	14	12	14	11	8	9	11	10	12		10		11
Divisibility	11	14	12	14	12	12		11	10	12		11		11
Bases other than ten		15		15			(12)		9	(7)				
Modular system														
Symbols as 7+2 for 9			14	7	(8)		11	7	7	7				9
Variables		11	12	7	8	9	9	9	9	8	12	12	9	11
Equations		12	7	7	7	8	7	8	8	9	12	12	10	11
Probability					14							12		
Statistics		11	11	9	11	8	10	8	8	11	10	10	10	9
Mental arithmetic	8	8	11	8	8		9	10	9	9	10	8		9
Approximation, estimation		13	11	13	11	9	9	10	10	9	12	9	9	9
Use of calculators												10		

Discussion and conclusion

Set-theoretical concepts were not included in the textbook series in primary school in the 50s and 60s in Norway, entered with New Math in the 70s and partly disappeared again in the 80s. As such, we can distinguish a clear "New Math" period in Norwegian textbooks. Several other concepts (for instance associative laws) were introduced to students at a younger age in the 70s than before. Of these, some of the concepts stayed in the textbooks for the same age group in the next decade, while others returned to textbooks for students of a higher age. While textbooks series vary, there is a tendency for more of the terms be introduced explicitly earlier in the 70s.

Our study gives a somewhat different picture of New Math's role than Bjarnadóttir's (2017) study. As there were no new textbook series in Iceland for decades before New Math, the change was abrupt. In Norway, on the other hand, several textbook series were published in the decades before New Math took hold. This contributed to a more gradual change. One major difference was that in Norway, many New Math ideas entered textbooks in the 60s, but New Math (including set theory)

came in full force in 1971, with a new temporary curriculum. In Iceland, New Math (including set theory) was implemented in the 1960s, but some New Math ideas (such as statistics) only entered primary school in the 1970s as other New Math ideas waned.

Although New Math has been partly derided and ridiculed in later years, we do see that many concepts which are now taken for granted as a part of school mathematics, were introduced in the 1960s and 1970s. This applies, for example, to statistics, approximation and estimation, as Bjarnadóttir also concludes in her study.

When it comes to the textbook series that were published in the period of New Math, in Iceland and Norway, we see that there are some similarities in when students in the two countries are introduced to the different concepts. However, we also see large variations between the different textbook series published at about the same time. This variation is more apparent in our analysis than in Bjarnadóttir's, as there were fewer competing textbooks in Iceland.

Taken together, the detailed analyses provided by Bjarnadottir for Iceland and by us for Norway, provides a foundation for a more nuanced discussion on the legacy of New Math in the two countries. If such analyses are done for more countries, we may see even more what is special with how New Math has been implemented in different countries.

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