A digital artefact based on visual programming for the learning of axial symmetry in primary school

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In this work we used visual programming in order to mediate the learning of axial symmetry in primary school. More in detail, we designed and implemented an a-didactical activity in the Scratch programming environment in which students have to symmetrize figures with respect to oblique axes. We tested the learning activity with fifth-grade students with the aim of understanding to what extent algorithmics and visual programming influenced the rise of new solving strategies by students. Preliminary results from a qualitative analysis seem to show that students were able to improve their strategies in the programming dimension and to evoke some aspects of axial symmetry.

Keywords: Elementary school mathematics, programming, symmetry.

Introduction and theoretical background

In recent years, several national and international institutions promoted the introduction of algorithmics and computer programming in the school. For example, the Italian Ministry of Education (MIUR, 2018) and the European Commission (https://ec.europa.eu/education/education-in-the-eu/digital-education-action-plan_en) encouraged the introduction of computational thinking, coding and the creative and critical use of digital technologies since primary school. Already in the 1980s the research in education was oriented towards the development of new programming languages and techniques, increasingly accessible to primary school students. In those years, Papert (1980) created the LOGO programming language, used, for instance, by Bideault (1985), Salem (1988) and Clements et al. (2001) for learning geometry in primary school. More recently, Bartolini Bussi and Baccaglini-Frank (2015) used the programmable robot Bee-Bot in order to deal with geometric aspects. Tchounikine (2016) underlines how visual programming and algorithmics are suitable tools for conveying content from different disciplines. In this view, many authors (for example, Benton et al., 2018; Forster et al., 2018; Zhang & Nouri, 2019) used the Scratch visual programming language (https://scratch.mit.edu/) for learning mathematical concepts. Scratch was also used in the French project EXPIRE (https://expire.univ-grenoble-alpes.fr/) to deal with mathematical contents in primary school (Chaachoua et al., 2018).

This work arises from the context described above. It is a part of a broader research project whose aim is to investigate students' difficulties related to symmetries (Dello Iacono & Ferrara Dentice, 2020), and to design suitable learning activities to try to overcome them. In particular, in this work, we decided to use Scratch programming language to mediate the learning of axial symmetry in primary school. Indeed, axial symmetry, like any non-identical geometric transformation, is associated with the idea of movement, as underlined by Ng & Sinclair (2015). Also Jagoda & Swoboda (2011) emphasize the importance of associating symmetries with movement, and they invite to provide students with tasks and tools that allow them to manipulate objects and to experience the action of geometric transformations and their results. In this regard, Scratch programming language could be a suitable tool as it allows students to interact dynamically with the environment. So, we designed an a-didactic situation (Brousseau, 1986) in the Scratch programming environment that involves the use of algorithmics in the students' task. Students have to work on a digital artefact, with figures to be symmetrized with respect to oblique axes. The choice of the oblique axis of symmetry wants to create a cognitive conflict in students (Fischbein, 1989), who are often accustomed to working with horizontal or vertical axes of symmetry. In this way, the intuitive model of an object symmetrical with respect to a horizontal or vertical axis can be replaced by the rigorous mathematical model of an object symmetrical with respect to an axis, regardless of the position of the latter with respect to a reference system such as a sheet of paper or a squared grid.

We experimented our a-didactic activity in a fifth-grade class. In this work, we show the preliminary findings of a qualitative analysis whose aim is to understand to what extent the algorithmic and the visual programming underlying the activity we designed influenced the emergence of new solving strategies by students, linked to the construction of symmetrical images with respect to an axis.

Design of the digital artefact

We designed an a-didactic activity requiring students to work on a digital artefact in the Scratch programming environment. The scene involves three students from a dance school, Piero, Isabella and Giada, moving across a stage to create choreographies for a show. Piero and Giada's movements are already pre-established, while students have to guide Isabella's movements through the creation of a program by dragging and encapsulating some available instruction blocks (see Figure 1).

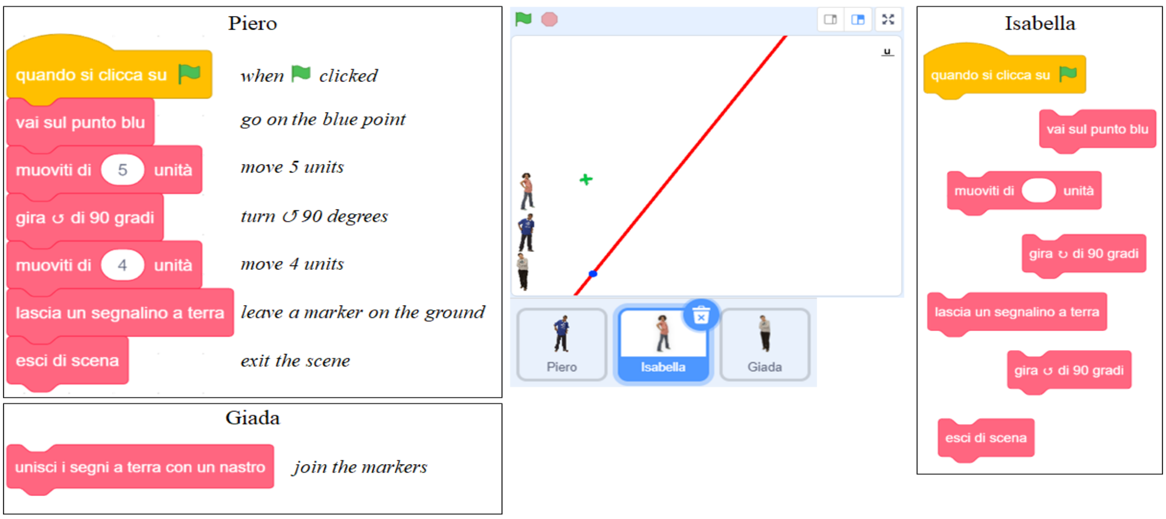


Figure 1: The Scratch visual programming artefact

The activity begins by clicking on the flag and executing Piero's program. Piero moves and performs a sequence of steps, leaving some markers on the stage (the markers represent the points to be symmetrized). Through the digital artefact, students have to provide instructions to Isabella, so that she can perform a choreography symmetrical of Piero's one with respect to a line crossing the stage transversely. By clicking on the flag again, students simultaneously display Piero and

Isabella's movements on the stage. In this phase, they can carry out a first visual check on the correctness of the program she created for Isabella, by observing if the simultaneous Piero and Isabella's movements create symmetrical choreographies. Finally, students execute Giada's instruction "join the markers". So, the artefact returns in feedback an image obtained by joining the markers left on the stage by Piero and Isabella with a broken line. In this way, students can verify if the geometric figure is symmetrical or not with respect to the line (see Figure 2). Therefore, they can check (in two different moments and in two different ways, that is by looking at the characters' movements or by visualizing the figure obtained by joining the markers) if the instructions given to Isabella's character are correct or not. If students are not satisfied with the program created for Isabella, they can make changes at any time.

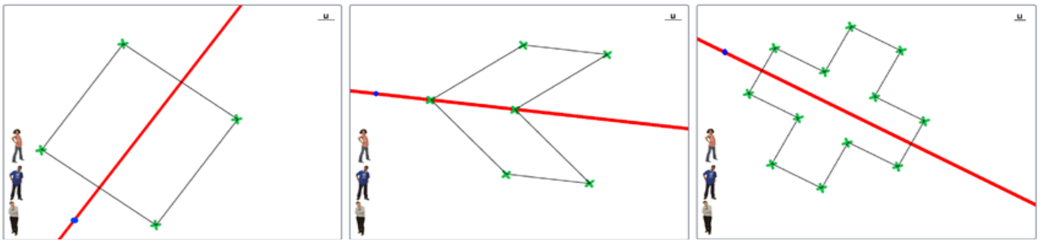


Figure 2: Examples of figures to be obtained on the stage at the end of the activity

The instruction blocks were created in the Scratch environment specifically for this activity by collecting different standard instruction blocks, in order to relieve students of the aspects related to programming and to not significantly affect their cognitive load. The text in the instruction blocks was designed to bring out mathematical meanings related to the concept of axial symmetry.

Methodology

The a-didactic activity, involving the use of the digital artefact described above, was experimented with 21 students of a fifth-grade class of the primary school of the "Istituto Comprensivo San Giovanni Bosco", near Benevento (Italy). They had already carried out classroom activities with their teacher on axial symmetry, but only with vertical/horizontal lines, and they had never handled activities related to computer programming.

The experiment took place in the classroom, during 3 curricular lessons of 90 minutes each, in the presence of a researcher and the mathematics teacher of the class. It was conducted during the Covid-19 pandemic and the students respected the rules of distancing and the use of masks. In the first lesson, the students became familiar with the digital artefact. The researcher presented the characteristics of the Scratch environment by means of a video projection and she invited the students, divided in groups, to manipulate Scratch on the class computer. In the next two lessons, the students were divided into pairs based on their closeness in the classroom (only one group consisted of 3 members) and each student worked on her laptop.

Each lesson consisted of two main moments: an a-didactic moment and a moment of collective discussion, at the end of which the researcher carried out the institutionalization. During the a-didactic moments the students acted on the digital artefact and, for each choreography, they carried out the following task: "Create a Scratch program for Isabella to replicate Piero's movements on the other side of the red line. Then ask Giada to join the markers”. Students communicated with each other to verbally validate their strategies.

Afterwards, the researcher delivered to each group a paper sheet with the screenshot of the stage on the PC as it appeared immediately after the execution of Piero’s program, that is, with the markers left on the ground by Piero. The students, communicating with each other to agree on shared answers, first carried out the following task: “Draw the marker(s) left by Isabella on the other side of the line. Then join the markers”. Then, they answered the following open questions: “What do you observe looking at the figure?”, “What do you observe looking at the programs of Piero and Isabella?”. In answering the questions, they could visualize the work done with the digital artefact.

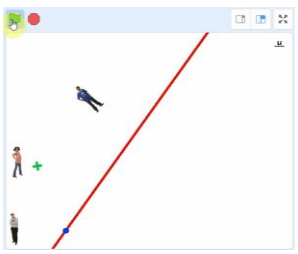
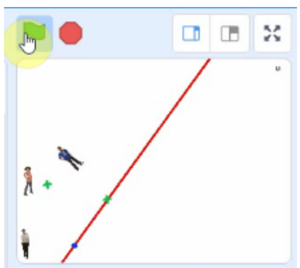
We collected the following data: the video recordings of the screens of the PC used by the students; the paper sheets relating to each choreography filled in by students after the activity with the digital artefact; for each group, audio recordings of the whole activity with the digital artefact, as well as of the collaborative work moments related to the filling in of paper sheets; the audio recordings of the collective discussions guided by the researcher; the notes collected in class by the researcher and the teacher. To analyze video/audio data, we identified critical events, and transcribed and coded them to construct the storyline (Powell et al., 2003).

Preliminary findings

In this section, we show the preliminary findings of a qualitative analysis aimed to point out the students’ programming strategies during the learning activity. In the analysis, we took into account explicit references to the digital artefact - or its characteristics - appeared when the students worked with it, or in students’ oral and written productions. Out of respect for ethical requirements, in the following analysis students’ names are fictional.

The first programming strategy we observed is as follows: the students first visualized the execution of Piero’s program, then they created a program for Isabella and, finally, they simultaneously performed both Isabella’s and Piero’s programs. Only once the students thought they had obtained the right programs, they clicked on the flag in order to visualize their execution. This strategy was employed by most students in the initial phase of the activity with the artefact. Table 2 shows an example of application of this strategy realized by Sabrina.

Table 1: The first programming strategy used by Sabrina

			
min. 2.02: Sabrina clicks on the flag and she	*min. 6.06*: She clicks on Isabella's icon.	*min. 10.33*: She creates Isabella’s program.	*min. 11.11:* She launches both Piero and Isabella’s

visualizes the execution of Piero's program.			programs.

Sabrina visualized the execution of Piero's program only once (see Table 1, min. 2.02) before constructing Isabella's program (from min. 6.06 on). Later, after completing Isabella's program, she ran both programs simultaneously (see minute 11.11). Nevertheless, after checking the execution of Isabella's program, she understood that the program she created did not satisfy the required task (Isabella's movements on the stage did not correspond to Piero's ones on the other side of the line). At time 11.43, her groupmate Arianna, who had already completed the task, intervened in order to help Sabrina in programming, and, while observing Sabrina's screen, she suggested a new strategy:

1	Arianna:	Let's compare it with Piero, I copied from Piero, but… just I changed turn in that way, because she has to turn here, do you see? So, go on Piero and look that these do not have to be here *(she refers to some instruction blocks of Isabella's program)*... So, go on Piero… [...]
2	Sabrina:	*(Sabrina clicks on Piero's icon)*
3	Arianna:	Well, do not touch Piero's program… *(Arianna clicks on the flag and she performs Piero's program)* This is Piero's program… Now you have to do the same thing with Isabella… Here it is *(Arianna scrolls the code area until she finds Piero's program)* Here it is… I copied everything from here, just I changed...
4	Sabrina:	But so did I...
5	Arianna:	Here it is, now go on Isabella…

The strategy suggested and clearly explained by Arianna was the following: to "copy" the instructions included in Piero's program, by dragging them, in the same order, in the code area of Isabella, reversing the direction of rotations ('I changed turn in that way, because she has to turn here', see line 1). After Arianna's explanation, Sabrina seemed to focus on the aspect related to the reversing of the direction of rotations (line 4), by saying that she considered this aspect during the construction of the program, too. Afterwards, still interacting with Arianna, Sabrina edited Isabella's program, by using the strategy suggested by her groupmate.
Table 2 shows an excerpt of application of this second strategy.

Table 2: The programming strategy used by Sabrina discussing with Arianna

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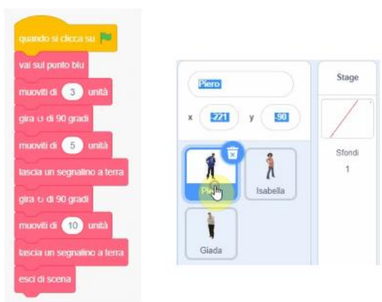
<tr>
<td>
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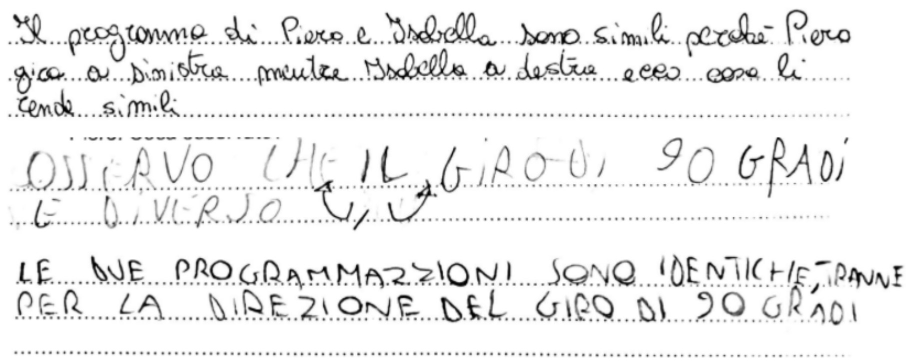
<tr>
<td>min. 13.03: Sabrina clicks on Piero's icon and she visualizes Piero's program.</td>
<td>min. 13.24: Sabrina, interacting with Arianna, modifies the number of steps in Isabella's program.</td>
<td>min. 14.09: Still interacting with Arianna and visualizing Piero's program, Sabrina correctly builds Isabella's program.</td>
</tr>
</table>

In this excerpt we see that Sabrina switches from viewing Piero’s program to correcting/constructing Isabella’s program. This passage from one character to another is repeated several times until Piero and Isabella’s programs appear “symmetrical”. Later in the activity, all the students in the class adopted this programming strategy, considered more effective.

In applying the first programming strategy (see Table 1) students focused on the visualization of Piero’s movements on the stage during the execution of the program. The absence of a grid made this strategy expensive, as the students had to establish the number of characters’ steps without a visual reference. On the other hand, the second strategy (see Table 2) was based on the direct visualization of Piero’s program, regardless of its execution.

As a result of using this second strategy, explicit references to the Scratch programs created by the students appeared in several students’ answers in the paper sheets delivered to them after working with the digital artefact. In particular, some students explicitly referred to some sentences displayed on the instruction blocks, such as “turn ... 90 degrees”, also reporting in written form the symbols of the arrows (see Figure 3). Furthermore, from some written productions relating to the question “Look at the program you created for Isabella and compare it with Piero’s program. What do you observe?”, it emerged that the students recognized that there was a sort of symmetry between the two programs.


Il programma di Piero e Isabella sono simili perché Piero gira a sinistra mentre Isabella a destra ecco cosa li rende simili

OSSERVO CHE IL GIRO DI 90 GRADI E DIVERSO ↺, ↻

LE DUE PROGRAMMAZZIONI SONO IDENTICHE, TRANNE PER LA DIREZIONE DEL GIRO DI 90 GRADI

Miriam: Piero and Isabella’s programs are similar because Piero turns to the left, while Isabella turns to the right

Valerio: I observe that the 90 degrees turning is different

Enrico: The two programmings are identical except for the direction of the 90 degrees turning

Figure 3: Some students’ written productions

As Figure 3 shows, the students spoke of “similarity” (e.g. Miriam) or even of “identicalness” (e.g. Enrico) between the programs of the two characters, except for the direction of rotations. Such observations became more and more frequent in the written and oral productions of students.

The use of the second strategy could indicate a purely reproductive (and not productive) students’ attitude. However, the reflection on aspects related to computer programming has allowed them to identify new strategies, better than the previous ones and more effective. In fact, the students, in using the first strategy, were not able to produce “symmetrical images”. It was precisely the failure of the strategy that led them to look for a new one, which allowed them not only to produce “symmetrical images” but also to identify aspects related to the definition of axial symmetry, such as the equidistance of corresponding points from the axis. This emerged, for example, during a moment of collective discussion, when a student, Francesco, referred to the movements of Piero and Isabella using the expression “the same distance run on one side and on the other side”.

Discussion and conclusions

This research aims to contribute to study how algorithmics and visual programming can be integrated into teaching practices as tools for learning mathematical concepts. Specifically, we designed an a-didactic activity based on the use of a digital artefact, realized in Scratch visual programming environment, for the learning of axial symmetry in the primary school. The task requires that three characters, Piero, Isabella and Giada, perform some choreographies in a show. The students visualize the predetermined Piero's movements, and they have to create a program for Isabella, by manipulating some suitable instruction blocks, so that Isabella realizes a choreography "symmetrical" to Piero's one with respect to an oblique line. The visualization of the execution of the predetermined Giada's program returns figures that students can verify as being symmetrical or not w.r.t. the line.

We experimented this activity with students of the fifth grade of primary school, who never handled classroom activities related to computer programming. The aim was to understand how the a-didactic activity, involving algorithmics and visual programming, influenced the rise of new solving strategies by students, linked to the construction of symmetrical images with respect to an axis. The qualitative analysis took into account the programming strategies adopted by students and the references to the algorithmic dimension in their oral and written productions. The preliminary findings showed the emergence of two main programming strategies. Firstly, most students created Isabella's program only by visualizing Piero's movements, and not the computer program which generated those movements. Such a strategy soon proved to be expensive and ineffective, in the sense that only few students were able to perform programs responding to the task. This encouraged the rise of a new strategy, also due to the collaboration among the students. The new strategy consisted of replying Piero's program for Isabella, by paying attention to reversing the directions of the rotations. Indeed, Piero and Isabella's programs had to allow their respective characters to move symmetrically w.r.t. the line. So, the directions of rotations had to be the one the reverse of the other. The emergence of this second strategy was linked to the identification of a sort of "symmetry" between the two programs. The new strategy was more effective than the previous one, both from an informatic point of view (i.e. students were able to realize "correct" programs) and, most importantly, from a didactic point of view (it allowed students to figure out some aspects linked to the axial symmetry, as, for instance, the equidistance of corresponding points from the axis). Therefore, visual programming could be a valuable learning tool, able to mediate mathematical meanings. Moreover, it could enable students to devise (new and more effective) solving strategies. In the future, we plan to test the a-didactic activity also with lower and higher secondary school students.

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