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# Logical inferences in the view of first-year students

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To control for logical ability in a broader study, the study presented here investigates the understanding of inferences. The tasks consist of a conditional sentence in A, the negated or not negated antecedent (the if-clause) or consequent (main clause) of this conditional in B, and a possible inference from A and B as in the following example: (A) *If the flowers are blue, the car is rolling.* (B) *The car is rolling. What do A and B imply mathematically-logically, if only the propositions A and B are known and nothing else? Cross the answer with which you agree most. □ Given only the propositions A and B, mathematical logic implies the flowers are blue. □ ...are not blue. □ One cannot infer with mathematical logic whether the flowers are blue or not.*

## Theoretical background

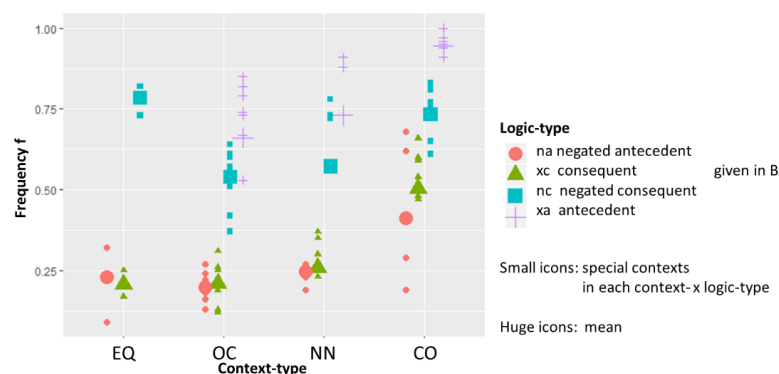
The analysis of conditionals is immensely broad (von Fintel 2011), being spread out into many disciplines – philosophy, linguistics, logic, and psychology. Perhaps, the root of the matter is the following: With the word “if” one or more possible (or even counterfactual) worlds are possibly entered depending on the context. Therefore, some points have to be considered in the construction of the items: (1) *Plausibility* (Pólya 1990, Vol. 2, Ch. XII & XIII) of antecedent and consequent, the link by the two in its (2) power of (*explanatory*) *necessity* (Bartelborth, 2007; Müller-Hill, 2017), (3) the formulation of the *question* ( $v_4$  in Durand-Guerrier 2003, p. 18; reasoning *from* or *about* a rule in Wason, 1968; Evans, 1997), (4) the *context* (e.g. deontic or not - Evans, 1997; Valiña & Martín, 2016), (5) implicit *quantification* (propositional, open or bounded sentences -  $v_7$  in Durand-Guerrier, 2003, p. 18), (6) reformulations without conditionals in predicative logic or as *restrictions* (Bartelborth, 2007), (7) *material or logical* implication in A.

## Present study

As a consequence of these seven issues, the items are constructed as follows: Plausibility of the antecedent and consequent (1) have to be unknown, the plausibility of the conclusion (1,2) is systematically varied by context-types, three choices for the answer are given (see example above; 3), only reasoning *from* a rule (3) in A, given as material implication (7) by abstract sentences in the present indicative (4), no quantifiers and avoiding the interpretation as open sentences in order to prevent implicit quantification (5, 6). Due to pilot studies, we did not use any negations in the conditional sentence and in a first step only contexts of every-day-life instead of mathematical ones (4, cf. Durand-Guerrier 2003, 18,  $v_1$ ; also to avoid differences in content knowledge). We used four cases of plausibility in A (*context-type*): CO (cogent) - The direction of the conditional is cogent, the logic converse is not. *If the brakes are broken, the car is out of order.* OC (not cogent) - The direction of the conditional is not cogent, whereas the converse is. Specifically, the converses of the CO-Items are used. *If the car is out of order, the brakes are broken.* EQ (equivalence) - Both directions of the conditional are cogent because it is a material equivalence. *If you are a pupil, you go to school.* NN (neutral) - No available truth value, no probability, arbitrary conditional. *If the train is coming,*

*the flower is yellow*. These cases are each combined with the following four logic types, realized by B: xa antecedent given, na negated antecedent, xc consequent, nc negated consequent. The convenience sample is comprised of N=593 students at the beginning of their university studies. Each combination of logic- and context-type used 2-8 contexts. Due to the multi-matrix booklet designs, a Rasch-model (N=593, infit $\leq$ 1.04, outfit $\leq$ 1.07, item-total-cor. $\geq$ .19, EAP-rel=.36) was applied.

## Results



Even if the contexts are extremely specified (see 1-7 above) and the logic-types and negation are controlled, the context has still a great impact on the variance of frequencies. The most abundant answer in each type coincide with those given if material implication in A had been mistaken as an equivalence - in contrast to the findings and interpretation of Durand-Guerrier (2003, p. 22).

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