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Silke Friedrich[1] and Elisabeth Rathgeb-Schnierer[2]

[1]University of Kassel, Germany; sfriedrich@mathematik.uni-kassel.de

[2]University of Kassel, Germany; rathgeb-schnierer@mathematik.uni-kassel.de

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Introduction

This study explores one approach of dealing with heterogeneous learning groups in mathematics instruction. In elementary school the diversity of students is a key challenge. Since elementary schools draw their student body from residential areas, a wide spectrum of ability emerges. In order to cope with heterogeneous learning groups, various approaches have been developed within mathematics education. In the German context, these approaches base on the concept of differentiation through open-ended tasks that allow multiple solutions on different achievement levels. This concept is specifically termed as 'natural differentiation' (literally translated) (Scherer, 2013). This concept assumes that students work on differentiating open-ended tasks according to their learning and performance level. Whether this assumption is also confirmed in practice has not yet been systematically empirically tested. This is the aim of the presented research project. The study will compare the individual learning potential of students and the performance level when working on an open-ended task that allows multiple solution (naturally differentiating tasks). We intend to analyze whether third grade students have worked on the open-ended task according to their learning potential.

Theoretical framework

The theoretical framework is the offer/take-up model (Angebots-Nutzungs-Modell) (Göbel & Helmke, 2010) developed by Helmke (2012) that describes the complex processes and influencing factors of teaching and learning in a simplified way. To explain the effects of teaching and learning success, the model gives a compact overview of the most important variable clusters.

In this project we offer students an open-ended task (see below). Open tasks allow multiple solutions at different levels and thus provide learning opportunities for students with different levels of proficiency (Häsel-Weide & Nührenbörger, 2021). Within those open-ended tasks students are encouraged to develop different ideas and solutions. They also have the option to talk about their different ideas and solutions. The use of open-ended tasks which allow multiple solution on different achievement represents an innovative approach of teaching and learning arithmetic in heterogeneous classes and inclusive education (Lindenskov & Lindhardt, 2020; Kosyvas, 2016). According to this concept, openness of solutions and working on different levels of difficulty is required.

Aim of the study

The study aims to determine to what extent the use of an open-ended task depends on prior knowledge and other variables of learning potential. Moreover, the effects of cognitive and personal learning and

performance-related characteristics on the usage of open-ended tasks will be investigated. In particular, the main question is whether the potential of the open-ended task "Kombi-Gleichungen" (invention of equations with multiple operations) is utilized by students according to their learning potential.

Methodology

The sample includes 100 third grades from different heterogeneous classrooms. Firstly, the learning potential of the students will be assessed by different instruments. We use a questionnaire to analyze students' attitudes towards learning mathematics (based on TIMSS 2007 (Bos, Bonsen, Kummer, Lintorf, & Frey, 2009); TIMSS 2015 (Wendt, et al., 2016); IGLU 2006 (Bos, et al., 2010); EDUCARE 2016 (de Moll, Bischoff, Lipinska, Pardo-Puhlmann, & Betz, 2016)). The achievement level in mathematics is measured by a standardized test (Keks Mathematik 3 (Hildenbrand, May, & Ricken, 2018)). The chosen instruments allow to survey cognitive learning prerequisites as well as motivational and volitional learning prerequisites of the students.

Secondly, we conduct two math lessons and offer all students the open-ended task to invent equations in which the equal sign is relationally used (e.g., $2 + 7 – 3 = 10 – 4$) (Harbour, Karp, & Lingo, 2016). It was developed according to the concept of 'natural differentiation'. With our open-ended task we intended a deeper understanding of arithmetic and algebraic relationships (Baireuther & Kucharz, 2007). Within these two lessons we encourage students to invent and systematically modify equations. Thereby, the students are supposed to use the equal sign in a relational sense. The open-ended task allows multiple solutions and enable students to work at their individual performance level. Students are invited to record their work by paper and pencil. Their worksheets provide the database for our analysis.

The investigation focuses on the level of students' solution process. To assess this level, we use a category system to classify the solution processes recorded on the worksheets (Friedrich & Rathgeb-Schnierer, 2020). Finally, we link level of the solution process to the learning prerequisites of the students.

Preliminary outcomes

The first results from the pilot study already show some very interesting correlations. These can be seen in the comparison of students' processing level and the values of the competence test.

After the analysis of the available documents, it appears the highest processing level is characterized by invention of an equations with many different arithmetic signs, multi-digit numbers, many calculation steps and transitions with a systematic procedure. It was possible to identify groups that showed similar ways of processing.

Inventing multiple systems with complex equations is a characteristic of a high level of processing. While high-performing students (as measured by the scores on the achievement test) attempted to form particularly complicated and long equations, the lower-performing students were also able to confidently handle the open-ended task. They were able to invent many equations, but rather those with few calculation steps and mostly with only one arithmetic sign.

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