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From classroom data consisting of transcripts from 20 videotaped lessons, we studied five Norwegian mathematics teachers' use of the word why. *It is our initial assumption that asking why in mathematics is a means to support students' mathematical reasoning. We found evidence for this assumption in our data. However, the analysis of the 114 occurrences of* why *indicated that the most common purpose for asking why was to make students re-state thinking they had already done, thus not prompting new mathematical reasoning. Moreover, when teachers used* why *in questions not about mathematics, they usually did so in a rhetorical and reproachful manner. We discuss these ambiguities and implications for teaching.*

Keywords: Mathematical reasoning, questioning, mathematical discourse, interactions.

Introduction

In the mathematics classroom, teachers' questioning is acknowledged as crucial in supporting students' learning (see e.g., Franke et al., 2009). Martino and Maher (1999) claim that there is "a strong relationship between (1) careful monitoring of students' constructions leading to a problem solution, and (2) the posing of a timely question which can challenge learners to advance their understanding" (p. 53). Recent studies have pointed out that *asking why* is a type of question that particularly holds the potential to encourage students to explain or reason (Ingram et al., 2016; Drageset, 2021). In those studies, the researchers first identified students' explanations in classroom data, and then considered what the teacher did to prompt the explanations. In contrast, we are taking as a starting point all teacher-initiations given in the form of a why-question, and we aim to categorize the possible implication of the question in the given situation, not unlike Enright et al.'s (2016) typology of questions according to the function they serve in instruction. We are particularly interested in the potential why-questions hold to challenge students to advance their reasoning, reflecting Martino and Maher's (1999) "timely questions" mentioned above.

In our study, we take a commognitive view of mathematics teaching and learning (Sfard, 2008). Within this tradition, learning is seen as increased participation in a discourse, and discursive behavior is characterized in terms of routines (Lavie et al., 2018). Teachers have the role of expert participants in the classroom discourse, and the theory of commognition states that imitation of expert participants is a central aspect of the learning process (Sfard, 2008). Martino and Maher (1999) point out that students do not naturally seek to build a justification or proof of the validity of a solution. Thus, routines related to reasoning and proving in mathematics must be made visible by the teacher. Therefore, teachers' use of why-questions influences students' routinization (i.e., individualization of routines in the community), regarding when it is a timely question to ask, and what kind of actions such a question requires.

Thus, to get the whole picture of how teachers' why-questions relate to students' mathematical work, we need to extend our knowledge beyond the why-questions identified by Drageset (2021) and Ingram (2016) that actually led to students' conceptual explanations. Other uses of why-questions will also affect how students routinize asking why in mathematical discourse. To this end, we employ the following research question: *How is the word "why" used by five middle-grade teachers in mathematical classroom discourse?* The data for the study consists of transcripts of 20 lessons given by the five teachers. As all occurrences of the word "why" in the data material are part of a question (this includes a few instances of implicit questions), we will use "why" and "why-questions" synonymously in this paper.

The routines of asking why

Within the commognitive framework, *routines* are important features of discourse in a community (Sfard, 2008). Routines are discursive patterns which are evoked whenever a participant finds herself in a situation similar to one she has experienced before, and acts thereafter (Nachlieli & Tabach, 2018). Routines consist of an initiation, a procedure, and a closure (Nachlieli & Tabach, 2018). The initiation is the clue that is recognized and thus prompts the participant to perform the expected actions, and the closure is "the conditions under which a procedure is considered complete" (Nachlieli & Tabach 2018, p. 3), together they form the "when" of a routine. The procedure is the action taken in between the initiation and the closure, also referred to as the "how" of the routine. Lavie et.al (2018) consider two types of discursive routines in mathematical discourse: Explorations and rituals. Explorations are outcome-oriented routines, where the aim is to produce or endorse new narratives (i.e., statements about mathematical objects and their relations). They are motivated by the question "What do I want to achieve?" and are often performed by the more experienced participants of the discourse. In contrast, rituals are process-oriented routines, guided by the question "How do I proceed?". Rituals are often performed by novices in the discourse, typically by imitating the teacher's actions, and the motivation is social acceptance. Routines can be nested – a new routine can be initiated, carried out and closed as a sub-routine of another routine (Nachlieli & Tabach, 2018).

As previously mentioned, we are particularly interested in routines that are connected to students' mathematical reasoning (MR). Therefore, we need to elaborate on what we mean by MR. We employ Jeanotte and Kieran's (2017) conceptual model of MR for school mathematics, which is constructed in coherence with the commognitive framework. The model consists of two interrelated aspects: The *structural aspect* addresses the underlying construction of an argument (e.g., deductive, inductive, abductive), while the *process aspect* capture activities related to searching for similarities and differences, validating, and exemplifying. A search for similarities and differences is aimed at producing new narratives, a validation is aimed at establishing the correctness (or wrongness) of narratives, while an exemplification works as a support for the first two types of MR processes. We note that all the MR processes – when considered as overall routines performed by students – are exploratory routines, because they seek to produce or endorse narratives. We thus use the term *MR exploration* for an exploratory routine involving one or more MR processes.

Both Drageset (2021) and Ingram et al. (2016) found that why-questions were connected to student explanations of reasoning. However, in their studies, it is not possible to separate students'

explorations and rituals, although it is likely that many of the explanations of reasoning were connected to an exploratory routine. Nor is it possible to see whether the teacher's question prompted the student to share his reasoning, or whether it invited the student to start to reason. This distinction between starting and explaining routines is also not evident in the typology of questions suggested by Enright et al. (2016). Our study aims to give more insights into these matters.

Methods

The study presented in this paper is part of an ongoing, large-scale study with a focus on reasoning and proving in primary education (ProPrimEd). As a first part of the ProPrimEd study, we collected classroom data (videotaped lessons, and interviews with some students and all teachers who participated in the lessons) to describe the "state of the art" of work with reasoning and proving in ordinary mathematics education. The data for the study reported here consist of the transcripts of the videotaped lessons. In total, we observed five teachers (T1, fifth grade; T2, sixth grade; T4, fourth grade; T5, seventh grade; T6, sixth grade) for two weeks each, a total number of 20 lessons of various lengths. All the five teachers were male. T1 and T2 worked at the same school, so did T4, T5 and T6. All were educated as primary school teachers with mathematics as one of their subjects. Some had additional professional development courses in mathematics. Their teaching experience varied between 2 and 17 years, with an average of 9 years. The researchers took the role of non-participating observants during the data collection and played no part in lesson planning. Topics of lessons were whole-number multiplication, addition and subtraction with rational numbers, problem solving involving geometrical shapes and multiplication, prime numbers, and operator precedence. Most classes consisted of a variety of activities. Although the data is limited to two schools and five teachers, we consider it to represent ordinary mathematics discourse in Norwegian classrooms.

The transcript documents consist of a total number of 143 448 words (including meta-data). From this material, we identified (using the search functionality in Microsoft Word) every occurrence of the word *why* used by a teacher in whole-class discussion and in discussion with (groups of) students. We found 114 occurrences.[1] For each teacher, we made a new document of cut-outs from the original transcript, consisting of a separate text block for each occurrence of *why* – including enough text before and after the word to make the context clear, so that we would be able to interpret the possible implication of the *why*. Sometimes, during the coding process, we would go back to the complete transcripts to review more of the context, and we also had the opportunity to consult the video recordings. We did not seek to identify differences between the teachers, and therefore treated all why-questions as a pool.

We were inspired by other studies on the use of certain words in mathematical discourse, such as Monaghan (1999) and Wagner and Herbel-Eisenmann (2008), who studied the words *diagonal* and *just*, respectively. These studies pointed to the influence of single words in mathematical discourse: Monaghan showed how an "informal" meaning of *diagonal* could possibly weaken the mathematical content at stake, while Wagner and Herbel-Eisenmann showed how a seemingly innocent word like

[1] The lessons, and thus the transcripts, were in Norwegian. We searched for the Norwegian word "hvorfor", which holds the same meaning as the English word "why". The data excerpts used in this paper are translated to English by the authors.

just could open or close discourse. In our study, we were guided by the commognitive concept of routines when analyzing and discussing data. The two authors read some of the material together and agreed on a temporary set of codes. These codes were:

- *initiating*, later renamed *routine-initiating*: *Why*'s that prompt new reasoning (based on the underlying assumption of the study)
- *recalling*, later renamed *routine-re-initiating*: *Why*'s that prompt students to share (aspects of) mathematical thinking they have already done
- *non-mathematical*: *Why*'s used in contexts not about mathematics.

Each author then coded the material separately. When discussing the coding afterwards, we found that we agreed for most occurrences and came to agreement on the few we had coded differently. We added a category *Other* for why's occurring within a mathematical context, but not fitting into the first two categories. We started the process of analysis with an open coding that was as close as possible to the data. As we interpreted our interim findings, we employed Sfard's and Jeannotte and Kieran's theories to further nuance the content of each category, and the categories were appropriately renamed. In the next section, we elaborate on some typical uses of *why* from each category.

Findings

Table 1 shows the number of occurrences of the word *why* in each of the categories routine-initiating, routine-re-initiating and non-mathematical (and other).

Table 1: Number of occurrences of the word *why*

routine-initiating	routine-re-initiating	non-mathematical	other
34	65	7	8

We note that the most common use of *why* is to recall mathematical thinking (routine-re-initiating). In the following, we provide examples from the four categories. The examples are chosen to indicate both "typical" usage of *why* and to demonstrate the variation within a category.

Teachers' use of routine-initiating why-questions

Routine-initiating why-questions invite the students to choose and take action. Almost all instances of routine-initiating why-questions in our data material called for an MR exploration. The data material provides examples that why-questions were used both to prompt students to search for similarities and differences, thereby producing a narrative, and to validate a conjecture. An example of the former is given below. Two students are working on a jigsaw puzzle of the multiplication table (the pieces in the puzzle consist of more than one number).

Teacher: What I'm wondering is why does it [a piece] fit so well here?
Student: Because here it says 36 and here it says 32
Teacher: Why do 36 and 32 fit together? And together with 28 and 24? What's going on with these numbers?
Student: Because (…) because. It's 4 between them.

In this example, it seems that the students first placed the piece in the puzzle because of its shape. The teachers' questions made them reason about the mathematical relationships, and eventually identify the pattern in the 4-times table. Next is an example of a why-question that is used to initiate a validation process. The excerpt is from a situation where the students are asked to calculate the surface area of the (glassed-in) walls of different buildings, all having the shape of rectangular prisms, and all having the same circumference of the ground floor and the same height.

Oline:	Like. Around the walls. It's the same for all around the glass? Isn't it the same for all?
Teacher:	Yes, why is it so?
Oline:	It's the same outline, it, it [Norw. outline is omriss]
Teacher:	The same circumference? [Norw. circumference is omkrets]
Oline:	Yes.
Teacher:	Yes. Why is the glass the same when it's the same circumference?
Oline:	I don't know, but it feels that way.

Here, the student claims that there is a relationship between the circumference of the ground floor and the area of the walls but has no ready-made reasoning to support the claim. Hence the two why-questions serve as possible initiators for a validation process of the student's hypothesis.

We conclude this section with two examples of routine-initiating *why*'s taken from a lesson on prime numbers. Common for these *why*'s is that they are not self-contained as routine initiators; they are a part of a more extensive initiation. In the first example, the students are investigating whether all whole numbers between 4 and 20 can be written as a sum of two prime numbers. During the students' group work the teacher reminds them of the task:

Teacher:	You'll get 3 more minutes. Remember that, although I claim, I claim that it's possible for all the numbers from 4 to 20, but it's not certain it's the case though. So it could be that you find some numbers where it doesn't work. But then maybe you have to say something about why it doesn't work. How can we be completely sure that it doesn't work? So we work for 3-4 more minutes (…)

Here, the *why* is a support for the process of validating a hypothesis, which is already going on in many students' work (thus, the *why* initiates a sub-routine nested within the routine initiated earlier, when the task was given). The students are working on a task, and this *why* clearly does not refer to an answer provided by a student. Later, the teacher discusses an answer with some students:

Teacher:	Yes, there you only have to explain why some numbers aren't prime numbers. Such that he said that 8 isn't a prime number, since you can get that by using the factors 2 and 4. You can just write down that computation.

The use of *why* here points to a validation, and the teacher add some information about the possible nature of this explanation. Thus, this *why* is connected to the structural aspects of a reasoning process. In our data material, we found very few examples of such *why*'s.

Teachers' use of routine-re-initiating why-questions

Routine-re-initiating why-questions invite students to recall, share and elaborate on the routine they have already performed. In most cases, this happens during a whole-class discussion after the students have worked on a task. As with the previous category, we find different uses of *why* within this category. In many cases, the why-questions were (mainly) addressing the procedure aspect, or the "how" of the routine performed. This is the case in the following example, taken from a lesson on negative numbers. The teacher is requesting details of a calculation:

Teacher:	Seven minus negative three. What will that be, Anja?
Anja:	Ten
Teacher:	That is ten. Why?
Anja:	Because it is the same as seven plus three.
Teacher:	It is the same as seven plus three. Yes.

In the next example, the teacher's why-question addresses the procedure of regrouping in addition with decimal numbers. Again, this excerpt addresses the "how" of the student's performed routine, but the "when" is also addressed as the teacher probably seeks to highlight conditions for taking this action (when to regroup: if we get ten tenths).

Teacher:	Yes, we get in a way zero tenths at the end. But why do we get zero tenths? What have you done then?
Anniken:	Because we transfer to the ones' place.
Teacher:	Yes. That is right. As soon as we get ten tenths, we have to regroup. That is the way the decimal system works.

In other cases, re-initiating why-questions were used mainly to highlight features related to the "when" of a routine. The next example, taken from a lesson on early multiplication, shows how a why-question seeks to emphasize the connection between the given context and the numerical expression, i.e., in what situations a multiplication procedure is called for.

Teacher:	Six times five. Ok. Adele, why have you written six times five?
Adele:	Because it [the worm] climbs six centimeter each hour.
Teacher:	Mm, it climbs six centimeters each hour. So it does.
Adele:	And it climbs for five hours.

There were also some examples where the teacher asked *why* to make students elaborate on a routine, but where the teacher (most likely) did not know what the student had done (e.g., when the student had provided a wrong answer). Why-questions used in this way have the potential to provide the student with an opportunity to revise his/her thinking, which would be a routine-initiating why. However, in our data we only find examples where the teacher guides the student to the correct answer by asking closed questions (like Drageset's (2014) *closed progress detail*). Thus, the student's possible contributions are limited to yes/no-replies and short answers, with few or no opportunities to engage in mathematical reasoning.

Non-mathematical and other uses of why

The few *why*'s in the *Other* category are either not part of a question to students, or meta-questions which we do not discuss further in this paper. Similarly, the non-mathematical why's are rare in the material, and occur in data from only two of the teachers. Yet, they are interesting because all of them are used in a rhetorical and even reproachful manner, such as:

Teacher: Six. Why don't you raise your hand when you know it?

This example, as well as the remaining six instances of this code, are rhetorical and reproachful because the teacher does not expect an answer and the aim of the questions is to address something unwanted in the student's behavior.

Discussion

We started this paper with an assumption that why-questions can serve as initiators of exploratory routines involving MR processes, in line with Martino and Maher's (1999) "timely question[s] which can challenge learners to advance their understanding" (p. 53). In our data, we found evidence that this indeed happens. Yet, we found that by far most examples of why-questions are routine-re-initiating. They are usually asked after the students have finished their work on a task. Thus, in commognitive terms, we could consider those why-questions initiators of subroutines that are nested within the closure of the outer routine, the task itself ("work on the task is done after the teacher has got all the details"). Still, they have an indisputable potential for developing students' mathematical reasoning: The student gets an opportunity to express her thinking; the (teacher and) other students get access to this reasoning; and the teacher is given the opportunity to emphasize for the whole class what he considers to be important ideas. Moreover, some of the routines expressed by the students may initially have been exploratory, and insights into explorations could model other students' later attempts at explorations.

So, is there a problem with this bias towards routine-re-initiating why-questions? Although earlier studies recognize the potential of why-questions as routine-initiating (Drageset, 2021; Ingram et al., 2016), we must remember that if students should interpret a why-question as a "signal" to start an MR exploration, then they must be exposed to why-questions used in this way. Lavie et al. (2018) write that "[t]he teacher's own mathematical discourse is the model for the learners to follow, and the question is whether its explorative nature gets through to the students" (p. 20). If why-questions are mostly restricted to the recalling of performed work (which may even not have been explorations), or to aspects that are non-mathematical, the connection between asking *why* and the need for MR explorations is not likely to be routinized by the students. Accordingly, we note from further reading of the transcripts that in many of the cases where routine-initiating *why*'s are used, explorations did not consequently occur. We do not elaborate on the reasons for that here – that would be outside the scope of the paper – but we remark that a possible reason could be that students simply does not interpret the question as a "signal" to start an MR exploration. One implication for teaching could be for teachers to be careful of how they use the word *why*. In situations where the aim is not to invite students to reason, but rather to invite them to share their thinking (and in particular when it concerns rituals and concepts rather than explorations), could there be other phrases to choose from? "What does that mean", "how does it work" and so on. But most important is probably that teachers should

stress to use *why* in connection with MR explorations, also when it is the teacher himself who takes the lead in the process.

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