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“Puncto è la cui parte non è, secondo i geometri dicono essel’ e immaginativo;”¹

Piero Della Francesca
De perspectiva pingendi (1576)

“Peindre qu’on ne voit pas”²

M. Proust on the painting by Monet “Bras de Seine près de Giverny”

“là où les âmes vivent avant de naître”³

A. Grossmann to J-M. Combes, about the second Riemann sheet

A jubilee of multifold research

Alex Grossmann left his mark on what is generally referred to as mathematical physics⁴. Those who knew Alex well remember that he did not like this hybrid term. On the contrary, he was able to bridge these disciplines and contribute to both, and to move later on to genomics.

This short and incomplete overview of Alex Grossmann’s work should suffice to convey the spirit of his eclectic thinking.

To summarize, Alex Grossmann’s scientific career can be divided into three phases. He started off in 1960 with quantum mechanics and the important analytical questions of the day. In the mid-nineties, he turned his attention to genetics and biology. Of course, starting in 1983, he contributed to the wavelet theory, of which he is one of the founding fathers. However, the success and uptake of wavelets both at the theoretical level and in its many applications should not let us forget the sheer variety of Alex Grossmann’s earlier scientific contributions. It is striking to note the number of ground-breaking works he made starting in 1960.

Except for his work on the quantum theory of fields, on physical statistics (together with J. Ginibre and D. Ruelle), relativity theory (with R. Coquereaux), and of course wavelets and biology, the majority of Alex Grossmann’s works was about quantum mechanics. One notes that, at a time when modern mathematical physics was being born, the ideas of Alex Grossmann influenced modern physics with a very personal and geometric vision of mathematical analysis.

His founding work on the now-famous wavelets could obscure his earlier works. For the sake of balance, it is necessary to document his earlier work, without trying to be exhaustive.

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¹ According to geometers who say it by imagination, the point is that part which is not;

² Painting that you don’t see

³ Where souls live before being born.

⁴ This text is an extended version of the translation made by Michael Grossmann of my paper *Alex Grossmann, un homme multidisciplinaire*, La Gazette des Mathématiciens **161** 78-81 (2019).

Quantum physics and related fields

The first important works of Alex Grossmann (1961-62) were about the theory of scattering. He published a series of articles with T.T. Wu on the analyticity properties of scattering amplitudes, where through rigorous analysis he came up with innovative results. This initial work has spurred much further research into this area.

A second series of works related rather to pure analysis. Trying to find a natural framework for the quantum theory of scattering, Alex Grossmann started to study a specific family of Hilbert spaces that are invariant under Fourier transform, and that may contain rapidly increasing functions, therefore non-tempered distribution. The prospect for providing a mathematically correct description of very singular operators guided his research for several years. While seeking spectral properties that are invariant between Hilbert spaces, Alex Grossmann came to introduce a new class of functional spaces called Nested Hilbert Spaces, which are sort of interweaved Hilbert Spaces. This construct, which was developed in 1966-1967, was meant to produce a satisfactory theory of resonances. This theory was generalized and somewhat simplified some ten years later in collaboration with J.P. Antoine, with the introduction of partial internal product spaces. This theory in turn led several years later to the method of complex dilations of J.M. Combes (with J. Aguilar): resonances are the eigenvalues of the “complexified” Schrödinger operator. A 1970 article together with G. Loupiau and E. Stein presents for the first time a semi-classical theory of the quantification process. This seminal work opened the way for significant research about pseudo-differential operators with the Planck constant, which are now commonly used both in physics and in mathematics. Towards the end of the seventies, Alex Grossmann returned to quantification problems, demonstrating the important role played by the parity operator (symmetry around the origin), moving in the phase-space, as an elementary bloc in the Weyl quantification process (a Wigner-Weyl Group). Numerous papers followed this idea, by, among others, P. Huguenin, H. Bacry, J. Zak and J. Reigner. It culminated with the doctoral thesis of I. Daubechies (1980). Much later, the Wigner transform became a standard tool in (mathematical) semiclassical analysis, and more recently, in kinetic equations theory. Alex Grossmann’s algebraic vision remains a source of constant inspiration in this area, which brings us to our next topic.

The linkage between analysis and algebra becomes nowadays increasingly present in theoretical and mathematical physics: the works of Jean Bellissard, inspired by those of A. Connes, can attest to this. It is important to mention, as J. Bellissard himself noted, that Alex Grossmann was the first one to exhibit a physical situation (solid-state physics with magnetic field) associated to a von Neumann algebra of Type II and not of usual the nType I. This ground-breaking work dates back to 1972, long before noncommutative geometry.

In parallel with these developments in mathematics, Alex Grossmann’s interest and work covered many areas of physics. Beyond quantum mechanics on phase-space, which we already mentioned, his work on solid-state physics deserves special mention, and in particular his mathematical formulation of the concept of point interaction. His bibliography clearly shows the evolution of his scientific thought that often brought him back to solid state questions (Bloch Hamiltonians, Kronig-Penney models, Fermi pseudo-potentials), such as the study of the spectral properties of concrete Hamiltonians: the one-dimensional point interaction model of Kronig-Penney was generalized into higher dimension, including Fermi pseudo-potentials, to which Alex Grossmann came back in the eighties with T.T. Wu. These, however had an unsatisfactory mathematical definition. In dimensions greater than one, the Dirac mass is too great a perturbation for the Laplacian, and it destroys its self-adjoint property. Non-standard analysis (a mathematical theory of the infinitely small) does provide a rigorous definition, but it is somewhat exotic and is difficult to handle. In an article with R. Høegh-Krohn and M. Mebkhout, the problem is somewhat demystified thanks to a functional analysis definition: one had to work on the resolvent.

On must also mention the quantization of canonical transforms (real or complex), the “leap-frogs”, the interaction between fields and atoms (with A. Tip), the theory of antennae (with T.T. Wu)- the list is long.

Wavelets

The second part of Alex Grossmann’s scientific careers is vowed into the history of wavelets. Having worked for a long time with the Weyl-Heisenberg group, Alex Grossmann was the go-to person for a geophysicist who had been wondering for years about the problem of sampling seismic signals. Such signals contain markers for each one of the geological layers. The problem was that they did not have a characteristic scale, ruining the efficiency of time-frequency methods associated to window of observation having a size fixed in advance. By proposing a time-scale analysis based on the basis of constant form wavelets, J. Morlet brought a potentially interesting solution, as was then supported by numerical tests. Alex Grossmann understood that this approach was equivalent to a decomposition into generalized coherent states, and that one had to give up the Weyl-Heisenberg group to use instead the “ $ax+b$ ” affine group. Translations are thus to be replaced by dilations, which are better suited to render the small-scale details of seismic signals.

We see here an epistemological very important aspect of the conceptual nature of Alex Grossmann’s thinking: it is not the coherent states of quantum mechanics which happened to be useful in signal theory, but, in contrast, it is them which allowed Alex Grossmann to perform the decisive step towards a deep theoretical, almost abstract, understanding of Jean Morlet’s empirical methods. A decisive step that gave rise to such a rich and *multidisciplinary* development, the wavelets. And a tool named abstraction which was at the

same time very profitable and elegant. For example, Alex adored explaining the inverse formula of the wavelet transform by saying that one just had to take the complex conjugate of the integral kernel of the transform itself and permute the variables: “the transform is unitary (on its range), its inverse is equal to its adjoint!”. Alex liked to praise the added value when a complicated calculation is replaced by a simple abstract argument⁵.

By building in the early eighties a rigorous framework for this time-scale decomposition, Alex Grossmann and Jean Morlet laid the foundations for continuous wavelet transforms, at the time known by their French name “ondelettes”. This theory was developed by Alex Grossmann, colleagues and students. In collaboration with I. Daubechies and Y. Meyer, he established the discrete twin of the wavelet decomposition: a signal can be described that way using a countable family of functions, all translated and dilated from an elementary function. Later came the fundamental discovery of orthogonal wavelets by Y. Meyer. The notion of multi-scale analysis introduced by S. Mallat allowed formalizing the building of basis functions. This led to the development of efficient algorithms, which by using compact wavelet basis functions (cf. I. Daubechies, with A. Cohen and J.C. Fauveau in their bi-orthogonal version), brought wavelets into the mainstream as a tool with multiple applications, such as the JPEG2000 compression standard.

Since the original works of Jean Morlet and Alex Grossmann, wavelets analysis has been used in such various domains as analysis and synthesis of signals, the detection of discontinuities, shape recognition problems, the analysis and processing of images, fractal theory, the inverse problem in potential theory (F. Moreau, G. Saracco), the analysis of turbulent phenomena (M. Farge), quantum mechanics (thesis of T. Paul under the direction of Alex Grossmann), complex dilations, Feynman integrals, etc.

Far from simply being a talented inspirer for research, Alex Grossmann would walk personally his ideas down the lane to their application in various fields. In signal analysis, Alex Grossmann did much to develop wavelets into a tool to analyze and synthesize sounds. His fruitful cooperation with R. Kronland-Martinet at the Mechanics and Acoustics Laboratory of Marseilles brought wavelets into the mainstream. It is noteworthy that his collaboration with M. Holschneider, R. Kronland-Martinet and J. Morlet on the detection of step changes in sound signals brought about another application of wavelet transforms: the characterization of fractal objects, and the ability to detect singularities in a function. This made wavelet analysis into a sort of mathematical microscope.

It is also thanks to Alex Grossmann that continuous wavelet transforms were generalized into higher dimensions. He suggested this idea to R. Murenzi, then advised him and oversaw his mathematical work to develop a wavelet transform associated with a Euclidian group in d-dimension with dilations (with J.P. Antoine). This in turn opened the way to a whole new field in images analysis (shape recognition, detection of edges and textures).

The combination of the continuous approach with progressive or directional wavelets (i.e. those whose basis of their Fourier transform is contained in a convex cone with a peak at the point of origin) is a Marseilles specialty, as Alex would muse. The use of such wavelets is used to define the concept of an “instant frequency”, which is local in scale also. This has many applications: Magnetic Nuclear Resonance, acoustical retrodiffusion, Milankovitch cycles, gravitational wave detection. The important contribution of the Marseilles group (PMA and CPT) in this area stands as proof of Alex’s acumen and the role he played during his Marseilles years.

Genomics

But there is also an Alex Grossmann from after wavelets.

After certain discussions on the wavelet analysis of DNA sequences, including those with A. Arnéodo, Alex Grossmann resolutely turned his attention to genetics and biology in the third part of his career, the witness of an exceptional dynamism and knowledge-based curiosity. In this context, he first used and then developed methods from data analysis, stochastic modelling, and then combinatorics, and theoretical computer science.

By studying correlations between phenotype data, he modelled together with A.S. Carpentier, A. Hénaut and B. Torrèsani the way in which each gene infers the likelihood that a chromosome will be compacted.

In an article with A. Hénaut, C. Devauchelle et B. Torrèsani, the evolution of genetic sequences (e.g. in proteins) was modeled as branching Markov chains. The nodes of these chains reveal divergences with respect to evolution by point mutations. This model reveals that, in many cases, evolutionary dynamics can be represented starting from a single universal stochastic matrix, and the divergence times can be modelled by a simple linear regression. In other words, one can reconstruct evolutionary history from simple comparisons of genetic sequences. Representing the evolutionary process as a tree was not new, but confrontation with data was definitely an original approach, very interesting in its simplicity. Alex was fascinated by the possibility of inferring evolution from current genomic sequences.

Alex Grossmann was very interested in the phenomena of growth and reconstruction of phylogenetic trees from genetic sequences using original methods phylogénétiques using combinatory and computation theory (with C. Devauchelle, A. Dress, S. Grünwald, A. Hénaut and J. Weyer-Menkhoff).

His sudden passion, when he was almost seventy, for processing real life data and the computational work this requires led him to write complex algorithms and thousands of lines of code.

⁵ Y. Avron, private communication

How to conclude such a short but mind-boggling overview?

As we can see, there is no trace of wavelets in Alex Grossmann's work in genetics. This properly reflects on his dynamic, cross-discipline thinking : instead of trying to apply his toolbox to new sets of problems, he went boldly into new fields to innovate from scratch.

During the more than fifty years of his career, he was able to combine abstract and concrete thinking, cultural and creative, rigorous and imaginative. The mark he left goes beyond the amount, however great, of material in articles that he published.

For all his students and those who worked under his guidance, speaking of him as a teacher is an understatement. His education started off in a Montessori school in Zagreb in the thirties, where learning was supposed to be driven by curiosity. He did not construct the knowledge of his students. Rather he subtly deconstructed formal knowledge, so that his students would find their own ways.

By playing between sciences, arts, literature, languages ... Alex Grossmann truly was a Rinascimento multidisciplinary man.

Postlude: a geometric existentialist way of thinking

It seems to be in order now to finish this text by wondering what was the core of such a multifold creation, created by a multifold thinking.

A word comes in mind whenever one looks at the mathematical way of thinking that Alex had: geometry. Alex was thinking geometrically, even when facing unknown, very dispersed scientific facts. But geometry does not mean abstraction - as many people think. After all, geometry surrounds all our life, sometimes hidden but never abstract.

Alex's geometrical perception was never abstract because Alex's way of thinking was fundamentally existentialist, a bit like was existentialist the way of painting of Piero della Francesca⁶ at a period of the renaissance where the geometry of perspective was just - or not even - built. Alex's work on the analyticity properties of scattering amplitudes illustrates this characteristic of Alex very much. Scattering amplitudes already existed in quantum mechanics when Alex, together with Tai Tsun Wu, studied their mathematical properties. But a few years later, the same property of analyticity became part of the axiomatic of quantum field theory, the theory supposedly essentialist, that is epistemologically "before" ordinary quantum mechanics, and a domain on which Alex never had a real interest. The same happened to be true for the discovery of a non-trivial algebra in solid-state physics: it is a situation of noncommutative geometry inside ordinary quantum mechanics, the one of everyday life, in contrast to the infinite number of papers nowadays dealing with quantum mechanics on noncommutative spaces.

It seems to me that it is this existentialist property which avoided the geometrical views of Alex to be abstract.

*“Je préfère penser au demi-plan de Poincaré en le représentant comme le demi-plan inférieur plutôt que le demi-plan supérieur comme il est d'usage. De cette façon, je peux l'imaginer comme la mer, avec l'horizon à l'infini et des bateaux, des bateaux qui deviennent de plus en plus petits au fur et à mesure qu'ils se rapprochent du bord.”*⁷

Alex Grossmann to Robert Coquereaux.

⁶ Who said this definitively existentialist definition of painting: “Painting is nothing but a representation of surfaces and solids foreshortened or enlarged, and put on the plane of the picture in accordance with the fashion in which the real objects seen by the eye appear on this plane.”

⁷ I prefer to think of the Poincaré half-plane by representing it as the lower half-plane rather than the upper half-plane as is customary. That way, I can imagine it as the sea, with the infinite horizon and boats, boats that get smaller and smaller as they get closer to the shore.