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Knight meets Vives: Financial Markets Where Traders Are Ambiguous About Market Crowdedness

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Abstract

I investigate the effect of ambiguity about market crowdedness on asset prices à la Vives (2014). I model uninformed traders as facing Knightian uncertainty about the total number of market participants. This uncertainty limits the ability of uninformed traders to infer information from prices. A key result is that uninformed traders endogenously believe there are more (less) informed traders trading in the market when observing usual (unusual) price. In financial market, this can make the equilibrium asset price exhibit under or over-reaction to news, excess volatility and result in higher trading volume and higher equity premium.

Keywords: Market Crowdedness, Knight Uncertainty, Ambiguity Aversion

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1 Introduction

In most models of asset pricing and market micro-structure, tractability requires that all market participants know the presence and the exact number of all of the other competing traders. However, in reality, it is often hard to observe the number of market participants in real time and traders have to make guesses about its level. For example, Stein (2009) notes that: “For a broad class of quantitative trading strategies, an important consideration for each individual arbitrageur is that he cannot know in real time exactly how many others are using the same model and taking the same position as him.” Schnitzlein (2002) provides experimental evidence to show that the equilibrium outcomes are qualitatively inconsistent with theoretical models when the number of participants is unknown.

In this paper, I study a competitive crowded markets using rational expectations equilibrium model à la Vives (2014) with ambiguity about the market crowdedness and I define market crowdedness as the number of the market participants. I allow uninformed traders to have incorrect beliefs about market crowdedness, and thus to misunderstand the mapping from price to other informed traders’ private information. Therefore, uninformed traders use a misspecified model of the world to extract information from prices, leading to misinference. Since uninformed traders trade on this incorrect information, the equilibrium price deviates from the benchmark result of the rational expectation equilibrium (REE) without ambiguity.

2 An REE Model Where Traders are Ambiguous about Market Crowdedness

In this section I develop a Rational Expectation Equilibrium (REE) model where traders are ambiguous about market crowdedness.

2.1 Setup of the Model

Our model is static. I consider a similar setup à la Vives (2014) and Mele and Sangiorgi (2015), after information acquisition costs have been incurred. I consider a
market for a single risky asset which is in fixed supply \( Z \). The economy is populated by two types of price-taking traders, the first one are \( M \) units of (identical) informed traders (denoted as \( I \)), and the second one are \( N \) units of (identical) uninformed traders (denoted as \( U \)). Both types of traders’ initial wealth are normalized to zero for convenience. The fundamental distinction between the informed traders and the uninformed traders is that only the informed traders receive a common noisy signal. Traders are assumed to be risk neutral. The profits of trader \( i \in \{I, U\} \) when the price is \( p \) are

\[
\pi_i = (f_i - p) x_i - \frac{k}{2} x_i^2, \quad i \in \{I, U\},
\]

where \( x_i \) is the individual quantity demanded by trader \( i \), \( f_i \) is the (fundamental) value idiosyncratic to the trader and \( kx_i \) is a marginal transaction, opportunity or limit to arbitrage cost (it could also be interpreted as a proxy for risk aversion\(^1\)).

I assume \( f_i \) has the following structure,

\[
f_I = \bar{f} + \theta_I + \epsilon, \quad (2)
\]

\[
f_U = \bar{f} + \theta_U + \epsilon, \quad (3)
\]

where \( \bar{f} \) is a positive constant, \( \theta_I \) and \( \theta_U \) are the idiosyncratic components of the traders’ valuation. \( \epsilon \) is the common shock, \( \epsilon \sim N (0, \tau^{-1}_\epsilon) \), and is independent of \( \theta_I \) and \( \theta_U \). I further assume \( \theta_I \sim N (0, \tau^{-1}) \), \( \theta_U \sim N (0, \tau^{-1}) \) and \( \theta_I, \theta_U \) are correlated with correlation coefficient \( \rho \in [0, 1] \). We therefore have \( \text{cov} (\theta_I, \theta_U) = \rho \tau^{-1} \).

Our information structure encompasses the case of a common value and also that of private value. If \( \rho = 1 \), the valuation parameters \( f_I \) and \( f_U \) are perfectly correlated and we are in a common value model. When \( 0 < \rho < 1 \), we are in a private value model. If \( \rho = 0 \), then the parameters are independent and we are in an independent values model. In this paper, our main focus is on the nontrivial case: \( 0 < \rho < 1 \).

At trading stage, each informed trader receives a common noisy signal \( s = \theta_I + u \), where \( u \sim N (0, \tau_{u}^{-1}) \). The precision of the signal \( \tau_u \) is exogenously given in this

\(^1\)We may directly assume that investors have CARA utility function. This alternative assumption complicates the analysis without producing any new economic implications. It is reassuring that this more elaborate setting gives similar results as in our current setting.
model. The informativeness of the signal is captured by the signal-to-noise ratio
\[ \lambda \equiv \frac{\text{cov}(s, \theta_I)}{\text{var}(s)} = \frac{\tau^{-1}}{\tau^{-1} + \tau}. \]
When forming their expectations about the valuation \( f_i \), informed traders use all the information available to them. The information set of informed traders at trading stage is \( F_I = \{s, p\} \), where \( p \) is the equilibrium price at trading stage. Uninformed traders receive no signals about the asset payoff. Hence, the uninformed traders' information set at trading stage is \( F_U = \{p\} \).

### 2.2 Ambiguity and Ambiguity Aversion

Our point of departure from the previous literature is the assumption that both informed traders and uninformed traders are ex ante uncertain about the total number (size) of the market participants. Throughout the whole paper, we assume that agents display preferences in the form of the maxmin expected utility (MEU) model of Gilboa and Schmeidler (1989). Specifically, we analyze the following case: (case \( \mathcal{A} - M \)) both informed traders and uninformed traders are only ambiguous about the total number (size) of the informed traders \( M \). They are unable to assess what \( M \) is, but they believe it belongs to some interval, \( M \in [M_1, M_2] \), with \( M_1 < M < M_2 \). We further assume that \( M_1 = M - \Delta M \) and \( M_2 = M + \Delta M \). I use the boldface of \( M \) to denote the true value of \( M \). \( \Delta M \) is an exogenous parameter that determines the ambiguity. (But here, the total number (size) of the uninformed traders \( N \) is known by all of the uninformed traders and this is common knowledge.)

To summarize, the tuple
\[ \mathcal{E} = (\vec{f}, \rho, \tau, \tau_\epsilon, M, N) \]
defines an economy. We are interested in the implications for asset prices, demand functions, price reactions, volatility, trading volume, etc.

### 2.3 Solution Concept

Our solution concept is (competitive) Rational Expectation Equilibrium (REE), where each trader optimizes while taking prices as given, as in the usual competitive equi-
librium, but infers from prices the relevant information. An REE is price and demand functions that satisfy the optimality (utility maximization) and the market clearing conditions.

**Definition 1. Competitive REE without Ambiguity**

Given the number of informed traders $M$ and the number of uninformed traders $N$, an REE is a set of functions $(p, x_I, x_U)$ such that:

1. (Optimization) The informed demand $x_I$ and the uninformed demand $x_U$ maximize the expected profits of the informed and uninformed traders respectively in the market;
2. (Market-Clearing) The price of the risky stock $p$ equates the supply and demand, $M \cdot x_I + N \cdot x_U = Z$.

**Definition 2. Competitive REE with Ambiguity (Case $A - M$)**

Given the number of informed traders $M$, the number of uninformed traders $N$, traders’ ambiguity about $M$ or $N$, an REE is a set of functions $(p, x_I, x_U)$ such that: an REE is a set of functions $(p, x_I, x_U)$ such that:

1. (Optimization) The informed demands of the risky asset $x_I$ maximize the expected profits of informed traders and the uninformed demands $x_U$ maximize the minimum expected profits of the uninformed traders in the market;
2. (Market-Clearing) The price of the risky asset $p$ equates the supply and demand of the risky asset, $M \cdot x_I + N \cdot x_U = Z$.

### 3 Benchmark: Financial Market Equilibrium without Ambiguity

In this section, I solve for the financial market equilibrium where traders do not suffer from the ambiguity and know the true number of each type of market participants for sure. Specifically, both informed traders and uninformed traders know the total number (size) of the informed traders $M$ and the total number (size) of the uninformed traders $N$ for sure, without ambiguity. The calculation details and proofs are in Appendix A.

**Proposition 1. Financial Market Equilibrium without Ambiguity**

When both the number of the informed trader $M$ and the number of the uninformed trader $N$ are known without the ambiguity, there exists a Rational Expectation Equilibrium (REE) in which the price function $p$ is a function of $s$,

$$p(s) = f + \left( \frac{M + N \rho}{M + N} \right) \lambda \cdot s - \frac{kZ}{M + N}$$

(4)
Proposition 2. Equilibrium Demand Function of Traders without Ambiguity

When both the number of the informed trader $M$ and the number of the uninformed trader $N$ are known without the ambiguity, the equilibrium demand function of informed traders and uninformed traders are characterized as:

$$x_I(s, p) = \frac{\bar{f} + \lambda \cdot s - p}{k}$$

$$x_U(p) = \frac{1}{k} \left[ \frac{M (1 - \rho) \bar{f} + \rho k Z}{M + N \rho} - \frac{M (1 - \rho)}{M + N \rho} \cdot p \right]$$

4 Financial Market Equilibrium Where Traders are Ambiguous about $M$

In this section, I characterize the financial market equilibrium where traders are only ambiguous about the total number (size) of informed traders $M$. To find the equilibrium, we need to first characterize the demand function of informed and uninformed traders who exhibit ambiguity aversion with maxmin utility function.

4.1 Demand Function of Informed Traders

By observing the realization of $s$, informed traders resolve their ambiguity straight away. They choose portfolio holdings $x_I$ to maximize the expected profits $\pi_I$

$$\max_{x_I} E \left[ (f_I - p) x_I - \frac{k}{2} x_I^2 \mid \mathcal{F}_I = \{s, p\} \right],$$

(6)
where $p$ is the observed asset price. Standard arguments yield


x_I(s, p) = \frac{E(f_I | s, p) - p}{k} \\
= \frac{E(\bar{f} + \theta I + \epsilon | s) - p}{k} \\
= \frac{\bar{f} + E(\theta I | s) - p}{k} \\
= \frac{\bar{f} + \frac{\text{cov}(s, \theta I)}{\text{var}(s)} s - p}{k} \\
= \frac{\bar{f} + \left( \frac{\tau^{-1}}{\tau^{-1} + \tau_{ul}^{-1}} \right) s - p}{k} \\
= \frac{\bar{f} + \lambda s - p}{k},

(7)

and the informativeness of the signal is captured by the signal-to-noise ratio:

\[
\lambda \equiv \frac{\text{cov}(s, \theta I)}{\text{var}(s)} = \frac{\tau^{-1}}{\tau^{-1} + \tau_{ul}^{-1}}
\]

(8)

4.2 Demand Function of Uninformed Traders

At trading stage, for any given $M$ and $N$, uninformed traders rationally conjecture that the price function is:

\[ p = \bar{f} + A(M; N) \cdot s - H(M; N) \]

(9)

where the function $A(M; N)$ and $H(M; N)$ will be endogenously determined in equilibrium. Since $N$ is known by the uninformed traders for sure, $N$ is an exogenous parameter here. For simplicity, we write $A(M) \equiv A(M; N)$ and $H(M) \equiv H(M; N)$ and the conjectured price function can be written as:

\[ p = \bar{f} + A(M) \cdot s - H(M) \]

(10)

Thus, the optimal demand of uninformed traders is determined by

\[
\max_{x_U} \min_{M \in [M_1, M_2]} \left( E_M \left[ (f_U - p) x_U - k \frac{1}{2} x_U^2 | F_U = \{p\} \right] \right),
\]

(11)

where $x_U$ is the asset demand of uninformed traders, and $E_M(\cdot)$ is the expectation operator taken under the belief that the size/total number of the informed traders is
The criterion underlying equation (11) is the maxmin expected utility (MEU) axiomatized by Gilboa and Schmeidler (1989). Since uninformed traders are ambiguous about $M$, they view the stock price $p$ as an ambiguous signal about $f_U$. This makes the inference problem of the uninformed traders more interesting and generates the results we will see in the following sections.\(^2\)

Under the belief that the size/total number of the informed traders is $M$, they map from the observed price $p$ to the extracted signal $s_M$:

$$s_M = \frac{p + H(M) - \bar{f}}{A(M)}$$  \(12\)

The conditional moments of $f_U$ taken under a particular belief $M$, are given by:

$$E_M[f_U | p] = E_M[f + \theta_U + \epsilon | p]$$
$$= \bar{f} + E_M[\theta_U | p] + 0$$
$$= \bar{f} + E_M[\theta_U | s = s_M]$$
$$= \bar{f} + \frac{\text{cov}(s, \theta_U)}{\text{var}(s)} s_M$$
$$= \bar{f} + \left(\frac{\rho \tau^{-1}}{\tau^{-1} + \tau_u^{-1}}\right) s_M$$
$$= \bar{f} + \rho \lambda \left[\frac{p + H(M) - \bar{f}}{A(M)}\right]$$
$$= \bar{f} + \rho \lambda \cdot G(M; p, \bar{f}, N)$$  \(13\)

For simplicity, we write

$$G(M) \equiv G(M; p, \bar{f}, N) = \frac{p + H(M) - \bar{f}}{A(M)}$$

and the objective function of an uninformed trader can be written as:

$$\min_{M \in [M_1, M_2]} \left( (E_M[f_U | p] - p) \cdot x_U - \frac{k}{2} x_U^2 \right)$$
$$\Rightarrow \min_{M \in [M_1, M_2]} \left( (\bar{f} + \rho \lambda \cdot G(M) - p) \cdot x_U - \frac{k}{2} x_U^2 \right)$$
$$= \begin{cases} 
  -\frac{k}{2} x_U^2 + [\bar{f} + \rho \lambda \cdot \{G(M)\}_{\min} - p] \cdot x_U, & \text{if } x_U > 0 \\
  0, & \text{if } x_U = 0 \\
  -\frac{k}{2} x_U^2 + [\bar{f} + \rho \lambda \cdot \{G(M)\}_{\max} - p] \cdot x_U, & \text{if } x_U < 0
\end{cases}$$  \(14\)

\(^2\)While formulating portfolio decisions, uninformed traders learn from the price and update each of their beliefs. This rule is known as full Bayesian updating. With full Bayesian updating, no learning occurs regarding the original set of priors, which means that the uninformed agents retain all their initial priors.
where \( \{ G(M) \}_{\text{min}} = \left\{ \frac{p + H(M) - f}{A(M)} \right\}_{\text{min}} \) and \( \{ G(M) \}_{\text{max}} = \left\{ \frac{p + H(M) - f}{A(M)} \right\}_{\text{max}} \) are the minimum and maximum of the function \( \{ G(M) \} \), given the value of \( p \), respectively.

Thus an uninformed trader’s demand function is:

\[
x_U(p) = \begin{cases} 
\bar{f} + \rho \lambda \cdot \left\{ \frac{p + H(M) - f}{A(M)} \right\}_{\text{min}} - p, & \text{if } \bar{f} + \rho \lambda \cdot \left\{ \frac{p + H(M) - f}{A(M)} \right\}_{\text{min}} - p > 0 \\
0, & \text{if } \bar{f} + \rho \lambda \cdot \left\{ \frac{p + H(M) - f}{A(M)} \right\}_{\text{min}} - p \leq 0 \leq \bar{f} + \rho \lambda \cdot \left\{ \frac{p + H(M) - f}{A(M)} \right\}_{\text{max}} - p \\
\bar{f} + \rho \lambda \cdot \left\{ \frac{p + H(M) - f}{A(M)} \right\}_{\text{max}} - p, & \text{if } \bar{f} + \rho \lambda \cdot \left\{ \frac{p + H(M) - f}{A(M)} \right\}_{\text{max}} - p < 0 
\end{cases}
\] (15)

**Proposition 3. Financial Market Equilibrium with Ambiguity about the Number of the Informed Traders**

When the number of the uninformed trader \( N \) is known and the number of the informed trader \( M \) is unknown with the ambiguity, there exists a competitive Rational Expectation Equilibrium (REE) in which the price function \( p \) is piecewise in \( s \) and \( M \),

\[
p(s, M) = \begin{cases} 
\bar{f} + \left[ \frac{M(M_1 + N\rho)}{M(M_1 + N\rho) + M_1 N(1-\rho)} \right] \lambda \cdot s - \left[ \frac{M_1 kZ}{M(M_1 + N\rho) + M_1 N(1-\rho)} \right], & \text{for } s \in (-\infty, s_1) \cup (s_3, +\infty) \\
\bar{f} + \left[ \frac{M(M_2 + N\rho)}{M(M_2 + N\rho) + M_2 N(1-\rho)} \right] \lambda \cdot s - \left[ \frac{M_2 kZ}{M(M_2 + N\rho) + M_2 N(1-\rho)} \right], & \text{for } s \in [s_1, s_2) \\
\bar{f} + \lambda s - \frac{kZ}{M}, & \text{for } s \in [s_2, s_3] 
\end{cases}
\] (16)

with \( s_1 < s_2 < s_3 \).

\[
s_1 = \frac{-kZ}{N\lambda (1-\rho)} \\
s_2 = \left[ \frac{1}{M} + \frac{\rho}{M_2 (1-\rho)} \right] \frac{kZ}{\lambda} \frac{M_2}{M_2} \\
s_3 = \left[ \frac{1}{M} + \frac{\rho}{M_1 (1-\rho)} \right] \frac{kZ}{\lambda} \frac{M_1}{M_1}
\] (17)

**Proposition 4. Equilibrium Demand Function of Traders with Ambiguity about the Number of the Informed Traders**

When the number of the uninformed trader \( N \) is known and the number of the informed trader
M is unknown with the ambiguity, the equilibrium demand function of informed traders and uninformed traders are characterized as:

\[
x_I(s, p) = \frac{\bar{f} + \lambda \cdot s - p}{k}
\]

\[
x_U(p) = \begin{cases} 
\frac{1}{k} \left[ \frac{M_1(1-\rho)\bar{f} + \rho kZ}{M_1 + Np} - \frac{M_1(1-\rho)}{M_1 + Np} \cdot p \right], & \text{Buy, for } p \in (-\infty, p_1) \\
\frac{1}{k} \left[ \frac{M_2(1-\rho)\bar{f} + \rho kZ}{M_2 + Np} - \frac{M_2(1-\rho)}{M_2 + Np} \cdot p \right], & \text{Buy, for } p \in [p_1, p_2) \\
0, & \text{Not Participate, for } p \in [p_2, p_3] \\
\frac{1}{k} \left[ \frac{M_1(1-\rho)\bar{f} + \rho kZ}{M_1 + Np} - \frac{M_1(1-\rho)}{M_1 + Np} \cdot p \right], & \text{Sell, for } p \in (p_3, +\infty) 
\end{cases}
\]

with three constants \( p_1 < p_2 < p_3 \).

\[
p_1 = \bar{f} - \left( \frac{1}{1-\rho} \right) \left( \frac{kZ}{N} \right)
\]

\[
p_2 = \bar{f} + \left( \frac{\rho}{1-\rho} \right) \left( \frac{kZ}{M_2} \right)
\]

\[
p_3 = \bar{f} + \left( \frac{\rho}{1-\rho} \right) \left( \frac{kZ}{M_1} \right)
\]

5 Properties of Equilibrium Outcomes

In this section I compare the properties of the price equilibrium with the ambiguity and benchmark result, and we discuss how the outcomes relate to empirical patterns in asset pricing. Specifically, we focus on the role of traders’ ambiguity and ambiguity aversion about the market crowdedness on concepts such as: price reaction, volatility, contrarians, trading volume, etc.

5.1 Over-reaction, Under-reaction and Excess Volatility

As shown in equation (15), for uninformed traders, ambiguity about the number of informed traders, and ambiguity aversion preference naturally lead to beliefs that
are more extreme than under fully rational inference when learning from the price. Therefore, ambiguity about the market crowdedness provides a micro-foundation to price over-reaction or under-reaction to news, as is clear from comparing equilibrium outcomes from proposition 1 and 3.

**Proposition 5. Over-reaction and Under-reaction to News**

The equilibrium price under case $A - M$ and benchmarks case is denoted as $p(s, M)$ and $p(s)$, respectively. Ambiguity about the number of informed traders $M$ makes the equilibrium price display both over-reaction and under-reaction to news,

\[
\begin{align*}
\frac{\partial p(s, M)}{\partial s} &\geq \frac{\partial p(s)}{\partial s}, \quad (\text{Over-reaction Region}), \text{ for } s \in (-\infty, s_1) \cup [s_2, +\infty) \\
\frac{\partial p(s, M)}{\partial s} &\leq \frac{\partial p(s)}{\partial s}, \quad (\text{Under-reaction Region}), \text{ for } s \in [s_1, s_2)
\end{align*}
\]

(20)

and the equality holds for true only if the true value of $M$ is equal to $M_2$.

Ambiguity about the market crowdedness speaks to the long-standing “excess volatility” puzzle (Shiller (1981)): it delivers excess volatility since beliefs are excessively volatile relative to fundamentals.

5.2 The Identity of Contrarians

**Proposition 6. Contrarians**

Uninformed traders are always contrarians (if participating the market) and informed traders always trade in the direction of the signal.

Following good news, informed traders know the asset is good they will trade in the direction of the private signal, while uninformed traders always make decision using worst-case analysis, and therefore think the asset is over-priced and trade in the opposite direction of the signal. Ambiguity about the market crowdedness speaks to the empirical findings of C. Luo et al. (2021): a very large fraction of retail investors trade as contrarians, in particularly contrarian buying.
5.3 Trading Volume

A well-documented fact in financial markets is that trading volume is far greater than what rational models can justify (Barberis (2018)).

**Proposition 7.** With ambiguity about the market crowdedness, trading volume is a function of the number of informed traders \( M \).

Proposition 9 highlights that with ambiguity about the market crowdedness, we do not need to rely on extreme signals to achieve excessive amounts of trading volume.

6 Extensions

6.1 Implementable REE: the Demand Schedule Game with Ambiguity

In this section, I discuss whether the (competitive) REE we obtained in proposition 3 is an implementable REE, which is the outcome of a well-specified game\(^3\). Following Vives (2014), the natural way to implement (competitive) REE in our context is to consider competition among demand functions in a market where each trader is negligible. Hence, we adopt the definition of Implementable REE with ambiguity as an REE which is associated with a Bayesian Nash equilibrium of the game in demand functions with ambiguity. I focus on the linear demand function equilibrium. We conjecture the strategies of the informed traders and the uninformed traders respectively are as:

\[
x_I(s,p) = \eta + \beta s - \gamma p
\]

\[
x_U(p) = \begin{cases} 
\phi_1 - \omega_1 p, & \text{Buy and believe } M = M_1 \text{ for } p \in (-\infty, p_1) \\
\phi_2 - \omega_2 p, & \text{Buy and believe } M = M_2 \text{ for } p \in [p_1, p_2) \\
0, & \text{Not Participate, for } p \in [p_2, p_3] \\
\phi_3 - \omega_3 p, & \text{Sell and believe } M = M_1 \text{ for } p \in (p_3, +\infty)
\end{cases}
\]  

\(^{\text{11}}\)

\(^3\)This is equivalent to find a trading mechanism that delivers the competitive REE.
**Definition 3. Demand Function Game Equilibrium with Ambiguity**

The demand functions $x_I(s, p)$ and $x_U(p)$ are a Bayesian Nash Equilibrium with ambiguity if:

1. (Best Response) Fixing uninformed traders’ strategies, for informed traders, $(\eta, \beta, \gamma)$ is a best response to the strategy profile $(\phi_1, \phi_2, \phi_3, \omega_1, \omega_2, \omega_3)$. Fixing informed traders’ strategies, for uninformed traders, $(\phi_1, \phi_2, \phi_3, \omega_1, \omega_2, \omega_3)$ is a best response to the strategy profile $(\eta, \beta, \gamma)$.

2. (Optimization) The informed demands of the risky asset $x_I^*$ maximize the expected profits of informed traders and the uninformed demands $x_U^*$ maximize the minimum expected profits of the uninformed traders in the market,

$$x_I^* \in \arg\max_{x_I} E \left[ (f_I - p) x_I - \frac{k}{2} x_I^2 \mid \mathcal{F}_I = \{s, p\} \right]$$

$$x_U^* \in \max_{x_U} \min_{M \in [M_1, M_2]} \left( E_M \left[ (f_U - p) x_U - \frac{k}{2} x_U^2 \mid \mathcal{F}_U = \{p\} \right] \right)$$

3. (Market-Clearing) For each $(s, p, M)$,

$$M \cdot x_I + N \cdot x_U = Z$$

**Proposition 8. Implementable REE**

The (competitive) REE of proposition 3 is an implementable REE and it can be implemented through a demand schedule game.

### 6.2 Separation of Ambiguity and Ambiguity Aversion: $\alpha$-Mamin Utility

The maxmin expected utility (MEU) of Gilboa and Schmeidler (1989) does not differentiate between the trader’s ambiguity aversion and the degree of ambiguity. However, they are different concepts: the former is a preference characteristic, while the latter is a characteristic of the environment. To differentiate ambiguity from ambiguity aversion, I consider the $\alpha$-maxmin expected utility model ($\alpha$-MEU) or the Hurwicz’s $\alpha$-pessimism rule. The objective function of uninformed traders is determined by

$$\max_{x_U} \left( \alpha \cdot \min_{M \in [M_1, M_2]} \left( E_M \left[ (f_U - p) x_U - \frac{k}{2} x_U^2 \mid \mathcal{F}_U = \{p\} \right] \right) \right)$$

$$+(1 - \alpha) \cdot \max_{M \in [M_1, M_2]} \left( E_M \left[ (f_U - p) x_U - \frac{k}{2} x_U^2 \mid \mathcal{F}_U = \{p\} \right] \right)$$

(25)
where parameter $\alpha \in (0,1]$ captures the trader’s attitude to ambiguity (see Ghirardato, Maccheroni, and Marinacci (2004)). The interpretation of equation (25) is that the trader lacks confidence to assign probabilities to possible scenarios, instead, he simplifies the problem and focuses on the weighted average of worst- and best-case scenarios. When $\alpha = 1$, equation (25) reduces to equation (11) in section 4.2. When $\alpha \to 0$, the trader becomes non-prudent and only focuses on the best-case scenario. The model is solved in Appendix E.

**Proposition 9. Financial Market Equilibrium with $\alpha$-maxmin Ambiguity about the Number of the Informed Traders $M$**

When the number of the uninformed trader $N$ is known and the number of the informed trader $M$ is unknown with the ambiguity, there exists a competitive Rational Expectation Equilibrium (REE) in which the price function $p$ is piecewise in $s$ and $M$,

$$p(s, M) = \begin{cases} 
\bar{f} + A^\bullet(M) \cdot s - H^\bullet(M), & \text{for } s \in (-\infty, s_a) \cup (s_c, +\infty) \\
\bar{f} + A^\Delta(M) \cdot s - H^\Delta(M), & \text{for } s \in [s_a, s_b) \\
\bar{f} + \lambda s - \frac{kZ}{M}, & \text{for } s \in [s_b, s_c] 
\end{cases} \quad (26)$$

where the expression of $A^\bullet(M)$, $A^\Delta(M)$, $H^\bullet(M)$, $H^\Delta(M)$ and the threshold value for the signal, $s_a, s_b, s_c$ are given in Appendix E.

**6.3 Alternative Setup: Financial Market Equilibrium Where Traders are Ambiguous about $N$**

In this section, I characterize the financial market equilibrium where traders are only ambiguous about the total number (size) of informed traders $N$. Specifically, both informed traders and uninformed traders are only ambiguous about the total number (size) of uninformed traders $N$. They are unable to assess what $N$ is, but they believe it belongs to some interval, $N \in [N_1, N_2]$, with $N_1 < N < N_2$. We further assume that $N_1 = N - \Delta N$ and $N_2 = N + \Delta N$. I use the boldface of $N$ to denote the true value of
\[ N. \Delta N \text{ is an exogenous parameter that determines the ambiguity. (But here, the total number (size) of the informed traders } M \text{ is known by all of the uninformed traders and this is common knowledge.) The solution method is quite similar with section 4. The calculation details and proofs are in Appendix F.} \]

**Proposition 10. Financial Market Equilibrium with Ambiguity about the Number of the Uninformed Traders } N**

When the number of the informed trader } M \text{ is known and the number of the uninformed trader } N \text{ is unknown with the ambiguity, there exists a Rational Expectation Equilibrium (REE) in which the price function } p \text{ is a function of } s \text{ and } N,

\[
p(s, N) = \bar{f} + \left[ \frac{M + N_2 \rho}{M + N + (N_2 - N) \rho} \right] \lambda \cdot s - \left[ \frac{(M + (N_2 - N) \rho) kZ}{M (M + N + (N_2 - N) \rho)} \right] \tag{27}
\]

**Proposition 11. Equilibrium Demand Function of Traders with Ambiguity about the Number of the Uninformed Traders } N**

When the number of the informed trader } M \text{ is known and the number of the uninformed trader } N \text{ is unknown with the ambiguity, the equilibrium demand function of informed traders and uninformed traders are characterized as:

\[
x_I(s, p) = \frac{f + \lambda \cdot s - p}{k} \\
x_U(p) = \frac{1}{k} \left[ \frac{(1 - \rho) f + \rho kZ}{M + N_2 \rho} - \frac{M(1 - \rho) kZ}{M + N_2 \rho} \cdot p \right] \tag{28}
\]

6.4 Alternative Setup: Two Dimension Ambiguity for Uninformed Traders

We now allow for two dimension ambiguity for uninformed traders. In this section, we assume uninformed traders are ambiguous about both the total number (size) of the informed traders } M \text{ and the total number (size) of the uninformed traders } N. \text{ The solution method is similar with section 4. The calculation details and proofs are in Appendix G.}

**Proposition 12. Financial Market Equilibrium with Uninformed Traders’ Ambiguity about both the Number of the Informed Traders } M \text{ and Uninformed Traders } N**
When the number of the informed trader $M$ and the number of the uninformed trader $N$ are both unknown with the ambiguity by uninformed traders, there exists a Rational Expectation Equilibrium (REE) in which the price function $p$ is piecewise in $s$, $M$ and $N$

$$p(s, M, N) = \begin{cases} 
\bar{f} + \left[ \frac{M(M_1+N_2\rho)}{M(M_1+N_2\rho)+NM_1(1-\rho)} \right] \lambda \cdot s - \left[ \frac{|M_1+(N_2-\rho)|\cdot kZ}{M(M_1+N_2\rho)+NM_1(1-\rho)} \right], & \text{for } s \in (-\infty, s'_1) \cup (s_3, +\infty) \\
\bar{f} + \left[ \frac{M(M_2+N_2\rho)}{M(M_2+N_2\rho)+NM_2(1-\rho)} \right] \lambda \cdot s - \left[ \frac{|M_2+(N_2-\rho)|\cdot kZ}{M(M_2+N_2\rho)+NM_2(1-\rho)} \right], & \text{for } s \in [s'_1, s_2) \\
\bar{f} + \lambda s - \frac{kZ}{M}, & \text{for } s \in [s_2, s_3] 
\end{cases}$$

(29)

with $s'_1 < s_2 < s_3$.

$$s'_1 = \frac{-kZ}{N_2\lambda(1-\rho)}$$

$$s_2 = \left[ \frac{1}{M} + \frac{\rho}{M_2(1-\rho)} \right] \frac{kZ}{\lambda}$$

$$s_3 = \left[ \frac{1}{M} + \frac{\rho}{M_1(1-\rho)} \right] \frac{kZ}{\lambda}$$

(30)

7 Conclusion

In this paper, I investigate the effect of ambiguity about market crowdedness on asset prices à la Vives (2014). I model uninformed traders as facing Knightian uncertainty about the total number of market participants. This uncertainty limits the ability of uninformed traders to infer information from prices. A key result is that uninformed traders endogenously believe there are more (less) informed traders trading in the market when observing usual (unusual) price. In financial market, this can make the equilibrium asset price exhibit under or over-reaction to news, excess volatility and result in higher trading volume and higher equity premium.
Appendices

A Proof For Section 3

A.1 Proof of Proposition 1

Proof. In this section, I solve for the financial market equilibrium where traders do not suffer from
the ambiguity and know the true number of each type of market participants for sure. To find the
equilibrium, we need to first characterize the demand function of informed and uninformed traders.

A.1.1 Demand Function of Informed Traders

By observing the realization of $s$, informed traders choose portfolio holdings $x_I$ to maximize the ex-
pected profits $\pi_I$

$$\max_{x_I} E \left[ (f_I - p) x_I - \frac{k}{2} x_I^2 | \mathcal{F}_I = \{s, p\} \right], \quad (31)$$

where $p$ is the observed asset price. Standard arguments yield

$$x_I(s, p) = \frac{E(f_I | s, p) - p}{k} = \frac{E(\bar{f} + \theta_I + \epsilon | s) - p}{k} = \frac{\bar{f} + E(\theta_I | s) - p}{k}$$

$$= \frac{\bar{f} + \frac{\text{cov}(s, \theta_I)}{\text{var}(s)} s - p}{k} = \frac{\bar{f} + \lambda s - p}{k}, \quad (32)$$

and the informativeness of the signal is captured by the signal-to-noise ratio:

$$\lambda \equiv \frac{\text{cov}(s, \theta_I)}{\text{var}(s)} = \frac{\tau^{-1}}{\tau^{-1} + \tau_u^{-1}} \quad (33)$$

A.1.2 Demand Function of Uninformed Traders

At trading stage, uninformed traders rationally conjecture that the price function is:

$$p = \bar{f} + A \cdot s - H \quad (34)$$

where $A$ and $H$ are two constants, which will be endogenously determined in equilibrium. Thus, the
optimal demand of uninformed traders is determined by

$$\max_{x_U} E \left[ (f_U - p) x_U - \frac{k}{2} x_U^2 | \mathcal{F}_U = \{p\} \right], \quad (35)$$
where \( x_U \) is the asset demand of uninformed traders. Since uninformed traders knows the true value of \( M \) and \( N \), they view the stock price \( p \) as a noisy signal about \( f_U \). They map from the observed price \( p \) to the extracted signal \( s_p \):

\[
s_p = \frac{p + H - \bar{f}}{A} \tag{36}
\]

The conditional moments of \( f_U \) are given by:

\[
E \left[ f_U \mid p \right] = E \left[ f + \theta_U + \epsilon \mid p \right] \\
= \bar{f} + E \left[ \theta_U \mid p \right] + 0 \\
= \bar{f} + E \left[ \theta_U \mid s = s_N \right] \\
= \bar{f} + \frac{\text{cov} \left( s, \theta_U \right)}{\text{var} \left( s \right)} s_p \\
= \bar{f} + \left( \frac{\rho \tau^{-1}}{\tau^{-1} + \tau_u^{-1}} \right) s_p \\
= \bar{f} + \rho \lambda \left[ \frac{p + H - \bar{f}}{A} \right] 
\tag{37}
\]

Thus an uninformed trader’s demand function is:

\[
x_U(p) = \frac{E(f_U \mid p) - p}{k} \\
= \frac{\bar{f} + \rho \lambda \left[ \frac{p + H - \bar{f}}{A} \right] - p}{k} \tag{38}
\]

We insert the equation (32) and (38) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,

\[
M \cdot \left( \frac{\bar{f} + \lambda s - p}{k} \right) + N \cdot \left( \frac{\bar{f} + \rho \lambda \left[ \frac{p + H - \bar{f}}{A} \right] - p}{k} \right) = Z
\]

Rearrange the terms and we get,

\[
\left( M + N - \frac{N \rho \lambda}{A} \right) p = \left( M + N - \frac{N \rho \lambda}{A} \right) \bar{f} + M \lambda s - \left( kZ - \frac{N \rho \lambda H}{A} \right)
\]

Using the undetermined coefficient method and match the coefficient, we get:

\[
A = \frac{M \lambda}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N} \tag{40}
\]

\[
H = \frac{kZ - \frac{N \rho \lambda H}{A}}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N}
\]
Hence, we can explicitly solve for the fixed point for $A$ and $H$,

$$A = \left( \frac{M + N \rho}{M + N} \right) \lambda$$

$$H = \frac{kZ}{M + N}$$

(41)

To summarize, when both the number of the informed trader $M$ and the number of the uninformed trader $N$ are known without the ambiguity, there exists a Rational Expectation Equilibrium (REE) in which the price function $p$ is a function of $s$,

$$p(s) = \bar{f} + \left( \frac{M + N \rho}{M + N} \right) \lambda \cdot s - \frac{kZ}{M + N}$$

(42)

A.2 Proof of Proposition 2

Proof. According to equation (32), we find the equilibrium demand function of informed traders,

$$x_I(s, p) = \frac{\bar{f} + \lambda \cdot s - p}{k}$$

Using the equation (38) and (41), we find the equilibrium demand function of uninformed traders,

$$x_U(p) = \frac{1}{k} \left[ M(1 - \rho)\bar{f} + \rho kZ - \frac{M(1 - \rho)}{M + N} \cdot p \right]$$

(43)

B Proofs For Section 4

B.1 Proof of Proposition 3

Proof. We define the value of $M$ that causes the function $G(M)$ to reach a minimum as $\underline{M}$ and the value of $M$ that causes the function $G(M)$ to reach a maximum as $\overline{M}$,

$$\underline{M} \equiv \arg \min_{M \in [M_1, M_2]} G(M) = \arg \min_{M \in [M_1, M_2]} \left[ \frac{p + H(M) - \bar{f}}{A(M)} \right]$$

$$\overline{M} \equiv \arg \max_{M \in [M_1, M_2]} G(M) = \arg \max_{M \in [M_1, M_2]} \left[ \frac{p + H(M) - \bar{f}}{A(M)} \right]$$

(43)

We define the values that $H(M)$ and $A(M)$ take on when $M = \underline{M}$ and $M = \overline{M}$ as:

$$H \equiv H(\underline{M}) \quad \overline{H} \equiv H(\overline{M})$$

$$\underline{A} \equiv A(\underline{M}) \quad \overline{A} \equiv A(\overline{M})$$

(44)
Note that $H$, $\overline{H}$, $A$ and $\overline{A}$ will be a constant. Hence, we can define the maximum and minimum of $G(M)$ as:

$$\left\{ \frac{p + H(M) - \bar{f}}{A(M)} \right\}_{\text{min}} = \bar{G} = \frac{p + H - \bar{f}}{A}$$
$$\left\{ \frac{p + H(M) - \bar{f}}{A(M)} \right\}_{\text{max}} = \bar{G} = \frac{p + H - \bar{f}}{A} \quad (45)$$

Hence, the optimal demand functions of the uninformed traders, equation (15), can be written as,

$$x_U(p) = \begin{cases} 
\bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p, & \text{if } \bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p > 0 \\
0, & \text{if } \bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p \leq 0 \leq \bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p \\
\bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p, & \text{if } \bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p < 0
\end{cases} \quad (46)$$

We evaluate the feasibility of the possible demand functions given in equation (48) case by case by inserting it into the market clearing condition.

I. First, suppose $x_U > 0$ and the premise

$$\bar{f} + \rho \lambda \cdot \frac{p + H - \bar{f}}{A} - p > 0 \quad (47)$$

is satisfied, then the demand function of informed traders and uninformed traders are:

$$x_I = \frac{\bar{f} + \lambda s - p}{k}$$
$$x_U = \frac{\bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p}{k} \quad (48)$$

We insert the equation (48) into the market clearing condition, $M \cdot x_I + N \cdot x_U = Z$ and we get,

$$M \cdot \left( \frac{\bar{f} + \lambda s - p}{k} \right) + N \cdot \left( \frac{\bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p}{k} \right) = Z$$

Rearrange the terms and we get,

$$p = \bar{f} + \left( \frac{M \lambda}{M + N - \frac{N \rho \lambda}{A}} \right) s - \left( \frac{kZ - \frac{N \rho \lambda H}{A}}{M + N - \frac{N \rho \lambda}{A}} \right) = \bar{f} + \left( \frac{M \lambda}{A(M)} \right) s - \left( \frac{kZ - \frac{N \rho \lambda H}{A}}{A(M)} \right) \quad (49)$$
Using the undetermined coefficient method and match the coefficient, we get:

\[ A(M) = \frac{M \lambda}{M + \left(1 - \frac{\rho \lambda}{A}\right) N} \]

\[ H(M) = \frac{kZ - \frac{N \rho \lambda H}{A}}{M + \left(1 - \frac{\rho \lambda}{A}\right) N} \]  \hspace{1cm} (50)

Hence, we can express the function \( G(M) \) as:

\[ G(M) = \frac{p}{A(M)} + H(M) - \bar{f} \]

\[ = \frac{p + \left[\frac{kZ - \frac{N \rho \lambda H}{A}}{M + \left(1 - \frac{\rho \lambda}{A}\right) N}\right] - \bar{f}}{\left[\frac{M \lambda}{M + \left(1 - \frac{\rho \lambda}{A}\right) N}\right]} \]
\[ = \frac{(p - \bar{f}) \left[M + \left(1 - \frac{\rho \lambda}{A}\right) N\right] + \left[kZ - \frac{N \rho \lambda H}{A}\right]}{M \lambda} \]  \hspace{1cm} (51)

We could find the partial derivative of \( G(M) \) w.r.t \( M \),

\[ \frac{\partial G(M)}{\partial M} = \frac{(p - \bar{f}) \left[\left(M + \left(1 - \frac{\rho \lambda}{A}\right) N\right] + \left[kZ - \frac{N \rho \lambda H}{A}\right]\right]}{M^2 \lambda} \]
\[ = \frac{-N \left[(p - \bar{f}) \left(1 - \frac{\rho \lambda}{A}\right) - \frac{\rho \lambda H}{A}\right] - kZ}{M^2 \lambda} \]
\[ = \frac{-NL - kZ}{M^2 \lambda} \]  \hspace{1cm} (52)

We define \( L = (p - \bar{f}) \left(1 - \frac{\rho \lambda}{A}\right) - \frac{\rho \lambda H}{A}. \)

Since at the beginning of this sub-case I, our main premise is that

\[ \bar{f} + \rho \lambda \cdot \frac{p + H - \bar{f}}{A} - p > 0 \]

This premise can be written as:

\[ - \left[(p - \bar{f}) \left(1 - \frac{\rho \lambda}{A}\right) - \frac{\rho \lambda H}{A}\right] > 0 \]

This implies that \( L < 0 \), and the sign of \( \frac{\partial G(M)}{\partial M} \) will be determined by the sign of \( -NL - kZ \).

1. Suppose \( -NL - kZ > 0 \), then \( \frac{\partial G(M)}{\partial M} > 0 \) and

\[ M_1 = M = \arg \min_{M \in [M_1, M_2]} G(M) \]
Inserting \( M = M_1 \) into equation (50), and we get,

\[
\Delta = A(M_1) = \frac{M_1 \lambda}{M_1 + \left(1 - \frac{\rho \lambda}{\Delta}\right) N}
\]

\[
H = H(M_1) = \frac{kZ - \frac{N \rho \lambda H}{\Delta}}{M_1 + \left(1 - \frac{\rho \lambda}{\Delta}\right) N}
\]

Solving for \( \Delta \) and \( H \), and we find the explicit expression:

\[
\Delta = \left(\frac{M_1 + N \rho}{M_1 + N}\right) \lambda
\] \hspace{1cm} (53)

\[
H = \frac{kZ}{M_1 + N}
\]

Inserting equation (53) into equation (49), and we find the price function,

\[
p = \bar{f} + \left(\frac{MA}{M + N - \frac{N \rho \lambda}{\Delta}}\right) \lambda - \left(\frac{kZ - \frac{N \rho \lambda H}{\Delta}}{M + N - \frac{N \rho \lambda}{\Delta}}\right)\]

\[
= \bar{f} + \left[\frac{M (M_1 + N \rho)}{M (M_1 + N \rho) + M_1 N (1 - \rho)}\right] \lambda \cdot s - \left[\frac{M_1 kZ}{M (M_1 + N \rho) + M_1 N (1 - \rho)}\right]
\] \hspace{1cm} (54)

We need to check that the two premise, equation (47) and \((-NL - kZ) > 0\), should be satisfied. Equation (47) and equation (53) imply that,

\[
p < \bar{f} + \left(\frac{\rho}{1 - \rho}\right) \left(\frac{kZ}{M_1}\right)
\] \hspace{1cm} (55)

\((-NL - kZ) > 0\) and equation (53) imply that,

\[
p < \bar{f} - \left(\frac{1}{1 - \rho}\right) \left(\frac{kZ}{N}\right)
\] \hspace{1cm} (56)

When equation (55) is satisfied, equation (56) will be automatically satisfied. Substituting equation (54) into equation (55), we get:

\[
s < \frac{-kZ}{N \lambda (1 - \rho)}
\] \hspace{1cm} (57)

To sum up, in this subsection, when \( p < \bar{f} - \left(\frac{1}{1 - \rho}\right) \left(\frac{kZ}{N}\right) \), or say, when \( s < \frac{-kZ}{N \lambda (1 - \rho)} \), the equilibrium price function is

\[
p = \bar{f} + \left[\frac{M (M_1 + N \rho)}{M (M_1 + N \rho) + M_1 N (1 - \rho)}\right] \lambda \cdot s - \left[\frac{M_1 kZ}{M (M_1 + N \rho) + M_1 N (1 - \rho)}\right]
\] \hspace{1cm} (58)

2. Suppose \((-NL - kZ) < 0\), then \( \frac{\partial G}{\partial M} < 0 \) and

\[
M_2 = M = \arg \min_{M \in [M_1, M_2]} G(M)
\]
Inserting $M = M_2$ into equation (50), and we get,

$$A = A(M_2) = \frac{M_2 \lambda}{M_2 + \left(1 - \frac{\rho \lambda}{M_2}\right) N}$$

$$H = H(M_2) = \frac{kZ - \frac{N \rho \lambda H}{M_2}}{M_2 + \left(1 - \frac{\rho \lambda}{M_2}\right) N}$$

Solving for $A$ and $H$, and we find the explicit expression:

$$A = \left(\frac{M_2 + N \rho}{M_2 + N}\right) \lambda$$

$$H = \frac{kZ}{M_2 + N}$$

(59)

Inserting equation (59) into equation (49), and we find the price function,

$$p = \tilde{f} + \left(\frac{M \lambda}{M + N - \frac{N \rho \lambda}{A}}\right) s - \left(\frac{kZ - \frac{N \rho \lambda H}{A}}{M + N - \frac{N \rho \lambda}{A}}\right)$$

$$= \tilde{f} + \left[\frac{M (M_2 + N \rho)}{M (M_2 + N \rho) + M_2 N (1 - \rho)}\right] \lambda \cdot s - \left[\frac{M_2 kZ}{M (M_2 + N \rho) + M_2 N (1 - \rho)}\right]$$

(60)

We need to check that the two premise, equation (47) and $(-NL - kZ) < 0$, should be satisfied. Equation (47) and equation (59) imply that,

$$p < \tilde{f} + \left(\frac{\rho}{1 - \rho}\right) \left(\frac{kZ}{M_2}\right)$$

(61)

$(-NL - kZ) < 0$ and equation (59) imply that,

$$p > \tilde{f} - \left(\frac{1}{1 - \rho}\right) \left(\frac{kZ}{N}\right)$$

(62)

Substituting equation (60) into equation (61), we get

$$s < \left[\frac{1}{M} + \frac{\rho}{M_2 (1 - \rho)}\right] \frac{kZ}{\lambda}$$

(63)

Substituting equation (60) into equation (63), we get

$$s > \frac{-kZ}{N \lambda (1 - \rho)}$$

(64)

To sum up, in this subsection, when $\tilde{f} - \left(\frac{1}{1 - \rho}\right) \left(\frac{kZ}{N}\right) < p < \tilde{f} + \left(\frac{\rho}{1 - \rho}\right) \left(\frac{kZ}{M_2}\right)$, or say, when $\frac{-kZ}{N \lambda (1 - \rho)} < s < \left[\frac{1}{M} + \frac{\rho}{M_2 (1 - \rho)}\right] \frac{kZ}{\lambda}$, the equilibrium price function is

$$p = \tilde{f} + \left[\frac{M (M_2 + N \rho)}{M (M_2 + N \rho) + M_2 N (1 - \rho)}\right] \lambda \cdot s - \left[\frac{M_2 kZ}{M (M_2 + N \rho) + M_2 N (1 - \rho)}\right]$$

(65)
II. Second, suppose $x_U = 0$ and the premise

$$\bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p \leq 0 \leq \bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p$$  \hspace{1cm} (66)$$

is satisfied, then the demand function of informed traders and uninformed traders are:

$$x_I = \frac{\bar{f} + \lambda s - p}{k}$$

$$x_U = 0$$  \hspace{1cm} (67)$$

We insert the equation (67) into the market clearing condition, $M \cdot x_I + N \cdot x_U = Z$ and we get,

$$p = \bar{f} + \lambda s - \frac{kZ}{M}$$  \hspace{1cm} (68)$$

Hence, in this situation,

$$A(M) = \bar{A} = \bar{A} = \lambda$$

$$H(M) = \frac{kZ}{M}$$

Hence, we can express the function $G(M)$ as:

$$G(M) = \frac{p + H(M) - \bar{f}}{A(M)}$$

$$= \frac{p + \left( \frac{kZ}{M} \right) - \bar{f}}{\lambda}$$  \hspace{1cm} (69)$$

It is obvious that $\frac{\partial G(M)}{\partial M} < 0$ and

$$M_2 = \bar{M} = \arg \min_{M \in [M_1, M_2]} G(M)$$

$$M_1 = \underline{M} = \arg \max_{M \in [M_1, M_2]} G(M)$$

Hence we can find the explicit expression for $H$ and $\bar{H}$,

$$H = H(M_2) = \frac{kZ}{M_2}$$

$$\bar{H} = H(M_1) = \frac{kZ}{M_1}$$

To satisfy the premise, equation (66), the following two equations show hold for true.

$$\bar{f} + \rho \lambda \cdot \left( \frac{p + \frac{kZ}{M_2} - \bar{f}}{\lambda} \right) - p \leq 0$$

$$\bar{f} + \rho \lambda \cdot \left( \frac{p + \frac{kZ}{M_1} - \bar{f}}{\lambda} \right) - p \geq 0$$

This implies that,

$$\bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_2} \right) \leq p \leq \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_1} \right)$$  \hspace{1cm} (70)$$

Substituting equation (68) into equation (70), we get
\[
\left[ \frac{1}{M} + \frac{\rho}{M_2 (1 - \rho)} \right] kZ \frac{1}{\lambda} \leq s \leq \left[ \frac{1}{M} + \frac{\rho}{M_1 (1 - \rho)} \right] kZ \frac{1}{\lambda}
\]  
(71)

To sum up, in this subsection, when \( \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_2} \right) \leq p \leq \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_1} \right) \), or say, when \( \left[ \frac{1}{M} + \frac{\rho}{M_2 (1 - \rho)} \right] kZ \frac{1}{\lambda} \leq s \leq \left[ \frac{1}{M} + \frac{\rho}{M_1 (1 - \rho)} \right] kZ \frac{1}{\lambda} \), the equilibrium price function is

\[
p = \bar{f} + \lambda s - \frac{kZ}{M}
\]  
(72)

III. Third, suppose \( x_U < 0 \) and the premise

\[
\bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{\lambda} \right) - p < 0
\]  
(73)

is satisfied, then the demand function of informed traders and uninformed traders are:

\[
x_I = \frac{\bar{f} + \lambda s - p}{k} \\
x_U = \frac{\bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{\lambda} \right) - p}{k}
\]  
(74)

We insert the equation (74) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,

\[
M \cdot \left( \frac{\bar{f} + \lambda s - p}{k} \right) + N \cdot \left( \frac{\bar{f} + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{\lambda} \right) - p}{k} \right) = Z
\]

Rearrange the terms and we get,

\[
(M + N - \frac{N \rho \lambda}{A}) \cdot p = \left( M + N - \frac{N \rho \lambda}{A} \right) \bar{f} + M \lambda s - \left( kZ - \frac{N \rho \lambda H}{A} \right)
\]

\[
p = \bar{f} + \left( \frac{M \lambda}{M + N - \frac{N \rho \lambda}{A}} \right) s - \left( \frac{kZ - \frac{N \rho \lambda H}{A}}{M + N - \frac{N \rho \lambda}{A}} \right)
\]  
(75)

Using the undetermined coefficient method and match the coefficient, we get:

\[
A(M) = \frac{M \lambda}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N}
\]  
(76)

\[
H(M) = \frac{kZ - \frac{N \rho \lambda H}{A}}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N}
\]
Hence, we can express the function $G(M)$ as:

$$G(M) = \frac{p + H(M) - \bar{f}}{A(M)}$$

$$G(M) = p + \left[ \frac{kZ - \frac{N\rho\Pi}{A}}{M + \left(1 - \frac{\rho\lambda}{A}\right)N} \right] - \bar{f}$$

$$= \left[ \frac{M\lambda}{M + \left(1 - \frac{\rho\lambda}{A}\right)N} \right]$$

$$= \frac{(p - \bar{f}) \left[ M + \left(1 - \frac{\rho\lambda}{A}\right)N \right] + \left[ kZ - \frac{N\rho\Pi}{A} \right]}{M\lambda}$$

(77)

We could find the partial derivative of $G(M)$ w.r.t $M$,

$$\frac{\partial G(M)}{\partial M} = \frac{(p - \bar{f}) M - \left\{ (p - \bar{f}) \left[ M + \left(1 - \frac{\rho\lambda}{A}\right)N \right] + \left[ kZ - \frac{N\rho\Pi}{A} \right] \right\}}{M^2\lambda}$$

$$= -N \left[ (p - \bar{f}) \left(1 - \frac{\rho\lambda}{A}\right) - \frac{\rho\lambda\Pi}{A} \right] - kZ$$

$$= \frac{-NL - kZ}{M^2\lambda}$$

(78)

We define $L = (p - \bar{f}) \left(1 - \frac{\rho\lambda}{A}\right) - \frac{\rho\lambda\Pi}{A}$.

Since at the beginning of this sub-case III, our main premise is that

$$\bar{f} + \rho\lambda \cdot \frac{p + \Pi - \bar{f}}{A} - p < 0$$

This premise can be written as:

$$- \left[ (p - \bar{f}) \left(1 - \frac{\rho\lambda}{A}\right) - \frac{\rho\lambda\Pi}{A} \right] < 0$$

This implies that $L > 0$, and the sign of $\frac{\partial G(M)}{\partial M}$ will be determined by the sign of $(-NL - kZ)$, which is negative. It is obvious that $\frac{\partial G(M)}{\partial M} < 0$ and

$$M_1 = \bar{M} = \arg \max_{M \in [M_1, M_2]} G(M)$$

Inserting $\bar{M} = M_1$ into equation (76), and we get,

$$\bar{A} = A(M_1) = \frac{M_1\lambda}{M_1 + \left(1 - \frac{\rho\lambda}{A}\right)N}$$

$$\bar{H} = H(M_1) = \frac{kZ - \frac{N\rho\Pi}{A}}{M_1 + \left(1 - \frac{\rho\lambda}{A}\right)N}$$

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Solving for $\bar{A}$ and $\bar{H}$, and we find the explicit expression:

$$A = \left( \frac{M_1 + N\rho}{M_1 + N} \right) \lambda$$

(79)

$$H = \frac{kZ}{M_1 + N}$$

Inserting equation (79) into equation (75), and we find the price function,

$$p = f + \left( \frac{M\lambda}{M + N - N\rho\lambda} \right) s - \left( \frac{kZ - N\rho H}{A} \frac{1}{A} \right)$$

(80)

$$= f + \left[ \frac{M(M_1 + N\rho)}{M(M_1 + N\rho) + M_1 N (1 - \rho)} \right] \lambda \cdot s - \left[ \frac{M_1 kZ}{M(M_1 + N\rho) + M_1 N (1 - \rho)} \right]$$

We need to check that the premise, equation (73), should be satisfied.

Premise (73) and equation (79) imply that,

$$p > f + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_1} \right)$$

(81)

Substituting equation (80) into equation (81), we get

$$s > \left[ \frac{1}{M} + \frac{\rho}{M_1 (1 - \rho)} \right] \frac{kZ}{\lambda}$$

(82)

To sum up, in this subsection, when $p > f + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_1} \right)$, or say, when $s > \left[ \frac{1}{M} + \frac{\rho}{M_1 (1 - \rho)} \right] \frac{kZ}{\lambda}$, the equilibrium price function is

$$p = f + \left[ \frac{M(M_1 + N\rho)}{M(M_1 + N\rho) + M_1 N (1 - \rho)} \right] \lambda \cdot s - \left[ \frac{M_1 kZ}{M(M_1 + N\rho) + M_1 N (1 - \rho)} \right]$$

(83)

**Finally**, to summarize, combing the results of sub-case I, II, III, when the number of the uninformed trader $N$ is known and the number of the informed trader $M$ is unknown with the ambiguity, there exists a Rational Expectation Equilibrium (REE) in which the price function $p$ is piecewise in $s$ and $M$,

$$p(s, M) = \begin{cases} 
\tilde{f} + \left[ \frac{M(M_1 + N\rho)}{M(M_1 + N\rho) + M_1 N (1 - \rho)} \right] \lambda \cdot s - \left[ \frac{M_1 kZ}{M(M_1 + N\rho) + M_1 N (1 - \rho)} \right], & \text{for } s \in (-\infty, s_1) \cup (s_3, +\infty) \\
\tilde{f} + \left[ \frac{M(M_2 + N\rho)}{M(M_2 + N\rho) + M_2 N (1 - \rho)} \right] \lambda \cdot s - \left[ \frac{M_2 kZ}{M(M_2 + N\rho) + M_2 N (1 - \rho)} \right], & \text{for } s \in [s_1, s_2) \\
\tilde{f} + \lambda s - \frac{kZ}{M}, & \text{for } s \in [s_2, s_3] 
\end{cases}$$

(84)

with

$$s_1 = \frac{-kZ}{N\lambda (1 - \rho)}$$

$$s_2 = \left[ \frac{1}{M} + \frac{\rho}{M_2 (1 - \rho)} \right] \frac{kZ}{\lambda}$$

(85)

$$s_3 = \left[ \frac{1}{M} + \frac{\rho}{M_1 (1 - \rho)} \right] \frac{kZ}{\lambda}$$
B.2 Proof of Proposition 4

Proof. 1. When \( p < f - \left(\frac{1}{1-p}\right) \left(\frac{k Z}{N}\right) \), or say, when \( s < \frac{k Z}{N\lambda(1-\rho)} \), using equation (46) and (84), we find that the demand function of uninformed trader is:

\[
x_U(p) = \frac{1}{k} \left[ \left(1 - \frac{\rho\lambda}{A}\right) \left(f - p\right) + \frac{\rho\lambda H}{A} \right] = \frac{1}{k} \left[ \frac{M_1 (1-\rho) \bar{f} + \rho k Z}{M_1 + N\rho} - \frac{M_1 (1-\rho)}{M_1 + N\rho} \cdot p \right] > 0 \tag{86}
\]

2. When \( f - \left(\frac{1}{1-p}\right) \left(\frac{k Z}{N}\right) < p < f + \left(\frac{\rho}{1-p}\right) \left(\frac{k Z}{M_2}\right) \), or say, when \( -k Z < s < \frac{k Z}{M_2(1-\rho)} \), using equation (46) and (84), we find that the demand function of uninformed trader is:

\[
x_U(p) = \frac{1}{k} \left[ \left(1 - \frac{\rho\lambda}{A}\right) \left(f - p\right) + \frac{\rho\lambda H}{A} \right] = \frac{1}{k} \left[ \frac{M_2 (1-\rho) \bar{f} + \rho k Z}{M_2 + N\rho} - \frac{M_2 (1-\rho)}{M_2 + N\rho} \cdot p \right] > 0 \tag{87}
\]

3. When \( f + \left(\frac{\rho}{1-p}\right) \left(\frac{k Z}{M_2}\right) \leq p \leq f + \left(\frac{\rho}{1-p}\right) \left(\frac{k Z}{M_1}\right) \), or say, when \( \frac{k Z}{M_1(1-\rho)} \leq s \leq \frac{k Z}{M_2(1-\rho)} \), using equation (46) and (84), we find that the demand function of uninformed trader is:

\[
x_U(p) = 0 \tag{88}
\]

4. When \( p > f + \left(\frac{\rho}{1-p}\right) \left(\frac{k Z}{M_1}\right) \), or say, when \( s > \frac{k Z}{M_1(1-\rho)} \), using equation (46) and (84), we find that the demand function of uninformed trader is:

\[
x_U(p) = \frac{1}{k} \left[ \left(1 - \frac{\rho\lambda}{A}\right) \left(f - p\right) + \frac{\rho\lambda H}{A} \right] = \frac{1}{k} \left[ \frac{M_1 (1-\rho) \bar{f} + \rho k Z}{M_1 + N\rho} - \frac{M_1 (1-\rho)}{M_1 + N\rho} \cdot p \right] < 0 \tag{89}
\]

Finally, to summarize, combing the results of 1, 2, 3, 4, when the number of the uninformed trader \( N \) is known and the number of the informed trader \( M \) is unknown with the ambiguity, in equilibrium, the demand function of uninformed traders is characterized as,

\[
x_U(p) = \begin{cases} 
\frac{1}{k} \left[ \frac{M_1 (1-\rho) \bar{f} + \rho k Z}{M_1 + N\rho} - \frac{M_1 (1-\rho)}{M_1 + N\rho} \cdot p \right], & \text{Buy, for } p \in (-\infty, p_1) \\
\frac{1}{k} \left[ \frac{M_2 (1-\rho) \bar{f} + \rho k Z}{M_2 + N\rho} - \frac{M_2 (1-\rho)}{M_2 + N\rho} \cdot p \right], & \text{Buy, for } p \in [p_1, p_2) \\
0, & \text{Not Participate, for } p \in [p_2, p_3] \\
\frac{1}{k} \left[ \frac{M_3 (1-\rho) \bar{f} + \rho k Z}{M_3 + N\rho} - \frac{M_3 (1-\rho)}{M_3 + N\rho} \cdot p \right], & \text{Sell, for } p \in (p_3, +\infty)
\end{cases} \tag{90}
\]
with $p_1$, $p_2$ and $p_3$ are three constants,

$$p_1 = f - \left( \frac{1}{1 - \rho} \right) \left( \frac{kZ}{N} \right)$$

$$p_2 = f + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_2} \right)$$

$$p_3 = f + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_1} \right)$$

\[ (91) \]

\[ \square \]

\section{Proof For Section 5}

\subsection{Proof of Proposition 5}

\textbf{Proof.} From proposition 1 and 3, we find that,

$$\frac{\partial p(s)}{\partial s} = \left( \frac{M + N\rho}{M + N} \right) \lambda$$

\[ (92) \]

$$\frac{\partial p(s,M)}{\partial s} = \left[ \frac{M(M_1 + N\rho)}{M(M_1 + N\rho) + M_1 N(1 - \rho)} \right] \lambda, \quad \text{for } s \in (-\infty, s_1) \cup [s_3, +\infty)$$

\[ (93) \]

$$\frac{\partial p(s,M)}{\partial s} = \lambda, \quad \text{for } s \in [s_1, s_2)$$

\[ (94) \]

It is easy to show that,

$$\left[ \frac{M(M_1 + N\rho)}{M(M_1 + N\rho) + M_1 N(1 - \rho)} \right] \lambda - \left( \frac{M + N\rho}{M + N} \right) \lambda = \frac{N^2 \rho(1 - \rho)(M - M_1)}{[M (M_1 + N\rho) + M_1 N(1 - \rho)] [M + N]} \lambda \geq 0$$

\[ (95) \]

$$\lambda > \frac{M + N\rho}{M + N} \lambda$$

\[ (96) \]

\[ \square \]

\subsection{Proof of Proposition 6}

\textbf{Proof. I. Benchmark} Using proposition 1 and 2, we find that,

$$\frac{\partial X_I}{\partial s} = \frac{N(1 - \rho)\lambda}{k(M + N)} > 0$$

\[ (97) \]
Therefore, in the benchmark case, informed traders’ equilibrium asset demand is increasing in the signal \( s \), while uninformed traders’ equilibrium asset demand is decreasing in the signal \( s \).

II. Case \( A - M \)  
Using proposition 3 and 4, we find that,

\[
\frac{\partial x_u}{\partial s} = \frac{-M(1-\rho)\lambda}{k(M+N)} < 0 \tag{98}
\]

Therefore, in the case \( A - M \), informed traders’ equilibrium asset demand is increasing in the signal \( s \), while uninformed traders’ equilibrium asset demand is decreasing in the signal \( s \).  

C.3 Proof of Proposition 7

Proof. Trading volume is formally defined as: \( V = M |x_I| + N \, |x_u| \).

I. Benchmark  
Using proposition 1 and 2, we find that,

\[
V^\text{Benchmark} = M |x_I| + N \, |x_u| = M \left( \frac{1}{k(M+N)} \, |kZ + N(1-\rho)\lambda s| + N \, \frac{1}{k(M+N)} \, |kZ - M(1-\rho)\lambda s| \right) \tag{101}
\]

II. Case \( A - M \)  
Using proposition 3 and 4, we find that,
Hence we could find the expression of the trading volume in case $A - M$.

**Proof.** I focus on linear market equilibrium, where informed trader’s strategy and uninformed trader’s strategy are, and then show that this equilibrium result coincides with the equilibrium result of proposition 3.

### D.1 Proof of Proposition 8

To prove proposition 8, we need to first find the demand function game equilibrium with ambiguity following the definition 3 and then show that this equilibrium result coincides with the equilibrium result of proposition 3.

#### D.1 Proof of Proposition 8

**Proof.** I first solve for the demand function game equilibrium with ambiguity. Following Kyle (1989), I focus on linear market equilibrium, where informed trader’s strategy and uninformed trader’s strategy are,

$$x_I(s, p) = \eta + \beta s - \gamma p$$  \hspace{1cm} (104)

$$x_U(p) = \begin{cases} 
\phi_1 - \omega_1 p, & \text{Buy and believe } M = M_1 \text{ for } p \in (-\infty, p_1) \\
\phi_2 - \omega_2 p, & \text{Buy and believe } M = M_2 \text{ for } p \in [p_1, p_2) \\
0, & \text{Not Participate, for } p \in [p_2, p_3] \\
\phi_3 - \omega_3 p, & \text{Sell and believe } M = M_1 \text{ for } p \in (p_3, +\infty) 
\end{cases}$$  \hspace{1cm} (105)

Hence we could find the expression of the trading volume in case $A - M$ as

$$V_{A - M} = M|x_I| + N|x_U|$$

\[\square\]
Using the method of undetermined coefficients and matching the coefficient of equation (7) and equation (104), we solve for the expression for \( \eta, \beta \) and \( \gamma \).

\[
\eta = \frac{f}{k}, \quad \beta = \frac{\lambda}{k}, \quad \gamma = \frac{1}{k}
\]  

(106)

In equilibrium, price will clear the market:

\[
\begin{align*}
M (\eta + \beta s - \gamma p) + N (\phi_1 - \omega_1 p) &= Z, \quad \text{for } p \in (-\infty, p_1) \\
M (\eta + \beta s - \gamma p) + N (\phi_2 - \omega_2 p) &= Z, \quad \text{for } p \in [p_1, p_2) \\
M (\eta + \beta s - \gamma p) &= Z, \quad \text{for } p \in [p_2, p_3] \\
M (\eta + \beta s - \gamma p) + N (\phi_3 - \omega_3 p) &= Z, \quad \text{for } p \in (p_3, +\infty)
\end{align*}
\]

(107)

\[
\begin{align*}
p &= \frac{M \beta}{M_\gamma + N \omega_1} \cdot s + \frac{M \eta + N \phi_1 - Z}{M_\gamma + N \omega_1}, \quad \text{for } p \in (-\infty, p_1) \\
p &= \frac{M \beta}{M_\gamma + N \omega_2} \cdot s + \frac{M \eta + N \phi_2 - Z}{M_\gamma + N \omega_2}, \quad \text{for } p \in [p_1, p_2) \\
p &= \frac{\beta}{\gamma} \cdot s + \frac{M \eta - Z}{M_\gamma}, \quad \text{for } p \in [p_2, p_3] \\
p &= \frac{M \beta}{M_\gamma + N \omega_3} \cdot s + \frac{M \eta + N \phi_3 - Z}{M_\gamma + N \omega_3}, \quad \text{for } p \in (p_3, +\infty)
\end{align*}
\]

(108)

For the uninformed trader, price \( p \) is informationally equivalent as \( s_M \).

\[
s_M = \begin{cases} 
\frac{\gamma p - \eta \beta}{\beta} + \frac{N \omega_1 p - N \phi_1 + Z}{M \beta}, & \text{for } p \in (-\infty, p_1) \\
\frac{\gamma p - \eta \beta}{\beta} + \frac{N \omega_2 p - N \phi_2 + Z}{M \beta}, & \text{for } p \in [p_1, p_2) \\
N/A, & \text{for } p \in [p_2, p_3] \\
\frac{\gamma p - \eta \beta}{\beta} + \frac{N \omega_3 p - N \phi_3 + Z}{M \beta}, & \text{for } p \in (p_3, +\infty)
\end{cases}
\]

(109)

Hence, for uninformed traders, the conditional moments of \( f_U \) taken under a particular belief \( M \), are given by:

\[
E_M [f_U \mid p] = E_M \left[ f + \theta_U + \epsilon \mid p \right] = f + E_M [\theta_U \mid p] + 0 = f + E_M [\theta_U \mid s = s_M] = f + \text{cov} (s, \theta_U) \frac{1}{\text{var}(s)} s_M = f + \left( \frac{\rho \tau^{-1}}{\tau^{-1} + \tau_u^{-1}} \right) s_M
\]

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The objective function of an uninformed trader can be written as:

\[
\min_{M \in [M_1, M_2]} \left( (E_M [fU | p] - p) \cdot xU - \frac{k}{2} x^2 U \right)
\]

\[
\Rightarrow \min_{M \in [M_1, M_2]} \left( (\bar{f} + \rho \lambda \cdot s_M - p) \cdot xU - \frac{k}{2} x^2 U \right)
\]

\[
= \begin{cases} 
-\frac{k}{2} x^2 U + [\bar{f} + \rho \lambda \cdot \{s_M\}_{\text{min}} - p] \cdot xU, & \text{if} \; xU > 0 \\
0, & \text{if} \; xU = 0 \\
-\frac{k}{2} x^2 U + [\bar{f} + \rho \lambda \cdot \{s_M\}_{\text{max}} - p] \cdot xU, & \text{if} \; xU < 0
\end{cases} (110)
\]

where \(\{s_M\}_{\text{min}}\) and \(\{s_M\}_{\text{max}}\) are the minimum and maximum of the function \(\{s_M\}\), given the value of \(p\), respectively. Thus an uninformed trader’s demand function is:

\[
xU(p) = \begin{cases} 
\bar{f} + \rho \lambda \cdot \{s_M\}_{\text{min}} - p, & \text{if} \; \bar{f} + \rho \lambda \cdot \{s_M\}_{\text{min}} - p > 0 \\
0, & \text{if} \; \bar{f} + \rho \lambda \cdot \{s_M\}_{\text{min}} - p \leq 0 \leq \bar{f} + \rho \lambda \cdot \{s_M\}_{\text{max}} - p \\
\bar{f} + \rho \lambda \cdot \{s_M\}_{\text{max}} - p, & \text{if} \; \bar{f} + \rho \lambda \cdot \{s_M\}_{\text{max}} - p < 0
\end{cases} (111)
\]

Inserting equation ?? into ??, and then using the method of undetermined coefficients, we find the demand function game equilibrium with ambiguity.

1. **Case I Uninformed traders buy and believe** \(M = M_1\) for \(p \in (-\infty, p_1)\)

Using equation (106), (109) and (111), solving for the \(xU\) and match the coefficients gives the solution for \(\phi_1\) and \(\omega_1\).

\[
\frac{\bar{f} + \rho \lambda \cdot \left( \frac{\gamma p - \eta}{\beta} + \frac{N\omega_1 p - N\phi_1 + Z}{M_1 \beta} \right) - p}{k} = \phi_1 - \omega_1 p
\]

\[
\Rightarrow \begin{cases} 
\frac{\bar{f} + \rho \lambda \cdot \left( \frac{\gamma p - \eta}{\beta} + \frac{N\omega_1 p - N\phi_1 + Z}{M_1 \beta} \right) - 1}{k} = -\omega_1 \\
\bar{f} + \rho \lambda \cdot \left( \frac{\gamma p - \eta}{\beta} + \frac{N\phi_1 + Z}{M_1 \beta} \right) = \phi_1
\end{cases} \]

\[
\Rightarrow \begin{cases} 
\omega_1 = \frac{1}{k} \left[ \frac{M_1 (1-p)}{M_1 + Np} \right] \\
\phi_1 = \frac{1}{k} \left[ \frac{M_1 (1-p) \bar{f} + \rho k Z}{M_1 + Np} \right]
\end{cases} (114)
\]

We need to check that two premise should be satisfied.

\[
\begin{cases} 
N\omega_1 p - N\phi_1 + Z < 0 \\
\bar{f} + \rho \lambda \cdot \left( \frac{\gamma p - \eta}{\beta} + \frac{N\omega_1 p - N\phi_1 + Z}{M_1 \beta} \right) - p > 0
\end{cases} (115)
\]
Inserting equation (106) and (114) into (115), we get

\[ p < \bar{f} - \left( \frac{1}{1 - \rho} \right) \left( \frac{kZ}{N} \right) \equiv p_1 \]  

(116)

2. Case II Uninformed traders buy and believe \( M = M_2 \) for \( p \in [p_1, p_2) \)

Using equation (106), (109) and (111), solving for the \( x_U \) and match the coefficients gives the solution for \( \phi_2 \) and \( \omega_2 \).

\[ \bar{f} + \rho \lambda \cdot \left( \frac{2p - \eta}{\beta} + \frac{N \omega_2 p - N \phi_2 + Z}{M_2 \beta} \right) - p = \phi_2 - \omega_2 p \]  

(117)

\[ \Rightarrow \begin{cases} \frac{\rho \lambda^2}{\beta} + \rho \lambda \frac{N \omega_2}{M_2 \beta} - 1 = -\omega_2 \\ \bar{f} + \rho \lambda \left( \frac{2p - \eta}{\beta} + \frac{N \omega_2 p - N \phi_2 + Z}{M_2 \beta} \right) = \phi_2 \end{cases} \]  

(118)

\[ \Rightarrow \begin{cases} \omega_2 = \frac{1}{k} \left[ \frac{M_2 (1 - \rho)}{M_2 + N \rho} \right] \\ \phi_2 = \frac{1}{k} \left[ \frac{M_2 (1 - \rho) f + \rho k Z}{M_2 + N \rho} \right] \end{cases} \]  

(119)

We need to check that two premise should be satisfied.

\[ \begin{cases} N \omega_2 p - N \phi_2 + Z > 0 \\ \bar{f} + \rho \lambda \cdot \left( \frac{2p - \eta}{\beta} + \frac{N \omega_2 p - N \phi_2 + Z}{M_2 \beta} \right) - p > 0 \end{cases} \]  

(120)

Inserting equation (106) and (119) into (120), we get

\[ p_1 = \bar{f} - \left( \frac{1}{1 - \rho} \right) \left( \frac{kZ}{N} \right) \leq p < \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_2} \right) \equiv p_2 \]  

(121)

3. Case III Uninformed traders sell and believe \( M = M_1 \) for \( p \in (p_3, +\infty) \)

Using equation (106), (109) and (111), solving for the \( x_U \) and match the coefficients gives the solution for \( \phi_3 \) and \( \omega_3 \).

\[ \bar{f} + \rho \lambda \cdot \left( \frac{2p - \eta}{\beta} + \frac{N \omega_3 p - N \phi_3 + Z}{M_1 \beta} \right) - p = \phi_3 - \omega_3 p \]  

(122)

\[ \Rightarrow \begin{cases} \frac{\rho \lambda^2}{\beta} + \rho \lambda \frac{N \omega_3}{M_1 \beta} - 1 = -\omega_3 \\ \bar{f} + \rho \lambda \left( \frac{2p - \eta}{\beta} + \frac{N \omega_3 p - N \phi_3 + Z}{M_1 \beta} \right) = \phi_3 \end{cases} \]  

(123)
\[
\begin{align*}
\omega_3 &= \frac{1}{k} \left[ \frac{M_1(1-\rho)}{M_1 + k\rho} \right] = \omega_1 \\
\phi_3 &= \frac{1}{k} \left[ \frac{M_1(1-\rho)\bar{f} + k\bar{Z}}{M_1 + k\rho} \right] = \phi_1
\end{align*}
\]

We need to check that two premise should be satisfied.

\[
\begin{align*}
N\omega_1 p - N\phi_1 + Z > 0 \\
\bar{f} + \rho \lambda \cdot \left( \frac{\gamma p - \eta}{p} + \frac{N\omega_1 p - N\phi_1 + Z}{M_1 p} \right) - p < 0
\end{align*}
\]

Inserting equation (106) and (124) into (125), we get

\[
p > \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( k\bar{Z} \right) M_1 \equiv p_3
\]

4. **Case IV** This case is trivial, for uninformed traders do not participate in stock markets. Hence, \( x_U = 0 \) in this case. We need to check that two premise should be satisfied.

\[
\begin{align*}
\bar{f} + \rho \lambda \cdot \left( \frac{\gamma p - \eta}{p} + \frac{N\omega_2 p - N\phi_2 + Z}{M_2 p} \right) - p < 0 \\
\bar{f} + \rho \lambda \cdot \left( \frac{\gamma p - \eta}{p} + \frac{N\omega_3 p - N\phi_3 + Z}{M_3 p} \right) - p > 0
\end{align*}
\]

Inserting equation (106), (119) and (124) into (127), we get

\[
p_2 \equiv \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( k\bar{Z} \right) M_2 < p < \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( k\bar{Z} \right) M_1 \equiv p_3
\]

To summarize, combining the results of case I, II, III, IV, we find the demand function game equilibrium with ambiguity. This demand function game equilibrium coincides with the proposition 3 and 4. Hence, we conclude that the (competitive) REE of proposition 3 is an implementable REE and it can be implemented through a demand schedule game.

\[\square\]

E  **Proof For Section 6.2**

In this section, I characterize the financial market equilibrium where uninformed traders exhibit \( \alpha \)-maxmin expected utility preference (\( \alpha \)-MEU). To find the equilibrium, we need to first characterize the demand function of both informed and uninformed traders.

### E.1 Demand Function of Informed Traders

By observing the realization of \( s \), informed traders resolve their ambiguity straight away. They choose portfolio holdings \( x_I \) to maximize the expected profits \( \pi_I \)

\[
\max_{x_I} E \left[ (\bar{f}_I - p) x_I - \frac{k}{2} x_I^2 \mid \mathcal{F}_I = \{s, p\} \right],
\]

\[\text{(129)}\]
where $p$ is the observed asset price. Standard arguments yield

$$x_I(s, p) = \frac{E(f_I \mid s, p) - p}{k}$$

$$= \frac{E(\bar{f} + \theta_I + \epsilon \mid s) - p}{k}$$

$$= \frac{\bar{f} + E(\theta_I \mid s) - p}{k}$$

$$= \frac{\bar{f} + \frac{\text{cov}(s, \theta_I)}{\text{var}(s)} s - p}{k}$$

$$= \frac{\bar{f} + \left(\frac{\tau^{-1}}{\tau^{-1} + \tau_u^{-1}}\right) s - p}{k}$$

$$= \frac{\bar{f} + \lambda s - p}{k},$$

(130)

and the informativeness of the signal is captured by the signal-to-noise ratio:

$$\lambda \equiv \frac{\text{cov}(s, \theta_I)}{\text{var}(s)} = \frac{\tau^{-1}}{\tau^{-1} + \tau_u^{-1}}$$

(131)

### E.2 Demand Function of Uninformed Traders

At trading stage, for any given $M$ and $N$, uninformed traders rationally conjecture that the price function is:

$$p = \bar{f} + A(M; N) \cdot s - H(M; N)$$

(132)

where the function $A(M; N)$ and $H(M; N)$ will be endogenously determined in equilibrium. Since $N$ is known by the uninformed traders for sure, $N$ is an exogenous parameter here. For simplicity, we write $A(M) \equiv A(M; N)$ and $H(M) \equiv H(M; N)$ and the conjectured price function can be written as:

$$p = \bar{f} + A(M) \cdot s - H(M)$$

(133)

Thus, the optimal demand of uninformed traders is determined by

$$\max_{X^{u}} \left( \begin{array}{c}
\alpha \cdot \min_{M \in [M_1, M_2]} \left( E_M \left[ \left( f_U - p \right) X^u - \frac{k}{2} X^u_2 \mid F_U = \{p\} \right] \right) \\
+ (1 - \alpha) \cdot \max_{M \in [M_1, M_2]} \left( E_M \left[ \left( f_U - p \right) X^u - \frac{k}{2} X^u_2 \mid F_U = \{p\} \right] \right) 
\end{array} \right)$$

(134)

where $x_{1u}$ is the asset demand of uninformed traders, and $E_M(\cdot)$ is the expectation operator taken under the belief that the size/total number of the informed traders is $M$. Since uninformed traders are ambiguous about $M$, they view the stock price $p$ as an ambiguous signal about $f_U$. This makes the inference problem of the uninformed traders more interesting and generates the results we will see in the following sections.  

Footnote 4: Under the belief that the size/total number of the informed traders is $M$, they map from the observed price $p$ to the extracted signal $s_M$:

$$s_M = \frac{p + H(M) - \bar{f}}{A(M)}$$

(135)

Footnote 4: While formulating portfolio decisions, uninformed traders learn from the price and update each of their beliefs. This rule is known as full Bayesian updating. With full Bayesian updating, no learning occurs regarding the original set of priors, which means that the uninformed agents retain all their initial priors.
The conditional moments of \( f_U \) taken under a particular belief \( M \), are given by:

\[
E_M[f_U \mid p] = E_M[\bar{f} + \theta U + \varepsilon \mid p] \\
= \bar{f} + E_M[\theta U \mid p] + 0 \\
= \bar{f} + E_M[\theta U \mid s = s_M] \\
= \bar{f} + \frac{\text{cov}(s, \theta U)}{\text{var}(s)} s_M \\
= \bar{f} + \left( \frac{\rho \tau^{-1}}{\tau^{-1} + \tau_u^{-1}} \right) s_M \\
= \bar{f} + \rho \lambda \left[ p + H(M) - \bar{f} \right] \\
= \bar{f} + \rho \lambda \cdot G(M; p, \bar{f}, N)
\]  

(136)

For simplicity, we write

\[
G(M) \equiv G(M; p, \bar{f}, N) = \frac{p + H(M) - \bar{f}}{A(M)}
\]

and the objective function of an uninformed trader can be written as:

\[
\alpha \cdot \min_{M \in [M_1, M_2]} \left( E_M \left[ (f_U - p) x_U - \frac{k}{2} x_U^2 \mid F_U = \{p\} \right] \right) \\
+ (1 - \alpha) \cdot \max_{M \in [M_1, M_2]} \left( E_M \left[ (f_U - p) x_U - \frac{k}{2} x_U^2 \mid F_U = \{p\} \right] \right)
\]

\[
\Rightarrow \alpha \cdot \min_{M \in [M_1, M_2]} \left( (E_M[f_U \mid p] - p) \cdot x_U - \frac{k}{2} x_U^2 \right) \\
+ (1 - \alpha) \cdot \max_{M \in [M_1, M_2]} \left( (E_M[f_U \mid p] - p) \cdot x_U - \frac{k}{2} x_U^2 \right)
\]

\[
\Rightarrow \alpha \cdot \min_{M \in [M_1, M_2]} \left( (\bar{f} + \rho \lambda \cdot G(M) - p) \cdot x_U - \frac{k}{2} x_U^2 \right) \\
+ (1 - \alpha) \cdot \max_{M \in [M_1, M_2]} \left( (\bar{f} + \rho \lambda \cdot G(M) - p) \cdot x_U - \frac{k}{2} x_U^2 \right)
\]  

(137)

\[
\begin{cases} 
- \frac{k}{2} x_U^2 + \left[ \bar{f} + \rho \lambda \cdot \{ \alpha G_{\min} + (1 - \alpha) G_{\max} \} - p \right] \cdot x_U, & \text{if } x_U > 0 \\
0, & \text{if } x_U = 0 \\
- \frac{k}{2} x_U^2 + \left[ \bar{f} + \rho \lambda \cdot \{ \alpha G_{\max} + (1 - \alpha) G_{\min} \} - p \right] \cdot x_U, & \text{if } x_U < 0
\end{cases}
\]

where \( G_{\min} = \left\{ \frac{p + H(M) - \bar{f}}{A(M)} \right\}_{\min} \) and \( G_{\max} = \left\{ \frac{p + H(M) - \bar{f}}{A(M)} \right\}_{\max} \) are the minimum and maximum of the function \( \{ G(M) \} \), given the value of \( p \), respectively.
Thus an uninformed trader’s demand function is:

\[
x_U (p) = \begin{cases} 
  \frac{\hat{f} + \rho \lambda \cdot \{aG_{\min} + (1 - \alpha)G_{\max}\} - p}{k}, & \text{if } \hat{f} + \rho \lambda \cdot \{aG_{\min} + (1 - \alpha)G_{\max}\} > p \\
  0, & \text{if } \hat{f} + \rho \lambda \cdot \{aG_{\min} + (1 - \alpha)G_{\max}\} \leq \hat{f} + \rho \lambda \cdot \{aG_{\max} + (1 - \alpha)G_{\min}\} \\
  \frac{\hat{f} + \rho \lambda \cdot \{aG_{\max} + (1 - \alpha)G_{\min}\} - p}{k}, & \text{if } \hat{f} + \rho \lambda \cdot \{aG_{\max} + (1 - \alpha)G_{\min}\} < p
\end{cases}
\]

\[\text{E.3 Proof of Proposition 9}\]

We define the value of \(M\) that causes the function \(G(M)\) to reach a minimum as \(\underline{M}\) and the value of \(M\) that causes the function \(G(M)\) to reach a maximum as \(\overline{M}\),

\[
\underline{M} \equiv \arg \min_{M \in [M_1, M_2]} G(M) = \arg \min_{M \in [M_1, M_2]} \left[ \frac{p + H(M) - \hat{f}}{A(M)} \right]
\]

\[
\overline{M} \equiv \arg \max_{M \in [M_1, M_2]} G(M) = \arg \max_{M \in [M_1, M_2]} \left[ \frac{p + H(M) - \hat{f}}{A(M)} \right]
\]

We define the values that \(H(M)\) and \(A(M)\) take on when \(M = \underline{M}\) and \(M = \overline{M}\) as:

\[
H \equiv H(M) \quad \overline{H} \equiv H(\overline{M})
\]

\[
\underline{A} \equiv A(M) \quad \overline{A} \equiv A(\overline{M})
\]

Note that \(H, \overline{H}, \underline{A}\) and \(\overline{A}\) will be a constant. Hence, we can define the maximum and minimum of \(G(M)\) as:

\[
\left\{ \frac{p + H(M) - \hat{f}}{A(M)} \right\}_{\min} \equiv \underline{G} = \frac{p + H - \hat{f}}{\underline{A}}
\]

\[
\left\{ \frac{p + H(M) - \hat{f}}{A(M)} \right\}_{\max} \equiv \overline{G} = \frac{p + \overline{H} - \hat{f}}{\overline{A}}
\]

Hence, the optimal demand functions of the uninformed traders, equation (138), can be written as,

\[
x_U (p) = \begin{cases} 
  \frac{\hat{f} + \rho \lambda \cdot \{aG_{\min} + (1 - \alpha)G\} - p}{k}, & \text{if } \hat{f} + \rho \lambda \cdot \{aG_{\min} + (1 - \alpha)G\} > p \\
  0, & \text{if } \hat{f} + \rho \lambda \cdot \{aG_{\min} + (1 - \alpha)G\} \leq \hat{f} + \rho \lambda \cdot \{aG_{\max} + (1 - \alpha)G\} \\
  \frac{\hat{f} + \rho \lambda \cdot \{aG_{\max} + (1 - \alpha)G\} - p}{k}, & \text{if } \hat{f} + \rho \lambda \cdot \{aG_{\max} + (1 - \alpha)G\} < p
\end{cases}
\]

We evaluate the feasibility of the possible demand functions given in equation (142) case by case by inserting it into the market clearing condition.

**I. First**, suppose \(x_U > 0\) and the premise

\[
\hat{f} + \rho \lambda \cdot \{aG_{\min} + (1 - \alpha)G\} - p > 0
\]

\[
\Rightarrow \hat{f} + \rho \lambda \cdot \left\{ a \left( \frac{p + H - \hat{f}}{\underline{A}} \right) + (1 - \alpha) \left( \frac{p + \overline{H} - \hat{f}}{\overline{A}} \right) \right\} - p > 0
\]

(143)

is satisfied, then the demand function of informed traders and uninformed traders are:

\[
x_I = \frac{\hat{f} + \lambda s - p}{k}
\]

(144)
\[ x_U = \frac{\bar{f} + \rho \lambda \cdot \{aG + (1 - \alpha)\bar{G}\} - p}{k} \]

We insert the equation (144) and (145) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,

\[ M \cdot \left( \frac{\bar{f} + \lambda s - p}{k} \right) + N \cdot \left( \frac{\bar{f} + \rho \lambda \cdot \{a \left( \frac{p + H - f}{A} \right) + (1 - \alpha) \left( \frac{p + H - f}{A} \right) \} - p}{k} \right) = Z \]

Rearrange the terms and we get,

\[ p = \bar{f} + \frac{M \lambda}{M + \left( 1 - \alpha \frac{\rho A}{A} \right) - (1 - \alpha) \frac{\rho A}{A}} N \quad s = \frac{kZ - \alpha \frac{N \rho A}{A} - (1 - \alpha) \frac{N \rho A}{A}}{M + \left( 1 - \alpha \frac{\rho A}{A} \right) - (1 - \alpha) \frac{\rho A}{A}} N \]  

Using the undetermined coefficient method and match the coefficient, we get:

\[ A(M) = \frac{\lambda}{M + \left( 1 - \alpha \frac{\rho A}{A} \right) - (1 - \alpha) \frac{\rho A}{A}} N \]  

\[ H(M) = \frac{kZ - \alpha \frac{N \rho A}{A} - (1 - \alpha) \frac{N \rho A}{A}}{M + \left( 1 - \alpha \frac{\rho A}{A} \right) - (1 - \alpha) \frac{\rho A}{A}} N \]

Inserting \( M \) or \( \bar{M} \) into equation (147), rearrange the terms and we get,

\[ A = A(M) = \frac{\lambda}{M + \left( 1 - (1 - \alpha) \frac{\rho A}{A} \right) N} \]

\[ \bar{A} = A(\bar{M}) = \frac{\lambda}{\bar{M} + \left( 1 - \alpha \rho N \right)} N \]

Hence, we could find the explicit expression of \( A \) and \( \bar{A} \),

\[ A = \left[ \frac{\bar{M}M + (1 - \alpha) M \rho N + \alpha \bar{M} \rho N}{\bar{M}M + NM - (\bar{M} - M) (1 - \alpha) \rho N} \right] \lambda \]

\[ \bar{A} = \left[ \frac{\bar{M}M + (1 - \alpha) M \rho N + \alpha \bar{M} \rho N}{\bar{M}M + NM + (\bar{M} - M) \alpha \rho N} \right] \lambda \]

Inserting \( M \) or \( \bar{M} \) into equation (148), rearrange the terms and we get,

\[ H = H(M) = \frac{kZ - \alpha \frac{N \rho A}{A} - (1 - \alpha) \frac{N \rho A}{A}}{M + \left( 1 - \alpha \frac{\rho A}{A} \right) - (1 - \alpha) \frac{\rho A}{A}} N \]
\[ H = H(\overline{M}) = \frac{kZ - \alpha \frac{N\rho \lambda H}{\Delta} - (1 - \alpha) \frac{N\rho \lambda \Pi}{A}}{\overline{M} + \left(1 - \alpha \frac{\rho \lambda}{\Delta} - (1 - \alpha) \frac{\rho \lambda \Pi}{A}\right) N} \] (153)

Hence, we could find the explicit expression of \( H \) and \( \overline{H} \),

\[ H = \frac{(\overline{M} + \Omega) kZ}{(\overline{M} + \Omega) (\overline{M} + \Omega) + \left(\frac{\alpha N\rho \lambda}{\Delta} \right) (\overline{M} + \Omega) + \left(\frac{(1 - \alpha) N\rho \lambda}{\Delta} \right) (\overline{M} + \Omega)} \] (154)

\[ \overline{H} = \frac{(\overline{M} + \Omega) kZ}{(\overline{M} + \Omega) (\overline{M} + \Omega) + \left(\frac{\alpha N\rho \lambda}{\Delta} \right) (\overline{M} + \Omega) + \left(\frac{(1 - \alpha) N\rho \lambda}{\Delta} \right) (\overline{M} + \Omega)} \] (155)

where \( \Omega \) is a constant,

\[ \Omega = \left(1 - \alpha \frac{\rho \lambda}{\Delta} - (1 - \alpha) \frac{\rho \lambda \Pi}{A}\right) N \]

To conclude in this subsection, we find the explicit expression of \( A, \overline{A}, H, \overline{H} \) as the function of the parameters \( \{M, \overline{M}, N, \alpha, \rho, \lambda\} \).

Using equation (147) and (148), we can express the function \( G(M) \) as:

\[ G(M) = \frac{p + H(M) - \overline{f}}{A(M)} \]

\[ = \frac{p + \left[ kZ - \alpha \frac{N\rho \lambda H}{\Delta} - (1 - \alpha) \frac{N\rho \lambda \Pi}{A} \right] - \overline{f}}{\left[ M + (1 - \alpha \frac{\rho \lambda}{\Delta} - (1 - \alpha) \frac{\rho \lambda \Pi}{A}) N \right]} \]

\[ = \frac{(p - \overline{f}) \left[ M + (1 - \alpha \frac{\rho \lambda}{\Delta} - (1 - \alpha) \frac{\rho \lambda \Pi}{A}) N \right] + \left[ kZ - \alpha \frac{N\rho \lambda H}{\Delta} - (1 - \alpha) \frac{N\rho \lambda \Pi}{A} \right]}{M \lambda} \] (156)

\[ = \frac{p - \overline{f}}{\lambda} + \frac{kZ + NL}{M \lambda} \]

where \( L \) is defined as \( L = (p - \overline{f}) \left(1 - \alpha \frac{\rho \lambda}{\Delta} - (1 - \alpha) \frac{\rho \lambda \Pi}{A}\right) - \alpha \frac{\rho \lambda H}{\Delta} - (1 - \alpha) \frac{\rho \lambda \Pi}{A} \).

Since at the beginning of this sub-case I, our main premise is that

\[ \overline{f} + \rho \lambda \cdot \{aG + (1 - a)\overline{G}\} - p > 0 \]

This premise can be written as:

\[ - \left[ (p - \overline{f}) \left(1 - \alpha \frac{\rho \lambda}{\Delta} - (1 - \alpha) \frac{\rho \lambda \Pi}{A}\right) - \alpha \frac{\rho \lambda H}{\Delta} - (1 - \alpha) \frac{\rho \lambda \Pi}{A} \right] \geq L \]

This implies that \( L < 0 \), and the sign of \( \frac{\partial G(M)}{\partial M} \) will be determined by the sign of \( (kZ + NL) \).

1. Suppose \( (kZ + NL) < 0 \), then \( \frac{\partial G(M)}{\partial M} > 0 \) and

\[ M_1 = \overline{M} = \arg \min_{M \in [M_1, M_2]} G(M) \]
\[ M_2 = \bar{M} = \arg \max_{M \in [M_1, M_2]} G(M) \]

Inserting \( M = M_1, \bar{M} = M_2 \) into equation (150), (151), (154), (155) and we get,

\[
\begin{align*}
\Delta^* &= \left[ \frac{M_1 M_2 + (1 - \alpha) M_1 \rho N + \alpha M_2 \rho N}{M_1 M_2 + N M_2 - (M_2 - M_1)(1 - \alpha) \rho N} \right] \lambda \\
\tilde{A}^* &= \left[ \frac{M_1 M_2 + (1 - \alpha) M_1 \rho N + \alpha M_2 \rho N}{M_1 M_2 + N M_1 + (M_2 - M_1) \rho N} \right] \lambda \\
H^* &= \frac{(M_2 + \Omega^*)(kZ)}{(M_2 + \Omega^*)(M_1 + \Omega^*) + \left( \frac{\alpha \rho \lambda}{A} \right)(M_2 + \Omega^*) + \left( \frac{1 - \alpha}{\rho \lambda} \right)(M_1 + \Omega^*)} \\
H^* &= \frac{(M_1 + \Omega^*)(kZ)}{(M_2 + \Omega^*)(M_1 + \Omega^*) + \left( \frac{\alpha \rho \lambda}{A} \right)(M_2 + \Omega^*) + \left( \frac{1 - \alpha}{\rho \lambda} \right)(M_1 + \Omega^*)} \\
\Omega^* &= \left( 1 - \alpha \frac{\rho \lambda}{A} \right) N
\end{align*}
\]

Hence, we find the price function

\[
p = \tilde{f} + \left( \frac{M \lambda}{M + (1 - \alpha) \frac{\rho \lambda}{A} - (1 - \alpha) \frac{\rho \lambda}{A} N} \right) s - \left( \frac{kZ - \alpha \frac{N \rho \lambda H^*}{A} - (1 - \alpha) \frac{N \rho \lambda H^*}{A}}{M + (1 - \alpha) \frac{\rho \lambda}{A} - (1 - \alpha) \frac{\rho \lambda H^*}{A} N} \right) = \Delta^*(M) \]

We need to check that the premise, equation \((kZ + NL) < 0\), should be satisfied. \((kZ + NL) < 0\) and equation (157) imply that,

\[
p < p_a \quad (159)
\]

where \(p_a\) is determined by the following equation,

\[
kZ + N \left( p_a - \tilde{f} \right) \left( 1 - \alpha \frac{\rho \lambda}{A} - (1 - \alpha) \frac{\rho \lambda}{A} \right) - \alpha \frac{\rho \lambda H^*}{A} - (1 - \alpha) \frac{\rho \lambda H^*}{A} = 0 \quad (160)
\]

Substituting equation (159) into equation (158), we get:

\[
s < s_a \quad (161)
\]

where \(s_a\) is determined by the following equation,

\[
p_a = \tilde{f} + \left( \frac{M \lambda}{M + (1 - \alpha) \frac{\rho \lambda}{A} - (1 - \alpha) \frac{\rho \lambda}{A} N} \right) s_a - \left( \frac{kZ - \alpha \frac{N \rho \lambda H^*}{A} - (1 - \alpha) \frac{N \rho \lambda H^*}{A}}{M + (1 - \alpha) \frac{\rho \lambda}{A} - (1 - \alpha) \frac{\rho \lambda H^*}{A} N} \right) = \Delta^*(M) \]

2. Suppose \((kZ + NL) > 0\), then \(\frac{\partial G(M)}{\partial M} < 0\) and

\[ M_2 = \bar{M} = \arg \min_{M \in [M_1, M_2]} G(M) \]

\[ M_1 = \bar{M} = \arg \max_{M \in [M_1, M_2]} G(M) \]
Inserting $\overline{M} = M_2$, $\overline{M} = M_1$ into equation (??), and we get,

$$
\begin{align*}
\textbf{A} &= \begin{bmatrix}
\frac{M_2 M_1 + (1-a) M_2 \rho + a M_1 \rho N}{M_2 M_1 + N M_1 - (M_1 - M_2)(1-a) \rho N} \\
\frac{M_2 M_1 + (1-a) M_2 \rho + a M_1 \rho N}{M_2 M_1 + N M_2 + (M_1 - M_2) \rho N}
\end{bmatrix} \lambda \\
\textbf{H} &= \frac{(M_1 + \Omega^*) k Z}{(M_1 + \Omega^*)(M_2 + \Omega^*)^2} + \left(\frac{\rho N \lambda}{\textbf{A}}\right)^2 + \left(\frac{(1-a) N \lambda}{\textbf{A}}\right)^2 (M_2 + \Omega^*)
\end{align*}

(163)

$$

Hence, we find the price function

$$
p = \tilde{f} + \left(\frac{M \lambda}{M + (1-a) \frac{\rho \lambda}{\textbf{A}} - (1-a) \frac{\rho \lambda}{\textbf{A}}} \right) s - \left(\frac{k Z - a \frac{\rho \lambda \textbf{A}}{\textbf{A}} - (1-a) \frac{N \rho \lambda \textbf{A}^*}{\textbf{A}^*}}{M + (1-a) \frac{\rho \lambda}{\textbf{A}} - (1-a) \frac{\rho \lambda}{\textbf{A}}} \right) N
\]

(164)

We need to check that the two premise, $(kZ + NL) > 0$ and equation (143), should be satisfied.

The premise $(kZ + NL) > 0$ and equation (163) imply that,

$$
p > p_a'
\]

(165)

where $p_a'$ is determined by the following equation,

$$
kZ + N \left(\frac{p_a' - \tilde{f}}{1-a} - (1-a) \frac{\rho \lambda}{\textbf{A}}\right) - a \frac{\rho \lambda \textbf{A}}{\textbf{A}} - (1-a) \frac{\rho \lambda \textbf{A}^*}{\textbf{A}^*} = 0
\]

(166)

Comparing equation (162) and (166), tedious calculation shows that

$$
p_a = p_a'
\]

(167)

Substituting equation (165) into equation (164), we get

$$
s > s_{a'}
\]

(168)

where $s_{a'}$ is determined by the following equation,

$$
p_{a'} = \tilde{f} + \left(\frac{M \lambda}{M + (1-a) \frac{\rho \lambda}{\textbf{A}} - (1-a) \frac{\rho \lambda}{\textbf{A}}} \right) s_{a'} - \left(\frac{k Z - a \frac{\rho \lambda \textbf{A}}{\textbf{A}} - (1-a) \frac{N \rho \lambda \textbf{A}^*}{\textbf{A}^*}}{M + (1-a) \frac{\rho \lambda}{\textbf{A}} - (1-a) \frac{\rho \lambda}{\textbf{A}}} \right) N
\]

(169)

Comparing equation (164) and (169), using the equation (167), tedious calculation shows that

$$
s_a = s_{a'}
\]

(170)

The premise, equation (143) and equation (163) imply that,

$$
p < p_b
\]

(171)
where \( p_b \) is determined by the following equation,
\[
\tilde{f} + \rho \lambda \left\{ \alpha \left( \frac{p_b + H}{\Delta} - f \right) + (1 - \alpha) \left( \frac{p_b + \overline{H}}{\Delta} - f \right) \right\} - p_b = 0 \tag{172}
\]
Substituting equation (172) into equation (164), we get
\[
s < s_b \tag{173}
\]
where \( s_b \) is determined by the following equation,
\[
p_b = \tilde{f} + \left( \frac{M \lambda}{M + \left( 1 - a \frac{\lambda}{\Delta} - (1 - a) \frac{\lambda}{\Delta} \right) N} \right) s_b - \left( \frac{kZ - a \frac{N \rho \lambda \Delta}{\Delta} - (1 - a) \frac{N \rho \lambda \Delta}{\Delta}}{M + \left( 1 - a \frac{\rho \lambda}{\Delta} - (1 - a) \frac{\rho \lambda}{\Delta} \right) N} \right) \tag{174}
\]
II. Second, suppose \( x_U = 0 \) and the premise
\[
\tilde{f} + \rho \lambda \cdot \{ a \overline{G} + (1 - a) \underline{G} \} \leq p \leq \tilde{f} + \rho \lambda \cdot \{ a \overline{G} + (1 - a) \underline{G} \}
\]
\[
\Rightarrow \tilde{f} + \rho \lambda \cdot \left\{ a \left( \frac{p + H - f}{\Delta} \right) + (1 - a) \left( \frac{p + \overline{H} - f}{\Delta} \right) \right\} \leq p \leq \tilde{f} + \rho \lambda \cdot \left\{ a \left( \frac{p + H - f}{\Delta} \right) + (1 - a) \left( \frac{p + \overline{H} - f}{\Delta} \right) \right\} \tag{175}
\]
is satisfied, then the demand function of informed traders and uninformed traders are:
\[
x_I = \frac{\tilde{f} + \lambda s - p}{k} \tag{176}
\]
\[
x_U = 0
\]
We insert the equation (176) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,
\[
p = \tilde{f} + \lambda s - \frac{kZ}{M} \tag{177}
\]
Hence, in this situation,
\[
A(M) = \Delta = \overline{\Delta} = \lambda
\]
\[
H(M) = \frac{kZ}{M}
\]
Hence, we can express the function \( G(M) \) as:
\[
G(M) = \frac{p + H(M) - \tilde{f}}{A(M)} = \frac{p + \left( \frac{kZ}{M} \right) - \tilde{f}}{\lambda} \tag{178}
\]
It is obvious that \( \frac{\partial G(M)}{\partial M} < 0 \) and
\[
M_2 = M = \arg \min_{M \in [M_1, M_2]} G(M)
\]
\[
M_1 = \overline{M} = \arg \max_{M \in [M_1, M_2]} G(M)
\]
Hence we can find the explicit expression for \( \overline{H} \) and \( \underline{H} \),
\[
\overline{H} = H(M_2) = \frac{kZ}{M_2}
\]
\[ \overline{\Pi} = H(M_1) = \frac{kZ}{M_1} \]

To satisfy the premise, equation (??), the following two equations show hold for true.

\[
\begin{align*}
\bar{f} + \rho \lambda & \left\{ a \left( \frac{p + H - f}{A} \right) + (1 - \alpha) \left( \frac{p + \overline{\Pi} - f}{A} \right) \right\} - p \leq 0 \\
\bar{f} + \rho \lambda & \left\{ a \left( \frac{p + H - f}{A} \right) + (1 - \alpha) \left( \frac{p + H - f}{A} \right) \right\} - p \geq 0
\end{align*}
\]

This implies that,

\[ p'_b \leq p \leq p_c \quad (179) \]

where \( p'_b \) and \( p_c \) are determined by the following equations,

\[
\begin{cases}
\bar{f} + \rho \lambda \left\{ a \left( \frac{p'_b + H - f}{A} \right) + (1 - \alpha) \left( \frac{p'_b + \overline{\Pi} - f}{A} \right) \right\} - p'_b = 0 \\
\bar{f} + \rho \lambda \left\{ a \left( \frac{p + H - f}{A} \right) + (1 - \alpha) \left( \frac{p + H - f}{A} \right) \right\} - p_c = 0 \\
A = \overline{\Pi} = \lambda \\
H = \frac{kZ}{M_2} \\
\overline{\Pi} = \frac{kZ}{M_1}
\end{cases}
\quad (180)
\]

Comparing equation \( \bar{f} + \rho \lambda \left\{ a \left( \frac{p'_b + H - f}{A} \right) + (1 - \alpha) \left( \frac{p'_b + \overline{\Pi} - f}{A} \right) \right\} - p'_b = 0 \) and equation (172), tedious calculation shows that

\[ p'_b = p_b \quad (181) \]

Substituting equation (177) into equation (179), we get

\[ s'_b \leq s \leq s_c \quad (182) \]

where \( s'_b \) and \( s_c \) are determined by the following equations,

\[ p'_b = \bar{f} + \lambda s'_b - \frac{kZ}{M} \quad (183) \]

\[ p_c = \bar{f} + \lambda s_c - \frac{kZ}{M} \quad (184) \]

Comparing equation (183) and (174), tedious calculation shows that

\[ s'_b = s_b \quad (185) \]

III. Third, suppose \( x_U < 0 \) and the premise

\[
\begin{align*}
\bar{f} + \rho \lambda \left\{ a \overline{G} + (1 - \alpha) \overline{G} \right\} - p < 0 \\
\Rightarrow \bar{f} + \rho \lambda \left\{ a \left( \frac{p + \overline{\Pi} - f}{A} \right) + (1 - \alpha) \left( \frac{p + H - f}{A} \right) \right\} - p < 0
\end{align*}
\]

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is satisfied, then the demand function of informed traders and uninformed traders are:

\[ x_I = \frac{\tilde{f} + \lambda s - p}{k} \quad (187) \]

\[ x_U = \frac{\tilde{f} + \rho \lambda \cdot \{aG + (1 - a) \bar{G}\} - p}{k} \]

\[ = \frac{\tilde{f} + \rho \lambda \cdot \{a \left(\frac{p+H-f}{A}\right) + (1 - a) \left(\frac{p+H-f}{A}\right)\}}{k} - p \quad (188) \]

We insert the equation (187) and (188) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,

\[ M \cdot \left(\frac{\tilde{f} + \lambda s - p}{k}\right) + N \cdot \left(\frac{\tilde{f} + \rho \lambda \cdot \{a \left(\frac{p+H-f}{A}\right) + (1 - a) \left(\frac{p+H-f}{A}\right)\}}{k} - p\right) = Z \]

Rearrange the terms and we get,

\[ p = \tilde{f} + \left(\frac{M\lambda}{M + \left(1 - a \frac{\rho A}{k} - (1 - a) \frac{\rho A}{N}\right) N}\right) s - \left(\frac{kZ - a \frac{N\rho H}{A} - (1 - a) \frac{N\rho H}{A}}{M + \left(1 - a \frac{\rho A}{k} - (1 - a) \frac{\rho A}{N}\right) N}\right) \quad (189) \]

Using the undetermined coefficient method and match the coefficient, we get:

\[ A(M) = \frac{M\lambda}{M + \left(1 - a \frac{\rho A}{k} - (1 - a) \frac{\rho A}{N}\right) N} \quad (190) \]

\[ H(M) = \frac{kZ - a \frac{N\rho H}{A} - (1 - a) \frac{N\rho H}{A}}{M + \left(1 - a \frac{\rho A}{k} - (1 - a) \frac{\rho A}{N}\right) N} \quad (191) \]

Inserting \( M \) or \( M \) into equation (190), rearrange the terms and we get,

\[ A = A(M) = \frac{(M + (1 - a) \rho N) \lambda}{M + \left(1 - a \frac{\rho A}{k}\right) N} \quad (192) \]

\[ \overline{A} = A(\overline{M}) = \frac{(\overline{M} + a \rho N) \lambda}{\overline{M} + \left(1 - (1 - a) \frac{\rho A}{k}\right) N} \]

Hence, we could find the explicit expression of \( A \) and \( \overline{A} \),

\[ A = \left[ \frac{MM + (1 - a) \overline{M} \rho N + aM \rho N}{MM + N \overline{M} - (M - \overline{M}) a \rho N} \right] \lambda \quad (193) \]

\[ \overline{A} = \left[ \frac{\overline{M}M + (1 - a) \frac{M}{M} \rho N + aM \rho N}{\overline{M}M + N \overline{M} + (\overline{M} - M) (1 - a) \rho N} \right] \lambda \quad (194) \]
Inserting $M$ or $\overline{M}$ into equation (191), rearrange the terms and we get,

$$H = H(M) = \frac{kZ - a\frac{N\rho\lambda\overline{H}}{\overline{A}} - (1 - \alpha)\frac{N\rho\lambda H}{\overline{A}}}{\overline{M} + \left(1 - a\frac{\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}\right)N} \tag{195}$$

$$\overline{H} = H(\overline{M}) = \frac{kZ - a\frac{N\rho\lambda\overline{H}}{\overline{A}} - (1 - \alpha)\frac{N\rho\lambda H}{\overline{A}}}{\overline{M} + \left(1 - a\frac{\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}\right)N} \tag{196}$$

Hence, we could find the explicit expression of $H$ and $\overline{H}$,

$$H = \frac{(M + \Omega)kZ}{(\overline{M} + \Omega)(\overline{M} + \Omega) + \left(a\frac{N\rho\lambda}{\overline{A}}\right)(M + \Omega) + \left(1 - a\right)\frac{(1 - \alpha)N\rho\lambda}{\overline{A}}(M + \Omega)} \tag{197}$$

$$\overline{H} = \frac{(\overline{M} + \Omega)kZ}{(\overline{M} + \Omega)(\overline{M} + \Omega) + \left(a\frac{N\rho\lambda}{\overline{A}}\right)(M + \Omega) + \left(1 - a\right)\frac{(1 - \alpha)N\rho\lambda}{\overline{A}}(M + \Omega)} \tag{198}$$

where $\Omega$ is a constant,

$$\Omega = \left(1 - a\frac{\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}\right)N$$

To conclude in this subsection, we find the explicit expression of $A$, $\overline{A}$, $H$, $\overline{H}$ as the function of the parameters $\{M, \overline{M}, N, a, \rho, \lambda\}$.

Using equation (190) and (191), we can express the function $G(M)$ as:

$$G(M) = \frac{p + H(M) - \overline{f}}{\overline{A}(M)}$$

$$= \frac{p + \left[\frac{kZ - a\frac{N\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{N\rho\lambda H}{\overline{A}}}{\overline{M} + \left(1 - a\frac{\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}\right)N}\right] - \overline{f}}{\overline{M} + \left(1 - a\frac{\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}\right)N}$$

$$= \frac{(p - \overline{f})\left[M + \left(1 - a\frac{\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}\right)N\right] + \left[kZ - a\frac{N\rho\lambda\overline{H}}{\overline{A}} - (1 - \alpha)\frac{N\rho\lambda H}{\overline{A}}\right]}{\overline{M} + \left(1 - a\frac{\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}\right)N} \tag{199}$$

$$= \frac{p - \overline{f}}{\lambda} + \frac{\rho\lambda}{\overline{M} + NL} = \frac{\rho\lambda}{\overline{M} + NL}$$

where $L$ is defined as $L = \left(p - \overline{f}\right)\left(1 - a\frac{\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}\right) - a\frac{\rho\lambda\overline{H}}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}$.

Since at the beginning of this sub-case III, our main premise is that

$$\overline{f} + \rho\lambda \cdot \{a\overline{G} + (1 - a)G\} - p < 0$$

This premise can be written as:

$$- \left[p - \overline{f}\right] \left(1 - a\frac{\rho\lambda}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}}\right) - a\frac{\rho\lambda\overline{H}}{\overline{A}} - (1 - \alpha)\frac{\rho\lambda H}{\overline{A}} < 0$$

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This implies that \( L > 0 \), and the sign of \( \frac{\partial G(M)}{\partial M} \) will be negative. Hence,

\[
M_2 = \overline{M} = \arg \min_{M \in [M_1, M_2]} G(M)
\]

\[
M_1 = \underline{M} = \arg \max_{M \in [M_1, M_2]} G(M)
\]

Inserting \( \underline{M} = M_2, \overline{M} = M_1 \) into equation (193), (194), (197), (198) and we get,

\[
\begin{align*}
\lambda &= \overline{\lambda}^{*} \\
\overline{\lambda}^{*} &= \left[ M_1 M_2 + (1-\alpha)M_1 \rho N + \alpha M_2 \rho N \right] \lambda = \overline{\lambda}^{*} \\
\overline{\lambda}^{*} &= \left[ M_1 M_2 + (1-\alpha)M_1 \rho N + \alpha M_2 \rho N \right] \lambda = \overline{\lambda}^{*} \\
H^{*} &= \frac{(M + \overline{\lambda}^{*}) k Z}{(M + \overline{\lambda}^{*}) (M + \overline{\lambda}^{*}) + \left( \frac{\alpha N \rho \lambda}{\overline{\lambda}^{*}} \right) (M + \overline{\lambda}^{*}) + \left( \frac{(1-\alpha) N \rho \lambda}{\overline{\lambda}^{*}} \right) (M + \overline{\lambda}^{*})} = H^{*} \quad (200) \\
\Omega^{*} &= \left( 1 - \alpha \frac{\rho \lambda}{\overline{\lambda}^{*}} - (1 - \alpha) \frac{\rho \lambda}{\overline{\lambda}^{*}} \right) N = \Omega^{*}
\end{align*}
\]

Hence, we find the price function

\[
p = f + \left( \frac{M \lambda}{M + \left( 1 - \alpha \frac{\rho \lambda}{\overline{\lambda}^{*}} - (1 - \alpha) \frac{\rho \lambda}{\overline{\lambda}^{*}} \right) N} = \overline{\lambda}^{*}(M) \right) s - \left( \frac{k Z - \alpha \frac{N \rho \lambda H^{*}}{\overline{\lambda}^{*}} - (1 - \alpha) \frac{N \rho \lambda H^{*}}{\overline{\lambda}^{*}}}{M + \left( 1 - \alpha \frac{\rho \lambda}{\overline{\lambda}^{*}} - (1 - \alpha) \frac{\rho \lambda}{\overline{\lambda}^{*}} \right) N} = H^{*}(M) \right) \quad (201)
\]

\[
= f + \left( \frac{M \lambda}{M + \left( 1 - \alpha \frac{\rho \lambda}{\overline{\lambda}^{*}} - (1 - \alpha) \frac{\rho \lambda}{\overline{\lambda}^{*}} \right) N} = \overline{\lambda}^{*}(M) \right) s - \left( \frac{k Z - \alpha \frac{N \rho \lambda H^{*}}{\overline{\lambda}^{*}} - (1 - \alpha) \frac{N \rho \lambda H^{*}}{\overline{\lambda}^{*}}}{M + \left( 1 - \alpha \frac{\rho \lambda}{\overline{\lambda}^{*}} - (1 - \alpha) \frac{\rho \lambda}{\overline{\lambda}^{*}} \right) N} = H^{*}(M) \right) \quad (201)
\]

The price function is same with the price function in equation (158).

We need to check that the premise, equation (186), should be satisfied. Premise (186) and equation (200) imply that,

\[
p > p'_c \quad (202)
\]

where \( p'_c \) is determined by the following equation,

\[
f + \rho \lambda \cdot \left\{ \left( \frac{p'_c + \overline{H}^{*} - f}{\overline{\lambda}^{*}} \right) + (1 - \alpha) \left( \frac{p'_c + \overline{H}^{*} - f}{\overline{\lambda}^{*}} \right) \right\} - p'_c = 0 \quad (203)
\]

Substituting equation (201) into equation (202), we get

\[
s > s'_c \quad (204)
\]
where \( s'_c \) is determined by the following equation,

\[
p'_c = \hat{f} + \left( \frac{M\lambda}{M + \left( 1 - a \frac{\rho}{A} - (1 - a) \frac{\rho}{A} \right) N} \right) s'_c - \left( \frac{kZ - a \frac{N\rho\lambda\Phi}{\Lambda} - (1 - a) \frac{N\rho\lambda\Phi}{\Delta}}{M + \left( 1 - a \frac{\rho}{A} - (1 - a) \frac{\rho}{A} \right) N} \right) s'_c
\]

(205)

Comparing equation (184) and (205), tedious calculation shows that

\[
s'_c = s_c
\]

(206)

Finally, to summarize, combing the results of sub-case I, II, III, when the number of the uninformed trader \( N \) is known and the number of the informed trader \( M \) is unknown with the ambiguity, there exists a Rational Expectation Equilibrium (REE) in which the price function \( p \) is piecewise in \( s \) and \( M \),

\[
p(s, M) = \begin{cases} \hat{f} + A^*(M) \cdot s - H^*(M), & \text{for } s \in (-\infty, s_a) \cup (s_c, +\infty) \\ \hat{f} + A^*(M) \cdot s - H^*(M), & \text{for } s \in [s_a, s_b) \\ \hat{f} + \lambda s - \frac{kZ}{M}, & \text{for } s \in [s_b, s_c] \end{cases}
\]

(207)

### F  Proof For Section 6.3

To find the equilibrium, we need to first characterize the demand function of uninformed traders who exhibit ambiguity aversion with maxmin utility function.

#### F.1 Demand Function of Informed Traders

By observing the realization of \( s \), informed traders resolve their ambiguity straight away. They choose portfolio holdings \( x_I \) to maximize the expected profits \( \pi_I \)

\[
\max_{x_I} E \left[ (f_I - p) x_I - \frac{k}{2} x_I^2 \mid F_I = \{s, p\} \right]
\]

(208)

where \( p \) is the observed asset price. Standard arguments yield

\[
x_I(s, p) = \frac{E(f_I \mid s, p) - p}{k} = \frac{E(\hat{f} + \theta_I + \epsilon \mid s) - p}{k} = \frac{\hat{f} + E(\theta_I \mid s) - p}{k} = \frac{\hat{f} + \frac{\text{cov}(s, \theta_I)}{\text{var}(s)} s - p}{k} = \frac{\hat{f} + \frac{t^{-1}}{t^{-1} + t_0} s - p}{k}
\]

(209)

\[
= \frac{\hat{f} + \lambda s - p}{k}
\]
and the informativeness of the signal is captured by the signal-to-noise ratio:

$$\lambda = \frac{\text{cov}(s, \theta)}{\text{var}(s)} = \frac{\tau^{-1}}{\tau^{-1} + \tau_u^{-1}}$$  (210)

### F.2 Demand Function of Uninformed Traders

At trading stage, for any given $M$ and $N$, uninformed traders rationally conjecture that the price function is:

$$p = \bar{f} + A(N; M) \cdot s - H(N; M)$$  (211)

where the function $A(N; M)$ and $H(N; M)$ will be endogenously determined in equilibrium. Since $M$ is known by the uninformed traders for sure, $M$ is an exogenous parameter here. For simplicity, we write $A(N) \equiv A(N; M)$ and $H(N) \equiv H(N; M)$ and the conjectured price function can be written as:

$$p = \bar{f} + A(N) \cdot s - H(N)$$  (212)

Thus, the optimal demand of uninformed traders is determined by

$$\max_{x_U} \min_{N \in [N_1, N_2]} \left( E_N \left[ (f_U - p) x_U - \frac{k}{2} x_U^2 \mid F_U = \{p\} \right] \right),$$  (213)

where $x_U$ is the asset demand of uninformed traders, and $E_N(\cdot)$ is the expectation operator taken under the belief that the size/total number of the uninformed traders is $N$. The criterion underlying equation (213) is the maxmin expected utility axiomatized by Gilboa and Schmeidler (1989). Since uninformed traders are ambiguous about $N$, they view the stock price $p$ as an ambiguous signal about $f_U$. Under the belief that the size/total number of the uninformed traders is $N$, they map from the observed price $p$ to the extracted signal $s_N$:

$$s_N = \frac{p + H(N) - \bar{f}}{A(N)}$$  (214)

The conditional moments of $f_U$ taken under a particular belief $N$, are given by:

$$E_N[f_U \mid p] = E_N[\bar{f} + \theta U + \varepsilon \mid p]$$

$$= \bar{f} + E_N[\theta U \mid p] + 0$$

$$= \bar{f} + E_N[\theta U \mid s = s_N]$$

$$= \bar{f} + \frac{\text{cov}(s, \theta_U)}{\text{var}(s)} s_N$$

$$= \bar{f} + \left( \frac{\rho \tau^{-1}}{\tau^{-1} + \tau_u^{-1}} \right) s_N$$

$$= \bar{f} + \rho \lambda \left[ \frac{p + H(N) - \bar{f}}{A(N)} \right]$$

$$= \bar{f} + \rho \lambda \cdot G(N; p, \bar{f}, M)$$  (215)

For simplicity, we write $G(N; p, \bar{f}, M) \equiv G(N)$ and the objective function of an uninformed trader can
be written as:
\[
\min_{N \in [N_1, N_2]} \left( (E_N [f u | p] - p) \cdot x u - \frac{k}{2} x^2 U \right)
\]
\[
\Rightarrow \min_{N \in [N_1, N_2]} \left( (\tilde{f} + \rho \lambda \cdot G(N) - p) \cdot x u - \frac{k}{2} x^2 U \right)
\]
\[
\begin{cases}
-\frac{k}{2} x^2 U + \left[ f + \rho \lambda \cdot \{G(N)\}_{\min} - p \right] \cdot x u, & \text{if } x u > 0 \\
0, & \text{if } x u = 0 \\
-\frac{k}{2} x^2 U + \left[ f + \rho \lambda \cdot \{G(N)\}_{\max} - p \right] \cdot x u, & \text{if } x u < 0
\end{cases}
\]

where \( \{G(N)\}_{\min} = \{ \frac{p + H(N) - f}{A(N)} \}_{\min} \) and \( \{G(N)\}_{\max} = \{ \frac{p + H(N) - f}{A(N)} \}_{\max} \) are the minimum and maximum of the function \( \{G(N)\} \), respectively.

Thus an uninformed trader’s demand function is:
\[
x u (p) = \begin{cases}
\frac{f + \rho \lambda \cdot \{p + H(N) - f\}_{\min}}{k} - p, & \text{if } \tilde{f} + \rho \lambda \cdot \{p + H(N) - f\}_{\min} - p > 0 \\
0, & \text{if } \tilde{f} + \rho \lambda \cdot \{p + H(N) - f\}_{\min} - p \leq 0 \leq \tilde{f} + \rho \lambda \cdot \{p + H(N) - f\}_{\max} - p \\
\frac{f + \rho \lambda \cdot \{p + H(N) - f\}_{\max}}{k} - p, & \text{if } \tilde{f} + \rho \lambda \cdot \{p + H(N) - f\}_{\max} - p < 0
\end{cases}
\]

\( 216 \) \( 217 \)

F.3 Proof of Proposition 10 and 11

Proof. We define the value of \( N \) that causes the function \( G(N) \) to reach a minimum as \( \underline{N} \) and the value of \( N \) that causes the function \( G(N) \) to reach a maximum as \( \overline{N} \),

\[
\underline{N} \equiv \arg \min_{N \in [N_1, N_2]} G(N) = \arg \min_{N \in [N_1, N_2]} \left[ \frac{p + H(N) - \tilde{f}}{A(N)} \right] \\
\overline{N} \equiv \arg \max_{N \in [N_1, N_2]} G(N) = \arg \max_{N \in [N_1, N_2]} \left[ \frac{p + H(N) - \tilde{f}}{A(N)} \right]
\]

\( 218 \)

We define the values that \( H(N) \) and \( A(N) \) take on when \( N = \underline{N} \) and \( N = \overline{N} \) as:

\[
\underline{H} \equiv H(\underline{N}) \quad \overline{H} \equiv H(\overline{N}) \\
\underline{A} \equiv A(\underline{N}) \quad \overline{A} \equiv A(\overline{N})
\]

\( 219 \)

Note that \( \underline{H}, \overline{H}, \underline{A} \) and \( \overline{A} \) will be a constant. Hence, we can define the maximum and minimum of \( G(N) \) as:

\[
\left\{ \frac{p + H(N) - \tilde{f}}{A(N)} \right\}_{\min} = \underline{G} = \frac{p + \underline{H} - \tilde{f}}{\underline{A}} \\
\left\{ \frac{p + H(N) - \tilde{f}}{A(N)} \right\}_{\max} = \overline{G} = \frac{p + \overline{H} - \tilde{f}}{\overline{A}}
\]

\( 220 \)
Hence, the optimal demand functions of the uninformed traders, equation (221), can be written as,

\[
x_U(p) = \begin{cases} 
  \frac{f + \rho \lambda \cdot \left( \frac{p + H - f}{A} \right) - p}{k}, & \text{if } \bar{f} + \rho \lambda \cdot \left( \frac{p + H - f}{A} \right) - p > 0 \\
  0, & \text{if } \bar{f} + \rho \lambda \cdot \left( \frac{p + H - f}{A} \right) - p \leq 0 \leq \bar{f} + \rho \lambda \cdot \left( \frac{p + H - f}{A} \right) - p \\
  \frac{f + \rho \lambda \cdot \left( \frac{p + H - f}{A} \right) - p}{k}, & \text{if } \bar{f} + \rho \lambda \cdot \left( \frac{p + H - f}{A} \right) - p < 0
\end{cases}
\]  

(221)

We evaluate the feasibility of the possible demand functions given in equation (221) case by case by inserting it into the market clearing condition.

I. First, suppose \( x_U > 0 \) and the premise

\[
\bar{f} + \rho \lambda \cdot \left( \frac{p + H - f}{A} \right) - p > 0
\]

is satisfied, then the demand function of informed traders and uninformed traders are:

\[
x_I = \frac{\bar{f} + \lambda s - p}{k} \\
x_U = \frac{f + \rho \lambda \cdot \left( \frac{p + H - f}{A} \right) - p}{k}
\]

(223)

We insert the equation (223) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,

\[
M \cdot \left( \frac{\bar{f} + \lambda s - p}{k} \right) + N \cdot \left( \frac{f + \rho \lambda \cdot \left( \frac{p + H - f}{A} \right) - p}{k} \right) = Z
\]

Rearrange the terms and we get,

\[
\left( M + N - \frac{N \rho \lambda}{A} \right) p = \left( M + N - \frac{N \rho \lambda}{A} \right) \bar{f} + M \lambda s - \left( kZ - \frac{N \rho \lambda H}{A} \right)
\]

\[
p = \bar{f} + \left( \frac{M \lambda}{M + N - \frac{N \rho \lambda}{A}} \right) s - \left( \frac{kZ - \frac{N \rho \lambda H}{A}}{M + N - \frac{N \rho \lambda}{A}} \right)
\]

(224)

Using the undetermined coefficient method and match the coefficient, we get:

\[
A(N) = \frac{M \lambda}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N}
\]

(225)

\[
H(N) = \frac{kZ - \frac{N \rho \lambda H}{A}}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N}
\]
Hence, we can express the function $G(N)$ as:

$$G(N) = \frac{p + H(N) - \bar{f}}{A(N)} = \frac{p + \left[ \frac{kZ - N_0\lambda H}{M + \left(1 - \frac{\rho\lambda}{A}\right)N} \right] - \bar{f}}{\left[ \frac{M\lambda}{M + \left(1 - \frac{\rho\lambda}{A}\right)N} \right]}$$

$$= \frac{(p - \bar{f}) \left[ M + \left(1 - \frac{\rho\lambda}{A}\right)N \right] + \left[ kZ - \frac{N_0\lambda H}{A} \right]}{M\lambda}$$

(226)

We could find the partial derivative of $G(N)$ w.r.t $N$,

$$\frac{\partial G(N)}{\partial N} = \frac{1}{M\lambda} \left[ (p - \bar{f}) \left( 1 - \frac{\rho\lambda}{A} \right) - \frac{\rho\lambda H}{A} \right]$$

We define $L = (p - \bar{f}) \left( 1 - \frac{\rho\lambda}{A} \right) - \frac{\rho\lambda H}{A}$.

Since at the beginning of this sub-case I, our main premise is that

$$\bar{f} + \rho\lambda \cdot \frac{p + H - \bar{f}}{A} - p > 0$$

This premise can be written as:

$$- \left[ (p - \bar{f}) \left( 1 - \frac{\rho\lambda}{A} \right) - \frac{\rho\lambda H}{A} \right] > 0$$

This implies that $L < 0$, $\frac{\partial G(N)}{\partial N} < 0$ and

$$N_2 = N = \arg\min_{N \in [N_1, N_2]} G(N)$$

Inserting $N = N_2$ into equation (225), and we get,

$$A = A(N_2) = \frac{M\lambda}{M + \left(1 - \frac{\rho\lambda}{A}\right)N_2}$$

$$H = H(N_2) = \frac{kZ - \frac{N_0\rho\lambda H}{A}}{M + \left(1 - \frac{\rho\lambda}{A}\right)N_2}$$

Solving for $A$ and $H$, and we find the explicit expression:

$$A = \left( \frac{M + N_2\rho}{M + N_2} \right) \lambda$$

$$H = \frac{kZ}{M + N_2}$$

(227)
Inserting equation (227) into equation (224), and we find the price function,

\[ p = \bar{f} + \left( \frac{MA}{M + N - N\rho} \right) s - \left( \frac{kZ - \frac{N\rho A}{M + N - N\rho}}{M + N - N\rho} \right) \]

\[ = \bar{f} + \left[ \frac{M + N\rho}{M + N + (N_2 - N)\rho} \right] \lambda \cdot s - \left[ \frac{(M + (N_2 - N)\rho)kZ}{M(M + N + (N_2 - N)\rho)} \right] \equiv \lambda \cdot s - \left[ \frac{(M + (N_2 - N)\rho)kZ}{M(M + N + (N_2 - N)\rho)} \right] \]

We need to check that the premise, equation (222), should be satisfied. This premise and equation (227) imply that,

\[ p < \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M} \right) \]

(229)

Inserting equation (228) into (229), this is equivalent with

\[ s < \left[ \frac{M + N\rho}{M + N + (N_2 - N)\rho} \right]^{-1} \left[ \frac{\rho}{M(1 - \rho)} + \frac{(M + (N_2 - N)\rho)}{M(M + N + (N_2 - N)\rho)} \right] \frac{kZ}{\lambda} \equiv \frac{kz}{M\lambda(1 - \rho)} \]

\[ (230) \]

II. Second, suppose \( x_U = 0 \) and the premise

\[ f + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p \leq 0 \leq f + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p \]

(231)

is satisfied, then the demand function of informed traders and uninformed traders are:

\[ x_I = \frac{\bar{f} + \lambda s - p}{k} \]

\[ x_U = 0 \]

(232)

We insert the equation (232) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,

\[ p = \bar{f} + \lambda s - \frac{kZ}{M} \]

(233)

Hence, in this situation, \( A(N) = \bar{A} = \overline{A} = \lambda \)

\[ H(N) = \bar{H} = \overline{H} = \frac{kZ}{M} \]

To satisfy the premise, equation (231), the following equation should hold for true.

\[ f + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p = 0 \]

This implies that,

\[ p = f + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M} \right) \]

(234)

This is equivalent with

\[ s = \frac{kz}{M\lambda(1 - \rho)} \]

\[ (235) \]

III. Third, suppose \( x_U < 0 \) and the premise

\[ f + \rho \lambda \cdot \left( \frac{p + H - \bar{f}}{A} \right) - p < 0 \]

\[ (236) \]
is satisfied, then the demand function of informed traders and uninformed traders are:

\[
x_I = \frac{\bar{f} + \lambda s - p}{k} \\
\]

\[
x_U = \frac{\bar{f} + \rho \lambda \cdot \left( \frac{p + \Pi - \bar{f}}{\lambda} \right) - p}{k}
\]

(237)

We insert the equation (237) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,

\[
M \cdot \left( \frac{\bar{f} + \lambda s - p}{k} \right) + N \cdot \left( \frac{\bar{f} + \rho \lambda \cdot \left( \frac{p + \Pi - \bar{f}}{\lambda} \right) - p}{k} \right) = Z
\]

Rearrange the terms and we get,

\[
\left( M + N - \frac{N \rho \lambda}{A} \right) p = \left( M + N - \frac{N \rho \lambda}{A} \right) \bar{f} + M \lambda s - \left( kZ - \frac{N \rho \lambda \Pi}{A} \right)
\]

\[
p = \bar{f} + \left( \frac{M \lambda}{M + N - \frac{N \rho \lambda}{A}} \right) s - \left( \frac{kZ - \frac{N \rho \lambda \Pi}{A}}{M + N - \frac{N \rho \lambda}{A}} \right)
\]

Using the undetermined coefficient method and match the coefficient, we get:

\[
A(N) = \frac{M \lambda}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N}
\]

\[
H(N) = \frac{kZ - \frac{N \rho \lambda \Pi}{A}}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N}
\]

(238)

Hence, we can express the function \( G(N) \) as:

\[
G(N) = \frac{p + H(N) - \bar{f}}{A(N)} = \frac{p + \left( kZ - \frac{N \rho \lambda \Pi}{A} \right) - \bar{f}}{M \lambda}
\]

(239)

We could find the partial derivative of \( G(N) \) w.r.t \( N \),

\[
\frac{\partial G(N)}{\partial N} = \frac{1}{M \lambda} \left[ (p - \bar{f}) \left( 1 - \frac{\rho \lambda}{A} \right) - \frac{\rho \lambda \Pi}{A} \right]
\]

We define \( L = (p - \bar{f}) \left( 1 - \frac{\rho \lambda}{A} \right) - \frac{\rho \lambda \Pi}{A} \).

53
Since at the beginning of this sub-case II, our main premise is that

\[ f' + \rho \lambda \cdot \frac{p + H - f}{A} - p < 0 \]

This premise can be written as:

\[- \left[ (p - f) \left( 1 - \frac{\rho \lambda}{A} \right) - \frac{\rho \lambda H}{A} \right] < 0\]

This implies that \( L > 0, \frac{\partial G(N)}{\partial N} > 0 \) and

\[ N_2 = \bar{N} = \arg \max_{N \in [N_1, N_2]} G(N) \]

Inserting \( \bar{N} = N_2 \) into equation (238), and we get,

\[ \bar{A} = A(N_2) = \frac{M \lambda}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N_2} \]

\[ \bar{H} = H(N_2) = \frac{kZ - \frac{N_2 \rho \lambda H}{A}}{M + \left( 1 - \frac{\rho \lambda}{A} \right) N_2} \]

Solving for \( \bar{A} \) and \( \bar{H} \), and we find the explicit expression:

\[ \bar{A} = \left( \frac{M + N_2 \rho}{M + N_2} \right) \lambda \]

\[ \bar{H} = \frac{kZ}{M + N_2} \]

(240)

Inserting equation xxx into equation xxx, and we find the price function,

\[ p = \bar{f} + \left( \frac{M \lambda}{M + N - \frac{N_2 \lambda}{A}} \right) s - \left( \frac{kZ - \frac{N_2 \rho \lambda H}{A}}{M + N - \frac{N_2 \lambda}{A}} \right) \]

\[ = \bar{f} + \left[ \frac{M + N_2 \rho}{M + N + (N_2 - N) \rho} \right] \lambda \cdot s - \left[ \frac{(M + (N_2 - N) \rho) kZ}{M + N + (N_2 - N) \rho} \right] \]

(241)

We need to check that the premise, equation (236), should be satisfied. This implies that,

\[ p > \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M} \right) \]

(242)

This is equivalent with

\[ s > \left[ \frac{M + N_2 \rho}{M + N + (N_2 - N) \rho} \right]^{-1} \left[ \frac{\rho}{M(1 - \rho)} + \frac{(M + (N_2 - N) \rho) kZ}{M(M + N + (N_2 - N) \rho)} \right] \]

(243)

To summarize, combing the results of sub-case I, II, III, when the number of the informed trader \( M \) is known and the number of the uninformed trader \( N \) is unknown with the ambiguity, there exists a
Rational Expectation Equilibrium (REE) in which the price function $p$ is a function of $s$ and $N$,

$$p(s, N) = f + \left[ \frac{M + N_2 \rho}{M + N + (N_2 - N) \rho} \right] \lambda \cdot s - \left[ \frac{(M + (N_2 - N) \rho) kZ}{M (M + N + (N_2 - N) \rho)} \right] (244)$$

G  Proof For Section 6.4

In this section, I characterize the financial market equilibrium where uninformed traders are ambiguous about both the total number (size) of informed traders $M$ and the total number (size) of uninformed traders $N$. Specifically, uninformed traders are unable to assess what $M$ is, but they believe it belongs to some interval, $M \in [M_1, M_2]$, with $M_1 < M < M_2$. We further assume that $M_1 = M - \Delta M$ and $M_2 = M + \Delta M$. Similarly, uninformed traders are unable to assess what $N$ is, but they believe it belongs to some interval, $N \in [N_1, N_2]$, with $N_1 < N < N_2$. We further assume that $N_1 = N - \Delta N$ and $N_2 = N + \Delta N$. I use the boldface of $M$ to denote the true value of $M$. I use the boldface of $N$ to denote the true value of $N$. $\Delta M$ and $\Delta N$ are exogenous parameters that determines the ambiguity.

To find the equilibrium, we need to first characterize the demand function of informed and uninformed traders who exhibit ambiguity aversion with maxmin utility function.

G.1 Demand Function of Informed Traders

By observing the realization of $s$, informed traders resolve their ambiguity straight away. They choose portfolio holdings $x_I$ to maximize the expected profits $\pi_I$

$$\max_{x_I} E \left[ (f_I - p) x_I - \frac{k}{2} x_I^2 \mid \mathcal{F}_I = \{s, p\} \right], \quad (245)$$

where $p$ is the observed asset price. Standard arguments yield

$$x_I(s, p) = \frac{E(f_I \mid s, p) - p}{k} = \frac{E(\hat{f} + \theta_I + \epsilon \mid s) - p}{k} = \frac{\hat{f} + E(\theta_I \mid s) - p}{k} = \frac{\hat{f} + \text{cov}(s, \theta_I) s - p}{k} = \frac{\hat{f} + \lambda s - p}{k} = \hat{f} + \lambda s - p$$

and the informativeness of the signal is captured by the signal-to-noise ratio:

$$\lambda \equiv \frac{\text{cov} (s, \theta_I)}{\text{var} (s)} = \frac{\tau^{-1}}{\tau^{-1} + \tau^2 u^{-1}} \quad (246)$$

and the informativeness of the signal is captured by the signal-to-noise ratio:
G.2 Demand Function of Uninformed Traders

At trading stage, for any given \( M \) and \( N \), uninformed traders rationally conjecture that the price function is:

\[
p = \tilde{f} + A(M, N) \cdot s - H(M, N)
\]

where the function \( A(M, N) \) and \( H(M, N) \) will be endogenously determined in equilibrium.

Thus, the optimal demand of uninformed traders is determined by

\[
\max_{x_u} \min_{M \in [M_1, M_2], N \in [N_1, N_2]} \left( E_{M,N} \left[ (f_u - p) x_u - \frac{k}{2} x_u^2 \mid F_u = \{p\} \right] \right),
\]

where \( x_u \) is the asset demand of uninformed traders, and \( E_{M,N}(\cdot) \) is the expectation operator taken under the belief that the size/total number of the informed traders is \( M \) and the size/total number of the uninformed traders is \( N \). The criterion underlying equation (248) is the maxmin expected utility axiomatized by Gilboa and Schmeidler (1989). Since uninformed traders are ambiguous about initial priors, \( \bar{x}_u \) occurs regarding the original set of priors, which means that the uninformed agents retain all their beliefs. This rule is known as full Bayesian updating. With full Bayesian updating, no learning occurs regarding the original set of priors, which means that the uninformed agents retain all their initial priors.

\[
s_{M,N} = \frac{p + H(M, N) - \tilde{f}}{A(M, N)}
\]

The conditional moments of \( f_u \) taken under a particular belief \( M \) and \( N \), are given by:

\[
E_{M,N} [f_u \mid p] = E_{M,N} [\tilde{f} + \theta u + \epsilon \mid p]
\]

\[
= \tilde{f} + E_{M,N} [\theta u \mid p] + 0
\]

\[
= \tilde{f} + E_{M,N} [\theta u \mid s = s_{M,N}]
\]

\[
= \tilde{f} + \frac{\text{cov}(s, \theta u)}{\text{var}(s)} s_{M,N}
\]

\[
= \tilde{f} + \left( \frac{\rho \tau_0}{\tau - 1} \right) s_{M,N}
\]

\[
= \tilde{f} + \rho \lambda \left[ p + H(M, N) - \tilde{f} \right] A(M, N)
\]

\[
= \tilde{f} + \rho \lambda \cdot G(M, N; p, \tilde{f})
\]

For simplicity, we write \( G(M, N; p, \tilde{f}) \equiv G(M, N) \) and the objective function of an uninformed trader can be written as:

\[
\min_{M \in [M_1, M_2], N \in [N_1, N_2]} \left( E_{M} [f_u \mid p] - p \right) x_u - \frac{k}{2} x_u^2
\]

\[
\Rightarrow \min_{M \in [M_1, M_2], N \in [N_1, N_2]} \left( \tilde{f} + \rho \lambda \cdot G(M, N) - p \right) x_u - \frac{k}{2} x_u^2
\]

\[
\begin{cases}
-\frac{k}{2} x_u^2 + [\tilde{f} + \rho \lambda \cdot \{G(M, N)\}_{\text{min}} - p] \cdot x_u, & \text{if } x_u > 0 \\
0, & \text{if } x_u = 0 \\
-\frac{k}{2} x_u^2 + [\tilde{f} + \rho \lambda \cdot \{G(M, N)\}_{\text{max}} - p] \cdot x_u, & \text{if } x_u < 0
\end{cases}
\]

5While formulating portfolio decisions, uninformed traders learn from the price and update each of their beliefs. This rule is known as full Bayesian updating. With full Bayesian updating, no learning occurs regarding the original set of priors, which means that the uninformed agents retain all their initial priors.
where \( \{G(M, N)\}_{\text{min}} = \{\frac{p + H(M, N) - \bar{f}}{A(M, N)}\}_{\text{min}} \) and \( \{G(M, N)\}_{\text{max}} = \{\frac{p + H(M, N) - \bar{f}}{A(M, N)}\}_{\text{max}} \) are the minimum and maximum of the function \( \{G(M, N)\} \), given the value of \( p \), respectively.

Thus an uninformed trader’s demand function is:

\[
x_u(p) = \begin{cases} 
\frac{f + \rho \lambda \left( \frac{p + H(M, N) - \bar{f}}{A(M, N)} \right)}{k} - p, & \text{if } \bar{f} + \rho \lambda \left( \frac{p + H(M, N) - \bar{f}}{A(M, N)} \right) > 0 \\
0, & \text{if } \bar{f} + \rho \lambda \left( \frac{p + H(M, N) - \bar{f}}{A(M, N)} \right) \leq 0 \leq \bar{f} + \rho \lambda \left( \frac{p + H(M, N) - \bar{f}}{A(M, N)} \right) \\
\frac{f + \rho \lambda \left( \frac{p - H(M, N) - \bar{f}}{A(M, N)} \right)}{k} - p, & \text{if } \bar{f} + \rho \lambda \left( \frac{p + H(M, N) - \bar{f}}{A(M, N)} \right) < 0
\end{cases}
\]  

(253)

G.3 Proof of Proposition 12

We define the value of \((M, N)\) that causes the function \(G(M, N)\) to reach a minimum as \((\bar{M}, \bar{N})\) and the value of \((M, N)\) that causes the function \(G(M, N)\) to reach a maximum as \((\overline{M}, \overline{N})\),

\[
(M, N) \equiv \arg\min_{M \in [M_1, M_2], N \in [N_1, N_2]} G(M, N) = \arg\min_{M \in [M_1, M_2], N \in [N_1, N_2]} \left[ \frac{p + H(M, N) - \bar{f}}{A(M, N)} \right] \\
(\overline{M}, \overline{N}) \equiv \arg\max_{M \in [M_1, M_2], N \in [N_1, N_2]} G(M, N) = \arg\max_{M \in [M_1, M_2], N \in [N_1, N_2]} \left[ \frac{p + H(M, N) - \bar{f}}{A(M, N)} \right]
\]  

(254)

We define the values that \(H(M, N)\) and \(A(M, N)\) take on when \((M, N) = (\bar{M}, \bar{N})\) and \((M, N) = (\overline{M}, \overline{N})\) as:

\[
\bar{H} \equiv H(\bar{M}, \bar{N}) \quad \bar{A} \equiv A(\bar{M}, \bar{N}) \\
\overline{A} \equiv A(\overline{M}, \overline{N})
\]  

(255)

Note that \(\bar{H}, \bar{A}, \overline{A}\) and \(\overline{A}\) will be a constant. Hence, we can define the maximum and minimum of \(G(M, N)\) as:

\[
\left\{ \frac{p + H(M, N) - \bar{f}}{A(M, N)} \right\}_{\text{min}} \equiv \bar{G} = \frac{p + H - \bar{f}}{\bar{A}} \\
\left\{ \frac{p + H(M, N) - \bar{f}}{A(M, N)} \right\}_{\text{max}} \equiv \overline{G} = \frac{p + \overline{H} - \bar{f}}{\overline{A}}
\]  

(256)

Hence, the optimal demand functions of the uninformed traders, equation (255), can be written as,

\[
x_u(p) = \begin{cases} 
\frac{f + \rho \lambda \left( \frac{p + H - \bar{f}}{A} \right)}{k} - p, & \text{if } \bar{f} + \rho \lambda \left( \frac{p + H - \bar{f}}{A} \right) > 0 \\
0, & \text{if } \bar{f} + \rho \lambda \left( \frac{p + H - \bar{f}}{A} \right) \leq 0 \leq \bar{f} + \rho \lambda \left( \frac{p + H - \bar{f}}{A} \right) \\
\frac{f + \rho \lambda \left( \frac{p - \overline{H} - \bar{f}}{\overline{A}} \right)}{k} - p, & \text{if } \bar{f} + \rho \lambda \left( \frac{p + H - \bar{f}}{A} \right) < 0
\end{cases}
\]  

(257)

We evaluate the feasibility of the possible demand functions given in equation (255) case by case by inserting it into the market clearing condition.
I. First, suppose $x_U > 0$ and the premise

$$\ddot{f} + \rho \lambda \cdot \frac{p + H - \ddot{f}}{A} - p > 0$$

(258)

is satisfied, then the demand function of informed traders and uninformed traders are:

$$x_I = \frac{\ddot{f} + \lambda s - p}{k}$$

$$x_U = \frac{\ddot{f} + \rho \lambda \cdot \left(\frac{p + H - \ddot{f}}{A}\right) - p}{k}$$

(259)

We insert the equation (259) into the market clearing condition, $M \cdot x_I + N \cdot x_U = Z$ and we get,

$$M \cdot \left(\frac{\ddot{f} + \lambda s - p}{k}\right) + N \cdot \left(\frac{\ddot{f} + \rho \lambda \cdot \left(\frac{p + H - \ddot{f}}{A}\right) - p}{k}\right) = Z$$

Rearrange the terms and we get,

$$\left(M + N - \frac{N \rho \lambda}{A}\right) p = \left(M + N - \frac{N \rho \lambda}{A}\right) \ddot{f} + M \lambda s - \left(kZ - \frac{N \rho \lambda H}{A}\right)$$

$$p = \ddot{f} + \left(\frac{M \lambda}{M + N - \frac{N \rho \lambda}{A}}\right) s - \left(\frac{kZ - \frac{N \rho \lambda H}{A}}{M + N - \frac{N \rho \lambda}{A}}\right)$$

(260)

Using the undetermined coefficient method and match the coefficient, we get:

$$A(M, N) = \frac{M \lambda}{M + \left(1 - \frac{\rho \lambda}{A}\right) N}$$

$$H(M, N) = \frac{kZ - \frac{N \rho \lambda H}{A}}{M + \left(1 - \frac{\rho \lambda}{A}\right) N}$$

(261)

Hence, we can express the function $G(M, N)$ as:

$$G(M, N) = \frac{p + H(M, N) - \ddot{f}}{A(M, N)}$$

$$= \frac{p + \left[kZ - \frac{N \rho \lambda H}{A}\right] - \ddot{f}}{M + \left(1 - \frac{\rho \lambda}{A}\right) N}$$

$$= \frac{p - \ddot{f}}{1} + \left[kZ - \frac{N \rho \lambda H}{A}\right] \cdot \frac{N}{M \lambda}$$

(262)
We define $L = \left( p - \bar{f} \right) \left( 1 - \frac{\rho \lambda}{A} \right) - \frac{\rho \lambda H}{A}$.

Since at the beginning of this sub-case I, our main premise is that

\[ \bar{f} + \rho \lambda \cdot \frac{p + H - \bar{f}}{A} - p > 0 \]

This premise can be written as:

\[ - \left( p - \bar{f} \right) \left( 1 - \frac{\rho \lambda}{A} \right) - \frac{\rho \lambda H}{A} > 0 \]

This implies that $L < 0$.

1. Suppose $(LN_2 + kZ) > 0$, then

\[ (M_2, N_2) = (M, N) = \arg \min_{M \in [M_1, M_2], N \in [N_1, N_2]} G(M, N) \]

Inserting $M = M_2$ and $N = N_2$ into equation (258), and we get,

\[ A = A(M_2, N_2) = \frac{M_2 \lambda}{M_2 + \left( 1 - \frac{\rho \lambda}{A} \right) N_2} \]

\[ H = H(M_2, N_2) = \frac{kZ - \frac{N_2 \rho \lambda H}{A}}{M_2 + \left( 1 - \frac{\rho \lambda}{A} \right) N_2} \]

Solving for $A$ and $H$, and we find the explicit expression:

\[ A = \frac{M_2 + N_2 \rho}{M_2 + N_2} \lambda \]

\[ H = \frac{kZ}{M_2 + N_2} \tag{263} \]

Inserting equation (263) into equation (260), and we find the price function,

\[ p = \bar{f} + \left( \frac{M \lambda}{M + N - \frac{N \rho \lambda}{A}} \right) s - \left( \frac{kZ - \frac{N_2 \rho \lambda H}{A}}{M + N - \frac{N \rho \lambda}{A}} \right) \]

\[ = \bar{f} + \left[ \frac{M (M_2 + N_2 \rho)}{M (M_2 + N_2 \rho) + NM_2 (1 - \rho)} \right] \lambda \cdot s - \left[ \frac{[M_2 + (N_2 - N) \rho] \cdot kZ}{M (M_2 + N_2 \rho) + NM_2 (1 - \rho)} \right] \]

We need to check that the two premise, equation (258) and $(LN_2 + kZ) > 0$, should be satisfied. Equation (258) and equation (264) imply that,

\[ p < \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_2} \right) \tag{265} \]

$(LN_2 + kZ) > 0$ and equation (264) imply that,

\[ p > \bar{f} - \left( \frac{1}{1 - \rho} \right) \left( \frac{kZ}{N_2} \right) \tag{266} \]
Substituting equation (264) into equation (265), we get
\[ s < \left[ \frac{1}{M} + \frac{\rho}{M_2(1-\rho)} \right] \frac{kZ}{\lambda} \quad (267) \]

Substituting equation (264) into equation (266), we get
\[ s > -\frac{kZ}{N_2\lambda(1-\rho)} \quad (268) \]

To sum up, in this subsection, when \( \bar{f} - (1-\rho) \left( \frac{kZ}{N_2} \right) < p < \bar{f} + \left( \frac{\rho}{1-\rho} \right) \left( \frac{kZ}{N_2} \right) \), or say, when
\[ -\frac{kZ}{N_2(1-\rho)} < s < \frac{1}{M + \frac{\rho}{M_2(1-\rho)}} \frac{kZ}{\lambda}, \]
the equilibrium price function is
\[ p = \bar{f} + \left( \frac{M (M_2 + N_2 \rho)}{M (M_2 + N_2 \rho) + N M_2 (1-\rho)} \right) \lambda \cdot s - \left[ \frac{[M_2 + (N_2 - N) \rho] \cdot kZ}{M (M_2 + N_2 \rho) + N M_2 (1-\rho)} \right] \quad (269) \]

2. Suppose \((LN_2 + kZ) < 0\), then
\[ (M_1, N_2) = (M, N) = \arg \min_{M \in [M_1, M_2], N \in [N_1, N_2]} G(M, N) \]

Inserting \( M = M_1 \) and \( N = N_2 \) into equation (270), and we get,
\[ A = A(M_1, N_2) = \frac{M_1 \lambda}{M_1 + (1 - \frac{\rho \lambda}{M}) N_2} \]
\[ H = H(M_1, N_2) = \frac{kZ - \frac{N \rho \lambda H}{M}}{M_1 + (1 - \frac{\rho \lambda}{M}) N_2} \]

Solving for \( A \) and \( H \), and we find the explicit expression:
\[ A = \left( \frac{M_1 + N_2 \rho}{M_1 + N_2} \right) \lambda \]
\[ H = \frac{kZ}{M_1 + N_2} \quad (270) \]

Inserting equation (270) into equation (260), and we find the price function,
\[ p = \bar{f} + \left( \frac{M \lambda}{M + N - \frac{N \rho \lambda}{M}} \right) s - \left( \frac{kZ - \frac{N \rho \lambda H}{M}}{M + N - \frac{N \rho \lambda}{M}} \right) \]
\[ = \bar{f} + \left( \frac{M (M_1 + N_2 \rho)}{M (M_1 + N_2 \rho) + N M_1 (1-\rho)} \right) \lambda \cdot s - \left[ \frac{[M_1 + (N_2 - N) \rho] \cdot kZ}{M (M_1 + N_2 \rho) + N M_1 (1-\rho)} \right] \quad (271) \]

We need to check that the two premise, equation (258) and \((LN_2 + kZ) < 0\), should be satisfied. Equation (258) and equation (264) imply that,
\[ p < \bar{f} + \left( \frac{\rho}{1-\rho} \right) \left( \frac{kZ}{M_1} \right) \quad (272) \]
When equation (273) is satisfied, equation (272) will be automatically satisfied. Substituting equation (271) into equation (273), we get:

\[ s < -\frac{kZ}{N_2\lambda(1 - \rho)} \]  

(274)

To sum up, in this subsection, when \( p < \bar{f} - \left(\frac{1}{1 - \rho}\right) \left(\frac{kZ}{N_2}\right) \), or say, when \( s < -\frac{kZ}{N_2\lambda(1 - \rho)} \), the equilibrium price function is

\[ p = \bar{f} + \lambda s - \frac{kZ}{M} \]  

(275)

II. Second, suppose \( x_U = 0 \) and the premise

\[ \bar{f} + \rho \lambda \left(\frac{p + \frac{H - \bar{f}}{A}}{A}\right) - p \leq 0 \leq \bar{f} + \rho \lambda \left(\frac{p + \frac{H - \bar{f}}{A}}{A}\right) - p \]  

(276)

is satisfied, then the demand function of informed traders and uninformed traders are:

\[ x_I = \frac{\bar{f} + \lambda s - p}{k} \]

\[ x_U = 0 \]  

(277)

We insert the equation (277) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,

\[ p = \bar{f} + \lambda s - \frac{kZ}{M} \]  

(278)

Hence, in this situation,

\[ A(M, N) = \bar{A} = \frac{A}{M} = \lambda \]

\[ H(M, N) = \frac{kZ}{M} \]

Hence, we can express the function \( G(M, N) \) as:

\[ G(M, N) = \frac{p + H(M, N) - \bar{f}}{A(M, N)} \]

\[ = \frac{p + \left(\frac{kZ}{M}\right) - \bar{f}}{\lambda} \]  

(279)

It is obvious that,

\[ \bar{H} = H(M_2, N) = \frac{kZ}{M_2} \]

\[ \bar{H} = H(M_1, N) = \frac{kZ}{M_1} \]

To satisfy the premise, equation (276), the following two equations should hold for true.

\[ \bar{f} + \rho \lambda \left(\frac{p + \frac{kZ}{M} - \bar{f}}{\lambda}\right) - p \leq 0 \]
\[ \bar{f} + \rho \lambda \cdot \left( \frac{p + \frac{kZ}{M} - \bar{f}}{\lambda} \right) - p \geq 0 \]

This implies that,
\[ \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_2} \right) \leq p \leq \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_1} \right) \]  

(280)

Substituting equation (278) into equation (280), we get
\[ \left[ \frac{1}{M} + \frac{\rho}{M_2(1 - \rho)} \right] kZ \leq s \leq \left[ \frac{1}{M} + \frac{\rho}{M_1(1 - \rho)} \right] kZ \]  

(281)

To sum up, in this subsection, when \( \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_2} \right) \leq p \leq \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_1} \right) \), or say, when
\[ s \leq \left[ \frac{1}{M} + \frac{\rho}{M_1(1 - \rho)} \right] kZ, \]

the equilibrium price function is
\[ p = \bar{f} + \lambda s - \frac{kZ}{M} \]  

(282)

III. Third, suppose \( x_U < 0 \) and the premise
\[ \bar{f} + \rho \lambda \cdot \left( \frac{p + \frac{H}{A} - \bar{f}}{\lambda} \right) - p < 0 \]  

(283)

is satisfied, then the demand function of informed traders and uninformed traders are:
\[ x_I = \frac{\bar{f} + \lambda s - p}{k} \]
\[ x_U = \frac{\bar{f} + \rho \lambda \cdot \left( \frac{p + \frac{H}{A} - \bar{f}}{\lambda} \right) - p}{k} \]  

(284)

We insert the equation (284) into the market clearing condition, \( M \cdot x_I + N \cdot x_U = Z \) and we get,
\[ M \cdot \left( \frac{\bar{f} + \lambda s - p}{k} \right) + N \cdot \left( \frac{\bar{f} + \rho \lambda \cdot \left( \frac{p + \frac{H}{A} - \bar{f}}{\lambda} \right) - p}{k} \right) = Z \]

Rearrange the terms and we get,
\[ \left( M + N - \frac{N \rho \lambda}{A} \right) p = \left( M + N - \frac{N \rho \lambda}{A} \right) \bar{f} + M \lambda s - \left( kZ - \frac{N \rho \lambda H}{A} \right) \]
\[ p = \bar{f} + \frac{M \lambda}{M + N - \frac{N \rho \lambda}{A}} \left( s - \frac{kZ - \frac{N \rho \lambda H}{A}}{M + N - \frac{N \rho \lambda}{A}} \right) \]  

(285)
Using the undetermined coefficient method and match the coefficient, we get:

\[
A(M, N) = \frac{M\lambda}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}
\]

\[
H(M, N) = \frac{kZ - \frac{N\rho\lambda\bar{\lambda}}{\bar{\lambda}}}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}
\]  \hspace{1cm} (286)

Hence, we can express the function \(G(M, N)\) as:

\[
G(M, N) = \frac{p + H(M, N) - \bar{f}}{A(M, N)}
\]

\[
= \frac{p + \left[\frac{kZ - \frac{N\rho\lambda\bar{\lambda}}{\bar{\lambda}}}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}\right] - \bar{f}}{\frac{M\lambda}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}}
\]

\[
= \frac{(p - \bar{f}) \left[1 - \frac{\rho\lambda}{\bar{\lambda}}\right] + \frac{kZ - \frac{N\rho\lambda\bar{\lambda}}{\bar{\lambda}}}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}}{\frac{M\lambda}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}}
\]

\[
= \frac{(p - \bar{f}) \left[1 - \frac{\rho\lambda}{\bar{\lambda}}\right] + \frac{kZ - \frac{N\rho\lambda\bar{\lambda}}{\bar{\lambda}}}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}}{\frac{M\lambda}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}}
\]

\[
= \frac{p - \bar{f}}{\frac{M\lambda}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}} + \frac{kZ - \frac{N\rho\lambda\bar{\lambda}}{\bar{\lambda}}}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}
\]

\[
= \frac{p - \bar{f}}{\frac{M\lambda}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}} + \frac{kZ - \frac{N\rho\lambda\bar{\lambda}}{\bar{\lambda}}}{M + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N}
\]

We define \(L = (p - \bar{f}) \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right) - \frac{\rho\lambda\bar{\lambda}}{\bar{\lambda}}\).

Since at the beginning of this sub-case III, our main premise is that

\[
\bar{f} + \rho\lambda \cdot \frac{p + \bar{\lambda} - \bar{f}}{\bar{\lambda}} - p < 0
\]

This premise can be written as:

\[
- \left[(p - \bar{f}) \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right) - \frac{\rho\lambda\bar{\lambda}}{\bar{\lambda}}\right] < 0
\]

This implies that \(L > 0\). Hence,

\[
(M_1, N_2) = (\bar{M}, \bar{N}) = \arg\max_{M \in [M_1, M_2], N \in [N_1, N_2]} G(M, N)
\]

Inserting \(\bar{M} = M_1\) and \(\bar{N} = N_2\) into equation (286), and we get,

\[
\bar{A} = A(M_1, N_2) = \frac{M_1\lambda}{M_1 + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N_2}
\]

\[
\bar{H} = H(M_1, N_2) = \frac{kZ - \frac{N_2\rho\lambda\bar{\lambda}}{\bar{\lambda}}}{M_1 + \left(1 - \frac{\rho\lambda}{\bar{\lambda}}\right)N_2}
\]
Solving for $\overline{A}$ and $\overline{H}$, and we find the explicit expression:

$$\overline{A} = \left( \frac{M_1 + N_2 \rho}{M_1 + N_2} \right) \lambda$$

$$\overline{H} = \frac{kZ}{M_1 + N_2} \tag{288}$$

Inserting equation (288) into equation (285), and we find the price function,

$$p = \bar{f} + \left( \frac{M \lambda}{M + N - \frac{N \rho A}{A}} \right) s - \left( \frac{kZ - \frac{N \rho \lambda A}{A}}{M + N - \frac{N \rho A}{A}} \right)$$

$$\Rightarrow p = \bar{f} + \left[ \frac{M (M_1 + N_2 \rho)}{M (M_1 + N_2 \rho) + N M_1 (1 - \rho)} \right] \lambda \cdot s - \left[ \frac{[M_1 + (N_2 - N) \rho] \cdot kZ}{M (M_1 + N_2 \rho) + N M_1 (1 - \rho)} \right] \tag{289}$$

We need to check that the premise, equation (283), should be satisfied. Equation (283) and equation (288) imply that,

$$p > \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_1} \right) \tag{290}$$

Substituting equation (289) into equation (290), we get

$$s > \left[ \frac{1}{M} + \frac{\rho}{M_1 (1 - \rho)} \right] \frac{kZ}{\lambda} \tag{291}$$

To sum up, in this subsection, when \( p > \bar{f} + \left( \frac{\rho}{1 - \rho} \right) \left( \frac{kZ}{M_1} \right) \), or say, when \( s > \left[ \frac{1}{M} + \frac{\rho}{M_1 (1 - \rho)} \right] \frac{kZ}{\lambda} \), the equilibrium price function is

$$p = \bar{f} + \left[ \frac{M (M_1 + N_2 \rho)}{M (M_1 + N_2 \rho) + N M_1 (1 - \rho)} \right] \lambda \cdot s - \left[ \frac{[M_1 + (N_2 - N) \rho] \cdot kZ}{M (M_1 + N_2 \rho) + N M_1 (1 - \rho)} \right] \tag{292}$$
References


