Laurent Nottale: Scale relativity and fractal space-time

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I have heard about scale relativity for many years, but never had the opportunity to actually get into it until I was asked to read Laurent Nottale's "Scale relativity and fractal space-time". The book consists in two parts, the first one, Chapters 1–8, in which the theory is exposed, and the second where applications are discussed, in Chapters 9–14.

The basic principle out of which the entire theory is developed in this book is that the particle trajectories need not be differentiable, following the original Feynman's idea whose path integral formulation of quantum mechanics can be recast in terms of continuous but nowhere differentiable trajectories with fractal dimension $D_{\mathrm{F}} = 2$. This propagator-based formulation is taken here seriously by assuming that particles do follow such trajectories (although later on in the course of the book, it is said that the particles do not, in fact, follow such trajectories, those being a mere representation of a geodesic flow leading to probabilities). This is the first idea developed in great length in Chapters 1–3.

The second idea, related to the first, consists in assuming that since the trajectories are fractals, their properties, as seen by an observer, are depending on the scale at which we observe them, i.e., on the accuracy of the measurement that are performed on them. Then, following the example of the invariance under Lorentz transformations in special relativity, the author proposes, in Chapter 3, a similar invariance with respect to scale transformations. This leads to a new kind of covariant derivative as well as to the existence of some special length scales, equivalent to the velocity of light in special relativity, which are identified to the Planck and cosmological constant scales, depending on the context.

P. Peter (✉)
Institut d'Astrophysique de Paris, UMR7095 CNRS, Université Pierre and Marie Curie,
98bis boulevard Arago, 75014 Paris, France
e-mail: peter@iap.fr

With the two ideas previously developed, it is argued in Chapter 5 that for a given particle whose trajectory is now a non-differentiable geodesic, the velocity field should be two-valued (left and right derivatives with respect to time being different) and recast into a single complex variable, later somehow to be identified with a wave function (although at this stage, trajectories are no longer part of the description). It is then claimed that it can also be implemented by means of coordinates having a classical part as usual and a stochastic (with vanishing mean) part, whose mean square provides a new constant, later to be related with the Planck constant: then, the trajectories are geodesics of a non-differentiable space. The classical geodesic (Newton-like trajectory) equation in these coordinates is then put in a form equivalent, under some arguable hypothesis, to a Schrödinger equation. It is then argued that the probability of finding a particle at a given position had to be proportional to the geodesic flow, hence leading to the Born rule.

Moving on to special relativistic trajectories in Chapter 6, the Klein–Gordon equation is obtained in a similar way, and after some considerations of algebra doubling, again based on the two-valuedness of derivatives, the Dirac equation is introduced. This aims at showing a geometrically-based theory of spin. Finally, the scale covariant derivative is used to produce gauge vectors, thus completing the set of known objects and forming the basis for a particle physics model (Chapter 7). Chapter 8 provides some ideas as to how to generalize what precedes by extending to fractal dimensions other than the usual quantum mechanical $D_{\mathrm{F}} = 2$, including scaling laws in a varying fractal dimension model.

The second part of the book is introduced in Chapter 9 and consists in applications ranging from ordinary laboratory experiments to astrophysics/cosmology and high energy particle physics, with a final chapter dedicated to biology and other human-related sciences. It begins in Chapter 10 with probably the most interesting issue, namely the transition between quantum and classical behaviors, which is said here to have a purely "geometric interpretation" (transition between fractal and non-fractal geodesics). An interesting example is a calculation of many possible trajectories in a 2-slit Young experiment or the treatment of the harmonic oscillator. All through this chapter, use is made of the quantum potential, defined in exactly the same way as in the ontological formulation of quantum mechanics.

Then comes Chapter 11, devoted to particle physics and the running of the coupling constants. The characteristic dilation–invariant scale is then taken to be the Planck length, which serves the same purpose that the usual renormalization scale normally assumes. Amusingly, this chapter also contains a prediction for the Higgs boson that should have been observed at $m_{\mathrm{H}} \simeq 113.7\,\mathrm{GeV}$ ... it would appear, according to the book itself, that the theory it describes would be already ruled out by LHC data!

When it comes to cosmology in Chapter 12, the theory now takes as its fundamental the cosmological constant length scale, although the Planck scale still plays a role close to the now reinterpreted primeval singularity. The new theory is said to solve the horizon puzzle without invoking a phase of inflation. Bizarrely, a practicing cosmologist will hardly recognize any equation in this chapter, which also does not mention primordial perturbations ... in the time of Planck ultimate observation of these fluctuations, this is disturbing.

Chapter 13 presents a fractal-based extended Newtonian-like theory of gravitation, leading to specific (and, as far as I know, not verified) new astrophysical predictions, ranging from the solar system (planet trajectories) to the formation of large scale structures in cosmology or star formation in galaxies. It is argued that the scale relativity approach permits to solve the dark matter question.

Last but not least, Chapter 14 discusses all other possible applications of scale relativity and fractal mathematics in biology (evolution theory for instance, but a new approach to the origin of life is presented, not invoking biological phenomena), history and sociology. Having no serious knowledge on these topics, I cannot pretend to argue in one way or another, but the presentation suggests some reformulation, possibly even useful, of data analysis.

Generally speaking, I found the book not clearly written, and it is always quite time consuming to understand almost any point which is raised. Out of 555 references covering all the topics discussed in the book, i.e. ranging from particle physics and cosmology to neuroscience (and with mention of historical papers such as 1905 and 1916 Einstein's), 95 are signed or co-signed by the author himself, including numerous proceedings: that amounts to a staggering almost 20 % self-references!

Apart from the fact that I have some doubts regarding the mathematical rigor of the discussion, I have found in many places either misleading or possibly erroneous statements concerning in particular the quantum mechanical treatment of particle physics, at least according to the widely accepted view shared by the vast majority of living physicists. This would not, by itself, be a problem for a monograph dealing with a highly controversial topic, as the book actually is, but then comes what is, in my view, the most important point: contrary to what the topic discussed should imply, it is clearly aimed at being a textbook, with over 700 pages, often repeating many times the same thing (which is definitely a good pedagogical method) and including exercises and problems to be worked out by students . . .

My personal opinion is that the topic is probably interesting (the quantum-to-classical transition treatment for instance is original and could lead to some yet-to-do developments), and could perhaps deserve more attention than it had until now. Basically, there is only a handful of people involved in these researches, and I cannot imagine the present book can in any way change this fact. Indeed, it is based on a highly controversial set of ideas that have not reached any level of acceptance by the physics community (as far as I can tell), and should thus be presented with much more caution than is done in the present work; a 150 page monograph at most might have been useful for practicing researchers. In particular, I would not suggest to any physics student to read this book.