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Jenni Ingram, Marcus Schütte, Maire Ní Ríordáin

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Language and communication in the mathematics classrooms

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MEETING THE CHALLENGES OF RESEARCHING LANGUAGE AND COMMUNICATION IN MATHEMATICS EDUCATION

Jenni Ingram, Marcus Schütte, Fengjuan Hu, Máire Ní Ríordáin, and Tran Vui
University of Oxford, UK; Technical University of Dresden, Germany; Capital Normal University, China; University College Cork, Ireland; Hue University, Vietnam

ICME-14 will bring together researchers from around the world and TSG 39 will include a range of researchers from different countries, theoretical perspectives, methodological approaches and with different foci. At ICME-13 Judit Moschkovich made four recommendations for research on language, interaction and communication in mathematics education and in this paper we return to these recommendations to explicate additional challenges that researchers in this TSG have faced as they seek to address these recommendations.

INTRODUCTION

Research in language and communication in mathematics education draws from a wide range of theoretical and methodological approaches that is clearly visible in the wide range of work shared at ICME conferences. Morgan (2006) talks of this as the 'turn to language' with the increased attention to the role and nature of language and communication in learning and doing mathematics. Yet as Pimm (2018) points out there is a long history of research focusing on language within mathematics education. What seems to be changing is the recognition of the complexity of the relationship between language (or language use) and the learning and teaching of mathematics. In TSG 39 language and communication are interpreted in the broadest sense and includes work focusing on classroom interactions, interactions between children at play, multimodal analysis, as well as research that focuses on the multi-semiotic nature of mathematical activities, or even the role of silences (e.g., Boistrup & Samuelsson, 2018; O'Connor et al., 2017). In this paper we return to Moschkovich's (2018) recommendations for research within TSG 39 and explicate the additional challenges we are encountering as we seek to address these recommendations.

RESPONDING TO RECOMMENDATIONS

At ICME-13 Judit Moschkovich (2018) gave four recommendations for research with an interest in language and communication in mathematics education and in this paper we return to these recommendations to consider where we are now. These recommendations were: using interdisciplinary approaches; defining central constructs; building on existing methodologies; and recognizing central distinctions while avoiding dichotomies.

To begin with we bring together the recommendations of using interdisciplinary approaches and building on existing methodologies as we see the two as interrelated. Research in language, communication and interaction within mathematics education draws from a wide range of theories and methodologies both within mathematics education but also from other disciplines such as sociology, psychology, anthropology, linguistics, semiotics and many more. The context of mathematics education and the focus on language, interaction and communication necessitates the bringing together of theories focusing on the teaching and learning of mathematics and theories of language, interaction and communication. This presents the challenge of ensuring the visibility of mathematics in linguistic analyses (Barwell et al., 2019) as well as articulating theories perhaps unfamiliar to many within mathematics education.

Research needs to not only recognise the complexity of language use but also to design research that explicitly considers this complexity. Furthermore, researchers interested in language, interaction and communication will also need to consider the complexity of mathematical learning, thinking and reasoning. Innovation within the field comes not only from developing new theories and methodologies, but also in the bringing together of existing theories and methodologies established in other fields.

The next recommendation relates to defining central constructs. Here Moschkovich focuses her recommendation on defining terms such as language, register and discourse, pointing to the complexity of the notions that lie between these terms, but also the different uses of these terms. Part of this issue arises because of the different theoretical and methodological approaches drawn upon. It is therefore important to not only define these terms, but also to use them in a way that is consistent with the approach to research being taken. A particular challenge when combining theories from different disciplines.

However, there is another issue of terminology that researchers need to confront. Different theories and methodologies use different terminology to describe similar notions. Research on language, interaction and communication in mathematics education has led to a plethora of terms whose distinctions, similarities and differences are not always clear. Further terms within the language research such as language repertoires or communication acts (as used for example in Andersson & Wagner, 2019 for example) as well as broader terms found in language, interaction and communication research such as knowledge, identity, or understanding (as used for example in Ingram & Andrews, 2019) reflect the different theoretical perspectives they originate from. Yet to what extent are we talking about the same, or similar, notions?

Finally, we turn to the recommendation of recognising central distinctions while avoiding dichotomies. Moschkovich emphasises the often-made dichotomy between quantitative and qualitative approaches in contrast to the many examples of work that combines both. Even where both quantitative and qualitative approaches are taken within a single study (e.g., O'Connor et al., 2017) they are often reported separately. Whilst in some cases making a distinction helps to clarify the approaches taken, in others there is more of a scale or spectrum. Take for example the scale at which researchers are working. For some it is the individual word use that may be of interest, for others it may be a sequence of interactions on a topic (using different definitions of topic), moving to interactions within an activity, or lesson, or across a sequence of lessons on a given topic (again with variations in the definition of topic), or at the macro end, discourses associated with cultures and practices within those cultures. This connects to another aspect worthy of further consideration, the nature of the attention to language in the research. This can often be treated as a dichotomy, with language being the focus of the research, or the medium through which researchers can gain access to mathematical thinking or learning. Yet language and mathematical activities are often intertwined making this distinction challenging to make (Andersson & Wagner, 2019).

FURTHER CHALLENGES

Researchers interested in language, interaction and communication in mathematics education are often also interested in other topics relevant to different TSGs at ICME-14. For many the interest is in the linguistic features of particular mathematical concepts such as multiplication (e.g., Sum & Kwon, 2018), geometry (e.g., Rønning & Strømskag, 2017) or trigonometry (e.g., Andresen & Dahl, 2018). For others the interest in is in particular contexts, such as word problems (e.g., Leiss et al., 2019) or textbooks (e.g., Dawkins et al., 2019), or mathematical activities such as explaining (e.g., Ingram et al., 2019), argumentation (e.g., Ferrari, 2017) or reasoning (e.g., Hunter, 2017). There is also considerable interest in multilingual classrooms and other language diverse contexts (e.g., Barwell et al., 2019; Planas, 2019; Ní Ríordáin et al., 2017), and other issues of social justice (Vogler et al., 2018). Many of the researchers concerned with language and communication in mathematics education are also interested in teacher professional development (e.g., Bakker et al., 2019). These different connections between and within fields require researchers to bring together different histories of research, relevant and commensurate theories and often complex research designs.

CONCLUSION

Whilst we have focused on the challenges researchers in language, interaction and communication in mathematics education face, these challenges are also opportunities. They are opportunities to innovate, and opportunities to influence the teaching and learning of mathematics in a variety of contexts and for a range of learners. By coming together as a TSG the differences and similarities in the research shared enable us all to take advantage of these opportunities and move the field forward.

References

Andersson, A., & Wagner, D. (2019). Identities available in intertwined discourses: mathematics student interaction. *ZDM - Mathematics Education*, *51*(3), 529–540. https://doi.org/10.1007/s11858-019-01036-w

Andresen, M., & Dahl, B. (2018). Classroom dialogue as a French braid: A case study from trigonometry. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 43–50). PME.

Bakker, A., Mackay, F., Smit, J., & Keijzer, R. (2019). Practicalizing principled knowledge with teachers to design language-oriented mathematics lessons: A design study. In M. Graven, H. Venkat, A. A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 57–64). PME.

Barwell, R., Bose, A., Moschkovich, J., & Phakeng, M. (2019). Researching mathematics education and language diversity: The theories we use and develop, and what they enable us to see, say or do. In M. Graven, H. Venkat, A. A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 85–105). PME.

Boistrup, L. B., & Samuelsson, J. (2018). Resistance from within the mathematics classroom: Silences, strategies, and subjectivities. In U. Gellert, C. Knipping, & H. Straehler-Pohl (Eds.), *Inside the Mathematics Class. Sociological Perspectives on Participation, Inclusion, and Enhancement* (pp. 63–82). Springer.

Dawkins, P. C., Inglis, M., & Wasserman, N. (2019). The use(s) of is in mathematics. *Educational Studies in Mathematics*, *100*(2), 117–137. https://doi.org/10.1007/s10649-018-9868-6

Ferrari, P. L. (2017). Language in argumentation and solution of problems with graphs. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 1276–1283). DCU Institute of Education and ERME.

Hunter, J. (2017). Developing interactive mathematical talk: investigating student perceptions and accounts of mathematical reasoning in a changing classroom context. *Cambridge Journal of Education*, *47*(4), 475–492. https://doi.org/10.1080/0305764X.2016.1195789

Ingram, J., & Andrews, N. (2019). What it means to do mathematics: The discursive construction of identities in the mathematics classroom. In M. Graven, H. Venkat, A. A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 392–399). PME.

Ingram, J., Andrews, N., & Pitt, A. (2019). When students offer explanations without the teacher explicitly asking them to. *Educational Studies in Mathematics*, *101*(1), 51–66. https://doi.org/10.1007/s10649-018-9873-9

Leiss, D., Plath, J., & Schwippert, K. (2019). Language and Mathematics - Key Factors influencing the Comprehension Process in reality-based Tasks. *Mathematical Thinking and Learning*, *21*(2), 131–153. https://doi.org/10.1080/10986065.2019.1570835

Morgan, C. (2006). What does Social Semiotics have to Offer Mathematics Education Research? *Educational Studies in Mathematics*, *61*(1–2), 219–245. https://doi.org/10.1007/s10649-006-5477-x

Moschkovich, J. N. (2018). Recommendations for research on language and learning mathematics. In J. N. Moschkovich, D. Wagner, A. Bose, J. Rodrigues Mendes, & M. Schütte (Eds.), *Language and communication in mathematics education* (pp. 37–47). Springer.

Ní Ríordáin, M., Flanagan, E., & Brilly, C. (2017). Ensuring methodological rigor for investigating the discourse development of bilingual mathematics learners. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the 10th Congress of Research in Mathematics Education* (pp. 1340–1347). DCU and ERME.

O'Connor, C., Michaels, S., Chapin, S., & Harbaugh, A. G. (2017). The silent and the vocal: Participation and learning in whole-class discussion. *Learning and Instruction*, *48*, 5–13. https://doi.org/10.1016/j.learninstruc.2016.11.003

Pimm, D. (2018). Sixty years (or so) of language data in mathematics education. In J. N. Moschkovich, D. Wagner, A. Bose, J. Rodrigues Mendes, & M. Schütte (Eds.), *Language and communication in mathematics education* (pp. 11–36). Springer.

Planas, N. (2019). Transition zones in mathematics education research for the development of language as resource. In M. Graven, H. Venkat, A. A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17–31). PME.

Rønning, F., & Strømskag, H. (2017). Entering the mathematical register through evolution of the material milieu for classification of polygons. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the 10th Congress of Research in Mathematics Education* (pp. 1348–1355). DCU and ERME.

Sum, E., & Kwon, O. N. (2018). An analysis of linguistic features of the multiplication tables and the language of multiplication. *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(7), 2839–2856. https://doi.org/10.29333/ejmste/90760

Vogler, A., Prediger, S., Quasthoff, U., & Heller, V. (2018). Students' and teachers' focus of attention in classroom interaction - subtle sources for the reproduction of social disparities. *Mathematics Education Research Journal*, *30*, 299–323. https://doi.org/10.1007/s13394-017-0234-2

TAU KĒ: A SOFTWARE SOLUTION FOR CAPTURING MULTIPLE REPRESENTATIONS OF PĀNGARAU LANGUAGE

Piata Allen
University of Auckland

For over 100 years, subjects such as mathematics in Aotearoa New Zealand have been taught exclusively in English as a consequence of English only schooling policy. As a result of this and many other assimilationist policies, by the late 1970s the indigenous language, te reo Māori, was considered an endangered language. In response, Māori speaking communities initiated Māori immersion schooling (Māori-medium). In order to teach mathematics in the vernacular, there has been rapid development in the past 30 years of the specialised pāngarau (mathematics) register. The register is now in common usage in pāngarau classrooms across Māori-medium settings, but generally restricted to the school domain where many students and teachers are second-language learners of te reo Māori. This study identifies the affordances of digital technology that address both the conceptual and linguistic challenges faced by Māori-medium teachers and students.

INTRODUCTION

Māori-medium education in Aotearoa New Zealand was initially established outside the state education system, in the 80s, as a challenge to the hegemonic outcomes of state education policies such as language and knowledge loss (McMurchy-Pilkington et al., 2013). Pivotal to the development of Māori-medium education has been the elaboration of the terminology to enable school subjects to be taught monolingually in te reo Māori (Trinick, 2015). At the forefront of this elaboration process has been pāngarau (mathematics), ironically one of the subjects that, in the medium of English, previously supported language and knowledge loss (Trinick, 2015). Despite advances in Māori-medium schooling, te reo Māori still remains a second language (L2) for many Māori-medium students and concerningly teachers. This has pedagogical implications for the teaching and learning of pāngarau.

Consequently, there is growing interest in the use of technology to alleviate these linguistic challenges. However, there is a paucity of research that examines the use of technology in Māori-medium pāngarau classrooms (Tiakiwai & Tiakiwai, 2010) that supports the development of language proficiency and conceptual understanding (Allen, 2015). There are very few interactive Māori language mathematics software applications (apps) and fewer still that are not based on drill and practice (Murphy & Reid, 2016; Trinick et al., 2016). In the absence of research, technology use often defaults to first language (L1) teaching pedagogy (Murphy & Reid, 2016). Applying English-medium L1 approaches to Māori-medium L2 settings can have unintended negative consequences. For example, restricting assessments to one language mode (particularly oral language) may limit L2 students' ability to fully demonstrate their mathematical thinking which can result in wrongfully attributing learning challenges to the mathematics content rather than the language (Allen, 2015). This paper outlines the design and testing of a software solution that addresses the dual goals of Māori-medium pāngarau education, the conceptual understanding of mathematics and revitalising Māori language and Māori knowledge. This paper also explores the hypothesis that Māori-medium pāngarau classrooms face distinct pedagogical and linguistic challenges requiring a specialised approach to software design and testing.

Universal design for learning (UDL) is a pedagogical framework that is being implemented in Aotearoa New Zealand classrooms to meet the needs of diverse learners, and particularly those with special learning needs (including L2 learners of the language of instruction) (Inclusive Education, 2019). The three principles of UDL are: multiple representations, multiple means for creative expression and multiple means of engagement (Rose & Meyer, 2002). Similarly, Anthony and Walshaw (2007), in their synthesis of effective pedagogies, identify the importance of task design that fosters original and creative thinking, mathematical communication, mathematical language and the use of a range of appropriate tools (including technology) and representations. Pimm (1987) also argues that supporting children's learning to communicate mathematically directly helps the learning of mathematical concepts. However, teachers and students need to know the specialised pāngarau terms and how to use them in order to communicate abstract concepts.

For learning environments where the language of instruction is an L2, Cummins (2000) advocates the judicious use of the L1, in this case English, to support the acquisition of cognitive academic language proficiency in an L2. In contrast, many Māori-medium programmes adhere to a language separation policy (Department of Internal Affairs, 1999). Arguments for limiting bilingual instruction in Māori-medium programmes include: the difficulty that L2 teachers and students may have in reverting back to their L2 once they have switched into L1 discussions (Trinick, 2015), a fear of reinforcing negative beliefs about heritage languages and their ability to express mathematical ideas (Jones, 2009) and the language of address directing the language of response (Moschkovich, 2007). Māori-medium programmes that teach monolingually seem to do so in order to maintain the central goal of language revitalisation.

Strengthening Māori language and culture is a core outcome of Māori-medium teaching and learning programmes including pāngarau. Therefore, any process used to design software for use in pāngarau classrooms must centralise Māori knowledge. Whiting, a prominent Māori artist, teacher and designer of Māori artifacts, promoted the importance of traditional Māori concepts and narratives to his design process (cited in Christensen, 2013). Whiting also uses collaboration with Māori communities as a way to strengthen the transmission of Māori knowledge traditions. In this way, Whiting's design process exemplifies the weaving together Māori knowledge traditions and personal creativity through collaboration (Christensen, 2013).

SOFTWARE DEVELOPMENT AND FINDINGS

Now in its third iteration, the first phase of this project was aligned to a professional learning and development programme tasked with designing an intervention for Māori-medium pāngarau students who were identified by their school as performing below national expectations for pāngarau. The phase 1 intervention design included selecting and testing mathematics software applications (apps) in five Māori-medium primary schools. A total of 53 students aged 7 to 12 years old participated in the project across the five schools. Phase 1 software use confirmed the limitations of drill and practice apps for use in Māori-medium pāngarau classrooms as they generally did not promote language development. There were also very few Māori language mathematics apps available. The students demonstrated the need to use multiple representations of pāngarau language to fully explain their mathematical thinking (see Allen, 2015; Trinick et al., 2016).

In phase 2, a literature review was undertaken to inform the design of a purpose-built software solution for use in Māori-medium pāngarau classrooms. Three key themes that emerged from the literature included the benefits of communicating mathematically using multiple representations, collaborating in the process of creating mathematical meaning and strengthening Māori language and Māori knowledge traditions. In particular, the work of Williamson-Leadley and Ingram (2013) with Show and Tell apps provided insights into how technology can be used to simultaneously support conceptual development and language. Show and Tell apps have a whiteboard feature that students can either draw-on or type into, the ability to capture or upload images and record audio or video explanations, thus allowing for multiple representations. Captured presentations of learning can then be revisited as many times as necessary to identify learning challenges, to consolidate learning, to highlight misconceptions and to share learning with others. Ingram and colleagues (2016) found that the use of Show and Tell software helped teachers to identify linguistic challenges and separate these from mathematical misconceptions. A further feature of Show and Tell software is the ability to share captured presentations of knowledge and pāngarau language with the wider community (Trinick et al., 2016), increasing familiarity and potentially the use of pāngarau language outside of the classroom. This led the project team to explore the development of a purpose-built software solution for Māori-medium pāngarau that had features similar to those of Show and Tell apps.

In collaboration with the University of Auckland, Department of Parallel and Reconfigurable Computing, a proof of the concept was developed for use on android tablets called Tau Kē. The app name is a play on words. Tau is the Māori word for number. Tau Kē means magnificent. While there are various Show and Tell apps available, the software designed for this project eliminates much of the language usually present in user interfaces. Instead, the software incorporates universal symbols to indicate the functions of buttons. This approach avoids the unnecessary introduction of English language into Māori-medium pāngarau classrooms and reduces language barriers for L2 learners of te reo Māori. The whiteboard feature also has a grid overlay that enables more accurate representations of mathematical concepts such as fractions. As part of the software development, user interface testing was conducted with a group of four students aged 10-13 years and their pāngarau teacher. Phase 2 user testing has shown that the software design provides opportunities to collaborate in the creation of mathematical presentations of knowledge. When two or more users worked together on the same android tablet, strategies and corrections were negotiated. Phase 2 user testing also showed the use of traditional Māori symbolism in drawn representations. Two of the users seemed to reference poutama imagery (stairway pattern) when representing the division of a quadrilateral into equal parts. These findings suggest that the software design supports the use of multiple representations of the pāngarau register, collaboration by multiple users and the inclusion of Māori representations.

In Phase 3, more in-depth data collection will be conducted in three Māori-medium classrooms with students aged 10-14 years old and their teachers. Phase 3 will also include interviews with pāngarau language experts exploring uniquely Māori ways of communicating mathematically. Phase 3 data analysis will be used to gain further insight into how the software can be used to support the development of conceptual understanding of Mathematics content and of Māori language.

References

Allen, P. (2015). *Te reo pāngarau: Communicating mathematically in Māori-medium classrooms* (Master's thesis, University of Auckland, New Zealand). http://hdl.handle.net/2292/26542

Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics/pāngarau: Best evidence synthesis iteration [BES]*. Ministry of Education.

Christensen, I. (2013). Cliff Whiting: He toi nuku, he toi rangi. He Kupenga Hao i te Reo.

Cummins, J. (2000). *Language, power and pedagogy: Bilingual children in the crossfire*. Multilingual Matters.

Department of Internal Affairs (1999). *Official version of Te Aho Matua o Ngā Kura Kaupapa Māori and an explanation in English.* The New Zealand Gazette, 32(Supp.), 734–746. https://www.dia.govt.nz//pubforms.nsf/NZGZT/Supplement_TeAho32Feb08.pdf/$file/Supplement_TeAho32Feb08.pdf

Inclusive Education. (2019) *Why UDL is valuable. An introduction to UDL and its value in an Aotearoa New Zealand context.* https://www.inclusive.tki.org.nz/guides/universal-design-for-learning/why-udl-is-valuable/

Ingram, N., Williamson-Leadley, S., & Pratt, K. (2016). Showing and telling: using tablet technology to engage students in mathematics. *Mathematics Education Research Journal*, *28*(1), 123–147.

Jones, D. (2009). Bilingual mathematics classrooms in Wales. In R. Barwell (Ed.), *Multilingualism in mathematics classrooms: Global perspectives* (pp. 113–127). Multilingual Matters.

McMurchy-Pilkington, C., Trinick, T., & Meaney, T. (2013). Mathematics curriculum development and indigenous language revitalisation: Contested spaces. *Mathematics Education Research Journal*, *25*(3), 341–360.

Moschkovich, J. (2007). Using two languages when learning mathematics. *Educational Studies in Mathematics*, *64*(2), 121–144. https://link.springer.com/article/10.1007/s10649-005-9005-1

Murphy, H., & Reid, D. (2016). *Ngā Hangarau Matihiko i ngā Kura Ara Reo Māori. Māori Medium ICT Research Report 2016: Digital Technologies*. Ministry of Education

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. Routledge and Keegan Paul

Rose, D. H., & Meyer, A. (2002). *Teaching every student in the digital age: Universal Design for Learning*. ASCD.

Tiakiwai, S., & Tiakiwai, H. (2010). *A literature review focused on virtual learning environments (VLEs) and e-learning in the context of te reo Māori and kaupapa Māori education*. Ministry of Education.

Trinick, A. (2015). *Te reo tātai: The development of a mathematics register for Māori-medium schooling* (EdD thesis, University of Waikato, Hamilton, New Zealand). http://hdl.handle.net/10289/9593

Trinick, T., Allen, P., Taplin B., & Pipi, A. (2016 July). *He puawaitanga harakeke – Using technology to accelerate learning in indigenous language schools* [Paper presentation]. 13th International Congress on Mathematical Education 2016, Hamburg, Germany.

Williamson-Leadley, S., & Ingram, N. (2013). Show and tell: Using iPads for assessment in mathematics. *Computers in New Zealand Schools: Learning, teaching, technology*, *25*(1–3), 117–137.

LANGUAGE-RESPONSIVE SUPPORT OF MEANING-MAKING PROCESSES FOR UNDERSTANDING MULTIPLICATIVE DECOMPOSITION STRATEGIES

Annica Baiker[a] and Daniela Götze[b]
[a]TU Dortmund University, Germany; [b]University of Siegen, Germany.

Multiplicative thinking involves the ability to coordinate composite units flexibly ('unitizing'; Lamon, 1994) and is fundamental for the understanding of multiplicative decomposition strategies. Specific phrases that are typically used in classroom discourse for talking about multiplicative situations and tasks (e.g., '3 times 4') might inhibit such meaning-making processes because they do not address the idea of unitizing. In the study presented in this paper, three second grade primary school teachers introduced multiplication in their classes (n = 66) in a language responsive manner by using meaning-related phrases of unitizing (e.g., '6 times 4 means 6 fours'). Fifty-eight second graders taught by teachers without this teacher program served as control group. The analyses of a multiplication post-test (after the intervention) and a follow-up test (3 months later) showed that the children of the intervention group developed a deeper understanding of decomposition strategies.

LANGUAGE-RESPONSIVE TEACHING OF MULTIPLICATIVE THINKING

Multiplication is fundamental for understanding topics such as fractions, proportionality, and percentage calculation (Downton & Sullivan, 2017; Prediger, 2019), and multiplicative thinking seems to be responsible for a 7-year span in Grades 5 to 9 students' general numeracy competences in each grade (Siemon, 2019). Consequently, the development of multiplicative thinking is considered a 'cutoff point' (Cawley et al., 2001). Nevertheless, even some students in upper grades have difficulties solving (two-digit) multiplication tasks (Siemon et al., 2006), providing a multiplication story even if the numbers are small (Moser Opitz, 2013), and explaining the meaning of multiplication (Prediger, 2019). As this knowledge is needed for understanding decomposition strategies, children often struggle in using simple multiplication tasks for solving difficult multiplication tasks (Baiker & Götze, 2019). Consequently, further research is needed on how meaning-making processes of understanding decomposition strategies can be supported. As recent studies have shown, such understanding can be fostered by relating mathematical representations with a focus on verbalizing multiplicative structures (Erath et al., 2021). Therefore, in the following section, multiplicative thinking as unitizing in relation to decomposition strategies and its language-responsive support will be illustrated.

Multiplicative thinking as unitizing

The transition from additive to multiplicative thinking constitutes an obstacle for many children (Downton & Sullivan, 2017; Moser Opitz, 2013); for example, they show difficulties in differentiating additive from multiplicative situations (Van Dooren et al., 2010). This might be due to the fact that multiplication is often introduced and therefore learned as repeated addition (e.g., 3 x 4 = 4 + 4 + 4). However, the interpretation of multiplication as repeated addition is insufficient, since it is not applicable to multiplication beyond natural numbers (Thompson & Saldanha, 2003). Thus, multiplicative thinking is not based on repeated addition, but rather implies the understanding of two core concepts. The first (1) is *understanding the different meanings of multiplicand and multiplier* (Singh, 2000) and 'dealing with composite units' (Steffe, 1992). This ability is often described as '*unitizing'* (Lamon, 1994) and is fundamental for the second concept: (2) the *understanding of decomposition strategies* based on multiplicative commutativity, associativity, and distributivity (Anghileri, 2000; Downton & Sullivan, 2017; Götze & Baiker, 2021). This means, for example, solving 7 x 4 using 5 x 4 and 2 x 4 and implies the ability to transfer a part-whole concept to composite units (Downton & Sullivan, 2017; Lamon, 1994). This ability is conceptually based if the distributive relationship between these multiplicative tasks can be explained (Mulligan & Watson, 1998): Why are 5 x 4 and 2 x 4 additive parts of 7 x 4?

For many children, however, the different meanings of the multiplier and the multiplicand do not become obvious, with the consequence that they do not develop multiplicative thinking as unitizing (Downton & Sullivan, 2017; Steffe, 1992). Furthermore, many children do not understand how multiplicative tasks are interconnected but solve most multiplicative tasks using counting strategies or repeated addition (Downton & Sullivan, 2017; Götze, 2019; Moser Opitz, 2013; Siemon, 2019). Some of these difficulties might be caused by specific expressions that are typically used in classroom discourse for talking about multiplicative structures. In Germany, more formal expressions like '3 times 4' and '3 multiplied by 4' are commonly used for talking about multiplicative structures in tasks, rectangular arrays, and situations. Some empirical findings have indicated that these typically used expressions inhibit the development of multiplicative thinking, as the core idea of unitizing remains linguistically unexpressed (Götze, 2019; Thompson & Saldanha, 2003). Expressions like 'groups of' are uncommon in Germany. Nevertheless, such expressions demonstrably support additive thinking because they focus on adding the groups step by step (Larsson et al., 2017).

Language-responsive support of multiplicative thinking as unitizing

Erath et al. (2021) stated that meaning-making processes require a common meaning-related language as a language for the classroom discourse. For the topic of multiplication, this means that phrases like '5 threes' are needed because

> students should not only translate the multiplication 5 x 3 into an array model with five rows of 3 points each, but also explain how to see the unitizing structure in the rows ('five threes' or 'five sets of three') in order to verbalize the meaning of multiplication as unitizing. (Erath et al., 2021, p. 249)

The crucial difference between '5 times 3' and '5 threes' is that the former tells a calculation to perform while the latter addresses directly the basic meaning of 5 x 3 and thus suggests something to imagine (Thompson & Saldanha, 2003). These phrases can then be used for decomposition strategies, for instance, '7 x 3 means 7 threes, 2 threes more than 5 x 3' or '7 threes are 7 ones more than 7 twos.' Combining these expressions with the concrete and graphical representations of rectangular arrays allows understanding of commutative, associative, and distributive properties, even beyond natural numbers. Different studies have shown that such language-responsive teaching approaches seem to support multiplicative thinking in young children (e.g., Götze & Baiker, 2021; Götze, 2019; Thompson & Saldanha, 2003). However, most of the studies have been single-case studies and did not investigate the impact of multiplicative meaning-making processes on children's multiplicative thinking as unitizing right from the start of the introduction of multiplication in the second grade and in whole classes. Moreover, these studies have not focused on fostering meaning-making processes for understanding decomposition strategies.

This is the starting point of an intervention study carried out by Götze and Baiker (2021). The analyses of 124 second graders' data showed that such a language-sensitive approach can support the development of multiplicative thinking as unitizing right from the start of the introduction of multiplication in the second grade in whole classes. The present paper gives further insights into the data of this study by focusing on the analyses of children's understanding of decomposition strategies and the following research question: To what extent can a language-responsive intervention have an impact on students' outcome in relation to their understanding of decomposition strategies?

METHODOLOGICAL BACKGROUND

In order to pursue the research question, a 3-month teaching intervention was carried out by the mathematics teachers of three second grade classes (students aged 7-8 years, n = 66). Three other classes in a nearby school served as control group (n = 58).

Teacher program and teaching intervention

Both schools worked with the textbook *Das Zahlenbuch* (translated *The Number Book*, Nührenbörger et al., 2017), which provides the connection of different representations and the development of decomposition strategies but does not use meaning-related phrases (see Figure 1).

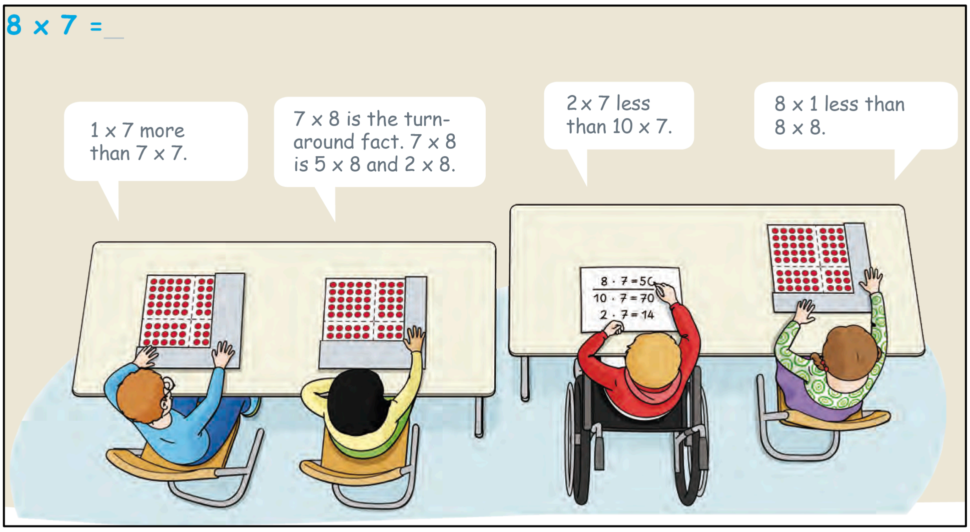
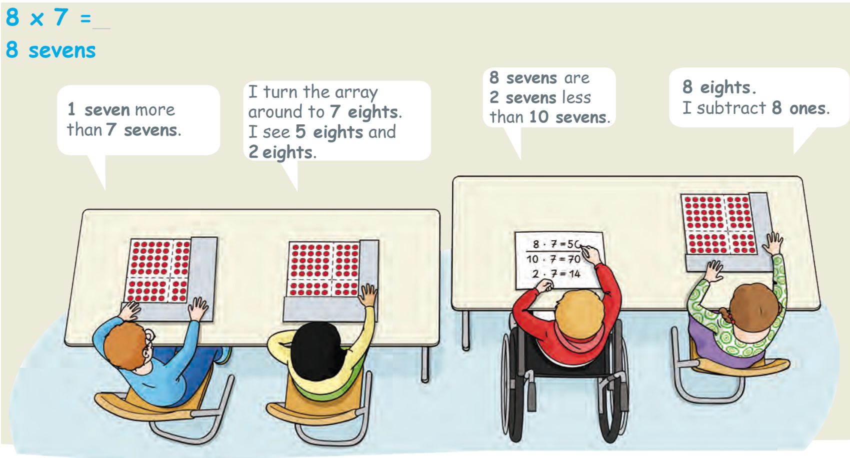


Figure 1: Translated example of the use of decomposition tasks from the textbook.

Therefore, for the teachers of the intervention group, the textbook materials were supplemented by hints on how to enhance rich discourses (see Figure 2). Additionally, these teachers learned in a 1-day teacher program how to start from children's everyday language and move on to multiplicative meaning-related phrases when talking about multiplicative structures in rectangular arrays or in multiplicative tasks and when explaining decomposition strategies. They were further instructed to encourage the children to use meaning-related phrases such as '3 fours' individually. During the intervention, we maintained an active exchange with the teachers of the intervention group. Once a week one of our research team visited them for an oral exchange. However, we did not collect any detailed insights of actions in the classroom. The teachers of the control group children did not join the teacher program, but they confirmed to us orally that they followed the tasks in the textbook.

- The textbook children can act as language models. Their explanations can be used as speaking frames in whole-class interaction.
- Connect these expressions with the interpretation of the arrays as multiplicative part-whole concept: "*Where do you see 7 sevens in the array? Why does the boy say '1 seven more'?*



- For explaining decompositions in general, establish the language of a multiplicative part-whole concept of composite units: "*Imagine the array. Which simple task(s) can you see in the array to help you solve 8 x 7? 8 sevens is 1 seven more than 7 sevens. You can also think about 7 eights and divide them into 5 eights and 2 eights*."

Figure 2: Translated example from the supplemental material for the intervention teachers

Implementation of the tests

In order to ensure comparability of the data, the general mathematical competences of all children were tested with the written standardized test Basis-Math 2+ (Moser Opitz et al., 2020). This test is the only instrument in Germany that tests both the procedural calculation skills and the conceptual knowledge of basic arithmetical topics of second graders. As it is conceptualized and validated for the end of second grade, it was implemented after the intervention. The results of the test showed no significant differences between the two groups ($t(122) = -1.22$, $p = 0.16$). Therefore, the basic arithmetical competences the two groups were considered comparable.

For testing children's multiplicative thinking as unitizing, a multiplication test was implemented as a post-test directly after the intervention. This test could not be used as pre-test because children had not been taught multiplication before the intervention. In addition, this test was used in a slightly modified version as follow-up test 3 months later in the third week of school after the summer holidays. At that time, the basic principles of multiplication had already been repeated in approximately 4 hours of teaching in both schools.

As there existed no standardized written German test that tested conceptual understanding of multiplication as unitizing, a test had to be developed. Taking the current state of the research and the two core multiplicative concepts mentioned above, three different item elements were developed deductively (for details, see Götze & Baiker, 2021). In the first item, the children had to solve three simple tasks with 2, 5, or 10 as one factor and one square number task. Additionally, they had to solve four more difficult tasks with 6, 7, 8, or 9 as factors. In the second item, students had to draw a picture that fit a task and explain why their picture fit this task. The third item was designed to reveal children's understanding of decomposition strategies. For this paper, the results of the decomposition item will be presented and discussed in detail.

This item consisted of two sub-items (see Figure 3). In each sub-item, a more difficult multiplication task (8 x 9 and 6 x 4) and three related simple tasks were provided. The children had to choose one of these three simple tasks for decomposition and then explain the decomposition.

Which task might help you solve 8 x 9 ?	Which task might help you solve 6 x 4 ?
8 x 10 ☐ 4 x 9 ☐ 8 x 8 ☐	5 x 4 ☐ 6 x 2 ☐ 3 x 4 ☐
How can this task help you solve 8 x 9 ? Explain.	How can this task help you solve 6 x 4 ? Explain.

Figure 3: Item for testing the understanding of decomposition strategies

Coding of the multiplication test

With the help of this item, we wanted to test whether the children were able to derive formally difficult tasks from simple tasks and thus explain distributive connections of multiplicative tasks. Hence, for the coding process it was irrelevant which simple task was chosen. Only the explanation of the decomposition was considered. The coding system was developed inductively. Meaning-related expressions that indicated multiplicative thinking (e.g., ‘I have to add 3 fours’) and more formal explanations (e.g., ‘I subtract eight’) were coded as *multiplicatively* with a score of 1. We added the more formal explanations to this category as well because these children also explained the connection of multiplicative tasks distributively, which indicates multiplicative thinking as unitizing (Downton & Sullivan, 2017). Commutative argumentations (meaning-related or formal) were coded with 1 as well. Answers that did not indicate multiplicative thinking as unitizing in any way, such as ‘8 x 10 is an easy task,’ wrong decompositions, and empty answers were coded as *inappropriate* with a score of 0. Table 1 gives a more detailed insight in the way of coding the explanations with some exemplary explanations of the children. As there were two sub-items, the children could reach up to 2 points in the whole item.

Category	Sub-category	Example
Multiplicatively (Score = 1)	Meaning-related expressions	- *8 times 10 is 8 tens, and for 8 times 9, every 10 becomes a 9, so I have to subtract 8 ones.* - *I have to add 3 fours.*
	Commutative argumentations	- *8 times 8 is 8 eights, and for 8 times 9, I need another 8.* - *I add 4 x 1.*
	Formal explanations	- *I take 8 times 8 and I add another 8.* - *5 x 4 = 20, just 1 four more.*
Inappropriate (Score = 0)	No multiplicative thinking	- *8 x 10 is an easy task.* - *5 x 4 is the fourth row.*
	Wrong decompositions	- *8 times 8 is 64 and 8 times 9 is one more.* - *5 x 4 is 20, so I just have to add 5 for 6 x 4.*
	Empty answers	

Table 1: Coding of the decomposition item

All tests were coded by one researcher. Additionally, 52 multiplication tests were coded by a second researcher so that 42.6% were coded by two researchers. The interrater reliability was quite high (κ = 0.96). For the analysis, first the percentages of children who reached 2, 1, or 0 points were compared in both groups for the post-test and the follow-up test. Second, means of scores and standard deviations of scores were calculated. This built the foundation for a one-sided paired *t*-test. Cohen's *d* values were then calculated to interpret the relevance of the changes (effects as follows: $d > 0.2$, small; $d > 0.5$, medium; and $d > 0.8$ large).

RESULTS

This section gives insight into the quantitative data analyses of the post-test and the follow-up test results of decomposition item for the intervention group (IG) and control group (CG). Table 2 presents the percentages of children at each level of total points on this item.

Total points	Post-test		Follow-up test	
	IG (n = 66)	CG (n = 58)	IG (n = 66)	CG (n = 58)
2	59.1%	39.7%	63.6%	27.6%
1	19.7%	20.7%	16.7%	24.1%
0	21.2%	39.7%	19.7%	48.3%

Table 2: Percentage of multiplicative explanations in the decomposition item

Considering the results of the post-test, in the intervention group almost 60% of the children explained both decompositions multiplicatively. This is approximately 20% more than in the control group, in which nearly 40% of the children explained both tasks multiplicatively. The same number (almost 40%) achieved no points in the control group, meaning that they had not explained the decomposition multiplicatively. By comparison, slightly more than 20% of the children in the intervention group reached 0 points. In both groups, around 20% of the children received 1 point.

Regarding the follow-up test, the percentage of children with 2 points increased slightly in the intervention group to 63.6% but decreased considerably in the control group to 27.6%. The percentage of children who achieved no points in the intervention group decreased slightly to almost 20%. However, around one half of the control group children (48.3%) achieved no points on this item and thus had difficulties explaining decomposition strategies. One-sided paired *t*-test analyses of the data of this item revealed the statistical significance of the differences (see Table 3).

	IG (n = 66)		CG (n = 58)		$t(122)$	p	Cohen's d
	M	SD	M	SD			
Post-test	1.38	0.81	1.00	0.89	2.46	0.008**	0.44
Follow-up test	1.44	0.80	0.79	0.85	4.33	< 0.001***	0.78

* $p < 0.05$ ** $p < 0.01$ *** $p < 0.001$

Table 3: t-test analyses of the decomposition item

In the post-test, the intervention group children reached 1.38 points on average, whereas the control group children reached 1.00 point on average. The *t*-test analyses of the decomposition item showed significant differences in the post-test, with a nearly medium effect size ($d = 0.44$). These differences increased in the follow-up test: The children in the intervention group reached 1.44 points on average, whereas in the control group the average score was 0.79 points. Thus, the average score increased in the intervention group and decreased in the control group. The *t*-test analyses showed that differences were still significant and the effect size increased from nearly medium to nearly high effect ($d = 0.78$).

CONCLUSIONS

In our study we focused in particular on how the core idea of multiplicative thinking as unitizing can be supported by a language-responsive teaching approach (Götze & Baiker, 2021). For this purpose, children's multiplicative thinking was tested using a multiplication test immediately after the intervention (post-test) and nearly three months later (follow-up test). The focus of this paper rests on the analyses of the test item that measures children's ability to explain decomposition strategies multiplicatively. The analyses of this item provide empirical evidence for a significant difference between the intervention and the control group in explaining decomposition strategies multiplicatively. The results of the follow-up test strengthened the results of the post-test, as the percentage of multiplicative explanations in the intervention group increased, whereas the amount in the control group decreased. As decomposition strategies were taught in both groups, one assumption is that the children in the control group had perhaps learned these strategies more procedurally. This might have caused the forgetting effect in the follow-up test. Some of the control group's written solutions such as 'and then one must calculate…' or 'you have to…' strengthen this assumption. In contrast, a higher number of the intervention group children showed conceptual understanding of decomposition strategies based on multiplicative thinking as unitizing in the follow-up test. This might be a hint that teaching decomposition strategies using meaning-related phrases and connecting them with different representations can support conceptual understanding of decomposition strategies. For this reason, primary school teachers should be more attentive to whether their students have understood multiplicative decomposition strategies conceptually. Even if they use multiplicative decomposition strategies after they have been introduced and trained, it must be determined whether this ability is based on multiplicative thinking as unitizing or on procedural skills without this understanding.

We are aware of some limitations of our study. As it was a written test, the solutions might suffer from information loss. This is why the results have to be treated carefully. Furthermore, we saw neither the intervention nor the control group's teaching units. Thus, we do not know exactly how the teachers have addressed multiplicative thinking within the classroom discourse. This could be the base for further investigations, for example, in observing children's learning pathways when multiplication is introduced with a focus on meaning-related phrases. However, our results indicate that language-responsive teaching can have an impact on students' long-term understanding of multiplicative decomposition strategies.

References

Anghileri, J. (2000). *Teaching number sense*. Continuum.

Baiker, A., & Götze, D. (2019). Distributive Zusammenhänge inhaltlich erklären können—Einblicke in eine sprachsensible Förderung von Grundschulkindern. In A. Frank, S. Krauss, & K. Binder (Eds.), *Beiträge zum Mathematikunterricht 2019* (pp. 69–72). WTM-Verlag.

Cawley, J. F., Parmar, R. S., Foley, T. E., Salmon, S., & Roy, S. (2001). Arithmetic performance of students. Implications for standards and programming. *Exceptional Children, 67*(3), 311–328. doi: 10.1177/001440290106700302

Downton, A., & Sullivan, P. (2017). Posing complex problems requiring multiplicative thinking prompts students to use sophisticated strategies and build mathematical connections. *Educational Studies in Mathematics, 95*(3), 303–328.

Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning—A review of the state of development and research. *ZDM Mathematics Education, 53*(2), 245–262. https://doi.org/10.1007/s11858-020-01213-2

Götze, D. (2019). Language-sensitive support of multiplication concepts among at-risk children: A qualitative didactical design research case study. *Learning Disabilities – A Contemporary Journal, 17*(2), 165–182.

Götze, D., & Baiker, A. (2021). Language-responsive support for multiplicative thinking as unitizing: Results of an intervention study in the second grade. *ZDM Mathematics Education, 53*(2), 263–275. https://doi.org/10.1007/s11858-020-01206-1

Lamon, S. (1994). Ratio and proportion: Cognitive foundations in unitizing and norming. In G. Harel, & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 89–122). State University of New York Press.

Larsson, K., Pettersson, K., Andrews, P. (2017). Students' conceptualization of multiplication as repeated addition or equal groups in relation to multi-digit and decimal numbers. *The Journal of Mathematical Behavior, 48*, 1–13.

Moser Opitz, E. (2013). *Rechenschwäche/Dyskalkulie. Theoretische Klärungen und empirische Studien an betroffenen Schülerinnen und Schülern*. Haupt.

Moser Opitz, E., Stöckli, M., Grob, U., Nührenbörger, M., & Reusser, L. (2020). *BASIS-MATH-G 2+. Gruppentest zur Basisdiagnostik Mathematik für das vierte Quartal der 2. Klasse und das erste Quartal der 3. Klasse*. Hogrefe.

Mulligan, J. T., & Watson J. M. (1998). A developmental multi-modal model for multiplication and division. *Mathematics Education Research Journal, 10*(2), 61–86.

Nührenbörger, M., Schwarzkopf, R., Bischoff, M., Götze, D., & Heß, B. (2017). *Das Zahlenbuch 2. Schülerbuch.* Klett.

Prediger, S. (2019). Mathematische und sprachliche Lernschwierigkeiten—Empirische Befunde und Förderansätze am Beispiel des Multiplikationskonzepts. *Lernen und Lernstörungen, 8*(4), 247–260. https://doi.org/10.1024/2235-0977/a000268

Siemon, D., Breed, M., Dole, S., Izard, J., & Virgona, J. (2006). *Scaffolding numeracy in the middle years. Project findings, materials and resources. Final report*. https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/scaffoldnum.aspx. Accessed 11 May 2021.

Siemon, D. (2019). Knowing and building on what students know: The case of multiplicative thinking. In D. Siemon, T. Barkatsas, & R. Seah (Eds.), *Researching and using progressions (trajectories) in mathematics education* (pp. 6–31). Brill Sense.

Singh, P. (2000). Understanding the concept of proportion and ratio constructed by two grade six students. *Educational Studies in Mathematics, 14*(3), 271–292.

Steffe, L. P. (1992). Schemes of action and operation involving composite units. *Learning and Individual Differences, 4*(3), 259–309. https://doi.org/10.1016/1041-6080(92)90005-Y

Thompson, P., & Saldanha, L. (2003). Fractions and multiplicative reasoning. In J. Kilpatrick, G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 95–113). National Council of Teachers of Mathematics.

Van Dooren, W., De Bock, D., & Verschaffel, L. (2010). From addition to multiplication... and back. The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction, 28*(3), 360–381.

LIFEWORLD CONNECTIONS IN MATHEMATICS EDUCATION – UNQUESTIONED, INDISPENSABLE, AND UNDEFINED?

Elisa Bitterlich
Technische Universität Dresden (Germany)

Within the mathematics classroom, connections to the everyday world are frequently used to catch the pupils' attention and to foster their understanding. Presumably, especially in primary school they are used with the aim of building bridges between the familiar 'everyday world' of the pupils and the more formal and abstract 'world of mathematics'. The presented study examines how language use looks like in situations with a lifeworld connection and to what extent this might have an impact on the pupils' supposed opportunities to learn. Within this paper, the focus is on the meaningfulness of the lifeworld connection and the assumed learning opportunities. Based on the interpretative paradigm of interpretative classroom research, interactional analyses of selected passages of everyday mathematics lessons will illustrate how mathematical classroom discourse is affected through the use of lifeworld connections and to what extent this could affect the learning of mathematics.

UNQUESTIONED – LEARNING IN SOCIAL INTERACTIONS

The presented study is based on the interpretative paradigm of interpretative classroom research (e.g., Bauersfeld, 1994; Krummheuer, 2000). This approach has its main basics in sociological theories of symbolic interactionism (e.g., Blumer, 1969) as well as ethnomethodology (e.g., Garfinkel, 1967) and combines sociological, social-constructivist and subject-specific educational theories of learning. One fundamental assumption of interpretative classroom research following this tradition is that

> meaning is negotiated in interactions between several individuals and that social interaction is thus be understood as constitutive of learning processes, speaking about mathematics with others is in itself to be seen as the 'doing' of mathematics and the development of meaning (Schütte et al., 2019, p. 104).

Following this, social interactions are at the center of attention and the 'place', where (mathematical) meaning is negotiated. Processes of interactive negotiation of meaning – here of mathematical meaning – take place between the participants. One fundamental theoretical assumption of this research project is that the use of language – and in result also the assumed opportunities to learn – differs depending on the situation. Therefore, pupils and teachers use resources from different language registers and participate in different manners during different situations of the mathematics classroom (Moschkovich, 2018). In this regard, language and discourse are central focal points of the empirical investigation as such interactive meaning-making processes mainly become visible through the participants' language use.

Through interactional processes, the participants' preliminary *interpretations of the situation* come closer to each other, resulting in an *interpretation taken as shared* (Jung & Schütte, 2018; Krummheuer, 1992). Krummheuer (1992) established the term 'taken as shared' to express that the participants' interpretations of the situation can never become completely congruent as everyone still has a subjective view on the situation based on his or her individual experience and knowledge. Following the approach of interpretative classroom research, the increasing adaption of the other participants' definitions of the situation through repeating negotiations could result in routinized definitions of the situations (Krummheuer, 1992). These so-called *framings* can be evoked again and again in similar situations. From this perspective, learning is understood as the "construction and modulation of framing stimulated by interaction" (Jung & Schütte, 2018, p. 1091). Following this, learning– here, the learning of mathematics – always takes place in a social context and is therefore mainly mediated and visible through language. The presented research views mathematics lessons as a discursive activity and establishes assumptions about the pupils' opportunities to learn mathematics which raise through the observed interactions. In this regard, different situations lead to different opportunities to use language. Additionally, it is widely accepted that meaningful contexts are also important for successful (and long-lasting) learning processes (Kaiser & Schwarz, 2010; e.g., Leuders et al., 2011). Following this, situations of mathematics lessons involving a real-world context should be meaningful for the pupils' lives outside the mathematics classroom, and moreover enable an authentic and rich engagement with the mathematical learning content on the one hand and the real-world experience on the other (Leuders et al., 2011).

INDISPENSABLE – RESEARCHING LANGUAGE USE IN DIFFERENT SITUATIONS

The reconstruction of the pupils' opportunities to learn mathematics therefore needs a deep qualitative view on interactional processes – especially language use – during 'daily' mathematics lessons. By 'daily', I mean that the observed lessons were held by the pupils' original teacher, preferably without influence by the researcher. The central purpose of the presented study is to examine language use and resulting opportunities to learn mathematics during different situations of the mathematics classroom. Of the multitude of language-influencing factors – like social formation, learning content, and (non-)existence of visual aids, I recognized scenes in which language use and its impact on the participants' supposed opportunities to learn were of a special character: Especially in the observed primary school mathematics lessons, the mathematical content was frequently and in different manners dressed up in a lifeworld connection (in the following abbreviated as LWC). Such moments in which the mathematical content is somehow connected to the pupils' 'lifeworld' outside the mathematics classroom might be characterized by their own language-specialties and in result also unique opportunities for learning. While most studies on such connections put the focus on written tasks or typical word problems, the presented research mainly considers such LWCs arising during (verbally) interactions.

To gain a broad impression of the pupils' language use during different situations of the mathematics lessons in which LWCs are used, it is intended to contemplate different school types and class levels in terms of these two central influencing factors. Therefore, mathematics lessons of the several classes (different school types and different class levels as well) in Germany have been video-recorded. In all observed classes from several school types LWCs could be identified. These, however, differ with regard to explicitness, scope, and grade of reality – what becomes visible through language use and might result in different opportunities to learn mathematics.

Selected passages with lifeworld connections were transcribed and later analyzed via *interactional analysis* (for a detailed English description of the approach, its basic concepts and analysis steps see Schütte et al., 2019) to illuminate to what extent LWCs within mathematics lessons have an impact on the pupils' use of language and their mathematical learning. After setting and structuring an interactional unit that should be analyzed in detail – here, these are scenes with a real-world context –

further step is to give a general description of the scene and to make a detailed sequential interpretation of the individual utterances. This detailed step-by-step analysis helps to look at the transcript scene in detail and to reconstruct the development of the interaction. It is often interwoven with the turn-by-turn analysis, where some interpretations of the sequential analysis are regarded as more or less applicable interpretations of an utterance. Finally, the summary of the interpretation helps to reduce the diversity of the interpretations that were made before and illustrates only these interpretations, which can best be justified. For reasons of space, the analysis below is limited to the description of the scene and the final summary of the interpretations.

UNDEFINED – LIFEWORLD CONNECTIONS IN MATHEMATICS EDUCATION

The Problem with Defining Lifeworld Connections

Investigating language use during lifeworld connections in mathematics lessons also means to set up a conceptual understanding of the term itself and its crucial characteristics. Concerning the existing literature in this field, such a conceptual understanding of LWCs turns out to be challenging, as there are several terms existing which are supposedly used synonymously, e.g., real-world connection, real-world context, realistic problem, contextualized task, modelling problem, authentic task, story problem or narrative (e.g., Kaiser & Schwarz, 2010; Karakoç & Alacacı, 2015; Larina, 2016; Palm, 2006; Zan, 2017; Zazkis & Liljedahl, 2009). In Germany where this study is located, the variety of existing terms is comparably extensive. In general, there is a lack of an established and universal specification of the term (Gainsburg, 2008; Larina, 2016). This missing universal definition and the lack of high-quality and empirically proved resources leads to mathematics teachers' freedom as well as their responsibility to select or construct LWCs by their own criteria – mainly guided by their individual beliefs, goals and knowledge (Gainsburg, 2008). Moreover, each term sets another focus and is used in different empirical research approaches. Within the frame of this research project, the term lifeworld connection is defined as follows:

In situations with lifeworld connection, the (mathematical) learning content is linked or related to a situation, thing or process of the pupils' experiences – based on assumptions of the teacher. This relation can be more or less explicit. Additionally, both spheres – learning content and real-world experiences – can be intertwined and reciprocally related to each other, as well as just function independently from one to another. The onset of the interactive negotiation can be either the learning content or the real-world experiences of the pupils.

Since the empirical data might reveal some characteristic features of LWCs themselves as well as of the language use during such situations in mathematics lessons, this definition is rather a scaffold with the opportunity for changes, modifications and differentiation than a final statement. Nevertheless, there are two central points the definition should emphasize: First, the connection between learning content and lifeworld experiences can be 1) intertwined as well as just 2) be effective from learning content to experiences or, vice versa, from experiences to learning content. Second, the final term and its definition should be neutral and not imply a particular pedagogical purpose or direction – as the term application might do (Gainsburg, 2008).

Two Sides of One Coin – Positive and Negative Aspects of Lifeworld Connections

In the following, a short elaboration of the pros and cons of LWCs in (mathematics) education will draw up the question of whether they are unequivocally appropriate for the teaching and learning of mathematics, or if there are also limitations (Notice: Regardless of the original term in the publications referred to, they are subsumed under the term lifeworld connection in this paper). Concerning mathematics education, previous studies document that teachers connect the mathematical learning content with real-world experiences mainly with the following aims (see amongst others Gainsburg, 2008; Karakoç & Alacacı, 2015; Pongsakdi et al., 2020; Verschaffel et al., 2020):

- providing a (supposedly easier) access to the learning content and enhancing understanding
- motivate to participate in the mathematics lesson
- to help apply mathematics in real-life situation and enable long-lasting and fruitful learning
- change the view on (school) mathematics

LWCs are frequently used in mathematics education to make the mathematical concept more meaningful and to illustrate the usefulness of specific mathematical ideas and skills. Pupils could be motivated at a higher level, as they seem to be more interested, attentive and open-minded to engage with a mathematical task, which becomes more subjective and personal for them (Boaler, 1993). In this regard, LWCs have the potential to be viewed as a bridge between the (more formal) mathematical learning content and real-world situations. For many – children as well as adults – the abstractness of mathematics is a synonym for a detached, inflexible and cold body of knowledge. Through the use of LWCs, the mathematical task becomes more subjective and personal in order to get involved with mathematics more easily (Boaler, 1993).

Following Sullivan, Zevenbergen and Mousley (2003), teachers need to develop a certain sensitivity and should be able to make judgments about the use of LWCs. This includes aspects of the (mathematical) suitability, the relevance to the pupils, the potential motivational and emotional impact, and even the possibility of negative effects for the pupils' learning. Because of the lack of a universal definition research suggests that LWCs therefore are used infrequently and cursory (Gainsburg, 2008). Often, a LWC (and especially its presumably most common kind: the word problem) comes from textbooks and less from the teachers themselves. In this context, Crespo and Sinclair (2008) point out that (prospective) teachers need rich practice of posing interesting and good connections to lifeworld, because also as students of mathematics they had fewer opportunities to construct them. Given the opportunity to construct their own lifeworld connection,

> it is reasonable to assume they will generate artifacts consistent with their normative understandings of what a school mathematics problem should look like […] students of different ages tend to construct familiar single-step story problems, namely, problems that invite quick, accurate responses and that re-formulate existing problems in ways that narrow rather than open the mathematics involved or required by the problem (Crespo & Sinclair, 2008, p. 396 shortened by the author E.B.).

Particularly, this stereotyped nature might be the most disadvantageous aspect of an LWC. In most cases, a connection to the lifeworld has an unrealistic, numerically clean nature including all necessary data, is always solvable with a single-step in only a single way, has exactly one solution, contains superficial features like key-words, and directly matches with the precede mathematical concept or operation (see also Gainsburg, 2008; Greer, 1997; Nesher, 1980; Zan, 2017). These shortcomings often result in an unreflecting treatment with the mathematical content within the presented LWC: Pupils become trained in solving a context task exactly in the expected way. They often believe what they are told without questioning the distance of the mathematical task from reality (Boaler, 1993). In this regard, many pupils could not sufficiently identify with the presented LWC, as they had not made compatible experiences (e.g., paying bills and wage slips); or they fail to recognize the purpose of the task and the mathematical content within (e.g., Boaler, 1993; Sullivan et al., 2003; Zan, 2017). Taking also into account these aspects, LWCs could also be seen as obstacles for understanding mathematics and recognizing the usefulness of mathematics for life.

Open Questions on LWCs – Meaningfulness, Language Use and Learning Opportunities

Unfortunately, there is few research analyzing the pupils' language, when LWCs are used in mathematics classrooms, so elaborations on their opportunities to learn mathematics are widely missing. Considering the related literature, many studies and literature reviews focus mainly on the usage of LWCs in higher grades (e.g., Gainsburg, 2008), or on linguistic characteristics in general (e.g., Nesher, 1980; Pongsakdi et al., 2020) – but not on their effects on learning and the language use during classroom interaction. But especially in primary school we expect that the relation between the everyday world experiences and the mathematical learning content could be problematic, particularly for young students: While experiences made in their everyday or real-world are mostly unquestionable and plausible for them, the world of mathematics seems to reveal a 'new way' of thinking about this everyday world, as nearly everything could be the topic of a mathematical task (Neth & Voigt, 1991). Presumably, especially primary school children take the presented connection to the lifeworld too much into account and fail to identify the mathematical content of it. While the teacher who presents the LWC puts his or her focus on the mathematical content, the students might think more directly and become emotional with connection to their experiences (Neth & Voigt, 1991). It can be assumed that the handling of LWCs noticeably influences the language use during such situations. In result, this might also have consequences for the resulting opportunities to speak and to learn. As a result of the previous elaboration on learning in interaction and lifeworld connections, the following research guiding question can be derived: *How does language use look like in situations with a lifeworld connection in mathematics education and to what extent might this have an impact on the pupils' supposed opportunities to learn?* In previous papers (Bitterlich, 2020; Bitterlich & Schütte, 2018), I already gave research examples illustrating the possible risk that the mathematical learning content is hidden and that the posed LWC could be too narrow-minded to illustrate the whole structure of the mathematical content – also outlining the impact on the pupils' opportunities to learn mathematics. In the following, another example will reveal new features of LWCs.

RESEARCH EXAMPLE – RESULTS AND FURTHER IMPLICATIONS

The following scene is from a third-class mathematics lesson. Beforehand the class spoke about different currencies. Additionally, as this is the first mathematics lesson after Easter, the teacher asked the pupils about what they had been given as a present for Easter. In detail, she asked how many children got a book – four children – and who got money –approximately ten children. Before the scene starts, the teacher places four books for children with self-made price tags on the board.

1 T: Close your books and come to me. There were four children who received a book for Easter and some received money and would like to buy a book from their

money. And I have just chosen four different books [Turning to the books]. One is even a special offer. And now let us assume that … Mike [Turning to the class and looking at Mike] now has received … nine Euro [Shrugging] fifty and wants to buy a book … If we then take a look at the books, he has some leftover money everywhere, hasn't he? [Looking at the pupils] but how much he has left

2 Olli: [Says something incomprehensible to the teacher]

3 T: [Looks at Olli, then to the books] No, not everywhere. [With a slight shake of the head] Exactly [Nods]. One book does not work and the other one maybe also does not work, but for some he has a remainder. And how can I now calculate this in the long form with decimals? We have never done this before. This is what we want to look at today. [Looks at Mike] Which book of these would you choose now, Mike?

4 Benjo: [Whispering to Mike]

5 Mike: [Whispering to Benjo. Looking at the books for 4 seconds, saying nothing]

6 T: Do you have one that you like maybe, of these books? Come forward.

7 Mike: The special offer [Goes to the board and tips on the book for 9,45€]

8 T: The special offer book [Takes the book] … costs how much?

9 Mike: Nine Euro forty-five…

10 T: Then you can hold it [Grinning. Gives the book to Mike. Then she turns one side of the board. There one can see play money stuck to the board. The total sum is 9,75€, assembled of a 5€ bill and coins of 2€, 2€, 50ct, 20ct, 5ct] And now here I already prepared the money for Mike. Let's assume this is the money he received.

In the subsequent scene, the teacher instructs the pupils to do the long form of subtraction with particular attention to correctly writing one amount of money below the other with respect to the position and the comma. After this first example being done in a class discussion, the teacher encourages the children to go back to their places and to do another long form of subtraction with the presented amounts of money on their own. The situation of buying a book is no longer of any interest and Mike is not asked to speak again.

With the help of interactional analysis it is possible to look at language use in interaction in detail and to reconstruct characteristics of the established LWC as well as the supposed learning opportunities. From interactional analysis, inferences can be drawn about the way participants define and interpret a situation. Only if teachers as well as learners understand and interpret an LWC in roughly the same way, those learning opportunities intended by the teaching staff can arise. For reasons of space, this analysis is limited to the analysis of the lifeworld connections' meaningfulness and the resulting assumed opportunities to learn.

In total, the LWC of the presented scene can be characterized as semi-meaningful: On the one hand, the situation of buying a book might be an experience all pupils already made. On the other hand the presented connection to the lifeworld evokes a lot of questions and seems to be radically limited concerning its didactical purpose. Why and how did the teacher pre-select four books and are their prices referring to reality? Who is the bookseller and where is the place of the deal? Why is the total sum such an unusual amount and not 10 €? Additionally, during the oral presentation of the LWC, the teacher speaks of 9,50 € while the sum on the board is 9,65 €, but nobody complains about this inconsistency. Since the pupils nearly do not speak as they are not asked to do so, we can hardly assume anything about their personal view on the situation and the presented LWC. Mike seems to rather be directed by the teacher than acting as an autonomous person, as he just ‘can hold’ the book (line 11), but is not allowed to explain why he chose the special offer book, let alone to take a look into the book. But neither Mike nor the other pupils seem to be distracted by the teachers’ predominance in guiding the interaction and setting the focus. It seems, as if they are already used to such scenes in which such lifeworld experiences and situations were used as initial moments for the further negotiation with a (mathematcical) learning content. These open questions and school-typical aspects result in a less authentic LWC, as the pupils might very fast realize that the established connection to their experiences is only a ‘decorative dress’ for the teachers’ didactical goal of the lesson – namely the calculation with decimal numbers, as she already pointed out in line 3. The established connection between buying a book and calculating the change with the long form of subtraction fades more and more from the spotlight. Finally, the presented LWC is less meaningful for the pupils and it remains open if they are able to identify the usefulness of the underlying learning content – namely the long form of subtraction of decimal numbers – for their own lives outside the mathematics classroom. This illuminates that teachers should critically question and rethink the use of lifeworld connections carefully, as they do not automatically lead to successful and meaningful learning processes. There is a risk that the mathematical core of the learning content is blurred, as the connection to the lifeworld situation is somehow contradictory and pupils might not identify with the presented LWC, as also Nesher (1980) points out:

> Word problems [here: lifeworld connections] at school do not resemble problems in real life situations, and they are not considered by the children themselves to be related to the real world. […] children are engaged in the special activity of ‘solving word problems’ without relating them to any real life experiences but rather accepting them as a part of a school ritual (p. 41; shortened by the Author E.B.).

References

Bauersfeld, H. (1994). Theoretical Perspectives on Interaction in the Mathematics Classroom. In R. Biehler, R. W. Scholz, R. Strässer, & B. Winkelmann (Eds.), *Didactics of Mathematics as a Scientific Discipline* (pp. 133–146). Kluwer Academic Publishers.

Bitterlich, E. (2020). About real-world contexts in mathematics education and their impact on language and learning. In J. Ingram, K. Erath, F. Rønning, & A. Schüler-Meyer (Eds.), *Proceedings of the 7th ERME Topic Conference on Language in the Mathematics Classroom* (pp. 131–138). ERME/HAL.

Bitterlich, E., & Schütte, M. (2018). Language use in different situations in the mathematics classroom: Everyday academic language and mathematical discourse. In N. Planas & M. Schütte (Eds.), *Proceedings of the Fourth ERME Topic Conference ‘Classroom-Based Research on Mathematics and Language’* (pp. 34–40). Technical University of Dresden, Germany, and ERME.

Blumer, H. (1969). *Symbolic Interactionism*. Prentice-Hall.

Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education*, *11*(5), 395–415.

Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *Journal of Mathematics Teacher Education*, *11*(3), 199–219.

Garfinkel, H. (1967). *Studies in ethnomethodology*. Prentice-Hall.

Greer, B. (1997). Modelling reality in mathematics classrooms: The case of word problems. *Learning and Instruction*, *7*(4), 293–307.

Jung, J., & Schütte, M. (2018). An interactionist perspective on mathematics learning. Conditions of learning opportunities in mixed-ability groups within linguistic negotiation processes. *ZDM - International Journal on Mathematics Education*, *50*, 1089–1099.

Kaiser, G., & Schwarz, B. (2010). Authentic Modelling Problems in Mathematics Education—Examples and Experiences. *Journal Für Mathematik-Didaktik*, *31*, 51–76.

Karakoç, G., & Alacacı, C. (2015). Real World Connections in High School Mathematics Curriculum and Teaching. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *6*(1), 31–46.

Krummheuer, G. (1992). *Lernen mit "Format". Elemente einer interaktionistischen Lerntheorie. Diskutiert an Beispielen mathematischen Unterrichts*. [Learning with 'format'. Elements of an interactionist theory of learning]. Deutscher Studien Verlag.

Krummheuer, G. (2000). Interpretative Classroom Research in Primary Mathematics Education. Some Preliminary Remarks. *Zentralblatt für Didaktik der Mathematik*, *5*, 124–125.

Krummheuer, G. (2011). Die Interaktionsanalyse. *Online Fallarchiv. Uni Kassel*, 1–11.

Larina, G. (2016). Analysis of Real-World Math Problems: Theoretical Model and Classroom Application. *Voprosy Obrazovaniya/ Educational Studies. Moscow*, (3), 151–168.

Leuders, T., Hußmann, S., Barzel, B., & Prediger, S. (2011). Das macht Sinn! Sinnstiftung mit Kontexten und Kernideen. [That makes sense! Meaning making with contexts and core ideas]. *Praxis Mathematik*, *53*(37), 2–9.

Moschkovich, J. (2018). Recommendations for Research on Language and Learning Mathematics. In J. Moschkovich, D. Wagner, A. Bose, J. Rodrigues Mendes, & M. Schütte (Eds.), *Language and Communication in Mathematics Education. International Perspectives. ICME-13 Monographs* (pp. 37–47). Springer International Publishing.

Nesher, P. (1980). The Stereotyped Nature of School Word Problems. *For the Learning of Mathematics*, *1*(1), 41–48.

Palm, T. (2006). Word problems as simulations of real-world situations: A proposed framework. *For the Learning of Mathematics, 26*, 42–47.

Pongsakdi, N., Kajamies, A., Veermans, K., Lertola, K., Vauras, M., & Lehtinen, E. (2020). What makes mathematical word problem solving challenging? Exploring the roles of word problem characteristics, text comprehension, and arithmetic skills. *ZDM Mathematics Education, 52*, 33–44.

Schütte, M., Friesen, R.A., & Jung, J. (2019). Interactional analysis: A method for analysing mathematical learning processes in interactions. In G. Kaiser & N. Presmeg (Eds.), *Compendium for Early Career Researchers in Mathematics Education* (pp. 101–129). Springer International Publishing.

Schütte, M., & Krummheuer, G. (2017). Mathematische Diskurse im Kindesalter. In U. Kortenkamp & A. Kuzle (Eds.), *Beiträge zum Mathematikunterricht 2017* (pp. 877–880). WTM.

Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: a survey. *Zentralblatt für Didaktik der Mathematik*, *52*, 1–16.

Zan, R. (2017). The crucial role of narrative thought in understanding story problems. *Didattica Della Matematica. Dalla Ricerca Alle Pratiche d'aula*, (2), 46–57.

Zazkis, R., & Liljedahl, P. (2009). *Teaching Mathematics as Storytelling*. Sense Publishers.

INTERACTIONAL OBLIGATIONS IN PAIR AND GROUP WORK

Rachel-Ann Böckmann and Marcus Schütte
Leibniz Universität Hannover, Germany

From an interactionist perspective, learning of mathematics can be described as the increasingly autonomous participation in processes of collective argumentation. This study aims at describing empirically within student interactions which interactional obligations for bringing forth warrants or backings within collective argumentations emerge. Specifically for this research, children were filmed when learning collaboratively in multi-age groups, as it increases the diversity of the learners abilities and therefore helps to contrast various interactional obligations at work within collaborative learning situations. Besides describing three interactional obligations – contradictions, mistakes and certain types of questions – changes in interpreting interactional obligations within an interaction are shown.

INTERACTIONIST THEORY OF LEARNING

From social-constructivist theories, learning is viewed as an individual process which is constituted in social processes of negotiating meaning (e.g. Vygotski, 1978). More specifically, this research is based on interactionist approaches describing learning as the development or modification of framings which become visible in an increasingly autonomous participation in collective argumentation (Krummheuer & Brandt, 2001; Jung & Schütte, 2018). As one of the ensuing demands is to incorporate more group and pair work into mathematics classrooms, the focus of this research is how learning takes place in student interaction when students collaboratively work together. Hereafter, two basic concepts of the interactionist theory of learning 'framing' and 'collective argumentation' are explained and connected to the concept of 'interactional obligations' before the research question for this study is presented.

Framing

The interactionist theory of learning is based on Symbolic Interactionism which views meaning as a social product which develops in interactions between people. Different participants of an interaction have their own interpretations of a situation on the basis of their individual experiences – called *definitions of the situation* (Schütte et al., 2019). While the participants negotiate these different definitions of the situation, ideally these become more aligned with each other. When brought forth repeatedly in the interaction, the individual definitions of situations can outlast the situation, which Bauersfeld et al. (1988) call *framings*. The process of developing, expanding or modifying individual mathematical framings is when learning takes place. However, often framings are not in alignment with each other. In order to continue the process of negotiating meaning, the *differences in framing* between the participants need to be coordinated. While these differences can make it more difficult for the participating individuals to adjust their definitions of the situation to fit each other, they also provide the "'motor' of learning" (Schütte, 2014) since, on the interactional level, they generate a necessity for negotiation within collective argumentation (Schütte et al., 2019). For this reason, videos were taken in multi-age learning groups with students learning together in different combinations of grade levels one through six (ages 6 to 12), in order to analyze student groups with very diverse interpretations of a situation.

Collective Argumentation

Krummheuer (1995) takes the idea of *collective argumentation* from a sociological learning theory by Miller (1986), who sees participating in collective argumentation as essential for fundamental learning of young learners. Contrary to Miller, for whom a collective argumentation is a communicative type of action which serves to solve a socio-cognitive conflict by bringing forth different arguments collectively and negotiating them, this study agrees with Krummheuer for whom there is no need for an explicit socio-cognitive conflict. From an ethnomethodological perspective, participants always indicate the rationality of their behavior in the interaction (for more details Jung & Schütte, 2018). To reveal, how the participants of an interaction contribute to a collective argumentation, Toulmin's analysis of argumentation (1969) identifies which utterances or actions contribute to which of the four functional categories of an argumentation: data (undoubted statements), conclusion (inference together with the data), warrant (contribution to the legitimation of the inference) or backing (undoubtable basic convictions which refer to the permissibility of the warrant). According to Krummheuer and Brandt (2001), one of the aspects which increases the possibility of mathematical learning is seen when processes of argumentation with a complete 'core' of an argumentation - meaning data, conclusion and warrant (or even backings) - are produced. If collective argumentation is so central to the learning process, the question arises, when are collective argumentations brought forth?

Interactional Obligations

According to Voigt (1994), participants within an interaction do not act independently of other participants' utterances and actions. Rather, individual utterances and actions create *interactional obligations* connecting them with each other. Because of these obligations, individual participants develop expectations of how others will react – e.g. give a warrant. However, these expectations typically remain implicit unless conflict arises between the participants. Whether or not a person interprets an utterance or action as creating an interactional obligations for him- or herself to respond to and in what way he or she should respond, is very much based on how he or she interprets the situation. Thus one can only reconstruct an interactional obligation by how the interaction unfolds. As interactional obligations are also crucial for initiating collective argumentations, the question arises "Which interactional obligations for bringing forth warrants or even backings within collective argumentations can be reconstructed in student interaction during group and pair work?"

INTERACTIONAL OBLIGATIONS FOR WARRANTS AND BACKINGS

Methodologically, this work is located within interactionist approaches of classroom research (Krummheuer & Brandt, 2001). The interactions of the pupils are transcribed and analyzed using the interactional analysis developed by Bauersfeld et al. (1988; Schütte et al., 2019) and the analysis of argumentation (Toulmin, 1969). Three interactional obligations will be summarized briefly in the next chapter as they have already been presented in Friesen & Schütte (2020) in detail. For this paper, the main focus is on how students interpret utterances and actions as interactional obligations for bringing forth a warrant or backing and how this changes within an interaction.

The following interaction takes place after two pairs – Lia (grade 2) and Lara (grade 1), Rebekka and Dana (grade 3) – work on finding as many pentominoes as possible and are now comparing with each other the ones they found. (Pentominoes are shapes consisting of five squares joined together at their sides.) So far, they have compared three pentominoes on Lia and Lara's worksheet.

202		Rebekka	this/ [*points to the fourth pentomino on Lia's and Lara's worksheet*] . <it is exactly the s- .. it is exactly the same [*points with her thumb to the fourth and with her index finger to the third pentomino*]	data & conclusion
203	<	Dana	eh is this supposed to be with/ *[points to the same pentomino]*	questions data
204		Lara	yes . > just turned around	warrant
205	>	Lia	really/	questions conclusion
206		Dana	yes < look it is only simply turned around [*points with her right index finger and thumb to the pentomino and then turns the palm of her hand upwards*]	repeats and clarifies warrant with gestures
207	<	Rebekka	yes turned around [*points with her right index finger to the fourth pentomino*]	repeats warrant
208	>	Lia	no	contradicts inference
209	>	Rebekka	first like this [*holds her left hand over the pentomino and then turns it upward*] . yes . this one [*holds her index finger and thumb over pentomino*] is first turned around like this <[*turns her left hand upward again*] and then like this [*folds her hand 90° towards herself*]	repeats and clarifies warrant with gestures in more detail
210	<	Lia	true	agrees with inference

Table 1: Transcript

In (202), Rebekka interprets the fourth pentomino as being the same as the third pentomino which they just finished comparing. She therefore interprets it as **a mistake.** As shown in Friesen and Schütte (2020), the participant interpreting a previous utterance or action as a mistake often comments on the mistake by including a warrant. This suggests that students can see pointing out a mistake as an interactional obligation for giving a warrant. However, in (202) Rebekka either does not feel obligated to do so or does not know how to meet this interactional obligation as she gives no warrant for her inference that pentomino three and four are the same. In (204), Lara then adds a warrant to this collective argumentation by saying it is "turned around". This suggests that she feels the interactional obligation as not having been met. After Lia expresses astonishment "really/" (205), Dana and Rebekka simultaneously repeat Lara's warrant and Dana clarifies it by adding gestures (206/207).

Later on, two similar moments arise in the interaction. First Rebekka and then Dana each notice two pentominoes on Lia and Lara's worksheet being the same. This time, they both immediately include the warrant "turned" or "turned around" verbally when pointing out the mistake. In (241), Rebekka says "Lia this here is also the same [*leans over to Lia's and Lara's worksheet and points back and forth at ninth and twelfth pentomino*] . turned . yes". After Lia agrees (243), Dana points out another mistake by saying "and this there . and this there is the same one as this [*points back and forth at the tenth pentomino and the forth pentomino on Lia's and Lara's worksheet*] simply just turned around" (244) which Lia again agrees to (246). Overall, Rebekka does not include a warrant in the first scene (202) when pointing out a mistake but later in the interaction she does (209/241). This can be interpreted as her not feeling obligated to do so in (202) and through the interaction identifying the interactional obligation she is expected to meet. Another interpretation could be that she does feel obligated in (202) but does not know how to explain further by giving a warrant. This implies that through the interaction she learns how to give a warrant for pointing out this particular mistake.

The two other two interactional obligation identified so far can also be found in this interaction. One is a **contradiction** to a previous utterance or action which then leads to the other student bringing forth a warrant or a backing. An example is Rebekka repeating the warrant (209) after Lia's contradiction in (208). The other interactional obligation is so far summarized as **certain types of questions**. Maybe the most obvious of which is explicitly requesting a justification. Another is suggesting a solution but phrased as a question because of uncertainty. In this transcript, a third type of question is identified. Lia asks "really/" (205) which seems to be a follow-up questions showing that the previous conclusion has not been fully convincing. Both Dana and Rebekka then repeat Lara's warrant (206/207).

CONCLUSION

So far three interactional obligations have been identified after which students seem to feel obligated to give a warrant or backing: contradictions, mistakes and certain types of questions. Overall, this paper shows how students interpretation of certain utterances and actions as interactional obligations for bringing forth a warrant or backing can change within an interaction. In this example, this change seems to be encourage by Lara adding the warrant Rebekka does not provide for her conclusion and by Lia pointing out that she is not convinced of Rebekka's conclusion yet. Rebekka learns that she is expected to add a warrant when pointing out a mistake. If, however, she simply did not know how to give a warrant, she learns how to reason regarding this specific mathematical topic. Furthermore, there seems to be a learning potential for the other students as well. Lia, for example, is able to follow the specific collective argumentation regarding the mathematical topic in the end which she could not at the beginning. In the future, these findings will be compared to other interactions in order to describe these interactional obligations in more detail and possibly describe further obligations as well.

References

Bauersfeld, H., Krummheuer, G., & Voigt, J. (1988). Interactional theory of learning and teaching mathematics and related microethnographical studies. In H.-G. Steiner, & A. Vermandel (Eds.), *Foundations and Methodology of the Discipline Mathematics Education* (pp. 174-188). Antwerp: University of Antwerp.

Friesen, R.-A., & Schütte, M. (2020). Interactional obligations within collaborative learning situations bringing forth deeper collective argumentation. In J. Ingram, K. Erath, F. Rønning, & A. Schüler-Meyer (Eds.), *Proceedings of the Seventh ERME Topic Conference on Language in the Mathematics Classroom* (pp. 147–154). Montpellier, France.

Jung, J., & Schütte, M. (2018). An interactionist perspective on mathematics learning: Conditions of learning opportunities in mixed-ability groups within linguistic negotiation processes. *ZDM - International Journal on Mathematics Education*, 50, 1089–1099.

Krummheuer, G. (1995). The Ethnography of Argumentation. In P. Cobb, & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: interaction in classroom cultures* (pp. 229-270). Hillsdale, NJ: Lawrence Erlbaum.

Krummheuer, G., & Brandt, B. (2001). *Paraphrase und Traduktion: Partizipationstheoretische Elemente einer Interaktionstheorie des Mathematiklernens in der Grundschule* [Paraphrasing and traducing: Participation theory elements of an interactional theory of learning mathematics in primary school]. Weinheim, Basel: Beltz Verlag.

Miller, M. H. (1986). *Kollektive Lernprozesse: Studien zur Grundlegung einer soziologischen Lerntheorie* [Collective learning processes: Studies on the basis of a sociological learning theory]. Frankfurt, Main: Suhrkamp.

Schütte, M. (2014). Language-related learning of mathematics: a comparison of kintergarten and primary school as places of learning. *ZDM Mathematics Education*, 46, 923–938.

Schütte, M., Friesen, R.-A., & Jung, J. (2019). Interactional analysis: A method for analysing mathematical learning processes in interactions. In G. Kaiser, & N. Presmeg (Eds.), *Compendium for Early Career Researchers in Mathematics Education. ICME-13 Monograph* (pp. 101-129). Cham: Springer.

Toulmin, S. E. (1969). *The uses of argument.* Cambridge: Cambridge University Press.

Voigt, J. (1994). Negotiation of Mathematical Meaning and Learning Mathematics. In P. Cobb (Ed.), *Learning Mathematics. Constructivist and Interactionist Theories of Mathematical Development* (pp. 171-194). Dordrecht: Kluwer Academic Publishers.

Vygotski, L. S. (1978). *Mind in Society.* Cambridge: Harvard University Press.

ACHIEVING MEANINGFUL STATISTICS CLASSROOM LEARNING THROUGH BILINGUALISM AND MULTILINGUALISM: A CASE OF SELECTED GRADE 10 STUDENTS IN MARIKINA CITY

Mary Jane A. Castilla[1] and Catherine P. Vistro-Yu[2]
University of Santo Tomas[1] and Ateneo de Manila University[2]

Recognizing language as a "master tool" (Engeström, 1993) and as "tool of tools" (Cole & Engeström, 1993), the present study examined the linguistic interactions that took place in a Statistics classroom. As part of a bigger study that analyzed Statistics classroom activities using the framework of the second generation Cultural Historical Activity Theory (CHAT), this paper focused on the unique features of Statistics classrooms in the Philippines. In particular, it considered bilingualism and multilingualism as a potential primary CHAT construct rather than just an artefact that mediates the attainment of learning outcomes as it recognized the specific contexts of Filipino learners. It looked into how the varying roles of students' alternating use of the Filipino and English languages combined with their mathematical language contributed to the students' participation in the activity. A major finding revealed that language, particularly for bilinguals and multilinguals, played a prominent role in the students' achievement of learning goals and made the learning more meaningful.

BACKGROUND AND SIGNIFICANCE OF THE STUDY

Learning and teaching resources have been developed to accommodate the needs and challenges brought by the new educational set-up in the Philippines. By studying students' classroom activities using the lens of Cultural Historical Activity Theory (CHAT), teachers could understand better socio-cultural components that make up group activities, particularly students' actions and conversations, and how these contribute to students' full realization of the goals of learning. This study highlights the unique contexts of Filipinos with respect to the language of instruction in mathematics; mathematics is taught in English but for most Filipinos, English is a second language. The researcher's version of the Second Generation CHAT by Engeström (1987) offers a new perspective to understanding how learning occurs in a Statistics class when students engage in and work on an activity and considers the context of Filipino learners (see Figure 1). In this study, the subject is a group of Grade 10 students working on activities on measures of position and measures of variation to achieve the intended learning goals (object) leading to a meaningful learning of the concepts (outcome). The content and performance standards as stated in the curriculum guide are the learning goals and competencies realized through concrete actions comprising of computing, explaining, illustrating, organizing, interpreting and concluding towards meaningful learning. Achieving the short-term object helps in completing the long-term outcome of the students.

Aligned with Radford's (2011) activity perspective in mathematics, the collective pursuit by this group of students stimulates each learning activity in which interaction becomes an avenue for exchange of meaning. Here, the interaction during the learning activity as a complex process of being with others

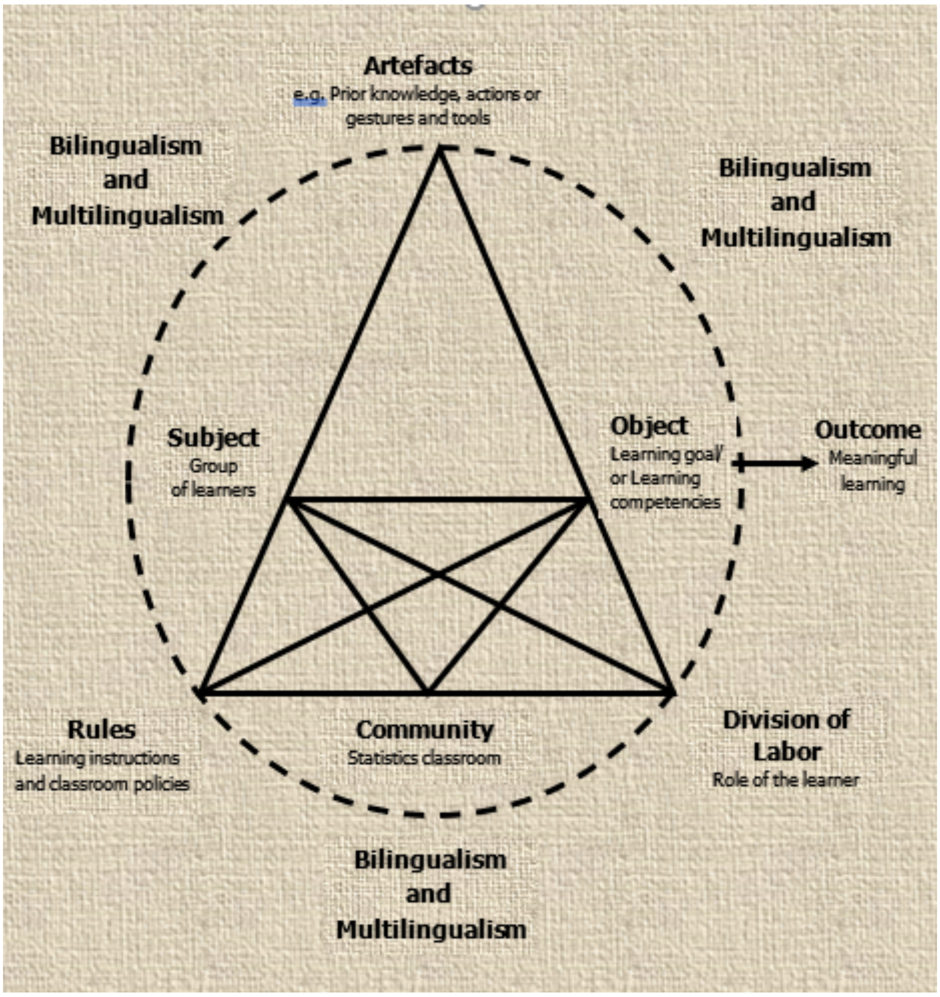


Figure 1: Researcher's Proposed Framework based on Second Generation CHAT by Engeström (1987)

comes with the purpose of realizing a collectively motivated learning goal and could lead to create or gain new artefacts or cultural tools. With Roth and Lee's (2007) view of learning as an evolving activity consisting of a complex structure of mediated and collected human agency, this study considers each learning activity as a unit of analysis that captures the dynamic interaction between the group of learners and the other CHAT constructs, namely, artefacts, rules, community and division of labor (see Figure 1). Prior knowledge, gestures or actions and tools are artefacts which may either mediate or interfere in the attainment of learning outcomes in a statistics classroom. How the group of learners relate their past experiences to their new insights during the activity constitutes prior knowledge that contributes to the students' mediated action. The actions or gestures of the students that result from and are marked by cultural-historical processes (Radford & Roth, 2011) reflect their emotions and feelings as they engage in the activity. As illustrated in figure 1, the learners reinforce their actions and the activity through emotions and feelings that are always tied to the goals of learning. Artefacts in the form of instructional materials or learning resources in doing the statistics classroom activity are called tools. These tools are not conveniently handed to the group of learners and are usually invented, purchased, discarded or replaced in the activity. In this study, the tools in Statistics class include activity sheets, calculators, pens, scratch papers, notebook, textbook, black board and chairs. The present study also accounts for the socio-historical aspects of mediated action in a Statistics class. First, the learning instructions and the classroom policies that may guide or restrict the group of students in doing their Statistics classroom activities are the rules. These rules as important resource for situated actions (Roth & Lee, 2007) guide the students on the correct procedures and acceptable interactions to take with other members of the group. Next, the Statistics classroom is a community which recognizes the social others such as the teacher, classmates or the members of the group. In this social group, the learners are working to accomplish a shared task while participating in the activity. As the activity unfolds, the learners acknowledge the others and position themselves by agreeing and disagreeing (Radford, 2018). Finally, how the students perform their assigned tasks during the statistics classroom activity pertains to the division of labor. In most cases, when a learner copes with the task at hand he/she also gives some part of him/herself to the other members of the group (Radford & Roth, 2011).

Taking the context of the Filipino learners and recognizing language as a “master tool” (Engeström, 1993) or “tool of tools” (Cole & Engeström, 1993) and as an established cultural artefact that regulates the interaction among learners and the other CHAT constructs, the researcher believes that bilingualism and multilingualism as represented by the dashed circle with a houndstooth background seem to play varying roles in the Statistics classroom at certain times and under certain circumstances. Perhaps, examining the role of bilingualism and multilingualism during the interaction of all the CHAT constructs would help in understanding the students’ engagement in different Statistics class activities. Drawing on Halliday’s view on the links between language, learning and social context as well as Graven and Robertson’s (2018) work on mathematics classroom talk, the researcher examines the linguistic interactions taking place in the statistics classroom with bilingualism and multilingualism as primary CHAT construct that mediate the attainment of learning outcome/s among the students. The alternating use of two languages describes bilingualism whereas using two or more languages (i.e., does not only include spoken and written, but also the mathematical language) in doing the activity pertains to a multilingual mathematics context. This study delved into how the learners’ choice and use of language in the statistics class affects their participation in the activity and investigated how learners use the different CHAT constructs during the learning activity with Statistics group talk as focal point. Thus, this would allow us to consider some perspectives of how learning really occurs in the classroom.

Many studies on different facets of language and learning mathematics were conducted but this study delved into how the students’ using Filipino language (i.e. language of their generation, and jargon or colloquial) and English language combined with the Mathematical language regulated their interaction in the meaning-making process. Some learning problems related to language and the cognitive challenges in a statistics class were addressed in this study.

MAIN PROBLEM

The present study aimed to analyze the interaction between a group of Grade 10 students and classroom activities in a statistics class using the lens of CHAT with language as a potential primary CHAT construct. It investigated how bilingualism and multilingualism might prominently help in the achievement of learning goals and desired outcome. Specifically, the study sought to answer this question: *How do bilingualism and multilingualism mediate in the interaction between the students and statistics classroom activities?*

METHODOLOGY

This study utilized qualitative data based on the accounts by a group of Grade 10 students taking Statistics and Probability. The participants of this study were a single group of five students from one of the city public schools in the Philippines' National Capital Region. Among these five students, there were three females and two males. All students were 16 years old during the conduct of the study. The five students were selected based on their mathematics grade and upon the recommendation of their mathematics teacher. The students must have had at least a satisfactory performance in mathematics during the previous quarter and must be at least bilingual (meaning they can speak two or more languages). The students were identified as bilinguals or multilinguals using the screening questions that were included in the background information of the respondents. The video and audio-taped recorded observations were conducted to capture the five students' interactions or group talks during the learning activity. The students were interviewed upon the consent of their parents. After accomplishing each activity, the students and their teacher answered the Activity Self-Reflection Sheet for Students and Teachers (ASRS/T) to assess their affective learning outcomes. The ASRSS consisted of 5 items inquiring about the students' experiences in doing a particular activity. It is an adapted version of Mathematics Self-Reflection Sheet (MSRS) that was designed by the Mathematics Assessment Project (MAP) to nurture self-reflection habits and skills (Fan, 2011). The items in ASRT were made parallel and aligned with the items in ASRSS to gauge the teacher's perspectives on students 'experiences in doing a particular activity. The teacher used the Rubrics for Cognitive Learning outcome by Clarke (2011) to assess the students' written outputs in the activity. A video recorded interview for each participant and the teacher was scheduled to gain an in-depth knowledge of the students' experiences, goals, perceptions, attitudes and beliefs towards the learning activity. There were four sets of activities given to the group of students namely: Activity on Quartiles for Ungrouped Data (AQUD), Activity on Deciles and Percentiles for Ungrouped Data (ADPUD), Activity on Measures of Position (AMP) and Activity on Measures of Variation (AMV). Each activity was given to students as part of their regular Statistics classroom activity. The schedule for the four sessions of the Statistics class activity was set by the teacher and approved by the school head. Each session was conducted with at least a two-day interval to give enough time for the teacher to prepare and brief the students. After each activity session, both the teacher and students answered the Activity Self-Reflection instrument. It took almost a month for students to complete all the four sessions. An interview with each of the five students and the teacher followed the last activity session.

This study is a part of a bigger study that highlighted the learning that took place in social and cultural context (for more information on the method of analysis in this study, see Castilla 2019). It focused on the experiences of learners within the learning activity from the point of view of the learner and using CHAT as the overarching framework. The analysis of data in this study involved clustering of objects or characteristics using the transcribed video-taped interviews, classifying both written and verbal data and, finding patterns. From the common threads in the interview data and the video analysis of concrete episodes during group talks or interactions, the themes that connected the categories were identified. The researcher integrated her interpretations and explanations based on CHAT in the context of the Filipino learners and other relevant theories on learning activity in Statistics classrooms. A triangulation method was also used in this study which involved verifying data obtained from the students' written output, videos of group talks and interviews.

MAJOR FINDINGS

Based on all the data gathered from the said instruments and recordings, the findings of this study show that language played a prominent role in the students' activities. In particular, bilingualism and multilingualism played different roles in mediating the attainment of meaningful learning serving as the thread that linked all the constructs together. By using Filipino and their own generational language, the students were able to achieve a deep understanding of the concepts because the procedures in the activities were made clearer and all the tasks were carried out successfully. Notably, these students used a "language" to provide a context for the terms that were too abstract to them. By using Filipino as the main language in their group talks and interspersed with the Mathematical language they know, the students' roles were clarified, their actions done correctly, and a further understanding was achieved.

The following are some excerpts showing the students' alternating use of the Filipino and English languages combined with their Mathematical languages:

a) Nicole: *Mas maganda yun kay Tristan*. [The handwriting of Tristan is better] *Susunod yun kay Cañete.* [Next, the handwriting of Cañete] (Nicole points to Princess then Princess smiles).

Roda: *Ako ganyan lang*. [My handwriting is like this] (Roda shows some hand gestures describing her handwriting).

Nicole: Median. (Nicole describes the handwriting of Roda)

Princess: Median

Roda: *Median lang ako*. [My handwriting is Median]

Tristan: *Lage na lang median is equals to one-half*. [Median is always equal to one-half]

Princess: Ahhhh

Roda: *one- half lang ako*. [I am always one-half.]

Princess: 50 / 100. So (inaudible)

Roda: Yeah.

The students uttered 'Median' several times and associated it with one-half or 50/100. However, it appears that they appropriated the term to describe the handwriting of a particular student as presented in excerpt above. The students capitalized on their first language to contextualize terms like one-half and median that ordinarily refer to mathematical concepts. The words were enriched and even invented, or their representation extended. The students also created particular meanings of these terms, which only they could understand. The said results are in line with Moschkovich's (2007) observations about using the preferred language that facilitated the ways students provided clear and appropriate explanations and justifications.

b) Nicole: *Sabaw.* [Soup]

Tristan: *Sabaw*![Soup!] *Wow soup*!

Princess: *Magkaiba lang yan sa..* [It is different in terms of..]

Tristan: *Magkaiba lang ng sagot yan kase..* [The answers are different]

c) Nicole: *Ano ba yun nakalagay*? [What does it say?]

Tristan: *Wala kasing sinabe talaga eh*. [It did not say anything]
Roda: *Girl paano?* [How?]
Princess: *Di ko gets eh*. [I don't get it]

The group talks above (b and c) showed how the students used the language of their current generation to express their ideas and feelings while engaging in the activity. For instance, the students used 'sabaw' to describe the stage of learning they were at that time. In common parlance, 'sabaw' means soup or a liquid component in any dish. In Filipino slang, 'sabaw' is an expression used for a person who is drained mentally in a given activity. Perhaps, the students were confused and they were confronted with a challenging situation and by uttering 'sabaw' the students understood one another and consequently were appeased. The use of Filipino made it easier for the students to follow the procedures that were carried out. This was observed when a student uttered 'di ko gets', it meant that a concept or procedure was not clearly understood and more importantly, the phrase had a shared meaning that elicited sympathy and understanding from the others or a plea for help.

Despite the challenges encountered by the students (e.g., recalling the needed formula, not listening and being exhausted), they were able to illustrate and calculate the specified measures of positions and some measures of variation. They were also able to accomplish all four activities using the Filipino language mixed with English as well as the formal Mathematical language. In the activity self-reflections, most of the students considered the activity worthwhile because they were able to work with their groupmates. The teacher's reflections showed that the activity was worthwhile because students were given the opportunity to work on the problems by themselves. These results support Shimizu et al.'s (2010) perspective on worthwhile tasks that allow students to stimulate their learning. It is a reminder that besides content, the students' experiences in dealing with the task and working with others towards the learning goals also matter. According to Schleppegrell (2010), language helps in resolving tensions. It is a valuable tool for promoting the mathematical knowledge. Luis Radford said that "all human activities involve language and language is important because it is the most powerful tool in the sense-making process" (personal communication, May 8, 2018). In the present study, it is noteworthy that students used Filipino while engaging in the activity as seen in the video recorded group talks and during the interviews. Language played an important role as the students transformed ideas. Through language, the students were able to break down concepts and bring them into their contexts. This made the overall interaction of the different CHAT constructs more meaningful which resulted to the students' achievement of the learning goals.

The data showed that students' using Filipino and English languages combined with the mathematical language regulated their interaction with the other CHAT constructs in the statistics class. The use of a second and possibly third language enabled students to draw from previous knowledge, clarify rules, implement division of labor, communicate mathematically and more meaningfully.

MAJOR CONCLUSIONS

The coordination of all the CHAT constructs leads to the students' achievement of learning goals and meaningful learning outcomes. Actions and interactions act as means for the students to be able to accomplish the tasks and serve as opportunities for them to contribute in their process of meaning-making. Activity is shaped by the students' prior knowledge with respect to the tasks, goal, tools and constraints. Tools contribute in the meaning making process of students because they influence and support the learning of students. The socio-historical CHAT constructs namely rules, division of labor and community also play important roles in accomplishing the tasks. They guide the students in the correct and acceptable actions and interactions, promoting task-sharing and recognizing the role of social others in achieving the learning goals. Language, specifically bilingualism and multilingualism, plays a prominent role in students' achievement of learning goals and meaningful learning. It holds together all the CHAT constructs and ensures their coordination. Language must be treated as a separate primary construct (see Figure 2).

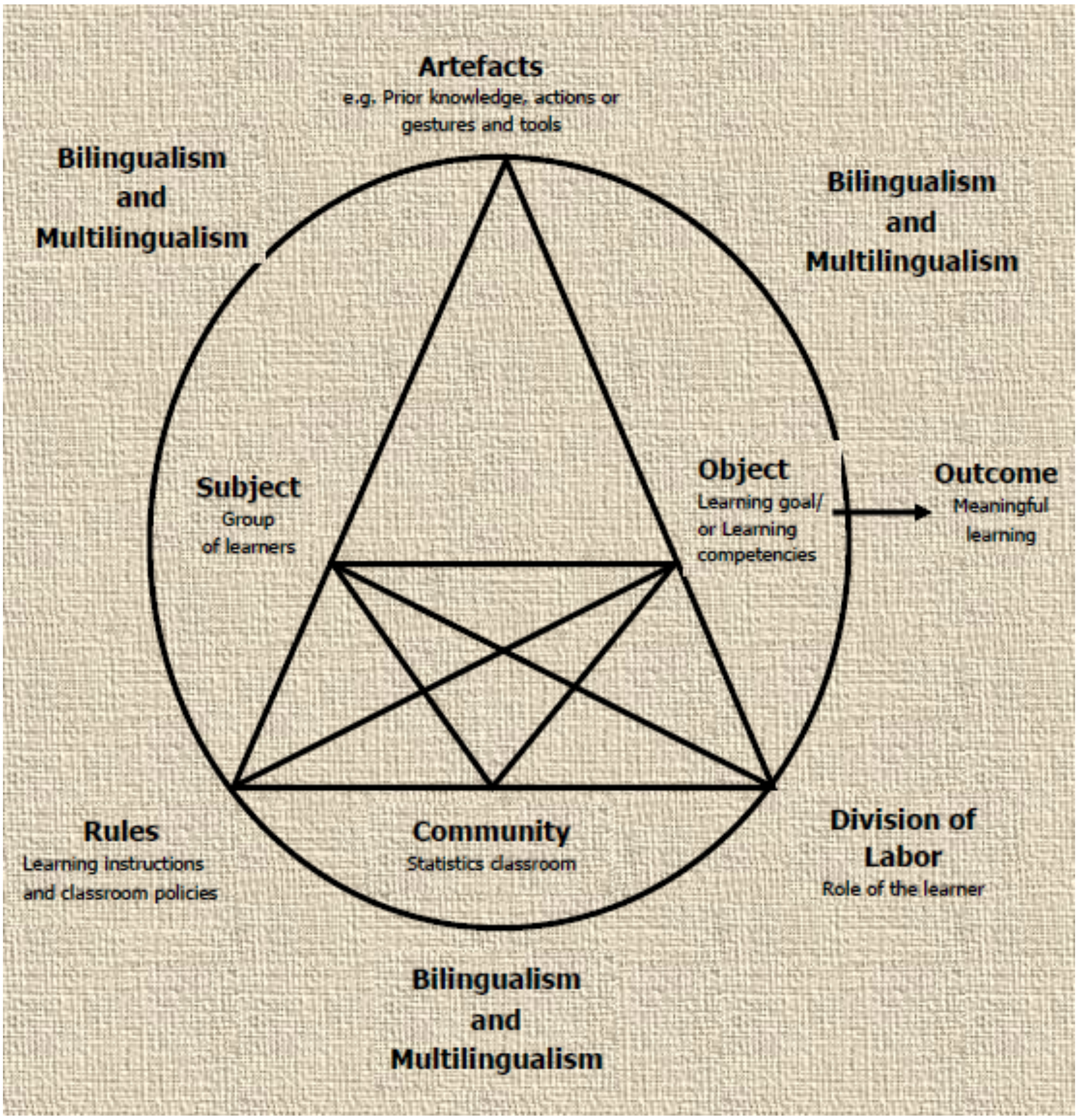


Figure 2: Researcher's Revised Framework based on Second Generation CHAT by Engeström (1987)

This study provides a theoretical and practical bases for improving and varying classroom activities in Statistics and possibly Mathematics as a whole in Junior High School, in order to help students learn concepts meaningfully. In addition, this study encourages teachers to support students' recognition and understanding of Statistics based on CHAT and with bilingualism and multilingualism playing a central role in the meaning-making process. The findings of this study will motivate supervisors, academic heads and leaders to implement and promote a curriculum that recognizes the students' use of language fundamental in making sense of mathematical meanings as well as in learning mathematics with understanding.

References

Castilla, M. J. A. (2019). *Meaningful statistics classroom activities in Junior High School : a reconceptualization of the cultural historical activity theory in mathematics* (Accession No. rlo.40067336) [Doctoral dissertation, Ateneo de Manila University]. Rizal Library's OPAC (Online Catalog) http://search.ebscohost.com/login.aspx?direct=true&db=cat02781a&AN=rlo.40067336&scope=site

Clarke, D. J. (2011). Open-ended tasks and assessment: The nettle or the rose. In B. Kaur & K. Y. Wong (Eds.), *Assessment in the mathematics classroom: Yearbook 2011, Association of Mathematics Educators* (pp. 131–163). World Scientific.

Cole, M., & Engeström, Y. (1993). A cultural-historical approach to distributed cognition. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 1–46). Cambridge University Press.

Engeström, Y. (1993). Developmental studies of work as a testbench of activity theory: The case of primary care medical practice. In S. Chaiklin & J. Lave (Eds.), *Understanding practice: Perspectives on activity and context,* (pp. 64–103). Cambridge University Press**.**

Fan, L. (2011). Implementing self-assessment to develop reflective teaching and learning in mathematics. In B. Kaur & K. Y. Wong (Eds.), *Assessment in the mathematics classroom: Yearbook 2011, Association of Mathematics Educators* (pp. 275–290). World Scientific.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical thinking and learning, 4*(2-3), 189–212.

Moschkovich, J. (2007). Using two languages when learning mathematics. *Educational Studies in Mathematics, 64*(2), 121–144.

Radford, L., & Roth, W. M. (2011). Intercorporeality and ethical commitment: An activity perspective on classroom interaction. *Educational Studies in Mathematics, 77*(2-3), 227–245.

Robertson, S. A., & Graven, M. (2018). Using a transdisciplinary framework to examine mathematics classroom talk taking place in and through a second language. *ZDM Mathematics Education, 50*(6), 1013–1027.

Roth, W. M., & Lee, Y. J. (2007). "Vygotsky's neglected legacy": Cultural-historical activity theory. *Review of Educational Research*, 77(2), 186–232.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly, 23*(2), 139–159.

Schleppegrell, M. J. (2010). Language in mathematics teaching and learning: A research review. In J. N. Moschovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research* (pp 73–112). Information Age Publishing.

Shimizu, Y., Kaur, B., Huang, R., & Clarke, D. (Eds.). (2010). *Mathematical tasks in classrooms around the world*. Brill.

A STUDY ON THE EVALUATING OF LEARNING OPPORTUNITIES IN MATHEMATICS CLASSES OF SECONDARY SCHOOLS BASED ON DISCOURSE ANALYSIS TECHNIQUES

Zhihui Chen and Yuting Tong
South China Normal University, Guangzhou; East China Normal University, Shanghai.

With the wide spread of the large-scale international academic assessment of mathematics, the development of literacy in young students has drawn public attention in China recently. How to cultivate students' ability based on the model of mathematical core competencies through teaching practice are becoming an increasingly urgent issue. To explore an evaluation method of such instruction, we analyze two video-based lessons with different teaching methods for the same course using discourse analysis techniques. Taking the teaching of Cross Multiplication as examples, we try to analyze the quality of learning opportunities (LO) in classes from the perspective of instructional tasks and classroom interactions. As results indicate, bases on the perspective of LO, we could deeply analyze how students engage in mathematics learning in the classroom. Strategies of task implement and leading the mathematical classroom dialogs for novice teachers are also discussed further.

INTRODUCTION

With the deepening and development of research on learning theory and instructional evaluation, traditional methods for evaluating the teaching is realized to be limited at providing targeted advice for improving teaching skills and professional development of teachers. For overcoming the shortage of such traditional methods, several educational reform programs, such as QUASAR (Silver et al., 1996) and IQA (Junker et al., 2005) has been initiated in The United States since last century. The term Learning Opportunity has been extended as an idea of classroom teaching quality evaluation instead of an indicator for measuring educational opportunities in different countries or regions. As indicated in the related research (Harskamp & Suhre, 1994; Jackson et al., 2013), subjective initiative of students with their engagement of thinking in the teaching process is fully investigated during the evaluation of teaching from the perspective of leaning opportunity. In terms of the development of students' literacy, the idea is obviously consistent with the purpose of current mathematics curriculum reform in China. Thus, the reference value of learning opportunity could not be ignored when exploring new methods of classroom teaching evaluation in the background of curriculum reform.

LITERATURE REVIEW

What is a learning opportunity in mathematics class?

As McDonnell (1995) noted, learning opportunity is one of a small set of generative concepts that has changed how researchers, educators, and policymakers think, which is used as an educational analytical concept in three ways from macro policy to micro student learning in international studies. Firstly, learning opportunity was considered as a determinant variable for exploring the relationships between effort on study and outcomes. In the mid-20th century, John Carroll originally used the term opportunity as a concept that influenced learning through the amount of time spent on. Building on

Carroll's theory of time use and Bloom's (1974) concept of mastery learning, Dahllöf (1971) developed a theoretical model which states that performance is a function of the initial ability status of the pupils, the curriculum objectives and the time spent on actual learning. In the PISA survey, learning opportunity of students was measured through a questionnaire about their familiarity with a range of topics within three mathematics domains (Number, Geometric Shapes and Measures, and Data Representation).

Later, the term was developed to describe the connection between teacher's teaching and students' learning in the mathematics classroom. Although the exposure of mathematics topics could make an impact on achievement, the issue that adaptability between content and students' current level is more significant in the term of mathematics learning. As Doyle (1983) noted, different instructional tasks provide various opportunities for students to learn, but the cognitive demand of tasks is the determined variant. Because instructional tasks not only draw students' attention to specific content but also determine the way they use information. In this sense, different cognitive levels will lead them to experience different thinking activities such as recall, inquiry, and reflection. Building on this foundation, Hiebert (1997) and Hiebert and Grouws (2007) added, when solving the task with high-level cognitive demand, teachers provide students with more opportunities to explain their reasoning and make connections between different concepts, problem-solving methods and representations.

Recently, learning opportunity has been extended as an idea for evaluating teaching quality, which stresses that students could be able to engage with mathematical thinking activities. How a task is introduced and presented in class appears to affect the progress work of teachers and the learning outcome of students, and classroom discourse interaction between teacher and students is also noticed to be the point of whether opportunities of learning tasks could be fulfilled. Boston (2012) forcefully argues that critical awareness of discourse practices in conjunction with teacher mediation of other affordances for learning within the classroom environment might engage students in mathematical practices such as problem-solving, explaining mathematical ideas, arguing for or against specific solutions to problems, and justifying mathematical thinking. In the study of Using the Instructional Quality Assessment (IQA) Toolkit to Assess Academic Rigor in Mathematics Lessons and Assignments (Boston et al, 2006), the indicators for measuring learning opportunities can be summarized as the following four aspects:1) cognitive tasks with cognitive challenges; 2) opportunities for students to participate in high-level thinking and reasoning in the implementation of tasks; 3) Students have the opportunity to explain the reasoning and thinking process during the discussion; 4) the teacher's learning expectations for the students (Boston, 2012).

In conclusion, learning opportunities can be understood as the possibility that students could be engaged in knowledge construction through their individual speech and cognitive thinking in the process of learning mathematics, which emphasizes that evaluation of teaching quality should focus on classroom mathematics tasks, teacher-student interaction. Based on the existing research, we develop a framework based on discourse analysis techniques for analysis of learning opportunity from two dimensions including feature of the Mathematical Task (MT) and feature of the Classroom Discourse (CD) as follows (Figure 1). The level of cognitive demands of instructional tasks will be considered as an indicator in the former dimension, while teacher's speech discourse (Initiated problem-posing & Feedback) that facilitate a meaningful understanding of mathematics (for now we can call them *Wise Speech*) for students will be focused on the latter feature.

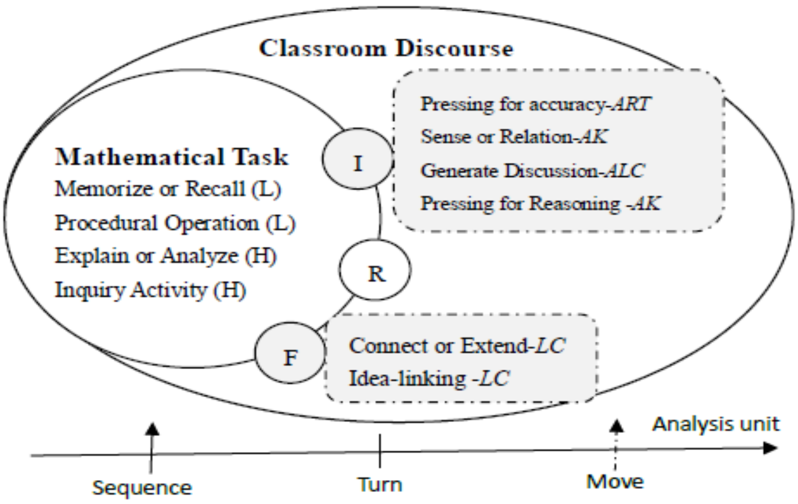


Figure 1 framework for analysis of learning opportunity in mathematics classroom

METHODS

Since the TIMSS series of research, video-based Lesson Study has become a valuable approach for improving teaching and promoting the professional development of teachers. Building on this idea, lessons of the same content with different teaching methods could provide multi-aspect, such as lesson structure, teaching events and classroom behavior, for viewing more details of classroom teaching to seek improvement. Due to evaluate students' opportunities to learn in class, two separate lessons of grade seven from 2 public junior high school, teaching with the same content named *Cross Multiplication*, were video-recorded from September to October 2017 in Shanghai China.

Participating teachers. In order to enhance the comparability of different teaching methods, we selected two teachers (T_1, T_2) with similar educational backgrounds from shanghai. They are both novice teachers who graduated from mathematics education major within 3 years. Before the classes, we have interviews with them to know the details of their teaching beliefs and instructional design in this lesson.

Data collection. With both teachers' permissions, we captured all the instructional processes in the classes with digital voice recorders and video cameras. After the data collection, we transcribed the instructional voice and video into text by turns of the teacher' and students' speech with the same format as lesson transcription.

Coding procedures. Before uploading the lesson transcription to the NVivo 11.0 software, we defined a codebook based on the framework of learning opportunity (Figure 1) and the analysis units of the transcription text. In this study, we defined that move is the basic analysis unit, a sequence refers to several turns that focus on a specific topic between teacher and students and each instructional task could be divided into several sequences depending on teaching pace.

Data analysis. When finished coding all the lesson transcription, the results that we concerned are including the frequency of each level of cognitive demand of mathematics tasks, the frequency of Wise Speech in teachers' turns. They provide us lots of empirical evidence to evaluate the quality of different of teaching and analysis how opportunities were created to facilitate students' mathematics learning.

FINDINGS AND DISCUSSIONS

Decomposition of mathematical tasks into interaction sequences. From the two cases of lesson video, we found that both teachers could cater for all students at different levels when implementing instructional tasks. However, same-level types of sequences are repeated frequently in the class when tasks with higher cognitive demands were decomposed. Although these kinds of events can provide opportunities for students to be familiarity with the operating procedures, they could not deepen their understanding of mathematical concepts and the relationship with prior knowledge. The cognitive demand of task sequences are not only the key conditions for ensuring that students at different levels to participate in problem-solving but also an important indicator for teachers to maintain task cognition challenge in the teaching process.

Productive Initiate problems and feedbacks. From the characteristics of the classroom discourse, two teachers frequently used a large number of questions to launch and implement the instructional tasks, which indicates the typical teacher-centered approach. Teachers need to raise more inspiring questions and give more constructive feedback in the process of knowledge construction. Giving students more space for discourse will help develop their mathematical expression and communication skills as well.

REFERENCES

Bloom, B. S. (1974). An introduction to mastery learning theory. In J. H. Block (Ed.), *Schools, society, and mastery learning* (pp. 3-14). Holt, Rinehart & Winston.

Boston, M., & Wolf, M. K. (2006). *Assessing Academic Rigor in Mathematics Instruction: The Development of the Instructional Quality Assessment Toolkit. CSE Technical Report 672*. Regents of the University of California.

Boston, M. (2012). Assessing instructional quality in mathematics. *The Elementary School Journal, 113*(1), 76–104.

Dahllöf, U. (1971) *Ability Grouping, Content Validity and Curriculum Process Analysis.* Teachers College Press.

Doyle, W. (1983). Academic work. *Review of Educational Research, 53*(2), 159–199.

Harskamp, E., & Suhre, C. (1994). Assessing the opportunity to learn mathematics. *Evaluation Review, 18*(5), 627–642.

Hiebert, J. (1997). *Making sense: Teaching and learning mathematics with understanding*. Heinemann.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics Teaching on students' learning. In F. Lester (Ed.), *Second handbook of research on mathematics Teaching and learning* (pp. 371–404). Information Age.

Jackson, K., Garrison A., Wilson, J., Gibons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education, 44*(4), 646–682.

Junker, B. W., Weisberg, Y., Matsumura, L. C., Crosson, A., Wolf, M., Levison, A., & Resnick, L. (2005). *Overview of the instructional quality assessment.* Regents of the University of California.

McDonnell, L. M. (1995). Opportunity to learn as a research concept and a policy instrument. *Educational Evaluation and Policy Analysis, 17*(3), 305–322.

Silver, E. A., & Stein, M. K. (1996). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education, 30*(4), 476-521.

MATHEMATICAL EXPRESSION IN DIFFERENT LANGUAGES: THE NEED FOR SYSTEMATIC DESCRIPTION

Cris Edmonds-Wathen
Charles Darwin University

There is a need for a systematic description of the variation in mathematical expression in different languages and the observed or speculated effects of this variation on mathematics education in those different languages. Such a description would decrease the risk of invalid generalizations about languages and mathematics. It would offer opportunities to explore more comprehensively the relationship between mathematics and language, with potential to illuminate mathematical conceptualization. It would also provide guidance for mathematical register development in languages which are new to the formal teaching of school mathematics. A systematic description could be informed by a functional typology approach, utilizing linguistic advances in describing the world's languages according to structural similarities and differences.

VARIATION IN MATHEMATICAL EXPRESSION IN DIFFERENT LANGUAGES

A body of research is growing on how mathematical expression in different languages varies, and how this variation affects the teaching, learning and practicing of mathematics in those different languages. But much remains to be done to bring together descriptions of how languages are used mathematically in a systematic cross-linguistic manner, to identify patterns and gaps in the data, and to flesh out further our understanding of the relationship between mathematics and language.

In this paper I advocate for a more systematic approach for several reasons. The first is that we can save ourselves and our colleagues from making the type of invalid generalizations that can be generated when we focus on only one or a few languages. The second is that this understanding is an essential part of furthering our theoretical perspectives on mathematics itself. And the third is that by understanding more about how the similarities and differences of how different languages express mathematics, we are better placed for mathematical register development in languages that do not yet have a formal mathematics register (but whose speakers would like one). I shall elaborate on each of these reasons, and then present some preliminary suggestions for how to proceed.

DEFINTIONS: LANGUAGE, MATHEMATICS, MATHEMATICS REGISTER, MATHEMATICAL EXPRESSION

At the outset, I would like to clarify what I include in my use of the terms *language*, *mathematics*, *mathematics register* and *mathematical expression*. Because my interest in is how mathematics is expressed in multiple, different languages, I restrict the scope of *language* to the syntax and semantics of natural languages in both spoken and written form. There might be some points at which this restriction is arbitrary or fuzzy, since for instance, I am interested in how variation in information structure of a text might impact mathematical thinking, and information structure may in some languages such as English be expressed using prosody in speech. However, I think it is wise to restrict the scope of my use of the term language in this way until more has been done to describe the variation in syntax and semantics from a perspective of how this variation impacts mathematics education. Similarly, while data about different languages is necessarily gathered primarily from instances of language-in-use, the goal of cross-linguistic description is a centripetal force to describe each language in a largely unitary way (Barwell, 2014).

In talking about *mathematics*, I favor a broad definition, beyond that of school mathematics or formal mathematics, incorporating for example Bishop's (1988) universal mathematical activities and Barton's (2009) QRS (quantity-relationship-space) systems. In the current case, however, it might be productive to attempt at least to delineate the scope of mathematics in a functional way, for example by focusing primarily on academic mathematics. I'm cautiously aware here of Moschkovich's admonition to "consider the spectrum of mathematical activity as a continuum, instead of reifying the separation between practices in out-of-school settings and the practices in school" (2018, p. 40) and thus to consider mathematical language to be the language that is used across this spectrum of activity. Nevertheless, for the purposes of comparing languages, it would be productive to begin with a list of mathematical activities as indicators of the language structures to investigate. By *mathematics register*, I mean the words and structures that are used to express mathematical meaning (Halliday, 1978) and to perform mathematical activity. This is not the narrower "set of words and phrases" that Moschkovich (2018, p.39) speaks against, but includes for example the structures of argument, comparison and modality (Edmonds-Wathen, 2017). *Mathematical expression* is a broader term used to minimize potential ambiguities that might be incurred using the term *mathematical language* when also talking about different languages. Mathematics register is taken to apply to language use that has been developed for educational purposes, which means we can say that there are languages that do not have a mathematics register, although all languages will have forms of mathematical expression.

RISKS OF OVERGENERALIZATION ABOUT MATHEMATICAL LANGUAGE

Whenever we talk about mathematical language without specifying for which language we run the risk of overgeneralizing or of our work being used by others to overgeneralize. This applies particularly to discussion of "the mathematics register" and its features. Schleppegrell's (2007) influential work focusses on linguistic challenges of mathematics teaching and learning in English, which is assumed rather than specified except where she discusses challenges for English language learners. Halliday (1978, 2004) described the mathematics register of English and its features such as frequent nominalization. While he identified nominalization as a common feature of scientific languages shared by Chinese and English, he also found differences in the nominalization practices in the two languages found (Halliday, 1996). Chinese and English both fall on the "analytic" side of the analytic-synthetic spectrum of languages (Haspelmath & Michaelis, 2017; not the same use of analytic-synthetic as in Prediger et al., 2019). Unless we are careful to specify "in English" (or which other language we are writing about) there is a risk that readers will take our work to refer to any language, not just the language of our own research context. The flip side of this need to specify which language we are talking about whenever we talk about mathematics registers or mathematical expression, is to query whether we are in fact positioned to say much at all about mathematical language in general if we do not have a body of comparative data to draw on. Linguistics has moved well past the audacity to make claims about "language" based only on data from what Whorf (1956) called Standard Average European. Similarly, we are starting to understand that the diversity of mathematical expression in different languages means that we need this comparative data before we can say with confidence what is common in mathematical expression in diverse languages and what is unique or unusual.

UNDERSTANDING MATHEMATICS THROUGH LANGUAGE

The nature of the relationship between mathematics and language is a point of major theoretical contestation but the trend is to recognize that the relationship is deep and productive (Morgan et al., 2014). This furthers the need to understand this relationship in terms of diverse languages:

> Once we recognise that the words we use and the ways in which they are combined grammatically play a constitutive role in the construction of mathematical thinking, we also need to be aware of how this role may be different depending on the specific (national) language that is being used (Morgan, et al., 2014, p. 850).

Evidence of the variation in mathematical thinking in different languages includes number systems (e.g., Miura et al., 1988). My contention is that the inverse is also true: once we recognize that the role that language plays in construing mathematical thinking is different for different languages, we are better placed to understand *how* language constitutes mathematical conceptualization. In fact, it is *necessary* to understand this diversity in order to understand the relationship between mathematics and language. Once we have more substantive descriptions of the variation of mathematical expression, we will be better placed to hypothesize about and test effects of the relationship between mathematics and language.

INFORMING MATHEMATICS REGISTER DEVELOPMENT

The final reason that I wish to give for the need for more extensive description of the variety of mathematical language is that it can help those who would like to formally develop a mathematics register, particularly in Indigenous languages in post-colonial contexts. Pimm (2014) points out that to understand "the mathematics register", we can learn from recent register development such as that undertaken for the Māori language (see Meaney et al., 2012), but it would also be beneficial to study the development of the now established mathematics registers of languages such as English, Russian, Mandarin, and Sanskrit. A more extensive, more systematic description of mathematics registers and their development would provide a more diverse range of examples on which to draw.

HOW TO PROCEED

Finally, I wish to describe briefly my suggestion for a methodology for systematic description of mathematical language. We want to be able to describe mathematical expression in different languages according to structural similarities and differences. This is similar to the goals of linguistic typology, a field of linguistics which aims to describe the world's languages in an analytically compatible manner (Evans & Dench, 2006). A typological perspective is an approach that enables the positioning of the many languages in which we are interested on equal terms for analysis. As I earlier indicated, it might be useful to define mathematical activity from a functional linguistic perspective in order to arrive at the units for analytical contrast. The language structures needed for mathematics should be first identified broadly in terms that are applicable to all or most languages, and then the range of how those language structures occur in specific languages described (Edmonds-Wathen, 2019). Such a functional typological approach would also help us to identify scope for future research when gaps are found. A concerted approach could help the wider theoretical implications of the work of many mathematics education research in specific linguistic contexts be substantially realised.

References

Barton, B. (2009). *The language of mathematics: Telling mathematical tales*. Springer.

Barwell, R. (2014). Centripetal and centrifugal language forces in one elementary school second language mathematics classroom. *ZDM, 46*(6), 911–922.

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Kluwer.

Edmonds-Wathen, C. (2017). Responding to the mathematics curriculum with language and culture. *Journal of Mathematics and Culture, 11*(3), 36–63.

Edmonds-Wathen, C. (2019). Linguistic methodologies for investigating and representing multiple languages in mathematics education research. *Research in Mathematics Education, 21*(2), 119–134.

Evans, N., & Dench, A. (2006). Introduction: Catching language. In F. Ameka, A. Dench, & N. Evans (Eds.), *Catching language: Issues in grammar writing* (pp. 1–39). Mouton de Gruyter.

Halliday, M. A. (1996). The analysis of scientific texts in English and Chinese. In M. A. K. Halliday & J. R. Martin (Eds.), *Writing science: Literacy and discursive power* (pp. 124–132). Falmer.

Halliday, M. A. K. (1978). *Language as social semiotic: The social interpretation of language and meaning*. Edward Arnold.

Halliday, M. A. K. (2004). *The language of science*. Continuum.

Haspelmath, M., & Michaelis, S. M. (2017). Analytic and synthetic. In I. Bushstaller & B. Siebenhaar (Eds.), *Language Variation-European Perspectives VI: Selected papers from the Eighth International Conference on Language Variation in Europe* (pp. 3-21). John Benjamins Publishing Company.

Meaney, T., Trinick, T., & Fairhall, U. (2012). *Collaborating to meet language challenges in indigenous mathematics classrooms*. Springer.

Miura, I. T., Kim, C. C., Chang, C.-M., & Okamoto, Y. (1988). Effects of language characteristics on children's cognitive representation of number: Cross-national comparisons. *Child Development, 59*(6), 1445–1450.

Morgan, C., Craig, T., Schuette, M., & Wagner, D. (2014). Language and communication in mathematics education: An overview of research in the field. *ZDM, 46*(6), 843–853.

Moschkovich, J. N. (2018). Recommendations for research on language and learning mathematics. In J. N. Moschkovich, D. Wagner, A. Bose, J. Rodrigues Mendes, & M. Schütte (Eds.), *Language and Communication in Mathematics Education: International Perspectives* (pp. 37–47). Springer.

Pimm, D. (2014). Authority, explanation, contention and register: Language data and the surface search for essence. *ZDM, 46*(6), 967–976.

Prediger, S., Kuzu, T., Schüler-Meyer, A., & Wagner, J. (2019). One mind, two languages–separate conceptualisations? A case study of students' bilingual modes for dealing with language-related conceptualisations of fractions. *Research in Mathematics Education, 21*(2), 188–207.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly, 23*(2), 139–159.

Whorf, B. L. (1956/2012). *Language, thought, and reality: Selected writings* (J. B. Carroll, S. Levinson, & P. Lee Eds. 2nd ed.). MIT Press.

IDENTIFYING LANGUAGE DEMANDS FOR UNDERSTANDING THE MEANING OF SIMILARITY

Kirstin Erath
Martin Luther University Halle-Wittenberg

The paper reports from a Design Research study that aims at developing a language-responsive teaching-learning arrangement for the topic of similarity. Following the macro-scaffolding approach and the call for promoting student participation in discourse practices for enhancing mathematical learning opportunities, language demands on the discourse level are identified along the mathematical learning opportunities. The definition of discourse practices by Interactional Discourse Analysis allows to differentiate discourse practices and thus to grasp the language demands in more detail. The topic of similarity, especially the transition from a dynamic to a static perspective, is linguistically challenging since it is necessary to explain meanings and describe general if-then-relationships.

Language is an important factor for learning mathematics. Thus, language is not only seen as a tool of thinking (Vygotsky, 1978) but increasingly also as a learning goal (Lampert & Cobb, 2003), especially for language diverse mathematics classrooms (Barwell et al., 2016). Furthermore, the crucial role of student participation in discourse practices for developing conceptual understanding of mathematics is highlighted (Moschkovich, 2015; Erath et al., 2018). But, as Wessel and Erath (2018) point out, there is a lack of Design Research in mathematics education that explicitly addresses the discursive perspective in its interplay with mathematical learning goals. This paper reports on the study MAGENTA that aims at closing this gap for the case of similarity and investigates the research question: Which language demands on the discourse level can be identified for this mathematical learning goal?

DISCOURSE PRACTICES AS LANGUAGE DEMANDS

Differentiating discourse practices in mathematics classrooms

The notion 'discourse' is often used in mathematics education with different definitions to capture various aspect of the intertwinement of mathematics and language (learning). For the presented research, it is functional to refer to the definition of Interactional Discourse Analysis (IDA) in order to identify language demands that can be aligned along the sequenced mathematical learning opportunities in the spirit of macro-scaffolding (Gibbons, 2002; see next section). IDA's conceptualization allows to differentiate between discourse practices like explanations, descriptions, reports, etc. (Erath et al., 2018, p. 164f.): "In this framework, discourse practices rely on patterns available in speech communities' knowledge, for instance, to solve the communication-related problem of conveying or constructing knowledge (explanations) or negotiating divergent validity claims (argumentation)." Furthermore, this definition points to the crucial role of interaction and is compatible with the notion of practices that is often used in mathematics education (Erath et al., 2018). Therefore, IDA provides a theoretical foundation for conceptualizing discourse practices. However, the concept of discourse practices needs to be further specified for the case of mathematics classrooms. For a deep understanding of mathematical meaning, it is important that students not only report on procedures but also particularly explain the meaning of concepts and operations, argue the validity of claims, and describe patterns in a general way (Prediger et al., 2019).

Sequencing language and mathematics learning opportunities

Following Wessel & Erath (2018), one important approach for designing language-responsive teaching-learning arrangements that explicitly take the discourse level into account is macro-scaffolding (Gibbons, 2002) with the idea of successively sequencing the intertwined conceptual and language learning opportunities. This idea of sequencing language learning opportunities from everyday language resources towards academic language in schools is already pursued in mathematics education (Adler & Ronda, 2015; Pimm, 1987; Pöhler & Prediger, 2015). The presented Design Research study particularly highlights the discourse level (for instance instead of focusing on the lexical level) and refers to genetic approaches (Brousseau, 1997; Freudenthal 1983) as a background on how to provide mathematical learning opportunities. This means that the discourse analytic point of reference is not only used to understand how students' language proficiency is intertwined with different mathematical learning opportunities (Moschkovich, 2015; Erath et al., 2018). It is additionally used to design teaching-learning arrangements and to offer insights not only to the sequenced mathematical learning opportunities but also for the counterpart in the language dimension.

The idea behind sequencing learning opportunities for discourse practices follows the thought of starting with less demanding discourse practices like reporting on procedures or describing visual impressions that students (aged around 15 in the presented study) are likely to have already acquired. In the following steps, the demand increases towards explaining meanings, arguing, and verbalizing generalizations. The question which discourse practices are adequate is linked to the mathematical topic that is discussed (Erath et al., 2018). In general, the language demand rises with the sophistication of the mathematical content and can be connected to the differentiation of procedural and conceptual knowledge (e.g., Setati, 2005; Erath, 2017). Whereas students can talk about procedures mainly by reporting, temporally structured sentences do not suffice the demands of talking about conceptual knowledge which makes it necessary to explain meanings or describe patterns in a general way. The definitions of discourse practices offered by IDA as solving different communicative problems takes on this idea of different language demands in different steps of knowledge construction. Together with the specification for mathematics classrooms as outlined in Prediger et al. (2019), it thus builds the basis for the presented analysis and design.

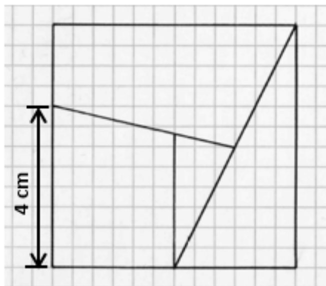
RESEARCH CONTEXT AND METHODS

The concept of similarity as mathematical learning goal

Like in many other instructional approaches to similarity (overview in Erath, 2019), the learning arrangement starts with eliciting students' intuitions in a more dynamic perspective while producing similar figures by uniform scaling. Referring to transformation geometry that means the process of enlarging or reducing a figure true to scale with a fixed magnification factor (scale) plus translating, rotating, or reflecting. The learning opportunities are then sequenced towards a more static perspective with an Euclidean definition of similar figures: "Similar rectilinear figures are those (which) have (their) angle separately equal and the (corresponding) sides about the equal angles proportional" (translated to English by Fitzpatrick, 2008, p. 156). In this case, the definition focuses on comparing and relating characteristics of two figures: Two figures are similar if corresponding angles and the length ratio of corresponding sides are the same.

In the presented study, the first elicitation of students' resources draws upon a variation of Brousseau's (1997) Tangram task (Figure 1) that is designed to activate students' individual ideas and promoting problem solving and discovering mathematics. It is particularly designed for students collaborating in small groups without direct teacher intervention in the first phases of working on the new mathematical topic. It thus makes demanding discourse practices necessary and working in a small group is crucial for accomplishing the task.

a) We would like to have a larger version of this figure.
In doing so, a length of 4 cm should become a length of 7 cm.



(1) Each choses a part of the figure.
(2) Execute on the following steps in individual work:
- Draw the enlarged version on squared paper
- Cut the figure out

b) Check in your group if the enlarged pieces fit together again.
What could be the reason?

Figure 1: Task 1 of the teaching-learning arrangement of enlarging figures (Erath, 2019)

Methodology and methods

The presented research is part of the project MAGENTA that is conducted in the methodological framework of Design Research in the learning process perspective (Prediger et al., 2015). In contrast to Design Research studies in the curriculum perspective, research in the learning process perspective "primarily aims at generating theory on teaching learning processes" and focuses on "producing local theories and paradigm cases that are meant to inform practitioners and researchers" (Prediger et al., 2015, p. 880). The project MAGENTA aims at developing a language-responsive teaching-learning arrangement for the topic of similarity in German grade 9 classrooms. It focuses on enabling all students to participate in meaningful discussions in small group work. On the research level, the focus is on empirical insights and contributions to local theories on learning and teaching processes connected to students' collective processes of knowledge construction in small group work in the case of similarity. This paper highlights the aspect of language demands on the discourse level and presents results from studying students' collective processes.

The presented empirical insights are based on video data collected in one classroom from a higher secondary school and groups of two to three students from a lower secondary school in urban quarters of the Ruhr area in Germany. All students were aged between 14 and 16 and worked on the concept of similarity for the first time. The design experiments were conducted by the author in the classroom from a higher secondary school and by pre-service teachers working on their Master's thesis in the case of the small groups from a lower secondary school. As part of the work in Design Research, the original version of the teaching-learning arrangement is currently undergoing a second deep revision.

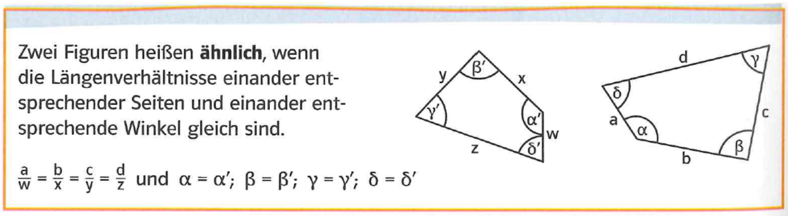
Table 1 shows the intended sequence of larger steps in the process of knowledge construction and related discourse practices and language means for talking about the respective mathematics. The categories of language demand for understanding similarity in the right column of Table 1 were derived in two steps. First, discourse practices (as identified by Prediger et al. 2019) were identified in videotaped students' talk. Second, since most groups did already struggle with developing a dynamic perspective, the identified discursive demands for the dynamic and static perspective were complemented by deduction from the mathematical topic and textbooks in an extra step. After the two-step analysis, the findings were joined and arranged along the sequenced mathematical learning opportunities. Furthermore, typical utterances and phrases (stylized from students' utterances and in some cases from teachers' utterances) serve to illustrate the needed lexical and grammatical language means (see Table 1; particularly important language means are italicized). Thus, the insights presented in Table 1 are mainly derived from the videotaped student talk. But particularly in the case of language demands for talking about the more advanced mathematical learning goals, students' utterances were enriched by teachers' utterances and further considerations.

LANGUAGE DEMANDS FOR UNDERSTANDING SIMILARITY

The analysis of video data shows that activating students' resources with the Tangram task succeeds particularly in eliciting descriptions of visual impressions and reports on procedures (first line in Table 1) and that many students lively contribute in these discourse practices (Erath, 2019; Erath, 2021). Students mainly describe to which extent the length of the sides of the enlarged figures do not fit together. Also, students' reports of how they proceeded most of the time are limited to how they enlarged the sides. In contrast, most of the groups do not address the question how to proceed with the angles in the process of enlarging the figures. Sometimes, the angles are more implicitly addressed when students start to describe that the enlarged figures changed the shape compared to the original. However, it is crucial that students start to talk about the shape of their enlarged figures since many initial strategies produce different angles compared to the original. Altogether, the language demands in this first step along the sequenced mathematical learning goals are rather low since the respective part of the task can be accomplished by describing visual impressions and reporting on procedures which seem to be discourse practices that students in the study are able to accomplish. Nevertheless, students struggle in identifying and talking about the mathematical relevant aspects of their observations (Erath, 2021).

For developing a dynamic perspective on similarity (second line in Table 1), students need to explain the meaning of enlarging. Especially the language means 'keep the shape' and 'modify uniformly' are key to grasp the mathematical core. However, particularly the meaning of 'modify uniformly' is hard to express for students and is often not explicitly explained in textbooks. Furthermore, students working in small groups without a teacher need to argue why an approach is correct or not and explain why pieces do not fit together on their way towards the mathematically sound idea of enlarging figures. Here, the deduction from the mathematical topic came into play during analysis since many groups did not succeed in rejecting their initial (mainly additive) ideas for enlarging. This means that students in the video data do not come to the point of explaining the meaning of 'modifying uniformly' (Erath, 2021). Thus, the language demand rises to explaining meaning for accomplishing the mathematical learning goal of transforming the initial, often mathematically not sound ideas towards the mathematically sound concept of enlarging true to scale. But, at least in the data analyzed so far, the students do not succeed in accomplishing these demands which hints at the crucial role of teacher support in this phase of systematizing students' thoughts and ideas. Furthermore, the first technical vocabulary 'magnification factor' needs to be introduced and explained (at least in line with many German textbooks) which also demands for a teacher-moderated phase in the learning process.

However, it is the transition from the dynamic to the static perspective on similarity (third line in Table 1) that challenges students mathematically and linguistically the most, even if supported by a teacher. No student group took this step without teacher intervention. Nevertheless, this transition is important since many textbooks have a strong focus on this perspective. Figure 2 shows the summary offered by the most popular textbook for higher secondary schools in Germany (version of North Rhine-Westphalia) as an example. For an extract from the most sold textbook for comprehensive schools in North Rhine-Westphalia see Erath (2019).



Two figures are called **similar**, if the length ratio of corresponding sides and corresponding angles are the same.

Figure 2: Summary on similarity in a book for higher secondary schools in Germany (Giersmehl et al., 2009, p. 48) and translation by the author

The discourse practice of describing general if-then relationships is very demanding and at the same time often the only way textbooks talk about similarity. But also, despite its appearance in textbooks, describing general if-then relationships is an important discourse practice in mathematics (classrooms) and should therefore be a language learning goal that can be addressed in the context of the mathematical learning goal of similarity. On the level of language means connected to talking about similarity in a static perspective, especially the meaning that is packed in the new technical vocabulary 'constant length ratio of corresponding sides', can be a sticking point for students. The new term includes two parts with mathematical meanings that need to be understood and connected: first the idea of length ratio and second the idea of corresponding elements in different figures. In addition, the term 'corresponding sides/angles' is a difficult construct in German ("einander entsprechende Seiten/Winkel") since "einander entsprechen" is not part of everyday language but part of technical vocabulary in mathematics classrooms and very specific for the topic similarity.

Mathematical learning goal	**Discourse practices and language means**
Activating students’ resources in mathematically rich situations	Describing visual impressions: • The puzzle doesn’t fit. Your side is shorter than mine, but it would need to be the same. • The large figure doesn’t look like the original. The triangle is still a triangle, but the angles don’t fit together. Reporting on procedures: • If you always add a number, then short sides in the original become too long in the new version. • For enlarging the figure, I kept the angles and calculated the new length of the sides by multiplying.
Developing a dynamic perspective on similarity	Explaining the meaning of enlarging: • Enlarging *keeps the shape* that means the *angles stay the same*. • Enlarging *keeps the shape*. That means, if I cut out the original and the new version, I can put corresponding angles exactly on top of each other. • The sides of the figure have to be *modified uniformly* so that the new version doesn’t look deformed. So, you need to do the same with all sides. Adding doesn’t work because then, for example, short sides in the original become too long in the new version and the angles don’t fit anymore. • For enlarging, all sides need to be multiplied with a constant factor to *modify uniformly*. I can calculate the factor by dividing the (given) length of the new version by the length of the original. Explaining the new technical vocabulary: • The number that all sides of the original are multiplied with is called *magnification factor*, no matter if the figure is enlarged or reduced.
Developing a static perspective on similarity	Describing the general if-then-relationship: • If two figures originate from each other by enlarging/reducing, then the two figures are *similar*. • Two figures are similar if *corresponding angles are the same* and the *length ratio of corresponding sides in the new version and the original is constant*. Explaining the new technical vocabulary: • *Constant length ratio of corresponding sides* means that there is a fix magnification factor that can be used to calculate the lengths of the new version from the given length of the original. • Enlarging with a magnification factor is also called *scaling up*.

Table 1: Language demands for understanding similarity

DISCUSSION AND OUTLOOK

Overall, the identification of language demands along the mathematical learning goals shows how increasing mathematical sophistication is linked to different and increasingly demanding discourse practices. Starting with describing visual impressions and reporting on procedures, explanations become more important in the second step and finally describing general if-then relationships is adequate for talking about the mathematics in view. Of course, in classroom interaction, not only the listed discourse practices are valuable. Especially, arguments and discussions between students are important parts on the path(s) towards understanding similarity and for example explaining what modifying uniformly means. Furthermore, the analysis shows that the means of working in small groups (a special focus of MAGENTA) are limited at the latest from the point of the transition from a dynamic to a static perspective. Many students did not yet acquire the language proficiency for further developing their mathematical thinking by describing general if-then-relationships. As outlined in Erath (2021), students need additional support on the level of demanding discourse practices and structuring small group interactions for succeeding in accomplishing the mathematical learning goal of collectively developing a mathematically sound concept of similarity. Furthermore, collective whole-class discussions are fruitful to support students' ideas and explain new technical vocabulary in between the different learning goals as sequenced in Table 1. Guiding this kind of whole-class discussions is very demanding for teachers. Therefore, further research in the project also aims at collecting ideas for organizing these discussions as well as teacher moves that support students in their mathematical and linguistic development.

Altogether, the information collected in Table 1 can help teachers in preparing their lessons on similarity by making the language demands explicit that come with the different mathematical learning goals. In this way, teachers can benefit from the theoretical insights on different discourse practices and language demands from Interactional Discourse Analysis adapted to mathematics education and further specified for the case of developing an understanding of the concept of similarity. Sensitizing teachers for the language demands and providing ideas, materials and examples for overcoming possible obstacles in the learning process (instead of going back to language learning arrangements that do not call for deep communication) are important parts of supporting teachers in their work in language-responsive mathematics classrooms. Thus, one main goal in the remaining project duration is to transfer the empirical insights into tasks, additional materials supporting different aspects of learning in the larger teaching-learning arrangement (e.g., role cards; Erath, 2021) and accompanying information for teachers.

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References

Adler, J., & Ronda, E. (2015). A framework for describing mathematics discourse in instruction and interpreting differences in teaching. *African Journal of Research in Mathematics, Science and Technology Education*, *19*(3), 237–254.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N., … Ubillus, M. V. (Eds.). (2016). *Mathematics education and language diversity. The 21st ICMI study*. Springer.

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Kluwer Academic Publishers.

Erath, K. (2017). Talking about conceptual knowledge. Case study on challenges for students with low language proficiency. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 321-328). PME.

Erath, K. (2019). Explorative study on language means for talking about enlarging figures in groups work. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 1632–1639). Freudenthal Group & Freudenthal Institute, Utrecht University, and ERME.

Erath, K. (2021). Enhancing students' language in collective processes of knowledge construction in group work. The case of enlarging figures. *ZDM – Mathematics Education, 53*(2), 317–335.

Erath, K., Prediger, S., Quasthoff, U., & Heller, V. (2018). Discourse competence as important part of academic language proficiency in mathematics classrooms. The case of explaining to learn and learning to explain. *Educational Studies in Mathematics*, 99(2), 161–179.

Fitzpatrick, R. (2008). *Euclid's elements of geometry*. http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf. Accessed 17 January 2020

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Kluwer Academic Publishers.

Gibbons, P. (2002). *Scaffolding language, scaffolding learning. Teaching second language learners in the mainstream classroom*. Heinemann.

Giersmehl, I., Jörgens, T., Jürgensen-Engl, T., Riemer, W., Sonntag, R. & Spielmans, H. (Eds.). (2009). *Lambacher Schweizer 9. Mathematik für Gymnasien Nordrhein-Westfalen*Ernst Klett Verlag.

Lampert, M., & Cobb, P. (2003). Communication and learning in the mathematics classroom. In J. Kilpatrick & D. Shifter (Eds.), *Research companion in the NCTM standards* (pp. 237–249). NCTM.

Moschkovich, J. (2015). Academic literacy in mathematics for English learners. *The Journal of Mathematical Behavior*, 40(A), 43–62.

Pimm, D. (1987). *Speaking mathematically. Communication in mathematics classroom*. Routledge.

Pöhler, B., & Prediger, S. (2015). Intertwining lexical and conceptual learning trajectories. A design research study on dual macro-scaffolding towards percentages. *Eurasia Journal of Mathematics, Science & Technology Education*, *11*(6), 1697–1722.

Prediger, S., Erath, K., & Moser Opitz, E. (2019). The language dimension of mathematical difficulties. In A. Fritz, V. Haase, & P. Räsänen (Eds.), *International Handbook of math learning difficulties. From the laboratory to the classroom* (pp. 437–455). Springer.

Prediger, S., Gravemeijer, K., & Confrey, J. (2015). Design research with a focus on learning processes. An overview on achievements and challenges. *ZDM Mathematics Education, 47*(7), 877–891.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education*, *36*(5), 447–466.

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.

Wessel, L. & Erath, K. (2018). Theoretical frameworks for designing and analyzing language-responsive mathematics teaching-learning arrangements. *ZDM Mathematics Education*, 50(6), 1053–1064.

EPISTEMIC (IN)JUSTICE IN MATHEMATICAL COMMUNICATION BETWEEN TEACHERS AND STUDENTS

Lauren Hickman McMahon
University of Michigan – Ann Arbor

Teacher-student communication is crucial for mathematics teaching and learning. It also has the potential to perpetuate or produce injustices faced by students from marginalized groups. Epistemic injustice is a form of injustice in which an individual is wronged specifically in their capacity as a knower (Fricker, 2007). In this paper, I describe two forms of epistemic injustice offered by philosopher Fricker—testimonial and hermeneutical—in the context of teacher-student communication about mathematics. Using an example from practice, I show how the frame of epistemic injustice provides insight into the ways everyday communication between teachers and students can produce or perpetuate marginalization. Finally, I discuss how this frame provides opportunities to approach teacher-student communication about mathematics in ways that push back against epistemic injustice for students from marginalized groups.

INTRODUCTION

Communication between teachers and students is at the heart of teaching and learning mathematics. Education scholars have dedicated substantial time and energy into understanding classroom dialogue (Mercer & Dawes, 2014), mathematical discourse and its unique characteristics (O'Halloran, 2015; Ryve, 2011; Schleppegrell, 2007), and the way communication operates in processes of mathematics thinking, teaching, and learning (Sfard, 2008). Much of this work highlights consequences of communication specifically for students from marginalized groups and demonstrates how discourse and communication at the interpersonal level can sustain (or challenge) injustice (Wagner et al., 2012; Martin, 2019).

In this paper, I consider a particular form of injustice, which philosopher Fricker (2007) calls *epistemic injustice,* in the context of mathematical communication between teachers and students. Epistemic injustice is a form of injustice in which an individual is wronged specifically in their capacity as a knower. Students from marginalized groups are particularly susceptible to experiencing epistemic injustice in mathematics, in part through interpersonal interactions. Mathematics as a discipline has historically and systematically excluded Black, Latinx, indigenous, women, poor, and students whose identities intersect multiple marginalized groups (Hottinger, 2016). It is also especially challenging to communicate about *mathematics*, because it demands the use of linguistic and other resources in ways that are unique to the discipline (Schleppegrell, 2007).

Communicating mathematics in ways that counter epistemic injustice and empower students as knowers is difficult work that takes place inside of complicated classroom environments, must be enacted in the moment, and requires specialized mathematical and communicative skill. In this paper, I use an example from practice to illustrate how the frame of epistemic injustice allows us to see how marginalization may be perpetuated in everyday mathematical communication between teachers and students. I then discuss how this frame provides opportunities to push back on epistemic injustice through teacher-student communication about mathematics. This requires certain epistemic sensibilities and an asset-based lens towards students that can support teachers in communicating mathematics in ways that honor and empower, rather than marginalize, students who are more likely to experience epistemic injustice in communication about mathematics.

TEACHER-STUDENT COMMUNICATION ABOUT MATHEMATICS

The interactions teachers have with students are situated within classrooms, schools, communities, and beyond, and these environments cannot be separated from the teacher-student interactions that take place within them (Ball, 2018). As Erickson (1986) describes,

> What the teachers do at the classroom and building level is influenced by what happens in wider spheres of social organization and cultural patterning. These wider spheres of influence must also be taken into account when investigating the narrower circumstances of the local scene (p. 122).

In considering wider spheres of influence, mathematics education in the United States is a "white patriarchal institutional space" (Martin, 2019) that historically and systematically excludes women, people of color, people who speak languages other than English, indigenous people, and people who are members of other marginalized groups (Hottinger, 2016; Martin, 2019). This exclusion occurs not only at structural levels but also at the level of interpersonal interactions between teachers and their students. Moreover, because of the roles teachers and students occupy, as well as racial, gender, and other identities, status hierarchies and deficit narratives are naturally at play within their interactions (Shah & Crespo, 2018). At the interpersonal level, communication between teachers and students is shaped by these broader social structures.

EPISTEMIC INJUSTICE

The challenges of communicating mathematics taken with the environments and deficit narratives that shape teacher-student interactions make this a site in which epistemic injustice is likely to occur. Epistemic injustice occurs when an individual is wronged specifically in their capacity as a knower (Fricker, 2007). Fricker describes two primary forms of epistemic injustice. The first, testimonial injustice, “occurs when prejudice causes a hearer to give a deflated level of credibility to a speaker’s word” (p. 1). Testimonial injustice occurs when a credibility deficit is afforded to a speaker by a hearer, causing the hearer to miss something the speaker has offered. In other words, when an individual approaches their interactions with another person through a deficit lens, they are less likely to focus on or notice that person's assets, causing the hearer to overlook or dismiss something being offered by the speaker. The second form of epistemic injustice, hermeneutical injustice, occurs “when a gap in collective interpretive resources puts someone at an unfair disadvantage when it comes to making sense of their social experiences” (p. 1). Hermeneutical injustice relates to the absence of a collective resource such as language, so that an individual is unable to articulate their experience and do so in a way that is understandable to others.

These two forms of injustice are not mutually exclusive. An example of hermeneutical injustice offered by Fricker (2007) is the case of women experiencing sexual harassment in the workplace prior to the existence of the term “sexual harassment.” Feelings of discomfort and threat that women experienced when men were overly flirtatious could not be understood until there was a collective interpretive resource—namely, the language of sexual harassment—that could allow women to not only articulate this experience but for others to make sense of it. At the same time, a testimonial injustice could occur: when a woman attempts to describe this experience, she may be also written off because of her gender, leading a hearer to give less credit to her and the situation she describes (i.e., her "believability" is questioned or denied by the hearer). Her inability to articulate her experience provides the hearer evidence in favor of credibility deficit, and the epistemic injustice continues.

EPISTEMIC INJUSTICE IN TEACHER-STUDENT COMMUNICATION

Students who are members of one or more of marginalized groups are more likely to be treated as less mathematically capable than their white male peers and their capabilities viewed through a deficit lens (Gutiérrez, 2013). If teachers approach interactions with students through a deficit lens, even inadvertently, they may miss out on mathematical knowledge students are offering. In other words, if a teacher expects a student to say things that are nonmathematical or not valuable contributions, then they will be less likely to be able to see or hear the mathematical contributions that student could be offering. This could lead to a testimonial injustice, because of the way knowledge gets overlooked or dismissed.

Hermeneutical injustice is also especially risky in mathematics teaching and learning, in part because of the uniqueness and complexity of mathematical discourse. More specifically, mathematics uses language, symbolism, and other discursive resources in ways that are unique to the discipline and to mathematics teaching and learning (Ball et al., 2008). To communicate mathematics requires working with mathematical objects, which are abstract by nature (Sfard, 2008). The number "five," for example, is a mathematical object that has been abstracted from the idea of quantity, describing the cardinality or size of a set of five objects. Although we might look at a set of concrete objects, such as apples, and say "there are five apples here," there is no concrete object to point to and say, "this is five," without using other communicative tools, like numerals (5), language (five), or symbolism (apples). The need to use multiple communicative resources together, such as language, diagrams, or gesture, is also a unique feature of mathematical discourse that creates additional challenges for communication (Schleppegrell, 2007).

Additionally, the "language of schooling" (Schleppegrell, 2007) often differs from the language and discursive practices students encounter in their lives outside of the classroom. In situations where students and teachers are operating with different discursive resources and expectations, there may not be collective hermeneutical resources available for students to articulate their thinking and for that thinking to be understandable to their teachers.

It is easy to see how these two forms of injustice might feed into each other through communication between teachers and students about mathematics. For example, a student in the U.S. whose first language is not English is likely to encounter situations where their status as an English learner causes their intelligence and ability to be underestimated by others, which would naturally create conditions where testimonial injustice is likely to occur. That student's familiarity with and use of language would naturally differ from the language of schooling, which means they might not have access to a *collective* or shared communicative resource that allows them to effectively convey their thinking to others—a hermeneutical injustice. They might instead use informal language, or a language with which they are more familiar. If, as a result, their thinking is not understandable to the teacher, this could provide the teacher with evidence that supports a credibility deficit; they might see the student's informal language as "wrong" or "incomplete," or "nonmathematical," which could lead the student's ideas to be overlooked or dismissed—a testimonial injustice.

Communicating mathematics in practice

To illustrate how epistemic injustice can manifest in the case of teacher-student communication about mathematics, I offer an example from a primary school classroom. Andrea[1] is a white 1st grade teacher in the United States (ages 6-7). All of her students are Black, which means they are members of a racially marginalized group in the United States. In this example, Andrea is working with a small group of four students, which she refers to as her “low group,” on the problem in Figure 1. While they are working, the rest of the class is at various centers around the room working independently or in groups.

She asks a student, Ashley, "Which circle has larger equal parts?" and Ashley responds with circle B. The following exchange occurs between Andrea and another student, Nathan.

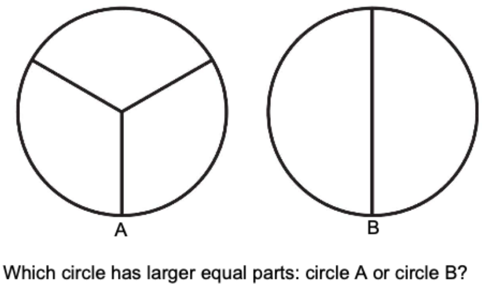

Figure 1: Task being discussed by the small group in Andrea’s 1st-grade classroom

Andrea: [to Nathan] How does Ashley know that the circle in B has larger equal parts than the circle in A?

Nathan: Because there’s two equal parts [pointing to circle B] and you have one, two [pointing to circle A] – and if you leave one out – that leaves two.

Andrea: So you’re showing me that there are two parts, and that they shaded one of them. You’re absolutely right. We know that it has larger equal parts, because we can see the parts and this one is obviously larger. There are less parts, and so that means each one of those parts is larger.

In an interview, I played this clip for Andrea and asked her about this moment. She offered her interpretation: “This kid is the most challenging kid in my entire class. He is often, he’s doing a lot of things, often not paying attention. Or often just trying to mess with you. He’s very conniving. But yeah, so he was having trouble, either he was intentionally—or he was having trouble understanding what problem we were even on.” After reviewing the video two more times, she said, “I’ll admit I was partly confused by what he was saying, I was like ‘yes there are two parts right there’…I just remember him pointing to the one that was right, which we had already established as a group was the correct one.” I also offer my own interpretation of Nathan’s reasoning, which appears to be based on the amount of the whole that is present when you consider two parts together: two equal parts of circle B makes a whole, but two equal parts of circle A would leave out part of the whole, so those parts must be smaller. Although it is difficult to say whether my interpretation is a correct representation of

[1] All names are psuedonyms

Nathan's thinking, I offer it to show how his thinking can be viewed as a mathematical contribution to their discussion.

Before discussing testimonial and hermeneutical injustice in the context of this example, I want to be clear that a generous and empathetic read on Andrea's teaching is warranted. I have the privilege of spending time with the video representation of this interaction, free from the additional demands of time and space that Andrea had to meet as she tried to communicate with Nathan in that moment. Andrea had to navigate this interaction in real time, drawing from her experience with this child. Additionally, the mathematical and pedagogical knowledge required in this moment is layered and complex. To hear Nathan's contribution in the discussion of circle A and circle B would require the ability to hear his explanation in 'kid language,' make sense of his explanation mathematically, and respond in a way that honors Nathan as a knower and contributor—all while keeping track of the other children at the table *and* the students working at other centers throughout the room. It is also likely that Nathan has a history of participating in ways that are potentially disruptive or distracting to Andrea. Paying attention to patterns in the classroom can help teachers respond when they are faced with a similar situation. Yet it is in these everyday moments where epistemic injustice can occur, which only highlights the need to understand what it takes to communicate within these complex social environments.

In the sections that follow, I consider the ways that testimonial and hermeneutical injustice may be at play in the example above. I also consider the question: Within an epistemic injustice frame, what might teachers do to push back against epistemic injustice in the work they do to communicate mathematics with students?

Testimonial Injustice

The above example of Nathan and Andrea demonstrates a case of a testimonial injustice, because of the way that the teacher appears "to miss out on knowledge offered by the child, but not heard by the adult" (Murris, 2013, p. 246). As described above, it is possible to view Nathan's response as a mathematical contribution to the discussion. He highlights the way that the same number of equal parts together (2 parts) gives either an entire whole (the circle divided into halves) or an incomplete whole (the circle divided into thirds). This demonstrates the important mathematical point that the whole stays the same regardless of how many equal parts it has been divided into, and that we can use the whole as a reference when comparing different sized parts of the same whole. This knowledge is available to be heard, seen, and acknowledged, but his teacher does not appear to pick up on it. Nathan's response to the teacher's question is apparently viewed through a lens of credibility deficit, perhaps by viewing him as a "low group" member, or "challenging" student; he is not afforded the degree of epistemic trust that would allow his reasoning to be taken seriously in their discussion. Because of this, for example, his pointing to circle B is interpreted by the teacher as pointing to the correct answer, rather than as a support for the reasoning he is attempting to express. This interpretation positions Nathan's response as something that had already been established by the group, and his mathematical contribution to their discussion seems to be overlooked.

Hermeneutical Injustice

Nathan's explanation also helps show how hermeneutical injustice can come into play in communication about mathematics. He uses important mathematical language by indicating "equal parts," and he motions to show the equal part being "left out." But he does not use any language that would indicate he is making a comparison, nor does he explicitly state that he is looking at the *wholes.* The language of comparison, for example "larger" or "more," is a collective hermeneutical resource for interpreting how much of circle A is present vis-à-vis circle B, and such language would make his thinking more visible and interpretable. Yet his language is understandably imprecise; he is, after all, a child. His 'child-ness' is one form of epistemic marginalization, as well as his treatment as a 'low' group member and 'troublemaker,' which enhances the likelihood that he has been excluded from participation in the mathematics classroom in the past. As a learner, it is not unexpected that he would not yet be fluent in the sort of mathematical discourse that would enable him to articulate his thinking more clearly for others.

PUSHING BACK ON EPISTEMIC INJUSTICE

How might teachers in Andrea's position take intentional action to communicate in ways that minimize the possibility of producing or perpetuating epistemic injustice? Within this frame, epistemic injustice is partially a consequence of credibility deficit in the case of testimonial injustice, and insufficient collective interpretive resources in the case of hermeneutical injustice. A logical counterforce to epistemic injustice would therefore be to approach communication with students through an asset-based lens that offers generosity towards students as they try to articulate their thinking. I posit that such a lens must include the following assumptions: a) that students are intelligent and capable of learning, knowing, and doing mathematics, b) that they have a genuine desire to learn, participate, and succeed, and c) that they are actively trying to do so, likely with limited hermeneutical resources to support their efforts. Each of these assumptions can be viewed as an extension of the scholarship of Martin (2019) and Gholson et al. (2012), taking as axiomatic the brilliance of Black students and broadening this stance to students from other marginalized groups. Under these assumptions, students' brilliance is taken as truth, and this brilliance includes not only ability but also motivation and effort.

What might be gained by approaching teacher-student with these asset-based assumptions? Let us return to the example of Nathan reasoning about the size of equal parts in a whole. The first assumption tells us that Nathan is intelligent and capable of learning, knowing, and doing mathematics. This actively pushes back against credibility deficit in how one might approach their interactions with Nathan. It also tells us that the label "low group" is an inappropriate and inaccurate description of him and his peers at the table. With the assumption that Nathan is intelligent and capable, one would know *before* he offers his ideas that those ideas are valuable and worthy of attention. His comment about the equal parts relative to the whole provides further evidence of his knowledge and ability, flipping the script on credibility deficit and interfering with the cycle of testimonial injustice.

The second and third assumptions tell us that Nathan has a genuine desire to learn, participate, and succeed, and that he is actively trying to do so. We see that he answers the question being asked, is involved in the discussion, and is actively engaging with the problem at hand. He is not trying to cause trouble or be disruptive; he is trying to learn and participate in the conversation. Through this lens, one can see how Nathan's actions provide evidence that confirms these assumptions, feeding into an asset-based view of this student. Again, we see a possible counterforce to testimonial injustice.

Hermeneutical injustice is caused by a lack of adequate resources for articulating one's thinking and for that thinking to be understood by others. In the assumptions described above, students are actively working to articulate their ideas, but they may be laboring with insufficient collective communicative resources. Rather than viewing this as a problem of Nathan's intelligence, motivation, etc., one might view this as a situation in which Nathan is on the receiving end of a hermeneutical injustice. Thus, the problem of communication arises not because *Nathan's ability* to communicate is inadequate; rather, it is a problem of inadequate *available collective resources*. A logical solution to this problem is therefore to search for, introduce, or develop additional resources that might offer Nathan new tools to articulate his ideas in a way that is understandable to others. Another possibility is to provide Nathan more opportunities to clarify, extend, or restate his thinking, which could give him a chance to use resources that he can access but may not have used in his initial response. Learning to use communicative resources such as symbols, diagrams, and language, is an important part of what it means to learn mathematics (Sfard, 2008). Rather than viewing a students' challenges as a problem situated within the student, one might view their challenges as a problem of available communicative or interpretive resources. This means working to develop shared understanding around those resources and their use, and supporting students in using those resources to express their thinking.

DISCUSSION

In this paper, I have offered the frame of epistemic injustice as a lens on teacher-student communication about mathematics. I presented an example from mathematics teaching, viewed through this lens, to show how such a frame may highlight opportunities to identify, understand, and push back on epistemic injustice in practice. As this example illustrates, everyday teacher-student communication is a site in which students can be wronged in their capacity as knowers and doers of mathematics, particularly students who are members of groups that have been systematically and historically excluded from the discipline of mathematics. Countering epistemic injustice for students requires knowledge, skill, and flexibility—both mathematical and communicative—in a given moment of teaching within a complicated classroom environment. In particular, it requires an asset-based approach to communication that emphasizes students' intelligence, motivation, and effort. Such a lens not only counters the credibility deficit that often produces testimonial injustice, but it also reframes problems of communication as a problem of resources, rather than a deficit lens towards students.

In this way, issues of communication, teaching, mathematics, and injustice are connected in practice and central to the work teachers must do in their everyday interactions with children. Attention to such concerns in research on mathematics communication shows promise for creating teacher-student interactions that push back on epistemic injustice to empower and honor students as knowers and doers of mathematics.

References

Ball, D. L. (2018, April 15). Just dreams and imperatives: The power of teaching in the struggle for public education. *American Education Research Association Presidential Address*, New York, NY.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.) *Handbook of Research on Teaching,* (3rd ed.). Simon & Shuster Macmillan.

Fricker, M. (2007). *Epistemic injustice: Power & the Ethics of Knowing*. Oxford University Press.

Gholson, M. L., Bullock, E. C., Alexander, N. N. (2012). On the brilliance of black children: A response to a clarion call. *Journal of Urban Mathematics Education, 5*(1), 1–7.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education, 44*(1), 37–68. https://doi.org/10.5951/jresematheduc.44.1.0037

Hottinger, S. N. (2016). *Inventing the mathematician: Gender, race, and our cultural understanding of mathematics*. SUNY Press.

Khisty, L. L., & Chval, K. B. (2002). Pedagogic discourse and equity in mathematics: When teachers' talk matters. *Mathematics Education Research Journal, 14*(3), 154–168. doi: 10.1007/BF03217360

Martin, D. B. (2019). Equity, inclusion, and antiblackness in mathematics education. *Race Ethnicity and Education, 22(*4), 459–478. https://doi.org/10.1080/13613324.2019.1592833

Mercer, N. & Dawes, L. (2014). The study of talk between teachers and students, from the 1970s until the 2010s. *Oxford Review of Education, 40*(4), 430–445. doi: 10.1080/03054985.2014.943087

Murris, K. (2013). The epistemic challenge of hearing child's voice. *Studies in Philosophy and Education, 32*, 245–259. doi: 10.1007/s11217-012-9349-9

O'Halloran, K. L. (2015). The language of learning mathematics: A multimodal perspective. *The Journal of Mathematical Behavior, 40*(A), 63–74. https://doi.org/10.1016/j.jmathb.2014.09.002

Ryve, A. (2011). Discourse research in mathematics education: A critical evaluation of 108 journal articles. *Journal for Research in Mathematics Education, 42(*2), 167–199. https://doi.org/10.5951/jresematheduc.42.2.0167

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading and Writing Quarterly*, 23(2), 139–159. doi: 10.1080/10573560601158461

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

Shah, N. & Crespo, S. (2018). Cultural narratives and status hierarchies: Tools for identifying and disrupting inequity in mathematics classroom interaction. In R. Hunter, et al. (Eds.) *Mathematical Discourse that Breaks Barriers and Creates Space for Marginalized Learners* (pp. 23-37). Sense Publishers.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–22. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Wagner, D., Herbel-Eisenmann, B., & Choppin, J. (2012). Inherent connections between discourse and equity in mathematics classrooms. In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.) *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 1-13). Springer.

DISCOURSES AS THE PLACE FOR THE DEVELOPMENT OF MATHEMATICAL THINKING – AN INTERACTIONIST PERSPECTIVE

Judith Jung, Marcus Schütte, Götz Krummheuer
Technische Universität Dresden, Leibniz University Hannover, Kassel

The article considers early childhood mathematics learning from an interactionist perspective. Here, mathematics learning is differentiated into two aspects: the acquisition of mathematical concepts and procedures in terms of learning mathematical content, and the development of mathematical thinking in terms of learning to reason. This contribution focuses on the second aspect. Based on interactionist perceptions of mathematical learning, the development of mathematical thinking is described as increasing participation in mathematical discourses. For a more detailed description of these discourses, the so far common focus of interactionist approaches to mathematics learning on the analysis of mathematical negotiation of meaning is expanded to include a description of emerging argumentative structuring of the mathematical negotiation processes. In the empirical analyses forming the basis of this article, different discourse styles are reconstructed utilizing this theoretical extension, which we call narrative, formal, and narratory discourses. In future research, they will be used as a theoretical basis for the reconstruction of the development of mathematical thinking.

THEORETICAL FOUNDATIONS: MATHEMATICS LEARNING IN INTERACTIONS

In constructing a theory for the development of mathematical thinking in pre-school and early primary school-aged children, we draw upon interactionist-constructivist ideas about the development of individuals (cf. Bauersfeld et al., 1988; Cobb & Bauersfeld, 1995; Schütte et al., 2019). A theoretical foundation of such a perspective on learning and the development of mathematical thinking requires basic theories focusing on the social or the collective rather than the individual during (early) human development. Fundamental theoretical assumptions of our investigation are to be found in the sociological theory of Symbolic Interactionism (Blumer, 1986). The conception of learning derived from Symbolic Interactionism and underlying here understands social interaction as the constituting starting point of learning processes. Following this theory, meanings of objects can be described as “social products, as creations that are formed in and through the defining activities of people as they interact” (Blumer, 1986, p. 5). The personal interpretations made by individuals within the collective negotiation process can be understood as an ever-changing “internalized social process” (Blumer, 1986, p. 5). Meanings of objects, and in particular those acquiring trans-situational validity, are thus neither contained in the objects nor in the subjects per se but are constructed in an intersubjective space. According to that, this also applies to mathematical meanings. Thus, for example the cube only becomes a cube when the persons involved in the interaction mutually negotiate its meaning as a cube with specific attributes and, with a trans-situational designation, label it ‘cube’ for better understanding.

In subject-specific – here, mathematical – interaction processes, the individual is ideally given the opportunity to participate in collective constructions of meaning that exceed his or her own abilities to construct meaning and thus represent the basis for individual learning. In concrete terms, this interplay of inside and outside can be described as follows: At the beginning of an interaction, the participants outline initial, provisional interpretations of the situation in which they find themselves, based on their individual experiences and knowledge. From an interactionist perspective, these individual definitions of a situation take place in anticipation of other participants' possible interpretation, and in adaptation to other interpretations emerging in the interaction within the process of collective negotiations of meaning. The mutual alignment of individual definitions of a situation between the participants can lead to the production of "taken-as-shared meaning" (Voigt, 1995, p. 172) – an interim of coordinated definitions of the situation (ICDS). The ICDS constitutes a 'stimulus potential' for the individual's trans-situational cognitive processes of construction and restructuring. Through repeatedly negotiated ICDS in interactions, the participants' definitions of situations change and solidify. The individual's trans-situational cognitive constructions are referred to as framings. This leads to the insight that mathematical learning is the trans-situational redesign or modification of framings with regard to mathematical contents or ways of argumentation (Krummheuer, 1992; Jung & Schütte, 2018). The levels of cognition and interaction seem to blur at this point since individual trans-situational cognitive processes of construction and restructuring are inevitably reciprocally linked to interactive collective mathematical negotiation processes (see also, for example, the concept of the Commognitive Framework of Sfard (2008)).

THE TWO COMPONENTS OF MATHEMATICS LEARNING

To help unpack the concept "mathematics learning", we use Bruner's (1983) learning theory of early native language acquisition. According to Bruner, successful native language acquisition depends on the existence of a "Language Acquisition Support System" (LASS). This refers to the full extent of dialogical activities taking place between the individuals with advanced skills in the interaction and the child. Bruner writes that, within the LASS, the learners' language acquisition takes place not only in the sense of "cracking […] a linguistic code" (Bruner, 1983, p. 14), but also in the more comprehensive sense of getting along with "the demands of the culture" (Bruner, 1983, p. 103). Applying this thought to early mathematical learning processes, two components of mathematics learning can be distinguished:

- The acquisition of mathematical concepts within mathematical content areas and related mathematical procedures;
- The development of mathematical thinking in the sense of being introduced to specifically mathematical negotiation processes – the mathematical culture of argumentation.

Because of the logical-argumentative structuring of mathematics, mathematics learning always represents a link between these two components of early mathematical development processes. Thus, in addition to the acquisition of mathematical knowledge via concepts and procedures, the learning of the argumentative structure of mathematics – that is, the approaches to argumentation linked to this mathematical content – gains particular significance. This second component of mathematics learning is henceforth described as the development of mathematical thinking.

THE DEVELOPMENT OF MATHEMATICAL THINKING – PARTICIPATION IN DISCOURSES

Based on the above considerations, learning how to argue – that is, acquiring the ability to 'feel at home' with negotiating the logical-argumentative and conceptual structure of mathematics – gains central significance for mathematics learning (Sfard, 2008). Drawing on interactionist approaches to mathematics learning, Krummheuer (1995) uses two models to describe mathematics learning – learning through participation in collective argumentation, and learning with formats. By collective argumentation, we understand the collaborative production of argumentation by multiple participants in an interaction. Thus, participants bring arguments into interactional processes and attempt, through negotiation of these arguments, to collaboratively develop an argument with a conclusion that is mutually accepted. According to Miller (2006), through the exchange with others and the collaborative search for collectively shared 'solutions' to problems, the individual systematically gains opportunities to exceed his or her particular 'limited' abilities. The second model relates to Bruner's (1983) concept mentioned above and is based on the idea of learning taking place in interactions of a group of participants with advanced technical skills. Bruner empirically reconstructs LASS formats as emerging patterns of interaction that repeat the same structure, in which the roles of child and adult shift over time, with the child acting increasingly autonomously in the interaction. Krummheuer (2011, p. 33) adapts the idea of formats to develop the concept of the "mathematics learning support system" (MLSS). This asserts that a child is not genetically predisposed to experience mathematics directly, and is not immediately able to ascribe mathematical meaning to, e.g., objects, concepts, or actions. Rather, the child needs a cognitively challenging social support system – the MLSS – in which questions are asked, hypotheses posed and tested, ideas argued, etc. This will allow him or her to develop a mathematical perspective step by step. These two models of the socially constituted conditions of learning are brought together in the concept of the "argumentation format" (Krummheuer, 1995, p. 253). Argumentation formats are to be understood as specific, argumentative kinds of formats which promote learning; they are specific patterns of interaction in which autonomy tends to shift towards the individuals less advanced in the interaction. Specific discourses around mathematical content can be understood as interactive realizations of an MLSS, produced between participants within one or several "interconnected" collective argumentations. The explicit elaboration of the discursive character of such collective processes of argumentation points towards a promising, theoretically consistent extension of existing theoretical approaches. It appears possible not only to describe how a child participates with increasing autonomy in such processes but also to reconstruct the argumentative structuring in which such participation unfolds – that is, ultimately, to reconstruct the style of rationalization practice that becomes established.

For instance, we will particularly examine how the objects of negotiation within the analyzed argumentations are determined and thematised, how the argumentation process is structured in its temporal or logical structure, whether rationales of participants have rather universal or particular validity, and to what extent acting subjects play a role within the emerging mathematical conversations. Therefore, we characterize the emerging negotiation processes as discourses with specific argumentative structuring, which we then call 'discourse style'.

According to this, we pursue the following research question: Which discursive structures can be reconstructed in collective argumentations during early mathematics learning? To answer this question, video recordings of play and discovery environments from the erStMaL project (early Steps in Mathematics Learning) of the Frankfurt IDeA Center were used as empirical data (for more information on the project, see Brandt et al., 2011). As part of the longitudinal study, children aged three to nine years were working in different group settings (two to four children) together with the project staff on mathematical play and discovery environments addressing all relevant mathematical content areas of early mathematics education (for a description of the content areas, see Clements & Sarama, 2007). The methodological procedure of analyzing the obtained data can be located in the field of interactionist works of interpretative classroom research in mathematics education (cf. Krummheuer & Naujok, 1999; Schütte et al., 2019; Voigt 1995). In analyzing the interactions, we are guided by a reconstructive-interpretive methodology and by a central element of the Grounded Theory research style – the methodological approach of comparative analysis (cf. Glaser & Strauss, 1967). The selected episodes were evaluated by means of interactional analysis (cf. Schütte et al., 2019).

RECONSTRUCTION OF DISCURSIVE STRUCTURES DURING MATHEMATICS LEARNING IN EARLY COLLECTIVE ARGUMENTATIONS

Our analyses of interactions in mathematical play and discovery environments lead us to the description of three different types of mathematical discourses (for a detailed account of the analyses results including illustrative empirical examples, see Schütte et al., 2021). On the one hand, there are discourses resembling narrations and focusing on personal experiences and intuitive explanations. And on the other hand, we find discourses with few references to personal experiences, which instead focus on object-related logical explanations. To reconstruct the argumentative structuring of the emerging mathematical discourses, we here apply the narrative principle of Bruner's (1996) psycho-cultural approach to learning and thinking. According to Bruner, two different diametral modes of thinking can be distinguished: narrative and logical-scientific thinking. Narrative thinking is characterized by the fact that people strive to think in terms of stories. This concerns the creation of context and meaning, and the ability to locate oneself in the world. Explanations provided by narrative thinking involve intuitive and specific frameworks. Logical-scientific thinking, on the other hand, focuses on physical objects. It concerns the ascertainment of facts, general rules, and laws of the world. Explanations provided by logical-scientific thinking are decontextualized and tend to focus on universal laws. Each kind of thinking, according to Bruner (1996), provides a different means of understanding the world. If the development of thinking can be described in terms of the development of different mathematical discourses, then, following Bruner (1996), it should also be possible to reconstruct narrative and logical-scientific (formal) levels of style in everyday negotiation of mathematical themes in early childhood development. Henceforth we will refer to these as narrative or formal discourses. By narrative discourse, we understand interrelating, sequentially structured sections of conversations, in which the participants produce collective argumentations which, taken together, produce a story, or at least resemble one in form due to their narrative character. Explanations emerging within this narration are strongly constrained by the context of the story.

According to our analyses, a narrative discourse is characterized by the following argumentative structuring:

- Everyday experiences are central to the negotiation process;
- the discourse is structured temporally-sequentially (and then... and then...);
- a sequential dramaturgy of a story or parts of a story unfolds in a narrative form;

- the objects' characteristic attributes are determined by the core of the story;
- particular, intuitive rationales are used or developed, which are based on experiential backgrounds in personal relation to the emergent 'story';
- there are acting subjects.

In contrast, in a formal discourse, the focus of negotiation is on the mathematical objects and the relations between them, without referring to their concrete, everyday aspects. A formal discourse is, according to our analyses, characterized by the following argumentative structuring:

- Relations between objects and characteristic attributes of objects are central to the negotiation process;
- the discourse is structured contentually-relationally and therefore also logically-sequentially[2] (e.g.: if..., then...);
- the objects' characteristic attributes are determined by a formal logic;
- universal rationales are used or developed.

Contrary to the statement of Bruner (1996), who dualistically separates logical-scientific thinking and narrative thinking, there are also discourses that question our theoretical subsumption relating to Bruner (1996) and cannot easily be classified into one of the two forms of discourse. A discourse style implicitly referring to a story, but not explicitly elaborating on it, and thus also containing aspects of a formal discourse since the plot of the story leads to specific formal relations among the objects, could be called quasi-narrative or quasi-formal discourse. On an empirical level, the existence of a discourse hybrid that is in a state of flux and thus cannot be definitely standardised is revealed. In the following, we will call this discourse hybrid narratory discourse[3]. According to our analyses, a narratory discourse is characterized by the following argumentative structuring:

- Participants continue to draw on stories experienced by themselves – the focus of the negotiation process is on the objects and the experiences associated with them;
- the experiences and the objects' characteristic attributes are no longer thematised in narrative form;
- the discourse is unspecifically structured: partly contentually-relationally or logically-sequentially, partly temporally-sequentially;
- the meaning of the sequentiality of what has been experienced is broken up in moments of contentual-relational or logical-sequential structure;
- particular, intuitive rationales and universal rationales are used or developed.

PROSPECTS: MATHEMATICS LEARNING AT THE INTERFACE OF DISCOURSES

In the reconstructions of interaction processes from episodes of play and discovery environments, it becomes apparent that purely narrative and purely formal discourses can be understood as theoretical endpoints of a continuum of teaching-learning arrangements, but do not necessarily appear in empirical situations in a distinctly separable way. The reconstructed mathematical discourses in the analyzed

[2] We speak of 'logical-sequential' to distinguish the characteristic of a formal discourse with logical implications from the characteristic of the temporal sequentiality of a narration, as, for example, in the following statement: "If the sum of the digits of a natural number n is divisible by three, then n is also divisible by three." This logical-sequential inference does not imply a before and after, as, for example in the following sentence: "Peter wakes up, then goes into the bathroom and then brushes his teeth.", which corresponds to a temporal-sequential structuring of a narrative discourse.

[3] Here, the term 'narratory' is used in its meaning as 'pertaining to a story/narrative'.

situations frequently move back and forth between the two forms of discourse; and narratory discourses also emerge. The torus depicted in the following graphic is intended to illustrate how the three discourse styles formal, narrative, and narratory blend into each other (see Figure 1).

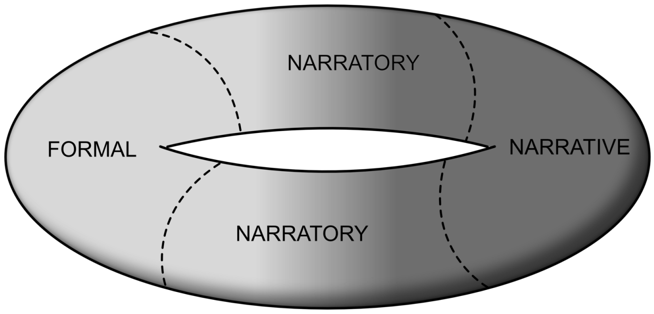

Fig. 1: The three discourse styles: narrative, narratory, and formal

The reconstruction of different discourse styles represents an important step on the way to the formation of a theory of development of mathematical thinking from an interactionist perspective. With regard to an increasingly empirically-based theory development, answering the focused research question creates the basis for being able to systematically describe the development of children's mathematical-argumentative abilities longitudinally in subsequent analyses. For further research steps, the question arises which forms of different individual developmental logics of mathematical thinking, indicated by a changing participation in mathematical discourses, can be reconstructed in children from pre-school to primary school age. Longitudinal comparative analyses give, for example, rise to the assumption that a developmental logic – which initially seems obvious – of an increasingly autonomous participation in narrative discourses towards an intensified participation in formal discourses does not seem the only way of a progressing development of children's mathematical thinking. To answer these further questions, a modified theoretical construct is required which allows to describe the longitudinal changes in children's participation, and to reconstruct the course of the development of their mathematical thinking from this. First approaches could be offered by the concept of the interactional niche in the development of mathematical thinking, which has already been devised for this purpose (cf. Krummheuer, 2011, 2012).

Without taking the reflections on the development of a theory of mathematical thinking any further in detail, a preliminary subsumption in terms of learning theory can be made based on the underlying examination's analyses that have already been conducted. Narrative discourses are mathematical discourses with a low access threshold for many children, in which negotiations about mathematical contents or problems take place, albeit sometimes hidden (for this purpose, see also the remarks on implicit learning in Vogler (2020)). Illustrative of further analyses we have already conducted, this observation highlights the importance of narrative discourses for everybody's collective learning. Examining the narrative discourse, however, it is also noticeable that the didactic approach of gently introducing children to the world of mathematics with a story close to everyday life can also make it difficult for some children to adopt or maintain a logical-mathematical perspective. In this way, for some children, a narrative approach will also possibly lead to a concealed access to the logical-argumentative structure of mathematics, which could eventuate in the limited possibility of further developing their existing formal abstract abilities. This, nevertheless, is not to be understood as a plea for banning references to reality from mathematical teaching-learning arrangements. For many children, an access to mathematics which is close to reality with the possibility of connecting to everyday experiences certainly represents a proper access to mathematics. For mathematics learning in groups with diverse learners, diverse approaches should be created. In this, we do not fundamentally assume that negotiations in formal discourses represent the goal of mathematical teaching-learning arrangements. With regard to later child development, both discourse styles might have different but not necessarily less important functions in mathematics learning. Our analyses also reveal that within a play and discovery environment, participants switch from one form of discourse to another in different ways. In many cases, the change took place suddenly due to the initiation of the project staff, for example, when she or he prevented the children's initial approaches to a task and introduced a story to explain the problem. To us, such abrupt changes do not seem very conducive to learning since learners are hardly given the possibility to build up their ability to participate in the interface of narrative and formal discourse.

This leads on to the consideration of the narratory discourse. For the goal of basic mathematical education, it seems reasonable that as many children as possible are given the opportunity to acquire competencies in mathematics classes in order to learn to successfully participate in narrative and formal discourses (cf. Barwell's (2016) approach to informal and formal mathematical discourses). In this respect, one could assume that the learning-theoretical significance of the narratory discourse might ostensibly lie in the possibility for a collective participation of different children in a mathematical discourse. However, if we take a closer look at the emerging narratory discourse, it seems challenging for all participants to obtain an ICDS at the interface of rationales arising from everyday experiences and formal logical rationales. This observation is contrary to the assumption of a facilitated opportunity for participation for all learners. Despite, or precisely because of it, we ascribe a fundamental developmental-theoretical function to the narratory discourse, which – admittedly – must first prove itself in further longitudinal observation Following this, we see a specific feature of optimized early mathematical negotiation processes in the hybrid function of the narratory discourse at the interface of what can be experienced in reality, on the one hand, and the description of what is detached from these contexts on the other. Seen from this angle, a narratory discourse might actually not be freer of access, but rather sets higher demands on children. In this way, a narratory discourse however also offers a special possibility for the development of specific argumentative skills – that is, the possibility of evoking modifications and redesigns of argumentative framings on the basis of emerging and negotiable framing differences between the participants on a larger scale than seems to be the case in narrative or formal discourses. Consequently, the interactional conditions for the possibility of the development of mathematical thinking are primarily given in the narratory discourse.

References

Bauersfeld, H., Krummheuer, G., & Voigt, J. (1988). Interactional theory of learning and teaching mathematics and related microethnographical studies. In H. G. Steiner, & A. Vermandel (Eds.), *Foundations and Methodology of the Discipline Mathematics Education* (pp. 174–188). University of Antwerp.

Barwell, R. (2016). Formal and informal mathematical discourses: Bakhtin and Vygotsky, dialogue and dialectic. *Educational Studies in Mathematics, 92*, 331–345.

Blumer, H. (1986). *Symbolic interactionism: perspective and method*. Prentice-Hall.

Brandt, B., Vogel, R., & Krummheuer, G. (2011). *Mathematikdidaktische Forschung am „Center for Individual Development and Adaptive Education* [Mathematics didactic research at the “Center for Individual Development and Adaptive Education”] (Vol. 1). Waxmann.

Bruner, J. (1983). *Child’s talk. Learning to Use Language*. Norton.

Bruner, J. (1996). *The culture of education.* Harvard University Press.

Clements, D. H., & Sarama, J. (2007). Early childhood mathematics learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 461–555). Information Age Publishing.

Cobb, P., & Bauersfeld, H. (1995). Introduction: The coordination of psychological and sociological perspectives in mathematics education. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning. Interaction in classroom cultures* (pp. 1–16). Lawrence Erlbaum.

Glaser, B. G. & Strauss, A. (1967). *The Discovery of Grounded Theory. Strategies for Qualitative Research*. Aldine Publishing.

Jung, J., & Schütte, M. (2018). An interactionist perspective on mathematics learning: conditions of learning opportunities in mixed-ability groups within linguistic negotiation processes. *ZDM Mathematics Education, 50* (6), 1098-1099.

Krummheuer, G. (1992). *Lernen mit „Format“. Elemente einer interaktionistischen Lerntheorie* [Learning with "format". Elements of an interactionist learning theory]. Deutscher Studien Verlag.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning. Interaction in classroom cultures* (pp. 229–269). Lawrence Erlbaum.

Krummheuer, G. (2011). Die empirisch begründete Herleitung des Begriffs der „Interaktionalen Nische mathematischer Denkentwicklung“ [The empirically based derivation of the term "interactional niche of the development of mathematical thinking"]. In B. Brandt, R. Vogel, & G. Krummheuer (Eds.), *Mathematikdidaktische Forschung am "Center for Individual Development and Adaptive Education"* (pp. 25-89). Waxmann.

Krummheuer, G. (2012). The “Non-canonical” Solution and the “Improvisation” as Conditions for Early Years Mathematics Learning Processes: The Concept of the “Interactional Niche in the Development of Mathematical Thinking” (NMT), *Journal für Mathematik-Didaktik, 33* (2), 317–338.

Krummheuer, G., & Naujok, N. (1999). *Grundlagen und Beispiele Interpretativer Unterrichtsforschung* [Basics and examples of interpretative research in teaching]. Leske + Budrich.

Miller, M. (2006). *Dissens. Zur Theorie diskursiven und systematischen Lernens* [Dissent. On the theory of discursive and systematic learning]. transcript Verlag.

Schütte, M., Jung, J., & Krummheuer, G. (2021). Diskurse als Ort der mathematischen Denkentwicklung - Eine interaktionistische Perspektive [Discourses as the place for the development of mathematical thinking - An interactionist perspective]. *Journal für Mathematik-Didaktik.* https://doi.org/10.1007/s13138-021-00183-6

Schütte, M., Friesen, R. A., & Jung, J. (2019). Interactional analysis. A method for analysing mathematical learning processes in interactions. In G. Kaiser, & N. Presmeg (Eds.), *Compendium for Early Career Researchers in Mathematics Education. ICME-13 Monograph* (pp. 101-129). Springer.

Sfard, A. (2008). *Thinking as communicating. Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

Voigt, J. (1995). Thematic patterns of interaction and sociomathematical norms. In P. Cobb, & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: interaction in classroom cultures* (pp. 163-201)..: Lawrence Erlbaum.

Vogler, A. M. (2020). *Mathematiklernen im Kindergarten* [Learning math in kindergarten]. Waxmann.

THE EFFECTS OF USING A MODIFIED FRAYER MODEL TO TEACH MATHEMATICS VOCABULARY TO JUNIOR-FORM ENGLISH LEARNERS IN A CHINESE MEDIUM-OF-INSTRUCTION SECONDARY SCHOOL

LI Wing-kwan and Simon S. Y. CHAN
The University of Hong Kong

Since the implementation of the fine-tuned medium-of-instruction policy (MOI) in 2009, Language Across the Curriculum (LAC) has been operated in Hong Kong secondary schools. Most Chinese-medium-of-instruction (CMI) schools adopt LAC at word level for non-language subjects. However, relatively little is known about how graphic organisers can help CMI students improve their academic vocabulary. The present study was set in a CMI school in which the Frayer Model was modified according to the multi-semiotic nature of Mathematics language to help thirty-three junior-form students to learn Mathematics vocabulary more effectively. The findings show that they had positive perception of the model. Meanwhile the study has shed insight into how the model impacts on students' learning of other subject-specific vocabulary.

BACKGROUND

The fine-tuned Medium of Instruction (MOI) policy has been implemented in Hong Kong secondary schools since 2009. Under the policy, schools can choose to teach the subjects in English by class or by sessions according to students' ability and teachers' English proficiency (Education Bureau, 2009). To support MOI arrangements, most Chinese-medium-of-instruction (CMI) schools adopt Language Across the Curriculum (LAC) across non-language subjects such as Mathematics in junior forms. Most schools implement LAC at word level by providing a glossary of vocabulary items in English and their Chinese meaning. However, such practice may not be empirically supported.

RESEARCH SETTING

This study was set in a CMI school which adopts the word-level LAC arrangement. Mathematics is taught in Chinese as well as one of the subjects which provide subject-specific vocabulary in English. Students learn Mathematics concepts in Chinese. Without explaining Mathematics vocabulary in detail, Mathematics teachers ask them to read aloud each item in English occasionally and copy both English and Chinese terms (e.g., radius (半徑)) for getting familiar with the vocabulary. Without being taught strategies to retain the vocabulary, most students are left to struggle by rote-learning it.

This study aimed to explore the impact of using a modified Frayer Model on learning Mathematics vocabulary and to investigate which components in the model work best for recalling the vocabulary.

MODIFIED FRAYER MODEL

To support Mathematics vocabulary instruction, graphic organisers are suggested (Bruun Diaz & Dykes, 2015). One common graphic organiser used in Mathematics is Frayer Model which helps English language learners "think deeply, determine relationships, and connect new concepts and words to what they already know" (Dunston & Tyminski, 2013, p.41). This model can reduce the linguistic demand on students (Smith & Angotti, 2012) who only need to “visually convey meaning without using complex language or complicated sentence structure” (Dunston & Tyminski, 2013, p.41). As it works like mapping of ideas, it “helps to activate and expand prior knowledge; it also helps students learn new words" (Johnson & Johnson, 1986, p. 625).

The Frayer Model (Figure 1) was modified by making connections between the Mathematics terms and the four components: Mathematics symbols/formulas, pictures/diagrams, sample sentence and related concepts/terms. Moreover, to reduce the participants’ linguistic demand, they could interpret the Mathematics terms using pictures/diagrams while extra related terms were provided by the researcher. To avoid decontextualisation, sample sentences with the Mathematics terms were given.

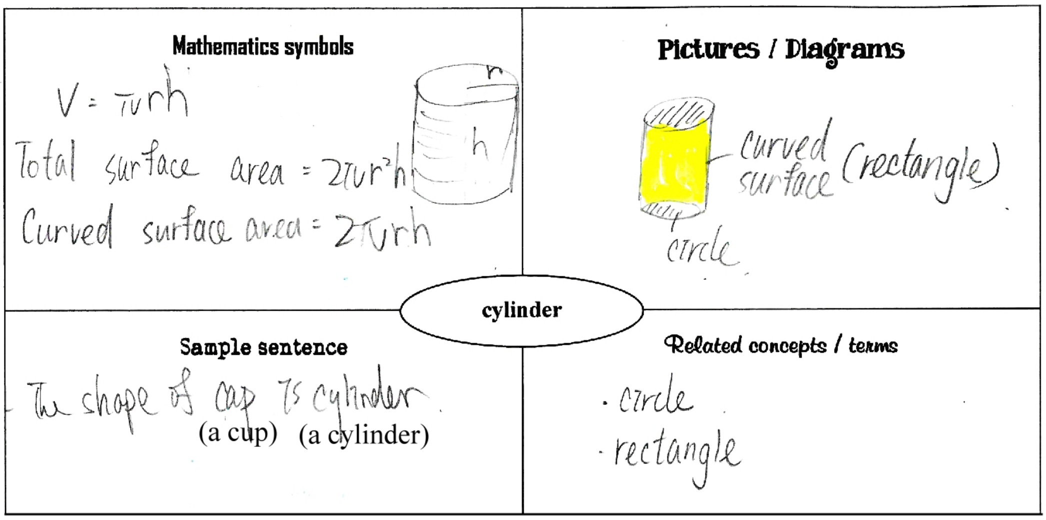

Figure 1: Modified Frayer Model

RESEARCH QUESTIONS

How do students perceive the modified Frayer Model when learning Mathematics vocabulary?

What components do students like most and least in the model, and what other components can be added to make the model more effective for learning Mathematics vocabulary?

METHODOLOGY

A two-week intervention with a total of ten sessions was conducted by the research with the thirty-three participants from a Secondary 2 class during lunch time. In each session which lasted for thirty to fifty minutes, the participants were taught one to two Mathematics vocabulary items by the researcher. After the intervention, the participants completed a questionnaire and a post-intervention quiz in the last session. Afterwards, six of them were invited to have a focus group interview.

MAJOR FINDINGS AND DISCUSSION

Statements	4 Strongly agree	3 Agree	2 Disagree	1 Strongly disagree	Mean	SD
The Frayer Model helped me memorise the mathematics vocabulary better.	42.42% (N=14)	54.55% (N=18)	3.03% (N=1)	0% (N=0)	3.39	0.55

The Frayer Model helped me learn extra Mathematics vocabulary.	33.3% (N=11)	63.6% (N=21)	3.0% (N=1)	0% (N=0)	3.30	0.52
I was able to learn Mathematics vocabulary better when it was presented in multiple ways such as pictures, Maths symbols and sample sentences.	69.7% (N=23)	27.3% (N=9)	3.0% (N=1)	0% (N=0)	3.67	0.53
The Frayer Model helped me relate one Mathematics vocabulary item to more vocabulary items.	36.36% (N=12)	60.61% (N=20)	3.03% (N=1)	0% (N=0)	3.33	0.53

Table 1: Major results of the questionnaire (Total N = 33 participants)

The post-intervention quiz assessed the participants' performance on learning the mathematics terms taught at word and sentence concept. Similar to their assessment method in school, Part A was a matching task in which they matched the mathematics terms with their Chinese translation. In Part B, they labelled a circle (Question 1) and a right-angled triangle (Question 3), and matched the shapes with their names (Question 2). In Part C, they completed the statements with the right Mathematics terms. Word boxes were given in Parts B and C so as to reduce spelling pressure on them.

	Part A	Part B (Question 1)	(Question 2)	(Question 3)	Part C
Full marks	10	6	8	4	6
Mean	8.24	5.00	4.36	2.91	3.94

Table 2: Results of Part A to Part C of the post-intervention quiz (N = 33 participants)

From the results of the questionnaire and the post-intervention quiz, the participants' good perception could be attributed to two major reasons.

Better retention of Mathematics vocabulary

According to Coggins et al. (2007), Mathematics is "wordy, abstract, and defined by symbols" (p.69). The modified Frayer Model satisfies this description by including (related) Mathematics words, concepts and symbols. One advantage of the intervention was the vocabulary instruction given by the researcher. She not only explained the relationships between the Mathematics terms and their components in English with occasional Cantonese, but also drew the participants' attention to the pronunciation of every term. Such explicit instruction was highly valued by the participants who are still struggling English and Mathematics learners. Moreover, the model could suit visual learners who could refer to the pictures / diagrams component if they forgot the meaning of the Mathematics terms.

Gains in Mathematics vocabulary and knowledge

According to Schleppegrell (2007), Mathematics "draws on multiple semiotic (meaning creating) systems to construct knowledge: symbols, oral language, written language, and visual representations" (p.141). These systems can enhance conceptual understanding of Mathematics (Anthony & Walshaw, 2009) through making associations among the four components in the modified Frayer Model. For example, under the topic of "areas and volumes", the participants used the modified Frayer Model to explore the links between different shapes (e.g., circles), formulas (e.g., calculating the circumference), and related terms (e.g., diameter and pi) with symbols (e.g., π). Moreover, most Mathematics terms taught in the intervention are already a Mathematics concept. That means when the participants studied a Mathematics term such as sine ratio, they not only learnt the term, but also its related vocabulary, formula and picture. In this way, the model could possibly expand the participants' pool of Mathematics vocabulary and deepen their understanding of concepts.

CONCLUSION AND RECOMMENDATIONS

This study has shown that that the participants perceived the modified Frayer Model as a better tool for learning and recalling Mathematics terms. These students may not have opportunities to apply Mathematics vocabulary in class or in their daily life, and to solve word problems in English. The model has raised their awareness of the terms, and the relationships between the terms and the four components, and expanded their Mathematics vocabulary and knowledge.

Mathematics vocabulary items should be treated as learning "tools rather than as facts to be memorised" (Nagy & Townsend, 2012, p.101). Therefore, Mathematics and LAC teachers can implement meaningful vocabulary instruction within CMI contexts with the use of the Frayer Model. Meanwhile, the instruction must have clear learning objectives which can range from helping students maintain Mathematics word meanings over time, making them recognise the newly and previously learned words to enabling them to be independent academic vocabulary learners.

References

Anthony G., & Walshaw, M. (2009). Characteristics of Effective Teaching of Mathematics: A View from the West. *Journal of Mathematics Education, 2(2)*, 147–164.

Bruun, F., Diaz, J. M., & Dykes, V. J. (2015). The language of mathematics. *Teaching Children Mathematics, 21,* 530–536. doi:10.5951/teacchimath.21.9.0530

Coggins, D., Kravin, D., Dávila Coates, G. & Dreux Caroll, M. (2007). *English Language Learners in the Mathematics Classroom.* SAGE Publications Inc.

Dunston, P., & Tyminski, A. (2013). What's the big deal about vocabulary? *Mathematics Teaching in the Middle School, 19,* 38–45. doi:10.5951/mathteacmiddscho.19.1.0038

Education Bureau (2009). *Fine-tuning of Medium of Instruction (MOI) for Secondary Schools.* http://www.edb.gov.hk/en/edu-system/primary-secondary/applicable-to-secondary/moi/support-and-resources-for-moi-policy/lsplmfs-sch/d-sch/ow/sp/index-2.html

Johnson, D. D., & Johnson, B. (1986). Highlighting vocabulary in inferential comprehension instruction. *Journal of Reading, 29,* 622–625.

Nagy, W., & Townsend, D. (2012). Words as tools Learning academic vocabulary as language acquisition. *Reading Research Quarterly*, 47, 91–108.

Schleppegrell, M. (2007). The Linguistic Challenges of Mathematics Teaching and Learning: A Research Review. *Reading & Writing Quarterly, 23*(2), 139–159.

RECONSTRUCTING DISSENT-CONSENSUS SITUATIONAL STRUCTURES IN COOPERATIVE MATHEMATICS AND COMPUTER SCIENCE LEARNING ENVIRONMENTS

Peter Ludes-Adamy and Marcus Schütte
Leibniz-Universität Hannover

Learning theories in general ask questions about how pupils – or people in general – learn. But exactly how and under which circumstances does the learning of the fundamentally new occur? To focus on this question, learning environments with core topics of mathematics and computer science are examined. Today, computer science is not a topic that is taught in primary schools in Germany. In the focus of digitalization, this topic and its connection to mathematics will play an important role in future curricula, making it an interesting object of investigation. The paper presents an ongoing study that examines, how the topic of computer science connected to mathematics learning can be approached in primary schools and what and how meanings are negotiated. The focus will be on the question, what roles consensus and dissent play in interactional processes of negotiation and how the learning of the fundamentally (Miller, 1986) new occurs in collective argumentation between pupils.

MATHEMATICS LEARNING AND THE LEARNING OF THE FUNDAMENTDALLY NEW

The understanding of learning is an interactionist one. Miller (1986) developed a sociological theory to distinguish himself from psychological approaches to the term learning. He focuses on collective learning processes of at least two individuals. Fundamental mathematical learning occurs in collective negotiation processes (Schütte, 2009). Based on a theory developed by Krummheuer and Brandt (Krummheuer, 2007) learning in mathematics is seen as an increasingly autonomous participation in collective argumentation processes. Hitherto, these theories focus on mathematics learning. To extend the theories a topic with a close relationship to mathematics is chosen but which is fundamentally new to primary school children: computer science. Mathematics can be connected to computer science on two levels: the content level, as almost all mathematics content is relevant for computer science, and the competence level. Especially the content-related mathematical competences can be linked to competences for computer science education in many ways (Ludes-Adamy & Schütte, 2019). The general idea is to transfer the findings in mathematics education to the learning of the - on primary level - fundamentally new but closely related topic of computer science and later on reconnect these new ideas to the learning of mathematics to possibly pave new ways of learning mathematics content.

STRUCTURES OF COOPERATION

According to Johnson & Johnson (1999) there are three organizational types of lessons called individualistic, competitive and cooperative. The first two types place the individual in the center of action, where no or negative interdependence occurs. The last structure, the cooperative one, follows the idea that pupils cooperate in a way that the group as well as the individual are responsible for the successful handling of the task, which is called positive interdependence, as the different members have to rely on each other's individual work. When working competitively and/or cooperatively different consensus and dissent situations arise on the basis of interpretational differences. On the basis of these, new mathematical meaning can be negotiated. The research question that will now be focused is how dissent-consensus situations are structured when primary school pupils learn a subjectively new topic (Schütte, 2014) and negotiate and construct computer science meaning collectively.

METHODOLOGY

Learning environments in the intersection of mathematics and computer science with the topics logic, algorithms, cryptography, programming, binary code have been designed. The tasks themselves are designed closely to the concept of natural differentiation (Krauthausen & Scherer, 2014). The children work on the tasks in groups of four. The data for the analyses originates from two pilot studies that have been run in small groups (11 and 13 pupils) in grade 3 and 4 and two main studies that took place with a larger number of pupils (48 and 47) in grade 4. The is transcribed and analyzed using methods of interpretive classroom research, primarily, interactional analysis (Schütte et al., 2019).

LOGIC – TASK AND ANALYSIS

Situations from computer science/mathematics learning environments have been described (Ludes-Adamy & Schütte, 2018), where a dissent-consensus situational structure emerge in which the collectively originated dissent transforms into a collectively negotiated consensus to construct situational-dependent new mathematical meaning (Krummheuer, 2015). The question now arises, whether this is the only structure that can be identified. The following examples will show other situations that indicate other situational structures as well. The task is a logical structure, with different statements that lead to logical conclusions. The 10 statements are as follows: 1. All numbers of this puzzle are integers. 2. The number D is 9 times A. 3. The sum of G and J is 100. 4. Three times D is G. 5. The fifth part of the number E is J. 6. H is half of D. 7. B is double of D. 8. If I subtract 2 from the quotient of E and J, I receive C. 9. The digit sum of all numbers is F. 10. I is double of C. 11. D is a number of the nine times table and less than 30. (Solution: A=2; B=36; C=3; D=18; E=230; F=62; G=54; H=9; I=6; J=46).

12 Kai	well, it is best, it is best . actually it is best/ well it can be nine . actually it is best if we get D
13 Jonas	is it now twenty-seven or eighteen . D/..
14 Kai	because with D there are a lot of tasks
15 Jonas	yes but then we have to find out B first
16 Kai	do we want to do the A eh do we want to (5) do we want oh man that is difficult (12)
17 L.	what did you want to say/ do we want/ ..
18 Kai	here . well (imcomprehensible) . write down the eighteen for D
19 Jonas	(a moment) D/ . but it could be twenty-seven or nine as well
20 Kai	I mean to try out
21 Jonas	moment (9)
22 Kai	so do we want to write down eighteen for D/ .
23 Jonas	mhh let's try ..
24 Kai	so eighteen .. so and A ehh . no that doesn't work
25 Jonas	wait . a moment/ .

26 Kai	because when this is eighteen A must be two
27 Jonas	B is that . a moment
28 Kai	no look, when you look at the second
29 Jonas	H is half of .. D so it muss be fourteen
30 Kai	no .
31 Jonas	if it is eighteen . .
32 Kai	nine /
33 Jonas	so .
34 Kai	H is nine if it is half of eighteen
35 Jonas	moment yes the eighteen is right . otherwise this would be . ehm . the sixth task one would not be able to solve it
36 Kai	so eighteen is right/
37 Jonas	yes

Throughout the task, Kai and Jonas work closely together. Kai's first utterance shows a logical deductive conclusion as he proposes to find the number D, because it is used in many other statements. He seems to understand that the possibility to find a successor increases, if a number that is often used in other statements is known. Jonas does not explicitly agree, but he seems to accept it as a good suggestion as he immediately proposes two possible suggestions for D. The two pupils show an almost perfect cooperation, as they work together without dividing tasks among themselves or discussion their collaboration. The utterances of the pupils get a little bit confusing in the course of the discussion, but they never seem to lose their goal out of sight. Kai and Jonas cooperate and deduct logically that eighteen is the only possible solution for D. In the 2018 findings the students transformed a dissent into a consensus through the development of a new mathematical term. In this first example, a general consensus situational structures seems to prevail throughout the cooperation which is merely spiked with *micro-dissents.* In this situation the two pupils do not explicitly construct new knowledge in the realm of computer science, but they construct an optimized solution based on existing competences.

A further example that illustrates a third type of consensus/dissent situational structure is taken from another learning environment with the topic *Algorithms*. After learning what an algorithm is and how it is connected to computer science the students have the task to find algorithms in mathematics. Most groups decide to use long forms of calculation that always follow a specific pattern. Theo and Karl try to find an example to illustrate the long form of a calculation before writing down a general algorithm.

01 Theo	let's say . . 500 minus 55
02 Karl	this is not the long ways Theo, I can do it in my head . . . 445
03 L.	What is the long way/
04 Karl	The long way is when you have 345 dividied by 7 and then you write down this numbers and underline them and then write the equal sign
05 Karl	Now, I say 345 divided by 7 then it works.

Theo proposes an example that is perfectly suitable to write down the algorithm for a long form of calculation. Karl on the other hand thinks that only an example, where the long form of calculation is compulsively necessary (mentally), qualifies. He does not consider Theo's example. An implicit dissent is created that is not resolved in any manner but rather kept on a meta level. The basis for moving on is just Karl's imprinting his opinion as correct onto the situation. This would be what we would call a dissent situational structure.

Relating back to Johnson & Johnson (1999) different basic forms of cooperation can be identified that describe different positive interdependencies between pupils when working on the fundamentally new. Thus, these different interdependencies have varying influences on the learning opportunities of the new. Currently, the situations are examined to see, how these basic structures form patterns that can be found throughout the situations can be categorized into a descriptive System, to find out in what way dissent and consensus and their resolution can built the basis for the negotiation of computer science/mathematics meaning und through this provide a basis for learning opportunities of the fundamentally new within this domain.

References

Johnson, D., & Johnson, R. (1999). *Learning together and alone: cooperative, competitive, and individualistic learning*. Allyn and Bacon.

Krauthausen, G., & Scherer, P. (2014). *Natürliche Differenzierung im Mathematikunterricht. Konzepte und Praxisbeispiele aus der Grundschule. Materialübersicht.* Kallmeyer in Verbindung mit Klett.

Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom. Two episodes and related theoretical abductions. *Journal of Mathematical Behavior*, *26*(1), 60–82. https://doi.org/10.1016/j.jmathb.2007.02.001

Krummheuer, G. (2015). Methods for Reconstructing Processes of Argumentation and Participation in Primary Mathematics Classroom Interaction. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to Qualitative Research in Mathematics Education* (pp. 51–74). https://doi.org/10.1007/978-94-017-9181-6_3

Ludes-Adamy, P., & Schütte, M. (2018). Cooperative learning in mathematics and computer science learning environments. In N. Planas & M. Schütte (Eds.), *Proceedings of the IV ERME Topic Conference "Classroom-based research on mathematics and language" (pp. 103-109)*. Retrieved from https://hal.archives-ouvertes.fr/hal-01856531

Ludes-Adamy, P., & Schütte, M. (2019). Programmieren mit Grundschullehrkräften: Fortbildung zu digitalen Medien im Mathematikunterricht und Programmieren mit dem Calliope mini. In D. Walter & R. Rink (Eds.), *Digitale Medien in der Lehrerbildung Mathematik – Konzeptionelles und Beispiele für die Primarstufe* (pp. 239–264). WTM-Verlag.

Miller, M. (1986). *Kollektive Lernprozesse: Studien zur Grundlage einer soziologischen Lerntheorie*. Suhrkamp.

Schütte, M. (2009). *Sprache und Interaktion im Mathematikunterricht der Grundschule. Zur Problematik einer Impliziten Pädagogik für schulisches Lernen im Kontext sprachlich-kultureller Pluralität*. Waxmann.

Schütte, M. (2014). Language-related Learning of Mathematics. A Comparison of Kindergarten and Primary School as Places of Learning. *Zentralblatt Für Didaktik Der Mathematik*, *46*, 923–938.

Schütte, M., Friesen, R.-A., & Jung, J. (2019). Interactional analysis: A method for analysing mathematical learning processes in interactions. In G. Kaiser & N. Presmeg (Eds.), *Compendium for early career researchers in mathematics education* (pp. 101–129). Springer.

THE PRACTICE AND EXAMINATION OF OPPORTUNITIES TO TRANSLATE REPRESENTATION THROUGH PROBLEM-SOLVING

Kunihiko Shimizu
Bunkyo University

The purpose of this research was to practice lessons based on hypotheses and examine opportunities to translate representation through classroom teaching. The research hypothesis was that children will be proactive in their use of diverse mathematical representations when they have questions and explorative tasks in learning problem-solving. The classroom practice involved high school students' learning of the triangular ratio, in which the learning objective was to find the formula of the area when a pair of corresponding angles and the included side are given. Based on this classroom practice, the process of representation was translated by exploring student questions, while students' feelings concerning their approach to mathematics were also important. Further, as students' inquiries deepened, their representation gradually became more sophisticated, and in the process, along with trial and error, a process of returning to representing thought was seen.

INTRODUCTION

In Japan, we are focused on the growth of representation skills, because some Japanese children have weaker thinking and representation skills than children in other countries. Solving mathematical problems is an important means of representation for children. However, it isn't clear why children use diverse representation. The purpose of this research was to practice lessons based on hypotheses and examine opportunities to translate representation through classroom teaching.

The research method was as follows: (1) I set up a research hypothesis and taught a lesson based on the hypothesis. The lesson was recorded on video. (2) I analyzed the video. (3) I interviewed students using different forms of representation. The analysis viewpoint differs between situations that use and do not use representation. About the procedure of the data analysis, I watched classroom teaching and student interview itself by video and proposed based on trends in the data.

HYPOTHESIS EXAMINATION

Representation is a possibility represented individual belief and concept accessed through individual's word and graphical product (cf. Duval, 2006). Thus, representation include understanding etc. This study focuses on mathematical writing based on this viewpoint. Mathematical writing are graphical representation, linguistic representation, and symbolic representation (Nakahara, 1995). Translation of representation in this study is to convert from one representation to another. Ichikawa's (2010) study aims to ensure substantial learning for all children, in classes involving understanding, utilization, thinking, and representation (Figure 1). Exploring children's questions is important for the processes of proactive problem-solving and proactive representation. The role of the problem-solving process in developing the qualities and abilities of children in representation, enforced in the 2018 New National Curriculum, was clarified (Figure 2). During problem-solving (Figure 2), representation activities are enriched as each process is reviewed, explored, evaluated, and improved.

Figures 1 and 2 share a common process for finding problems and questions, pursuing trial and error, and representing the former as thoughts. Thought and representation are mutually beneficial. Therefore, it was important to ask the children questions. Previous research has not specified the cause of the translation of mathematical representation.

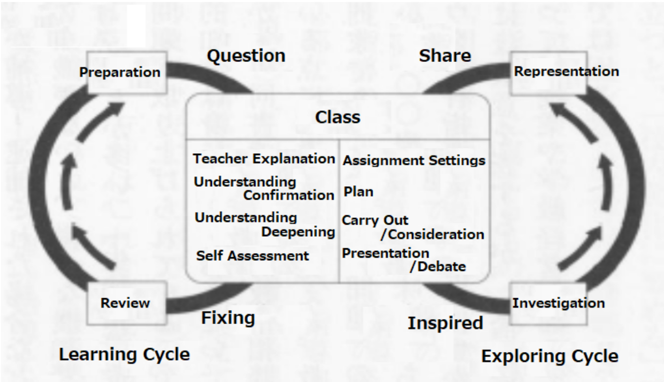

Figure 1: "Teaching and thinking lessons" as acquisition (Ichikawa, 2010, p.32)

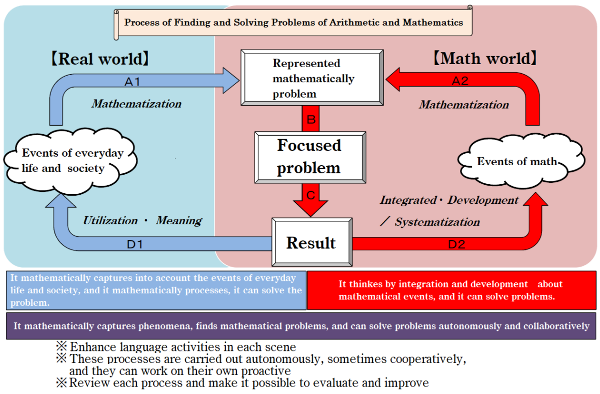

Figure 2: Process of finding and solving problems of arithmetic and mathematics (Ministry of Education, 2016, p.16)

The hypothesis of this research is that children, through exploratory learning and problem-solving, make proactive use of diverse mathematical representations. Exploratory learning is also dependent on teaching materials. This research examines exploratory learning based on the notion of the teacher as trigger and as children's questions increase, they lean toward exploration learning to answer them.

The following explains class to practice the hypothesis. In the Japanese educational content, the area of the triangular ratio when a pair of corresponding sides and the included angle are given is to be calculated. Depending on the Japanese textbook, students also learn Heron's formula as advancement content. However, Japanese textbooks do not describe and pick up the triangle's area when a pair of corresponding angles and the included side are given. But, in this case, a triangle is determined from the congruence condition of the triangle, so the area should exist. Therefore, in this case there is a formula. So, the problem in this class is as follows (Figure 3).

Problem

Represent the area of $\triangle ABC$ with a, B, C.

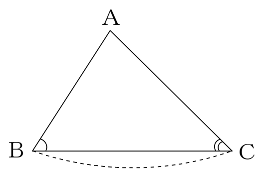

Figure 3. Problem content

STATE OF THE CLASS

Depending on the difficulty level of the assignment, there were students who proceeded with the problem-solving and students who did not progress. In the class, students were devoted to researching, exploring, and representing in various ways. The following are some statements by students (Table 1, Figures 4, 5, and 6).

Table 1. Conversation about Trial and Error in Collaborative and Explorative Activities of Students

SR: Is this different? (looking at MS's calculations)

MS: This answer seems to be leaving something out. The answer is 90% correct, but it is like we are forgetting something. I do not understand anything.

SR: Is this 180° correct at 90°? (silence)

MS: I see. （Silence, as MS looks at the textbook）

MS: Wrong. When sin is 180°, it remains sin. When sin is 90°, it feels like equaling cos something degrees.

SR: Hey. I assisted you.

MS: Oh, I understand! (in a cheerful voice)

MO: Oh. What do you mean?

MS: 180°-θ becomes sinθ because it becomes sin(B+C).

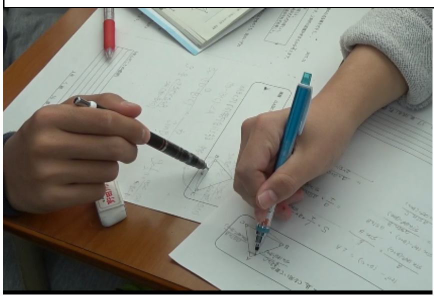

Figure 4: Explain by writing on the diagram.

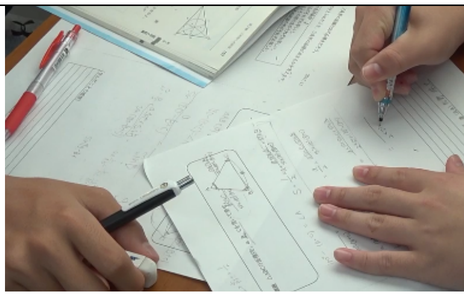

Figure 5: After the explanation, he said he understood. Write the trial and error results.

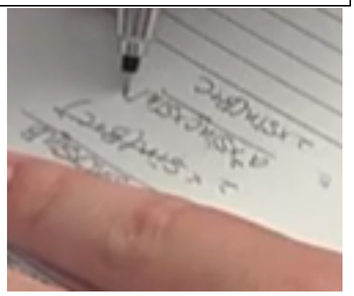

Figure 6: Rewrite with further indications

The students used diverse forms of representation during the exploration process, but this study focused on Student MS, who always attempted to solve his own questions. By exchanging thoughts and representations, learning deepened, and students helped one another create ideas that had not existed before and shared them with the other party, producing interaction.

Looking again at the protocol in Table 1, the student's question is resolved once Student MS says, "I understand!" Before and after saying "I understand!", Student MS uses diverse forms of representation, which is key. To clarify why Student MS says "I understand!" further analysis is conducted through the interview.

STUDENT INTERVIEW

The interview took place several weeks after class. After showing the video of the lesson to Student MS, I asked the question, "What was it you understood when you said, 'Oh, yes! I understand!'"? Student MS replied, "It was a textbook formula, finding that '180° -θ' equals 'θ' in a fluid way." After that, it is as follows.

Table 2. Interview with Student MS

T: How did you feel when you understood it?
MS: That I was able to solve it myself.
T: How do you feel about being able to solve it yourself?

MS: I was not thorough enough at that point.

T: What do you think now? Do you regret it? Wasn't your "Okay!" a bright voice?

MS: I was delighted to be able to solve it. There was more to solve, and I wanted to solve it myself.

T: Was this SR statement, "Is this 180° correct at 90°?" the key? After that, I began to look at the textbooks and was struggling. How did you feel during this part?

MS: For a time, I thought I couldn't do it in my head, so I decided to write by using my hand. MO and SMI drew a perpendicular line to solve the problem. When I imagined it in my head, I thought it would not be possible to solve if I can not replace it into a strange alphabet. But when I replaced it for now, the problem-solving proceeded well.

Student MS said, "I understand!" because he knew what he could and had to do: answer the questions "What happens to this? Is it better to make it simpler?" This resulted from a casual response that made the teacher feel students something was missing and led to further exploration. In the process, the solution was found thanks to Student SR's question "Is this 180° correct at 90°?", which was key. It pushed Student MS to solve the problem, and the answers to the questions gradually became clearer. I also think that Student MS enjoyed mathematics and the fun associated with exploration activities, with trial and error, and translating the representation method. In this way, it is thought that the translation of the representation method occurred because of the question and the inquiry to the question.

FINAL REMARKS

This study's analysis was based on lessons and an interview. The translation of the representation method occurred during the process of exploring student questions and their approach to mathematics. Students deepened their exploration activities, and the answers to questions became clearer. It is speculated that deepening of understanding is involved. In the beginning, their representations were not refined and organized, but in the end, students could see how their representations had gradually become more refined.

APPENDIX

This is a restructured paper presented at the 47th Research Presentation Meeting of the Japan Academic Society of Mathematics Education and the International Conference on Educational Research 2018. It was rewritten and restructured following the student interview by the researcher, and the video was analyzed again based on the guidance and advice of some researchers.

References

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(1-2), 103–131.

Ichikawa, S. (2010). Prospecting the lessons taught and thought, teaching and evaluation, 12, 32–35. Japan: Toshyobunkasya Publishing (in Japanese).

Ministry of Education. (2016). *Summary of deliberations in arithmetic and mathematics working group.* Ministry of Education: https://bit.ly/3AdsGeF

Nakahara, T. (1995). *A study of constructive approach in mathematical education*. Seibunsha.

Ooshima, T., Kakei, S., Kathou, H., Kawanaka, N., Enomoto, H., Oonishi, T., Kanono, T., Sakae, T., Tsuboi, T., Hattori, T., Hukaya, K., Matsunaga, M., Muneshige, T., & Yoshida, S. (2016). *Mathematics I* (Revised edition). Sukensyuppann Publishing.

HOW PRE-SERVICE PRIMARY TEACHERS ENGAGE IN LANGUAGE-RESPONSIVE MATHEMATICS TEACHING WHILE WORKING ON A SCRIPTWRITING TASK

Victoria Shure and Bettina Rösken-Winter
Humboldt-Universität zu Berlin, Germany.

Integrating academic language development into mathematics instruction is a challenge for most mathematics teachers. Such language-responsive mathematics teaching can be distinguished with respect to language demands on the word, sentence, or discourse level. Regarding the latter, practices encompass explaining meaning or reporting procedures. Utilizing the methodology of a scriptwriting task, we examine how pre-service teachers address language difficulties to support students in gaining mathematical reasoning competencies. Twelve pre-service primary teachers participated in the study, with six regarded as "low performers," having a score of sufficient on the course exam and six as "high performers," scoring excellent or very good. "High performers" were found to have a propensity for engaging students on the discourse level and "low performers" focused more on the word or sentence level. The outcomes of pre-service mathematics teachers' scripts are discussed with implications for future teacher education.

INTRODUCTION

The role of language in mathematics teaching and learning has gained a progressively greater focus in research, in particular concerning those learning the language of instruction in the mathematics classroom (Barwell, 2009; Chval et al., 2015; Moschkovich, 2005). The implementation of interventions for improved academic language development and second language instruction in all subject areas, and specifically in mathematics such that students improve their Cognitive Academic Language Proficiency (CALP) (Cummins, 2000), as well as their academic language, or "language of schooling" (Schleppegrell, 2004), is essential, but also presents teachers with a challenge. Education reforms in Berlin in 2007 integrated academic language development into teacher education in all subject areas. The study at hand aims at exploring how, after these reforms, pre-service mathematics teachers plan on supporting students who may struggle with language-related aspects of tasks involving conceptual understanding and mathematical reasoning.

THEORETICAL FRAMEWORK

Some studies provided empirical findings on how language-responsive mathematics teaching can be conducted effectively, in particular to develop conceptual understanding (cf. Wessel & Erath, 2018; Zahner et al., 2012). These studies underline that language support should be connected to students' mathematical thinking and knowledge acquisition (Moschkovich, 2013). The development of conceptual understanding and specifically of reasoning competencies is considered an important part of mathematics education (National Council of Teachers of Mathematics, 2000). In order for students to develop their mathematical reasoning competencies, pre-service mathematics teachers must learn to guide their students through the processes of mathematical reasoning, and in particular, to support them in "the process of searching for similarities and differences, validating, and exemplifying" (Ellis et al., 2019, p. 3). In regard to the language necessary to support students to engage in these processes, various discursive moves or practices have been identified such as explaining, arguing, reporting, and describing that allow for participation in meaning-making related to the mathematical concepts involved (Wessel & Erath, 2018; cf. Zahner et al., 2012; cf. Moschkovich, 2007). Amongst the different discourse practices, however, describing general patterns and explaining meanings have shown to be the most relevant for language learners, providing them with opportunities to build conceptual knowledge (Prediger, 2019; Moschkovich, 2013).

With the development of conceptual knowledge as a main goal of mathematics instruction, we thus draw upon a language-responsive mathematics teaching framework (Prediger, 2019), which differentiates between discourse practices that push students to explain meaning versus those to push students to report procedures. Prediger (2019) underlines the practices that focus on explaining meaning, including *explaining meaning of terms and operations, describing general relationships/patterns*, and *justifying decisions or relationships*; contrastingly, practices with the aim of reporting procedure comprise *explaining calculations and procedures*, with the sub-categories of *naming conventional rules, naming general procedures*, and *naming concrete solutions*. Lastly, practices that only focus on *correcting/highlighting sentence or grammatical structures* (on the sentence level) or *correcting or highlighting wording* (on the word level) are distinguished from those on the discourse level (Prediger, 2019), as they are considered subordinate practices that often result in too much focus on vocabulary work (Schleppegrell, 2004).

PRESENT STUDY & RESEARCH QUESTION

In order to determine how pre-service primary mathematics teachers support language learners in engaging in mathematical reasoning, this project employs a scriptwriting task (Zazkis et al., 2013) in a master's course. Pre-service teachers imagined interactions with fictional students, identified potential student difficulties that could arise, and wrote these in the form of a dialogue or script, thereby planning for verbal exchanges and considering their imagined action and the specific language the teacher might use to address student difficulties. Against the theoretical framework presented above, we are particularly interested in how high and low achieving pre-service teachers differ when addressing students' language difficulties and pursue the following research question: *How do high and low achieving pre-service primary mathematics teachers address student language difficulties in completing the scriptwriting task?*

Methodology

Participants & task. During the winter semester 2018/2019, twelve pre-service primary mathematics teachers at a Berlin-area university enrolled in a master's course "Foundations of Mathematics Instruction in Grades 5 and 6," participated in the study by completing a dialogue of their interaction with fictional students in reaction to a scriptwriting task (figure 1). The course, which had both a mathematical subject matter knowledge and mathematical pedagogical content knowledge focus, concentrated on, among other topics, operations with rational numbers. The textbook task below (figure 1, on the right side) served as the basis for the dialogue (also below) that the pre-service teachers had to complete:

Mia claims:

The product of $\frac{2}{5}$ and $\frac{1}{3}$ is bigger than the quotient of $\frac{2}{5}$ und $\frac{1}{3}$, because division always makes things smaller.

Is Mia right? Justify your response.

Ceyda: Okay, product means times. Two fifths times one third is two fifteenths, or one third times two fifths is also two fifteenths.

Azra: But two fifteenths is not as big as the other two numbers... With times, I thought we always get a big number.

Leonie: I'll show you guys. [Leonie draws a fraction circle → *Figure A*] We still have two fifths of a pizza from yesterday. There are three of us and I get one third of it... two fifteenths. That looks like this. [Leonie further partitions the pizza and shows her proportion of the remaining pizza → *Figure B*]

Figure A

Figure B

Azra: How did you know the new number? Did you calculate it?

Leonie: No, I cut the pizza more... to split it.

Azra: But that is completely backwards. Timesing makes things smaller...? **But you cut the pizza more, so did you divide the pizza?** So Mia is right, isn't she?

Leonie: Nope, I multiplied. Now we are going to divide it up... divide it.

Ceyda: It really doesn't matter, we can start with two fifths of the pizza or one third of it, it's timesing. She gets two fifteenths of the pizza.

Azra: Hmm... okay... with dividing... **two fifths divides into one third, right?**

...

Figure 1: The textbook task (adapted from Padberg & Wartha, 2017, p. 148) & dialogue beginning

Within the cohort of twelve pre-service primary mathematics teachers, six were regarded as "low performers," having a score of sufficient with a D equivalent or lower on the course exam and the other six as "high performers," scoring excellent or very good with an A or B equivalent.

Data analysis.

Utilizing a coding system (Kuckartz, 2018), we categorized their scripts with a mathematical language focus on the word level, the sentence level, or on the discourse level, either focusing on reporting procedure (explaining calculations and procedures) or explaining meaning, (specifically explaining meanings of terms and operations, describing general relationships/patterns, or justifying decisions or relationships) (adapted from Prediger, 2019).

Results & implications

The use of supports to assist the students in the scriptwriting task through their difficulties toward mathematical reasoning tended to reflect performance on the course exam. "High performers" had instances of supporting students engage on the discourse level, with some attempts to have students explain meaning, while the "low performers" tended to focus either only on the word or sentence level and, in occasions of language supports on the discourse level, generally pushed students to only report procedure. A "high performer" (C1_11) from this cohort, for example, attempted to support the fictional students in the dialogue in explaining the meaning of the multiplication of the two fractions on the discourse level by prompting the students to draw a new pictorial representation of the situation and to explain the meaning behind their drawing. The pre-service teacher asked such questions as: can you summarize what this means in relation to the original task? A "low performer" (C1_03) from the group provided impulses to the fictional students that either focused purely on a word level, asking for or stating technical vocabulary terms the students should be using or, when attempting to support the students on the discourse level, only focused on having the students report procedure without connecting to the meaning behind it.

All students, including academic language learners, should be able to participate in the mathematics classroom discourse and teachers need the skills to facilitate this exchange of ideas. As the first results of our study show how pre-service primary mathematics teachers perceive student challenges and how they plan to support the students in working through them, we can better prepare future teachers in planning and considering the different levels of language on which to focus, and additionally, contribute to the development of an instrument and approach that is increasingly utilized in teacher education **(**Zazkis & Herbst, 2018**)**. This research examines the impact of recent mathematics teacher education reform efforts in order to present recommendations to improve policy and, ultimately, help future mathematics teachers foster mathematical reasoning.

References

Barwell, R. (2009). *Multilingualism in mathematics classrooms: Global perspectives*. Multilingual Matters.

Chval, K.B., Pinnow, R.J., & Thomas, A. (2015). Learning how to focus on language while teaching mathematics to English language learners: A case study of Courtney. *Mathematics Education Research Journal, 27*(1), 103–127.

Cummins, J. (2000). *Language, power, and pedagogy: Bilingual children in the crossfire.* Multilingual Matters.

Ellis, A.B., Özgür, Z., & Reiten, L. (2019). Teacher moves for supporting student reasoning. *Mathematics Education Research Journal, 31*, 107–132. doi.org/10.1007/s13394-018-0246-6

Kuckartz, U. (2018). *Qualitative Inhaltsanalyse. Methoden, Praxis, Computerunterstützung*. [Qualitative content analysis. Methods, praxis, and computer support]. Juventa.

Moschkovich, J. (2005). Using two languages when learning mathematics. *Educational Studies in Mathematics, 64*(2), 121–144.

Moschkovich, J. (2007). Examining mathematical discourse practices. *For the Learning of Mathematics, 27*(1), 24–30.

Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education, 6*(1), 45–57.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. NCTM.

Padberg, F., & Wartha, S. (2017). *Didaktik der Bruchrechnung* [Didactics of rational numbers]. Spektrum.

Prediger, S. (2019). Promoting and investigating teachers' pathways towards expertise for language-responsive mathematics teaching. *Mathematics Education Research Journal, 31*, 367–392. doi.org/10.1007/s13394-019-00258-1

Schleppegrell, M. (2004). *The language of schooling: A functional linguistics perspective*. Lawrence Erlbaum.

Wessel, L., & Erath, K. (2018). Theoretical frameworks for designing and analyzing language-responsive mathematics teaching-learning arrangements. *ZDM, 50*(6), 1053–1064.

Zahner, W., Velazquez, G., Moschkovich, J., Vahey, P., & Lara-Meloy, T. (2012). Mathematics teaching practices with technology that support conceptual understanding for Latino/a students. *Journal of Mathematical Behavior, 31*(4), 431–446.

Zazkis, R., Sinclair, N., & Liljedahl, P. (2013). *Lesson play in mathematics education: A tool for research and professional development*. Springer.

Zazkis, R, & Herbst, P. (2018). *Scripting approaches in mathematics education: Mathematical dialogues in research and practice*. Springer.

SUPPORT SYSTEMS AS INTERSUBJECTIVE PROCESSES BETWEEN TEACHERS AND STUDENTS

Ann-Kristin Tewes
Leibniz Universität Hannover, Germany, ann-kristin.tewes@ifs.uni-hannover.de

The aim of the study is to reconstruct different support systems between students, primary school teachers and special needs education teachers in order to reconstruct the potential effects of these support systems for the participation in inclusive mathematic lessons. On the basis of an interactionist view on learning and with the perspective that learning is an intersubjective process between individuals in the zone of proximal development, a support system is located between the participants of the interaction. The main focus of this paper, on a theoretical level, is to locate support systems between the individuals as a dialectical process and, on an empirical level, to reconstruct a special support system between a teacher and a student with special needs.

VIEW ON LEARNING MATHEMATICS

Learning mathematics from an interactionist perspective

In this paper, the view of learning mathematics is from an interactionist perspective. The interactionist theory of mathematical learning is based on the theory of Symbolic Interactionism with the idea that the content to be learned or the topic of a lesson is negotiated between the participants of the interaction. Based on their experiences and knowledge, each individual has its own interpretations of a situation. This leads to a development of preliminary interpretations of the situation which, however, can be rejected or transformed in the process of an interaction (Blumer, 1969). The participants attempt to attune these to each other which can lead to a taken-as-shared meaning or working consensus (Goffmann, 1959). The working consensus is a condition which is created by the members of the interaction and also a basis on which the interaction can be continued (Krummheuer, 1992; Schütte et al., 2019). From an interactional perspective, learning is thus seen as a social interpretive act in which meanings are constructed through interactive negotiation processes (Blumer, 1969).

Learning mathematics as a social intersubjective process

From the interactionist perspective, the social interaction is fundamental for negotiation processes and thus for mathematical learning. Also Vygotskij (1934) and Jantzen (2005), members of the cultural-historical psychology, describe that learning or development situations arise in and through social interaction. Learning is located between individuals in the interaction in the zone of proximal development (ZPD). The ZPD is characterized as a place between individuals (interpsychic) and the inside of a person (intrapsychic):

> The ZPD is a special case of a general law: At first the higher psychic functions exist interpsychic in social intercourse and communication. They shift in the process of the social practice (historically, culturally and mediated) to inside, become intrapsychic (Jantzen, 2005).

According to this general law of development, it can be deduced that people always develop objects and meanings by communication with other individuals. The ZPD is the place where "learning and development become mediated" (Jantzen, 2005). With this perspective we can say, that (mathematical) learning is a social intersubjective process between individuals (Steffens, 2019; Vygotskij, 2003).

SUPPORT SYSTEMS IN INCLUSIVE MATHEMATICAS EDUCATION

Following the idea, that mathematical learning is a social interactive process, the individual has to participate in collective mathematical negotiation processes for learning mathematics. From an interactionist perspective, support can be located between the participants of the interaction. Support is negotiated in the interaction and is not to be seen as an activity of an individual person. A support intention alone, e.g., on the part of the adult, does not immediately become a support system. Utterances and actions are established as a support system if the members of the interaction accept them and orientate their interpretations by them (Tiedemann, 2013). Support has to establish itself in the interaction and becomes a support system if the participants in the interaction orientate themselves towards it with their interpretations. Hence, a support system is a dialectic intersubjective process between the teacher and the student which is designed together. According to Vygotskij (1934) and Jantzen (2005), a support system evolves between the individuals in the intersubjective space at the ZPD. Since the ratification of the Convention on the rights for persons with disabilities (CRPD), there is a legal right for persons with disabilities to a free choice of school and equal access to an inclusive education system (CRPD, 2007, article 24 (2)). For this reason, we have in Germany more and more children and young people, who have been taught at special schools, are now enrolled in regular schools (Neumann, 2019). The diversity of the students implies different opportunities for participation of each student in the mathematics classroom (Tewes, 2020) and thus different support systems for learning mathematics between the members of the interaction in the ZPD. In the following, the so far described theory is to be supported by the empirical reconstruction of a support system between a student with special needs and a teacher in the ZPD.

METHODOLOGY AND PROCEDURES

The greater research project is located in qualitative social research following a reconstructive-interpretative methodology (Bohnsack, 2007). The situation from the transcript below is recorded in an inclusive primary school mathematics classroom. I am guided by a broad concept of inclusion: inclusive work means that all students, with their differences, are recognized and valued, regardless of whether they need special support or not. To analyze the transcript concerning the support systems I am doing an interaction analysis in the first step. In this way, the negotiation processes in mathematics lessons can be examined (Krummheuer, 1992; Schütte et al., 2019). Afterwards, the support system, which establishes itself between the participants of the interaction, will be reconstructed (Tiedemann, 2012).

EMPIRICAL RECONSTRUCTION OF A SUPPORT SYSTEM

In the scene Jella (a student with special needs) and the teacher are playing a game. The game consists of a cork board, various templates on which there are different geometric shapes and the matching geometric shapes made of wood. According to the game instructions, the wooden shapes should be placed on the matching shapes on the templates and then fixed on them with a nail. In the following, two excerpts of the scene are analyzed and examined for the support system.

Excerpt 1

Teacher:	here is a red square [*takes a red square and holds it up with the left hand*] this is a square [*traces the edges of the square with the right index finger and moves the square in Jella's direction*]

Jella:	< [*takes the square with her hand*]
Teacher:	< [*taps with the right index finger on the red square on the template sheet at the top left*]
Jella:	[*place the square where teacher pointed it at the top left of the template*]

The teacher first names the shape and then takes a red wooden square. Then the teacher names the shape a second time and outlines it with her index finger. By tracing the wooden square, attention is drawn to the sides and to the outer shape. At the end of the excerpt, the teacher hands Jella the square and points with her finger to the place on the template sheet where Jella then places the square. After Jella and the teacher have fixed the square on the cork board, the scene continues as follows:

Excerpt 2

Teacher:	then take [*runs the left index finger on the right where the shapes are*] then take
Jella:	[*reaches for the geometric shapes*] there / [*smiles and looks at teacher and takes a yellow triangle and holds it up*]
Teacher:	what is that/
Jella:	[*takes the triangle and puts it with one corner to her forehead*] angly[4]
Teacher:	a yellow triangle
Jella:	hmm [*looks at the board and alternately takes the triangle in the left and right hand, then takes the right hand a little forward, looks at the triangle and then at the template, then takes the triangle in the left hand and places it on the template*]

In the second excerpt, the teacher asks Jella to take a new shape. Unlike in the first excerpt, Jella does not take a square but a yellow triangle. When asked by the teacher, Jella brings the wooden triangle to her forehead and answers "angly”. Jella can recognize and name the property angular. Feeling the shape on the forehead can be an additional stimulus and represent the physical transition between interphysic and intraphysic. Jella not only sees the shape but also feels it and can internalize it better. The teacher then names the shape and Jella places the triangle on the template sheet. The transcript shows that tracing or feeling the shape, as an action to determine the outline of the shape, is established in the interaction between Jella and the teacher. Jella takes over the support and orientates her action by the teacher’s action and can thus determine the property angular. Also, the teacher takes up Jella's statement and orientates herself by Jella. The mutual exchange creates a ZPD in which Jella and the teacher come together. They enter into a communicative intersubjective negotiation process and determine the properties of geometric shapes. This action and the resulting focused attention on the external form seems to establish itself as a support system between the teacher and Jella in the ZPD.

CONCLUSION

This analysis showed, how a support system is established between a teacher and a student with special needs in the inclusive mathematics classroom. It turns out that with the combination of both theoretical approaches, the emergence of support systems can be located and described more precisely. A support system is an interactive and intersubjective process between the teacher and the student, which is designed together in the ZPD. As the learners interact with different teachers in mathematics lessons, different ZPD and thus different support systems develop between them. Through the reconstruction of support systems, possible conditions for the participation of all students in mathematics lessons are

[4] Jella says a partially incomprehensible word, which is reminiscent of the German word for "corner".

worked out. The aim of the project is to reconstruct the support systems between learners and primary school teachers and learners and special needs teachers. With this perspective, the study ties in with the current educational-political discussion about cooperation between special needs teachers and primary school teachers in Germany (Neumann, 2019). The results of the study therefore make an important contribution to fundamental research in the field of inclusive mathematics education.

References

Blumer, H. (1969). *Symbolic Interactionism*. Prentice-Hall.

Bohnsack, R. (2007). *Rekonstruktive Sozialforschung. Einführung in qualitative Methoden* [Reconstructive social research. Introduction to qualitative methods]. Barbara Budrich.

Goffman, E. (1959). *The Presentation of Self in Everyday Life*. The Overlook Press.

Jantzen, W. (2005). Towards a new understanding of the "Zone of proximal development". *Lecture at First ISCAR Conference*. Seville. In B. Siebert (Ed.), Kulturhistorische Integrationspädagogik. Arbeiten zur integrativen Pädagogik im Kontext der Vygotskij-Schule.

Krummheuer, G. (1992). *Lernen mit "Format". Elemente einer interaktionistischen Lerntheorie. Diskutiert an Beispielen mathematischen Unterrichts*. [Learning with "format". Elements of an interactionist learning theory. Discusses using examples of mathematical lessons.] Deutscher Studien Verlag.

Neumann, P. (2019). *Kooperation selbst bestimmt?*. [Cooperation self-determined?] Waxmann.

Schütte, M., Friesen, R.-A. & Jung, J. (2019). Interactional analysis: A method for analysing mathematical learning processes in interactions. In G. Kaiser & N. Presmeg (Eds.) *Compendium for early career researchers in mathematics education.* Springer.

Steffens, J. (2019). Der Mensch lernt nicht mit einem Gehirn, sondern mit vielen Gehirnen in Gesellschaft. (Neuro-)Psychologische Grundlagen für die Gestaltung inklusiven Unterrichts [Humans do not learn with one brain, but with many brains in society. (Neuro-) psychological basics for the design of inclusive lessons.] In A. Langner, M. Ritter, J. Steffens, & D. Jugel, (Eds.), *Inklusive Bildung forschend entdecken* [Explore inclusive education through research] (pp. 31-76). Springer VS.

Tewes, A. K. (2020). Support systems for participation in collective argumentation in inclusive primary mathematics education. In J. Ingram, K. Erath, F. Rønning, A. Schüler-Meyer, & A. Chesnais (Eds.), *Proceedings of the Seventh ERME Topic Conference on Language in the Mathematics Classroom* (pp.179–186). ERME / HAL Archive.

Tiedemann, K. (2012). *Mathematik in der Familie. Zur familialen Unterstützung früher mathematischer Lernprozesse in Vorlese- und Spielsituationen* [Math in the family. For family support of early mathematical learning processes in reading and play situations]. Waxmann.

Tiedemann, K. (2013). How Families Support the Learning of Early Years Mathematics. In B. Ubuz (Ed.) *Proceedings of the Eighth Congress of European Research in Mathematics Education* (pp. 2218–2227). Middle East Technical University.

UN General Assembly, (2007). *Convention on the rights of persons with disabilities (CRPD)* (resolution adopted by the General Assembly). United Nations. https://www.un.org/development/desa/disabilities/convention-on-the-rights-of-persons-with-disabilities.html

Vygotsky, L.S. (1934). Thinking and speech. In R. W. Rieber & A. S. Carton (Eds.), *The Collected Works of L.S. Vygotsky* (Vol. n.). Plenum.

Vygotskij, L. S. (2003). Das Säuglingsalter [The infancy]. In J. Lompscher (Ed.), *Lev Vygotskij Band II−Ausgewählte Schriften. Arbeiten zur Entwicklung der Persönlichkeit* (pp. 53–90). Lehmanns Media-LOB.de.

A COMPARATIVE STUDY ON TEACHING LANGUAGE OF ALGEBRA CLASSROOM BETWEEN NOVICE TEACHERS AND EXPERT TEACHERS ——TAKING 'LINEAR EQUATION IN ONE UNKNOWN' AS AN EXAMPLE

Wang Si-kai[1] Ye Li-jun[2]

[1] East China Normal University

[2] Hangzhou Normal University

Teaching language directly affects the effect of mathematics classroom teaching. From the perspective of pragmatics, we compare teaching language of algebra teaching between the novice teacher and the expert teacher, and obtain some conclusions about the use of two teachers' teaching language. Then we get the inspiration that teachers how to optimize their teaching language in the classroom teaching.

THE REAEARCH BACKGROUND

At present, the use of teaching language has become one of the basic characteristics of identifying the main teaching behaviors of teachers in efficient mathematics classroom (Yang & Wang, 2011). Compared with novice teachers, expert teachers can use teaching language appropriately to ensure the classroom teaching efficient and smooth. Thus, it is necessary to compare novice and expert teachers' teaching language with the goal of improving mathematics teachers' ability of using teaching language.

THE RESEARCH DESIGN

The research object

Based on relevant research (Yan, 2009; Ye & Zheng, 2018), the selection criteria for novice teachers is: (1) be graduated from normal colleges and participated in educational internships, (2) be engaged in teaching for 1-3 years; and the selection criteria for expert teachers is: (1) have won provincial and municipal teaching honors, (2) get a senior professional title, (3) be engaged in teaching for more than 15 years. Thus we select the expert teacher A from Hangzhou C Middle School and the novice teacher B from Hangzhou W Middle School. And the teaching content is about "linear equation in one unknown".

The research steps

On the basis of TIMSS Video Study, the specific analysis is as following: (1) visit the classroom and shoot the video to collect relevant information; (2) record the teacher and students' behavior and its starting and ending time according to the video; (3) classify teaching language from the perspective of pragmatics and classify students' response; (4) collect and analyze the data on the number and time of teacher's teaching language and the number and time of students' response.

The type of teaching language

Teaching language implies the teacher's behavioral intention. According to the existing classification of teaching language (Ye et al., 2015), teaching language is divided into seven categories: (1) Affirmative language. The teacher puts forward its own interpretation and opinion, or quotes others' opinion. (2) Descriptive language. The teacher states the fact and incorporates existing content into the teacher and students' vision. (3) Commitment language. The teacher and students intend to do something, or the teacher organizes teaching activities. (4) Imperative language. Expect students to complete the teacher's instructions. (5) Query language. The teacher asks students about the knowledge or the steps to solve the problem, expecting students to answer. (6) Declarative language. The teacher gives feedback on student's opinion or behavior timely, in order to accept or clarify the student's answer. (7) Expressive language. The teacher evaluates students' classroom behavior through praise and encouragement.

The type of student's response

On the basis of teacher's questioning, student's response is divided: specific student's response, unclear personal response, multiple students' response together, and students answer together with the teacher.

THE RESEARCH RESULTS AND ANALYSIS

Both teachers value the use of teaching language, with teaching language time accounting for more than 50%.

As shown in Table 1, both teachers' teaching language time accounts for more than 50% and the teacher-student dialogue time accounts for more than 65%, indicating that both teachers attach importance to using teaching language to organize teaching activities. Moreover, the time of two teachers' teaching language is more than three times that of students' response, which reflects that students spend less time in answering teacher questions in algebra class.

teacher	teaching language			Students' response		
	number	time / s	Time percentage	number	time / s	time percentage
A	486	1588	66.36%	174	388	16.21%
B	494	1435	51.43%	199	396	14.19%

Table 1: The number and the time of teaching language and students' response

The novice teacher uses affirmative language more frequently than the expert teacher, the number of which is twice as many as the expert teacher.

As shown in Figure 1, the number of the novice teacher's affirmative language is twice that of the expert teacher. Worried that students have difficulty in understanding the problem, the novice teacher uses much more affirmative language directly to clarify the background and meaning of the problem to ensure that the teaching goes smoothly. As a result, students lack the necessary time to deliberate the intent of the problem and their mathematical thinking can't be effectively trained.

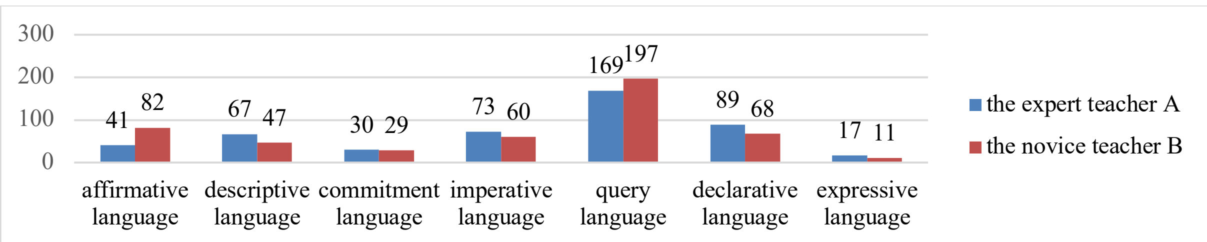

Figure 1: The number of teaching language during the algebra classroom teaching

The novice teacher's declarative language is mostly used to simply repeat students' answer when exploring new knowledge, taking a short time.

When exploring new knowledge, the number of the novice teacher's declarative language is 28 and the time is only 48 seconds, which is about half and one-third of the expert teacher's. The novice teacher uses declarative language to simply repeat students' answer, declaring whether the answer is correct or not; while the expert teacher gives students more feedback timely, or to accept the student's idea to express encouragement and affirmation, or to clarify the student's answer, reflecting that the expert teacher communicates with students more deeply and frequently on a certain issue.

The novice teacher uses too much teaching language when consolidating new knowledge, causing students having fewer opportunities to think.

As shown in Figure 2, the number of the novice teacher's teaching language is 248 when consolidating new knowledge, accounting for 50.20%, which is significantly higher than 17.49% of the expert teacher. The novice teacher uses teaching language directly to lead students applying new knowledge to solve problems, or to analyze students' answer immediately, especially when the answer is wrong. Excessive use of teaching language may help students temporarily master using new knowledge to solve problems, but doesn't give students necessary time to understand deeply.

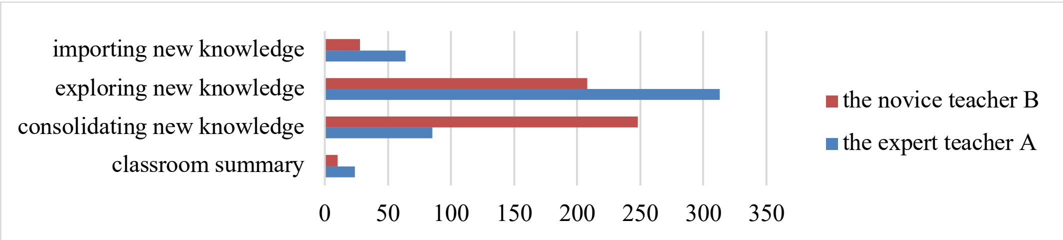

Figure 2: The number of teaching language in each link of the algebra teaching

THE RESEARCH INSPIRATION

Streamline teaching language and control teaching language time reasonably.

Only going through the proper thinking process and under the appropriate guidance of the teacher can student truly achieve the goal of mastering new knowledge. Teachers should streamline teaching language and regulate the use time of teaching language reasonably to ensure that students will not only be tired of the teacher's redundant teaching language, but also have sufficient time to deepen the process of derivation of knowledge to truly understand the connotation of new knowledge.

Make good use of affirmative language to prompt students and give necessary thinking time.

Excessive use of affirmative language will only directly replace students' thinking process and greatly reduce their thinking time. Teacher's' ahead of hint can only lead students to simply imitate, lacking the "intuitive" and "sudden sight" edification (Zhang, 2018). Teachers should give students necessary time to explore knowledge, and use affirmative language tactfully to prompt and guide students when students encounter difficulties or doubts, ensuring that students' mathematics thinking is effectively exercised.

Avoid retelling answer simply and use declarative language subtly to improve feedback quality.

Classroom feedback can effectively promote teaching. Teachers should use the declarative language ingeniously, or in a different tone, or to explain properly on the basis of student responses, especially when the questioning is difficult, which can not only indicate whether the teacher agrees with the answer, but also encourage students to reflect and clarify the answer, and attract other students to rethink the problem in order to deepen their understanding on knowledge.

Use teaching language reasonably to help students accumulate basic activity experience.

The process of applying new knowledge to solve mathematics problems is of great significance for students to grasp new knowledge, which puts high demands on the use of teaching language. Teachers should use all types of teaching language reasonably, such as using query language to stimulate students' problem awareness, so as to help students to go through the activity process and accumulate basic experience of applying new knowledge to solve mathematical problems.

References

Yan, J. (2009). Diversified Reaction to Classroom Teaching Between Junior and Senior Teachers and Its Impact to The Professional Development. *Journal of Wuhan Polytechnic*, *8*(2), 76–78.

Yang, T., & Wang, G. (2011). Case Study about High-effective Mathematics Teaching Behaviors Based on A Mathematics Teacher in Middle School. *Journal of Mathematics Education*, *20*(2), 37–40.

Ye, L., Li, Y., & Si, H. (2015). Comparative Study of the New and Experienced Teachers' Differences about Their Language of Instruction in the Mathematics Classroom. *Journal of Mathematics Education*, *24*(4), 40-43.

Ye, L., & Zheng, X. (2018). Study of the Expert Mathematics Teachers' Questioning Behavior on the Algebra Review Course——Such as the Cases of Linear Function and Inverse Proportional Function. *Journal of Mathematics Education*, *27*(2), 46-49.

Zhang, J. (2018). *Face-To-Face Talk with Well-Known Educators on Key Competencies of Mathematics*. Zhejiang Education Press.

POSTERS

THE MATHEMATICS CLASSROOM IN ITS DISCURSIVE ARENA FORM

Mozart Edson Lopes Guimarães and José Joelson Pimentel de Almeida
Universidade Estadual da Paraíba – UEPB

The second school, we find subjects on the roles of teacher and student, who have a well-defined social status and speech opportunities, a fact that generates ideological and dialogical conflicts between subjects and their social roles, clearly translated into power relations, in particular, when we consider the context of mathematics classes, a moment when the teacher uses the specific language mastery, as a way of confirming his hegemony before the subjects on the roles of students, who are placed in the accommodation and submission status. In this paper, our objective broaden our horizons of discussion and debates.

Teacher vs student

The speeches involve the society sign's are intricately focused on possible relationships between subjects who live in similar space and time, counting on the indispensable presence of the communication phenomenon. From this perspective, it is possible to see the school as a fraction of society along the lines of fractals, where the communicative characteristics, the definition of social functions and the hierarchy of roles as a part and as a whole are confused.

In this sense, let us analyze the hypothetical situation, experienced in a class of 9th grade of elementary school, reported below.

A mathematics teacher elaborated and gave the following question to his class: **Find the value of x in the equation $\mathbf{2x-6=4}$.** One of his students submitted the following answer: $\mathbf{x=5.}$

When examining the students answers the teacher was faced with a resolution different from the one previously expected and, for this reason, considered that there were not enough arguments from the student to validate his answer, thus, the question was considered wrong.

For Bakhtin, the discourse is composed of interlocutors, since the subjects involved alternate roles of listeners and speakers, gettering understandings from the speeches of others. However, in our hypothetical situation, we are faced with the subject in the role of student, thus imposed by the hegemonic speaker in the listening role, which sometimes leads to the status of accommodation in the face of situations in which, by their nature, have the potential to generate debate or, even, critical dialogues between the interlocutor in the role of teacher and the interlocutor in the role of student.

In other words, it is up to the teacher to provide an open environment for dialogue, reliving the student from a position of submission and discursive confinement. The premeditated prohibition of the student's voice in the classroom environment in situations of, theoretically, knowledge construction directs the class to a mere transmission of techniques, codes and algorithms, a fact that maximizes the difficulties in a process that should be of exchanges of ideas, experiences, knowledge and concerns in the search for understandings and useful or even apparently useless information.

References

Bakhtin, M. (2015). *Estética da criação verbal*(6th Ed.). Matins Fontes.

THE RELATIONSHIP BETWEEN THE PROBLEMS GIVEN BY THE TEACHER AND THE DEPTH OF COMMUNICATION BETWEEN TEACHERS AND STUDENTS: TAKING THE ZERO POINT OF FUNCTION AS AN EXAMPLE

Fengjuan HU
Capital Normal University, China

Teacher questioning-student answering is the dominant type of interaction in all four countries (e.g., Yu et al., 2019), we would like to describe the relationship between the problems given by the teacher and the depth of communication between teachers and students: taking the Zero Point of function as an example.

PROBLEM POSING

Different teachers teach the same content, the interaction between teachers and students is very different, and the learning effect is also very different. The important reason for this phenomenon is that the problems given by teachers are different. So what's the relationship between the problems given by the teacher and the depth of communication between teachers and students?

Methodology and Results

I chose four class videos of the Zero Point of function about four different teachers. This study makes a comparative study on the mathematical problems raised by different teachers in different processes, as well as the frequency and types of teacher-student interaction (Brown, 2000). The study also studies the interaction between the problems given by the same teacher in different teaching processes and teacher-student interaction. And found that the challenges produced by problems posed by teachers affected the depth of teacher-student interaction.

Discussion and Conclusion

The present results confirmed previous research results showing the coherence in mathematics. We find that the coherence of mathematical problems affects students' thinking and thus the depth of teacher-student interaction. For example, *one teacher let students to find the x ：* ① $x^2-5x+6=0$, ② $x^2-5x+6\geq 0$. *And compare the relationship between the two.*

The problem is so simple that students do not know what the purpose of the teacher is, so that few students respond. At the same time, we think that the study of teacher-student interaction can't be separated from the problem itself, not just the type of problem posing (Cai & Jiang, 2017), the type of teacher-student interaction.

References

Brown T. (2001) *Mathematics Education and Language: Interpreting Hermeneutics and Post-Structuralism.* Springer.

Cai, J., & Jiang, C. (2017) An Analysis of Problem-Posing Tasks in Chinese and US Elementary Mathematics Textbooks. *International Journal of Science and Mathematics Education. 15*(8), 1521–1540.

Yu G.， Cao, Y.， Clarke, D., & Chan, M. C. E. (2019). An Empirical Study of Teacher-Student Interaction: A Focus on Middle School Mathematics Classrooms in China， Finland， France and Australia. *Global Education*, *48*(1), 71-81.

SUPPORTING STUDENTS' INTERACTIONS THROUGH PAIR WORK

Eszter Kovács-Kószó, Viktória Kiss and József Kosztolányi
University of Szeged, Hungary

Frontal teaching methods are widespread and it is difficult to invite students into classroom discussions or mathematics-oriented partner talk. Our aim is to support teachers at the beginning of the implementation of an easy-to-apply method which can support 15-year-old students' involvement in interactions. A structured pair work method seems to be an effective tool to create discussions among students, which is one of our focuses.

AIM AND METHOD

The aim of this research is to encourage teachers to increase students' involvement in traditional classrooms. Teachers were asked to switch the daily applied individual work to structured pair work and leave everything else unchanged. We wonder if this could increase students' meaningful mathematics-oriented partner talk.

In the 'sage and scribe' structure students work in pairs with a single pen (Kagan, 2002). For the first task, the sage tells the scribe how to solve the problem and the scribe records the work, then they switch roles. It is the duty of the scribe to help or correct the sage when it is necessary, but he must not take over his/her role. This structure may not seem to be cooperative but it is useful for students who always worked alone in mathematical tasks and rarely participate actively in classroom discussions. It could help to think aloud in front of a classmate. The sage and scribe structure can balance the work, create situations when students get to know each other's thoughts and get reactions to theirs. It is also useful that students' thoughts become hearable for the teacher, who goes around the class.

After eleven pre-experiments, we had a two-month main experiment. We collected data through questionnaires and interviews with students and the teacher. We took audio recordings from each pair.

RESULTS

Analyzations of the data are still in progress, but it is clear that teachers find the method useful, and many students experienced benefits of cooperative working, like "it was interesting to hear how my partner thinks" and "I got immediate help". We are using grounded theory for analyzing the dynamics and the quality of partner talk and the role of the teacher in it (Glasser & Stauss, 1967).

Acknowledgement

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References

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Aldine.

Kagan, S. (2002). *The Instructional Revolution*. *https://www.kaganonline.com/free_articles/dr_spencer_kagan/ASK39-2.php* ,

TEACHING STUDY ON 6TH GRADERS' MATHEMATICAL COMMUNICATING REASONING COMPETENCY

Zhiling Wang
College of Education, Hangzhou Normal University, China

This article aimed at investigating the current status of mathematical communicating reasoning competency of Chinese 6th graders and exploring the task design and teaching strategies for promoting the development of Chinese 6th graders' mathematical communicating reasoning competency. The results showed that the overall development level of the 6th graders' mathematical communicating reasoning competency was generally low, and they had difficulty in the process of mathematical communicating reasoning. The tasks and teaching strategies were helpful to avoid the errors in 6th graders' process of mathematical communicating reasoning effectively.

INTRODUCTION

It is undeniable that "communicating reasoning" has always existed objectively in the practice of mathematics and mathematics education. Smarter Balanced Assessment Consortium (SBAC) regards "communicating reasoning" as one of the mathematical learning goals (2015). Based on the definition of communicating reasoning from SBAC, put Chinese student's ability in consideration, in the article, communicating reasoning competency is defined as follows: students can clearly and precisely construct viable arguments to support their own reasoning, and can use brief mathematical language to clarify their reasoning systematically, reasonably and accurately, and to critique the reasoning of others.

METHODS AND DATA

The test survey (306 6th graders' mathematical communicating reasoning competency tests were distributed) and interview (12 sixth-grade learners interview contents) were conducted to investigate the current status of mathematical communicating reasoning competency of 6th graders in China (RQ1). Teaching interventions, with mathematical tasks and teaching strategies were applied in the experimental class with total 52 students for one month, and, in total, 104 test sheets were distributed to all learners in the experimental class before and after interventions. Classroom observation and interviews were conducted (utilizing the researcher's classroom observation notes and students' interview contents) to explore whether these tasks and teaching strategies were helpful to avoid the errors in 6th graders' process of mathematical communicating reasoning effectively (RQ2).

RESULTS

The results showed that the 6th graders' mathematical communicating reasoning competency was generally low in China. Five teaching strategies, (1) create classroom climate for communicating reasoning; (2) classroom questioning; (3) group discussion; (4) mathematical language refinement strategy; (5) problem solving-based math communication reasoning study sheet were helpful to avoid the errors in 6th graders' process of mathematical communicating reasoning effectively.

References

Smarter Balanced Assessment Consortium (2015). *End of Grant Report*. https://files.eric.ed.gov/fulltext/ED572036.pdf

STUDYING STUDENT PARTICIPATION IN COLLABORATIVE MATHEMATICS PROBLEM SOLVING BASED ON ONE GROUP OF FOUR CHINESE STUDENTS

Shu Zhang[1], Man Ching Esther Chan[2] and Yiming Cao[1]
[1]Beijing Normal University, [2]The University of Melbourne

This poster reports on an investigation of student participation during collaborative problem solving in mathematics in terms of the roles that students took during the problem solving activity and the content of their contribution to the discussion. Based on the analysis of the group interaction, it was found that frequently taking a particular position, e.g., initiation, may not necessarily suggest a control of the discussion.

CONCEPTUALIZATION OF THE STUDY

Collaborative mathematics problem solving has been increasingly adopted as a daily classroom practice in many countries. We propose that student participation may be a crucial concept to understand how students are involved in collaborative problem solving activities. We view participation as a process of taking part in the student interaction and task completion through taking different positions and contributing to the content of group discussion during problem solving activities. We ask the question: what could student participation look like in collaborative problem solving in mathematics based on this conceptualization of student participation?

RESULTS AND DISCUSSION

The data reported in this paper came from the Australian government-funded research project: The Social Essentials of Learning (Chan, Clarke, & Cao, 2018). One group of four Chinese students in Beijing was examined, with two girls (S3 and S4) and two boys (S1 and S2), all in Grade 7 (12 to 13 years old), working collaboratively to solve an open-ended task.

It was found that students were changing their positions (initiation, response, evaluation and non-interactive) as they moved through a sequence of topics. For example, S1 participated in the discussion mostly through taking the evaluation role. S3 and S4 formed an interactive pair who talked to each other frequently. S2 shifted his positions from initiation to non-interactive which may suggest a diminished level of participation during the group discussion.

Regarding the discussion content, students could contribute to the content (about mathematics and task-related, non-mathematics but task-related, or off-task), through proposing topics for group discussion. They could revisit topics which had been proposed by themselves or other group members.

In our study, students who occupied the initiation position may not necessarily control the discussion. This is because students in the response position may also propose their own ideas, which could then be taken up by the whole group, particularly when the idea helps the students to solve the problem.

References

Chan, M. C. E., Clarke, D., & Cao, Y. (2018). The social essentials of learning: an experimental investigation of collaborative problem solving and knowledge construction in mathematics classrooms in Australia and China. *Mathematics Education Research Journal, 30*(1), 39–50.