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1 Introduction

Our study investigates the Electric Autonomous Dial-a-Ride Problem (E-ADARP) introduced by [1] which consists of scheduling electric autonomous vehicles (EAVs) to transport users from specific origins to specific destinations within predefined time windows. The E-ADARP consists of the following features that are different from the typical DARPs: (1) detour to recharging station on the route, (2) partial recharging at recharging station, (3) vehicle can locate at different origin depots and select from a set of optional destination depots, (5) no restriction for route duration time.

As with classical DARP, the exact methods cannot solve large-scale instances of E-ADARP within reasonable computational time [1]. Inspired by [2], we combine an exact method with heuristics with the objective of providing near-optimum solutions in reasonable computational time and improving the lower bound of previous literature ([1]). In this work, we propose a heuristic algorithm integrating deterministic annealing with column generation. In the hybrid algorithm, heuristic operators (including constructive and local search operators) are embedded in the column generation to identify negative reduced-cost columns. The solution obtained from column generation is further improved by deterministic annealing. We also implement a column management strategy to clean useless columns and speed up the calculation. Then, we evaluate the effectiveness of hybridization and conduct sensitivity analysis for selecting good parameters. Two benchmark instance sets from the literature are considered. For each instance set, we consider three scenarios, representing three different energy restriction level (i.e., low, medium, high energy restriction). The obtained solutions are compared to the solution in the literature (i.e., [1],[3]), which report the best results by using exact method/heuristic method, respectively. The new solutions found by the proposed algorithm are also presented in the paper.

2 Problem Formulation

The problem is considered on a complete directed graph, denoted as $G = (V, A)$, where V is the set of vertices and A is the set of arcs. Set V can be further partitioned into the set of customer $N = \{1, 2, \cdots, i, \cdots, 2n\}$ and the set of recharging stations $F = \{2n + 1, \cdots, 2n + f\}$. The customer set N is divided into the customer pick-up node set P^u and customer drop-off set D^u. The node in the customer set N can be visited exactly once, and the node in the recharging stations set F can be visited at most once (as defined in [1]). Each user request consists of two nodes, one pickup node i and its corresponding drop-off node $n + i$, which n is the number of requests. Each vehicle should depart from the original depot and end at the destination depot. Specifically, the vehicle can select the end depot from a set of destination depots. In other words, the destination depot will not be predefined at the beginning of the time horizon, and it will be decided during the service. It can be visited, therefore, at most once. The total number of vehicles is denoted as K.

2.1 Column generation framework

We decompose the block diagonal MIP model into a "master problem" and a set of subproblems. The master problem is defined as a set covering problem in which a set of feasible routes has covered the set of transportation requests. The objective function is formulated as the sum of cost (shown in equation 1). Let Ω denote the set of all feasible routes concerning the constraints defined in the MIP model. For each route $\omega \in \Omega$, we define c_ω as the cost for route ω. Additionally, we define $\theta_{i\omega}$ as a binary coefficient that equals one if the request i is visited by the route ω (zero otherwise). Let y_ω denote a binary variable that equals one if and only if the route $\omega \in \Omega$ is included in the solution (0 otherwise). The number of requests to be served is n, and the total vehicle number is K. To restrict the visit of the recharging station and the destination depot (as the destination set is optional), we define another binary coefficient $\phi_{f\omega}$, denoted whether the recharging station f is visited in route ω. The set covering problem is formulated as:

$$\min \sum_{\omega \in \Omega} c_\omega * y_\omega \tag{1}$$

subject to:

$$\sum_{\omega \in \Omega} \theta_{i\omega} y_\omega \geqslant 1, \forall i \in P^u \tag{2}$$

$$\sum_{\omega \in \Omega} \phi_{f\omega} y_\omega \leqslant 1, \forall f \in F \cup D^d \tag{3}$$

$$\sum_{\omega \in \Omega} y_\omega \leqslant |K| \tag{4}$$

$$y_\omega \in \{0, 1\}, \forall \omega \in \Omega \tag{5}$$

The constraint 2 ensures that each request is served at least once. Constraint 3 restricts the number of visits to recharging stations. Constraint 4 guarantees that the number of vehicles used cannot exceed the maximum number of vehicles. Constraint 5 restricts the value of variable y_ω. In order to retrieve the dual information, we solve the linear relaxation of the master problem, that is $y_\omega \geqslant 0$. The linearized master problem is called LMP.

Due to the large size of Ω, we restrict the master problem to a subset of Ω, denoted as Ω', and we solve the restricted LMP (abbreviated as RLMP). New columns that have negative reduced cost (i.e., have the chance to improve objective 1) are generated by solving the pricing sub-problem in which the objective is to minimize the reduced cost (presented in Equation 6). In the expression, the dual variable of constraint of constraint 2 and constraint 3 are denoted as $\pi_i (i \in P^u)$ and $\tau_f (f \in F \cup D^d)$. Another dual variable associated with constraint (4) is δ. The new columns with negative reduced costs are added into Ω', and the master problem is resolved using the updated column pool. The RLMP and sub-problem are solved iteratively until no more negative reduced cost column can be found, the column generation process converges.

$$\bar{c_w} = c_w - \sum_{i \in P^u} \theta_{i\omega} \pi_i - \sum_{f \in F \cup D^d} \phi_{f\omega} \tau_f - \delta \tag{6}$$

3 Results and Conclusions

We propose a hybrid column generation algorithm to solve the E-ADARP. The initial column pool is first populated by applying Deterministic Annealing with local search (hereafter DA with local search) described in our previous work [3]. The subproblem is first solved heuristically on the reduce graph through a constructive algorithm designed in the fashion of the greedy randomized search algorithm. This method is applied as long as columns with negative costs can be generated. The column generated by the constructive algorithm is then improved by a DA based column generator. Different from the mentioned algorithm for initializing the column pool, the DA-based column generator minimizes the reduced cost of the column rather than the actual route cost. Several operators have been adjusted in order to fit the context. If the constructive algorithm cannot find negative columns on the reduced graph, then the subproblem is solved on the complete graph and the columns found are further improved by the DA algorithm. Every N_{iter}, we manage the size of the column pool by cleaning the "useless" columns. The "useless" columns include the columns not used in the Linear Restricted Master Problem (denoted as "RLMP") and Integer Restricted Master Problem (denoted as "RIMP") resolution and the columns that the reduced costs are larger than a threshold value D_{max}. The column generation is iterated until no more columns can be identified by either of the heuristics or the maximum time limit is reached. Then, with the obtained solution from the final resolution of RIMP, we apply deterministic annealing, which aims to minimize the actual cost to improve the solution further. The hybrid algorithm returns the best solution of iterating DA algorithm N^{DA} times.

The proposed hybrid algorithm integrates exact and heuristic methods and is proven efficient in solving E-ADARP. Its efficiency does not deteriorate when solving medium-to-large-sized instances under high energy restriction level while the existing heuristic and exact method are impacted. On the same computer, the hybrid CG algorithm requires less than one-third of the computational time comparing to the existing heuristic method. Compared to the best reported results in the literature, the proposed algorithm improves the computational efficiency by up to 27.81%, and the average gap is 0.89%. Several new best solutions are found on the previously solved and unsolved instances, indicating the proposed algorithm's high performance. The strength of the proposed algorithm is more evident when comparing to the pure DA with local search in solving hard instances. A significant improvement in computational time and solution quality is observed.

References

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