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M Ahmad, X Pelorson, I A Fernández, O Guasch, Annemie van Hirtum. Low-strain effective Young's modulus model and validation for multi-layer vocal fold based silicone specimens with inclusions. Journal of Applied Physics, 2022, 131 (5), pp.054701. 10.1063/5.0080468 . hal-03538688

HAL Id: hal-03538688

<https://hal.science/hal-03538688>

Submitted on 21 Jan 2022

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Low-strain effective Young's modulus model and validation for multi-layer vocal fold based silicone specimens with inclusions

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(Dated: 13 January 2022)

A model of the effective low-strain elastic Young's modulus of multi-layer stacked composites is proposed which is capable to account for an arbitrary stacked inclusion. Geometrical and discretisation based model results are validated against measured effective Young's moduli (from 10 kPa up to 40 kPa) on 14 molded silicone specimens embedding a stiff (298 kPa) inclusion with variable size, position and stacking. Specimens without inclusion represent the muscle, superficial and epithelium layers in a human vocal fold with Young's moduli between 4 kPa and 65 kPa. The proposed model allows to predict the influence of a stiff inclusion, mimicking a structural abnormality or pathology somewhere within the vocal fold, on the low-strain effective Young's modulus. Quantifying the influence of an inclusion or local stiffening on the vocal fold bio-mechanics is a necessary step towards the understanding and mitigation of structural vocal folds pathologies and associated voice disorders.

I. INTRODUCTION

Phonation or voiced speech sound production is due to the fluid-structure interaction (FSI) between airflow coming from the lungs along the inferior-superior direction and the vocal folds tissues enveloping the airflow within the glottis¹. Although the physical mechanisms driving phonation are known²⁻⁴, systematic studies of the influence of the vocal folds structure, either normal or abnormal, on the FSI and subsequent vocal fold auto-oscillation are few^{5,6} as most physical studies focus on the glottal flow⁷⁻⁹. This is partly due to the lack of a low-cost computational model predicting the elasticity of deformable silicone vocal fold replicas mimicking, up to some extent, the multi-layer (ML) anatomical representation¹ of a human vocal fold without (Fig. 1) or with structural abnormality or pathology.

FIG. 1. Multi-layer representation of the anatomical structure of a human vocal fold in the medio-frontal plane, *i.e.* a coronal section.

In the linear low-strain region, the stress-strain relationship is described by Hooke's law so that the elasticity of a soft material is characterised by its low-strain or elastic Young's modulus. In¹⁰, a quasi-analytical model of the low-strain effective elastic Young's modulus $\mathcal{E}_{eff}$ was proposed for homogenised ML silicone composites with serial ($\perp$), parallel ($\parallel$) or combined ($\perp\parallel$) stacked layers with respect to the force direction, as depicted in Fig. 2. This means that composites are treated as n layers which are stacked either serial or parallel to each other. The model was validated (overall accuracy of ± 2.4 kPa) against measured $\mathcal{E}_{eff}$ from uni-axial tensile tests at room temperature 21 ± 2 °C using precision loading (PL) and a mechanical press. The validation was done on 13 bone-shaped molded silicone specimens containing two ($\perp$ or $\parallel$) or three ($\perp$, $\parallel$ or $\perp\parallel$) layers and $\mathcal{E}_{eff} \leq 40$ kPa. The Young's modulus $\mathcal{E}$ of the molded layers was varied considering two different silicone mixtures (Thinner-Ecoflex (TE) or Thinner-Dragonskin (TD)) at different mass mixing

FIG. 2. Layer stacking with respect to the force direction (black arrows) for three-layer ($n = 3$) composites with height h and length l : a) serial ($\perp$), b) parallel ($\parallel$), c) combined ($\perp\parallel$) and d) sought homogenised composite with effective Young's modulus $\mathcal{E}_{eff}$. Layers Young's moduli $\mathcal{E}_i$ and dimensions (height h_i , length l_i) are indicated with $i = 1 \dots n$. Positions of midway cross-section area measurements $\mathcal{A}_{x_s}$ are indicated (dash-dotted line) for each serial stacked equivalent layer $s = 1 \dots k$ so that in a) $k = 3$, b) $k = 1$ and c) $k = 2$.

ratios $\mathcal{M} = r_T : r_{E(D)}$ with constant $r_{E(D)} = 2$. Silicone mixtures, mixing ratios $\mathcal{M}$ and measured $\mathcal{E}$ commonly used to represent the muscle (Mu), superficial (Su) and epithelium (Ep) layer in ML silicone vocal fold replicas are given in Table I^{10–18}. The measured Young's modulus $\mathcal{E}$ of molded layers varies between 4 kPa and 65 kPa, which corresponds to the range associated with anatomical layers in a normal adult male human vocal fold^{19–25}.

FIG. 3. Molded bone-shaped three-layer reference specimen $A_{0,0}$ with serial stacking ($\perp$) without inclusion (dimensions in mm). The force $\mathcal{F}$ direction during uni-axial tension testing is shown (black arrows).

In the current work, it is aimed to propose a model approach predicting $\mathcal{E}_{eff}$ for multi-layered specimen containing a stiff inclusion. The model outcome is validated against uni-axial tensile test measurements (using precision loading¹⁰) on molded bone-shaped composite specimens. All experiments are done at room temperature 21 ± 2 °C. Specimens are obtained from a three-layer reference specimen (labelled $A_{0,0}$) to which a stiffer silicone inclusion (In) is inserted with constant elasticity $\mathcal{E}^{In}$ given in Table I. The reference specimen without inclusion is depicted in Fig. 3. It consists of three serial ($\perp$) stacked layers with a composition similar to the muscle, superficial and epithelium layer indicated in Table I. The length l_i of each layer $i = 1 \dots n$ in the force ($\mathcal{F}$) direction, with $n = 3$ serial stacked layers, is set so that the volume ratios for the muscle, the superficial and the epithelium layer with respect to the test section's volume match the volume ratios of a three-layer silicone vocal fold replica (MRI-replica^{14,16–18}), *i.e.* 69% (Mu, $i = 1$), 27% (Su, $i = 2$) and 4% (Ep, $i = 3$) respectively. The size, the position or the orientation of the inclusions are varied resulting in complex ML composites. From the ratio $\mathcal{E}^{In}/\mathcal{E}$ in Table I is seen that $\mathcal{E}^{In}$ is about 4.5 up to 75 times greater than $\mathcal{E}$ associated with the layers in $A_{0,0}$. The presence of a stiffer portion somewhere within the vocal fold structure is commonly reported in case of a vocal fold abnormality or pathology^{1,26–28}. Consequently, a validated model is sought for ML composite specimens embedding a stiffer inclusion, for which besides the Young's modulus also the size, position and stacking orientation is varied. This will contribute to the systematic prediction and understanding of the effect of an inclusion on the low-strain elasticity of silicone ML vocal folds replicas mimicking a local vocal fold structural abnormality or pathology. Concretely, six specimen types (resulting in 15 molded specimens) are experimentally assessed in order to complete the model validation with respect to the main inclusion characteristics: position,

TABLE I. Molded layer properties: mixture (Mix) TE (Thinner-Ecoflex) or TD (Thinner-Dragonskin), mass mixing ratio $\mathcal{M}$, measured Young's modulus $\mathcal{E}$, ratio between $\mathcal{E}$ and the value of the inclusion $\mathcal{E}^{In}$.

Molded layer	Mix	$\mathcal{M}$ [-]	$\mathcal{E}$ [kPa]	$\mathcal{E}^{In}/\mathcal{E}$ [-]
Muscle (Mu)	TE	1:2	23	13.0
Superficial (Su)	TE	4:2	4	74.5
Epithelium (Ep)	TD	1:2	65	4.6
Inclusion (In)	TD	0:2	298*	1.0

* $\mathcal{E}$ of the inclusion material is denoted $\mathcal{E}^{In}$.

size (length and height) and shape. This validation is pertinent to the representation with silicone-based vocal fold replicas of vocal fold pathologies characterised by an arbitrary-shaped inclusion of any size embedded somewhere within a single or multiple vocal fold layers. The specimen types with inclusions are presented in section II. Molded specimens are motivated. Next, the measurement of low-strain effective Young's moduli for the molded specimens is detailed in section III. The analytical model approach for the effective Young's modulus of ML composite specimens with an inclusion and serial, parallel or combined stacking is outlined in section IV. A discretisation and geometrical based model approach for arbitrary stacked layers is proposed. Measured and modelled effective Young's moduli for different inclusions are discussed in section V. The conclusion is formulated in section VI.

II. MOLDED SPECIMENS WITH INCLUSION

Fig. 4 schematically illustrates the different positions of beam-shaped inclusions with varying height h_{in} and length l_{in} inserted (striped region) in the test section of the reference specimen $A_{0,0}$ with serial layer stacking ($\perp$) depicted in Fig. 3. All inclusions have constant width $w_{in} = 15$ mm, which matches the width of the test section ($w_{in} = w$ with $w = 15$ mm) as illustrated for two specimens with inclusions in Fig. 5. Thus, the inclusion size is fully characterised by its height h_{in} and its length l_{in} and its position is fully defined by the side views provided in Fig. 4. Six different ML composite specimen types – A, B, C, D, E and F – are considered based on the position and size of the inclusion. Concretely, the size, in terms of height h_{in} and length l_{in} , of experimentally assessed inclusions is summarized in Table II:

- (A) specimens $A_{h_{in}/h}$ (Fig. 4(a)) are obtained for 5 inclusions with constant length $l_{in} = l_2$ and varying height $0.1 \leq h_{in}/h \leq 1$ placed at the side of the superficial layer (Su, $i = 2$) of $A_{0,0}$. In general, these specimens have four layers ($n = 4$) with combined stacking ($\perp||$) as the inclusion in the superficial layer results in two adjacent layers with parallel stacking. This is illustrated for $A_{0,3}$ ($h_{in}/h = 0.3$) in Fig. 5(a). In the extreme case, that the inclusion replaces the superficial layer ($h_{in}/h = 1.0$), a three-layer ($n = 3$) specimen $A_{1,0}$ with serial stacking ($\perp$) is obtained as illustrated in Fig. 5(b).

- (B) specimens $B_{h_{in}/h}$ (Fig. 4(b)) are obtained for 3 of the 5 inclusions with constant length $l_{in} = l_2$ and varying height $0.1 \leq h_{in}/h \leq 0.6$ considered in type A, but now translated (in

FIG. 4. Side view of molded ML composite types with inclusions (striped region) of size h_{in} , l_{in} and constant width $w_{in} = 15$ mm (dimensions in mm). In a,b,c,f) $l_{in} = l_2$. In d,e) $l_{in} < l_2$. The clamping ends are dashed.

the transverse direction) to the center of the superficial layer (Su, $i = 2$) in $A_{0,0}$. Comparing the elasticity of $A_{h_{in}/h}$ and $B_{h_{in}/h}$ allows to assess the influence of the transverse inclusion position (side versus center) within the superficial layer of $A_{0,0}$. These specimens are com-

posed of five layers ($n = 5$) with combined stacking ($\perp\parallel$) as the inclusion in the superficial layer results in three adjacent parallel stacked layers.

- (C) specimen $C_{h_{in}/h}$ (Fig. 4(c)) is obtained using the inclusion with constant length $l_{in} = l_2$ and height $h_{in}/h = 0.8$ positioned in both the superficial and the muscle layer of $A_{0,0}$. This specimen can be considered as a seven-layer ($n = 7$) specimen with combined stacking ($\perp\parallel$) as the inclusion has parallel stacking with respect to the adjacent portions of the superficial layer and the muscle layer.
- (D) specimens $D_{h_{in}/h}^{l_{in}/l_2}$ (Fig. 4(d)) are obtained for 5 inclusions with varying length $0.2 \leq l_{in}/l_2 \leq 0.6$ and varying height $0.3 \leq h_{in}/h \leq 0.8$ placed at the side of the superficial layer and at the interface with the muscle layer. These specimens are considered to have five layers ($n = 5$) with combined stacking ($\perp\parallel$) as the inclusion has a parallel stacking with respect to the adjacent portion of the superficial layer with the same height. Thus, the inclusion length is reduced so that comparing type A and type D allows one to assess the influence of the

FIG. 5. Molded ML bone-shaped specimens (dimensions in mm) with inclusion (In) inserted in the superficial (Su) layer of the reference specimen $A_{0,0}$ depicted in Fig. 3: a) four-layer specimen $A_{0,3}$ with combined stacking ($\perp\parallel$), b) three-layer specimen $A_{1,0}$ with serial stacking ($\perp$). The force $\mathcal{F}$ direction during uni-axial tension testing is shown (black arrows).

inclusion length.

- (E) specimen $E_{h_{in}/h}^{l_{in}/l_2}$ (Fig. 4(e)) is obtained by inclining the inclusion with angle α ($\alpha = 46^\circ$ is experimentally assessed). The inclined inclusion is placed in the superficial layer at the interface with the epithelium layer. Due to the inclination, the stacking in the superficial layer is arbitrary (Arb) and it is neither serial ($\perp$) nor parallel ($\parallel$) with respect to the adjacent superficial layer portion. The equivalent length l_{eq} of the inclusion corresponds to its length in the force direction and $l_{eq} < l_{in}$ due to the inclination.
- (F) specimen $F_{h_{in}/h}$ (Fig. 4(f)) is obtained by bending the inclusion with length $l_{in} = l_2$ and placing it in the muscle layer. As for type E, the stacking orientation within the muscle layer is arbitrary (Arb) since the bent portion of the inclusion is stacked neither serial ($\perp$) nor parallel ($\parallel$) with respect to the adjacent muscle layer portion. Again, the equivalent length l_{eq} of the inclusion corresponds to its length in the force direction and $l_{eq} < l_{in}$ due to the bending.

A total of 15 silicone specimens are molded following the mixture procedure outlined in^{10,29}.

TABLE II. Inclusions geometry (constant width $w_{in} = 15$ mm): h_{in} , l_{in} , ratios h_{in}/h and l_{in}/l_2 with $h \approx 10$ mm and $l_2 \approx 22$ mm. ML molded specimens with these inclusions and their layer stacking: serial ($\perp$), combined ($\perp\parallel$) and arbitrary (Arb). Reference specimen $A_{0.0}$ corresponds to $h_{in}/h = 0$.

Inclusion geometry		Dimension ratios		Specimens & stacking		
h_{in} [mm]	l_{in} [mm]	h_{in}/h [-]	l_{in}/l_2 [-]	($\perp$)	($\perp\parallel$)	Arb
0.0	—	0.0	0.0	$A_{0.0}$		
1.3	22.0	0.1	1.0		$A_{0.1}$ $B_{0.1}$	
2.6	22.0	0.3	1.0		$A_{0.3}$ $B_{0.3}$	$F_{0.3}$
5.8	22.0	0.6	1.0		$A_{0.6}$ $B_{0.6}$	
7.8	22.0	0.8	1.0		$A_{0.8}$ $C_{0.8}$	
10.0	22.0	1.0	1.0	$A_{1.0}$		
2.6	5.0	0.3	0.2		$D_{0.3}^{0.2}$	
7.8	5.0	0.8	0.2		$D_{0.8}^{0.2}$	
4.6	13.4	0.5	0.6		$D_{0.5}^{0.6}$	$E_{0.5}^{0.6}$

The size and position of the inclusion and the resulting ML specimen type (Fig. 4) and associated layer stacking – serial ($\perp$), combined ($\perp\parallel$) or arbitrary (Arb) – is summarised in Table II. Firstly, inclusions are molded using horizontal 3D-printed specimen molds (Stratasys ABS-P430, accuracy 0.33 mm), which are filled with the inclusion silicone mixture indicated in Table I up to each inclusion's height h_{in} whereas their width is determined by the width of the mold so that $w_{in} = 15$ mm. The molded inclusion sheets are then cut to match the desired inclusion length l_{in} . Next, vertical 3D-printed specimen molds are used to build the ML specimens layer-by-layer with the appropriate silicone mixture detailed in Table I. Inclusions are inserted during the molding process so that they are fully embedded. The length of each molded layer is measured with a laser transceiver (Panasonic HL-G112-A-C5, wavelength 655 nm, accuracy 8 μ m) whereas the inclination angle is derived from the spatial position of the inclined mold (accuracy 0.1 mm) using trigonometry. The overall molding accuracy for all 45 molded layers with lengths $l_1 \approx 55$ mm (muscle), $l_2 \approx 22$ mm (superficial) and $l_3 \approx 3$ mm (epithelium) results in a mean and a standard deviation of 0.1 ± 0.8 mm, which is within the molding accuracy of ± 1.5 mm previously reported¹⁰.

III. UNI-AXIAL TENSION TESTS AND $\mathcal{E}_{eff}$ ESTIMATION

A. Uni-axial tension test data on molded specimen

In order to experimentally estimate the linear or low-strain effective Young's moduli $\mathcal{E}_{eff}$ of the 15 molded composite specimens, uni-axial tension tests using precision loading (PL) are performed¹⁰. Firstly, the tested specimen is placed vertically and fixed from one clamping end. Then, a weight of mass m (Vastar 500G X 0.01G, accuracy 0.01 g) is added to the other clamping end in order to control the force increment. The weight is increased at a single rate for each specimen. Overall, *i.e.* considering all specimens, the weight is incremented with 10.9 (6.9) g (mean (and standard deviation)). Total added weight ranges from 46.8 g to 425.9 g resulting in a total applied loading force $\mathcal{F}$ ranging from 0.5 N to 4.2 N. The applied force $\mathcal{F}$ causes an elongation Δl_s of each equivalent serial stacked layer with initial length l_s . The total elongation $\Delta l = \sum_{s=1}^k \Delta l_s$ is then measured at every added weight increment with an accuracy of 0.05 mm where k indicates the number of equivalent serial stacked layers. Total elongation measured for all specimens ranges from 23.0 mm up to 131.0 mm. From these measurements, force-elongation relationship $\mathcal{F}(\Delta l)$

can be obtained. Examples of two force-elongation diagrams for two different specimens $A_{0,0}$ and $A_{0,8}$ are plotted in Fig. 6(a). Additionally, the midway cross-sectional area of each equivalent serial stacked layer $\mathcal{A}_{x_s}$ perpendicular to the forcing direction is measured with an accuracy of 0.02 mm as illustrated in Fig. 2. For each specimen, cross-sectional areas $\mathcal{A}_{x_s}$ are measured at a constant weight increment amounting to 26.7 ± 15.9 g for all specimens, which corresponds to an elongation increment of 10.6 ± 4.5 mm. The specimen cross-sectional area $\mathcal{A}$ is then calculated from the arithmetic mean of its measured cross-sectional areas weighted by their respective lengths as

$$\mathcal{A} = \frac{\sum_{s=1}^k (l_s + \Delta l_s) \mathcal{A}_{x_s}}{l + \Delta l}. \quad (1)$$

A quadratic fit (coefficient of determination $R^2 \geq 99\%$) to the area-elongation data $\mathcal{A}(\Delta l)$ is then used in order to have a continuous approximation of the area-elongation relationship $\mathcal{A}^q(\Delta l)$ for each specimen. An example of resulting data points $\mathcal{A}(\Delta l)$ and their continuous fit $\mathcal{A}^q(\Delta l)$ for two different specimens $A_{0,0}$ and $A_{1,0}$ is plotted in Fig. 6(b).

FIG. 6. Examples of uni-axial tension testing data: a) force-elongation data $\mathcal{F}(\Delta l)$ for specimens $A_{0,0}$ and $A_{0,8}$ with $m \leq 102$ g and $m \leq 231$ g, respectively, b) area-elongation data $\mathcal{A}(\Delta l)$ and quadratic fits $\mathcal{A}^q(\Delta l)$ with $R^2 = 99\%$ (lines) for specimens $A_{0,0}$ and $A_{1,0}$.

B. Experimental $\mathcal{E}_{eff}$ estimation from tensile test data

In order to estimate the linear low-strain effective Young's modulus $\mathcal{E}_{eff}$ for each specimen, stress-strain curves $\sigma_t(\epsilon_t)$ are firstly calculated from the measured instantaneous elongation Δl

and associated force $\mathcal{F}(\Delta l)$ and area $\mathcal{A}^q(\Delta l)$. The true stress σ_t and true strain ε_t are then given as

$$\sigma_t = \frac{\mathcal{F}}{\mathcal{A}^q}, \quad (2)$$

$$\varepsilon_t = \ln \left(\frac{l + \Delta l}{l} \right). \quad (3)$$

The sought $\mathcal{E}_{eff}$ is obtained by fitting the low-strain region of the stress-strain curves to a linear model whose slope equals $\mathcal{E}_{eff}$, as the relationship between stress and low-strain is governed by Hooke's law

$$\mathcal{E}_{eff} = \frac{\sigma_t}{\varepsilon_t}. \quad (4)$$

For each specimen, the low-strain region is determined as the range for which R^2 is maximum, where $R^2 \geq 98\%$ expresses the goodness of fitting a linear model to the stress-strain curve with a lower bound at $\varepsilon_t = 0$. For all specimens tested, the mean (and standard deviation) of the upper bound of the low-strain region is $\varepsilon_t = 0.26$ (0.02), which corresponds to a total elongation of 24.5 (1.7) mm. Examples of fitting a linear model to a low-strain region are plotted in Fig. 7 for three different specimens. Fig. 7(a) shows the different fits, and hence slopes $\mathcal{E}_{eff}$, obtained for two specimens $A_{0,0}$ and $A_{0,6}$, without and with an inclusion, respectively. Fig. 7(b) shows similar fits, and hence slopes $\mathcal{E}_{eff}$, obtained for two specimens $A_{0,6}$ and $B_{0,6}$ with the same inclusion ($h_{in}/h = 0.6$ and $l_{in}/l_2 = 1$) but positioned at either the side ($A_{0,6}$) or the center ($B_{0,6}$) of the superficial layer in $A_{0,0}$.

IV. EFFECTIVE YOUNG'S MODULUS $\widehat{\mathcal{E}}_{eff}$ MODEL

A. Serial, parallel and combined layer stacking

For the composite specimens in Table II and in Fig. 4 with serial ($\perp$) or combined ($\perp||$) stacked layers – *i.e.* of type A (Fig. 4(a)), type B (Fig. 4(b)), type C (Fig. 4(c)) or type D (Fig. 4(d)) – the effective Young's modulus $\widehat{\mathcal{E}}_{eff}$ is modelled considering an equivalent homogeneous composite (*e.g.* in Fig. 2(d)) as outlined in¹⁰ for serial, parallel or combined stacked layers.

For k serial ($\perp$) stacked layers (*e.g.* in Fig. 2(a)), the Reuss hypothesis of homogeneous stress³⁰ between the stress in the equivalent homogeneous composite σ and the stress $\sigma_{i=1\dots k}$ in each layer is applied so that $\sigma = \sigma_{i=1\dots k}$. The effective Young's modulus of the equivalent homogeneous

FIG. 7. Experimental stress-strain curves $\sigma_t(\epsilon_t)$ for three specimens and linear fits (lines) to the low-strain region ($R^2 \geq 98\%$) with slope $\mathcal{E}_{eff}$: a) specimens $A_{0.0}$ and $A_{0.6}$ (low-strain region $\epsilon_t \leq 0.25$ and $\epsilon_t \leq 0.27$), b) specimens $A_{0.6}$ and $B_{0.6}$ (low-strain region $\epsilon_t \leq 0.27$).

composite is then obtained as the harmonic mean of the layers Young's moduli $\mathcal{E}_i$ weighted with their lengths l_i so that

$$\widehat{\mathcal{E}}_{eff}^{\perp} = \frac{\sum_{i=1}^k l_i}{\sum_{i=1}^k \left(\frac{l_i}{\mathcal{E}_i} \right)} \quad (5)$$

with $l = \sum_{i=1}^k l_i$ being the length of the equivalent homogeneous composite. Eq. (5) is independent of the layer order in the serial stack.

For k parallel ($\parallel$) stacked layers (*e.g.* in Fig. 2(b)), the Voigt hypothesis of homogeneous strain³¹ between the strain in the equivalent homogeneous composite ϵ and the strain $\epsilon_{i=1\dots k}$ in each layer is applied so that $\epsilon = \epsilon_{i=1\dots k}$. The effective Young's modulus of the equivalent homogeneous composite is then obtained as the arithmetic mean of the layers Young's moduli $\mathcal{E}_i$ weighted with their heights h_i , so that

$$\widehat{\mathcal{E}}_{eff}^{\parallel} = \frac{\sum_{i=1}^k h_i \mathcal{E}_i}{\sum_{i=1}^k h_i} \quad (6)$$

with $h = \sum_{i=1}^k h_i$ the height of the equivalent homogeneous composite. Eq. (6) is independent of the layer order in the parallel stack.

FIG. 8. Influence of the height ratio $0 \leq h_{in}/h \leq 1$ and length ratio $0 \leq l_{in}/l_2 \leq 1$ of an inclusion in the superficial layer on modelled $\hat{\mathcal{E}}_{eff}$. Values for molded specimens ($\circ$, $\square$) are shown: a) specimens type A (or B) ($\circ$) and type D ($\square$), constant length ratio $l_{in}/l_2 = 1.0$ (horizontal dashed line) and constant height ratio $h_{in}/h = 0.3$ (dashed vertical line), b) detail for these constant length and height ratios.

Specimens with combined ($\perp\parallel$) stacking (*e.g.* in Fig. 2(c)) contain both serial and parallel stacked layers. The effective Young's modulus of the equivalent homogeneous composite $\mathcal{E}_{eff}^{\perp\parallel}$ of specimens with combined stacking is then modelled using first Eq. (6) to homogenise parallel stacked layers followed by applying Eq. (5) to the remaining equivalent stack of serial layers. As layers with high $\mathcal{E}_i$ contribute more to the arithmetic mean (Eq. (5) in $\hat{\mathcal{E}}_{eff}^{\parallel}$) than to the harmonic mean (Eq. (6) in $\hat{\mathcal{E}}_{eff}^{\perp}$) the subsequent steps in the homogenisation procedure of the layer stack (from combined stacking to equivalent serial stack to homogeneous equivalent) can not be permuted.

Inclusions associated with type A, type B, type C and type D in Fig. 4 result in serial ($\perp$) or combined ($\perp\parallel$) stacked layers as summarised in Table II. The effective Young's modulus $\hat{\mathcal{E}}_{eff}$ of the molded specimens from these types are thus modelled as $\hat{\mathcal{E}}_{eff} = \hat{\mathcal{E}}_{eff}^{\perp}$ or $\hat{\mathcal{E}}_{eff} = \hat{\mathcal{E}}_{eff}^{\perp\parallel}$. Following this model approach, shifting the same inclusion from the side to the center of the superficial layer does not affect the model outcome. Thus $\hat{\mathcal{E}}_{eff}$ for type A and type B specimens containing an inclusion with the same height ratio h_{in}/h have equal value. Indeed as $\mathcal{E}_i$ are similar, modelled $\hat{\mathcal{E}}_{eff}$ depend solely on the height ratio $0 \leq h_{in}/h \leq 1$ and length ratio $0 \leq l_{in}/l_2 \leq 1$ of the inclusion and not on its position. The influence of h_{in}/h and l_{in}/l_2 on $\hat{\mathcal{E}}_{eff}$ is illustrated in Fig. 8. Model values for molded specimens are indicated (symbols). Extending the size of the inclusion in the superficial layer increases modelled $\hat{\mathcal{E}}_{eff}$ from $\hat{\mathcal{E}}_{eff} = 10$ kPa for the reference specimen $A_{0,0}$

without inclusion up to about $\hat{\mathcal{E}}_{eff} = 33$ kPa for specimen $A_{1,0}$, for which the inclusion occupies the entire superficial layer. Nevertheless, Fig. 8(a) shows that $\hat{\mathcal{E}}_{eff}$ increases more rapidly with l_{in}/l_2 than with h_{in}/h . In particular, this is the case for $h_{in}/h \geq 0.2$. This is due to the arithmetic mean associated with parallel stacked layers in Eq. (6), which tends to mask low $\mathcal{E}_i$ in favor of the large $\mathcal{E}_i$ of the inclusion (Table I) which is not the case for the harmonic mean associated with serial stacking in Eq. (5). Therefore, it is mainly the length ratio l_{in}/l_2 of the inclusion in the superficial layer what affects the mean value $\hat{\mathcal{E}}_{eff}^{\parallel}$ of the equivalent homogenised superficial layer and hence $\hat{\mathcal{E}}_{eff}$ of the homogenised specimen. This is further illustrated for $l_{in}/l_2 = 1$ in Fig. 8(b) as an increase of h_{in}/h from 0.1 to 1 only increases $\hat{\mathcal{E}}_{eff}$ 27%, from 26 kPa up to 33 kPa, whereas for constant $h_{in}/h > 0.2$ ($h_{in}/h = 0.3$ is plotted) an increase of l_{in}/l_2 from 0.1 to 1 increases $\hat{\mathcal{E}}_{eff}$ with about 270% from 11 kPa up to 30 kPa. Thus the most notable variation of $\hat{\mathcal{E}}_{eff}$ for the molded specimens plotted in Fig. 8(a) is predicted to occur for specimens with different inclusion lengths $l_{in}/l_2 \in \{0.0, 0.2, 0.6, 1\}$ associated with $\hat{\mathcal{E}}_{eff} \in \{10, 12, 17, 30\}$ kPa.

B. Arbitrary layer stacking

Specimens of type E with an inclined inclusion ($\alpha > 0^\circ$) in the superficial layer (Fig. 4(e)) or of type F with a bent inclusion in the muscle layer (Fig. 4(f)) are not serial or parallel stacked with respect to adjacent layers. Instead, their stacking is arbitrary (Arb). However, using the model outlined in section IV A for serial, parallel and combined stacking, two approaches are proposed to model the effective Young's modulus $\hat{\mathcal{E}}_{eff}$ for arbitrary stacked layers either based on spatial discretisation along the force direction (section IV B 1) or on geometrical approximation (section IV B 2). A comparison of modelled values with both approaches for molded specimens with arbitrary stacking $E_{0.5}^{0.6}$ and $F_{0.3}$ is provided in section IV B 3.

1. Discretisation

The model approach outlined in section IV A is applied to arbitrary stacking by discretising the specimen portion containing the inclusion with equivalent length l_{eq} into multiple shorter layers with discretisation step length l_j . The inclusion within each discretised layer is then approximated by a rectangle with height h_j set either to height h_j^U of the largest rectangle enveloped within the inclusion or to height h_j^O of the smallest rectangle enveloping the inclusion in that discretised layer

as schematically depicted in Fig. 9 for specimens of type E and type F. Consequently, each discretised layer with rectangular inclusion approximation is represented as parallel stacked layers so that $\hat{\mathcal{E}}_{eff,j}$ of each homogenised discretised layer is modelled using Eq. (6). The sought $\hat{\mathcal{E}}_{eff,j}$ of each discretised layer with length l_j is thus underestimated (U) as $\hat{\mathcal{E}}_{j,U} = \hat{\mathcal{E}}_{eff}^{\parallel}$ using height h_j^U or overestimated (O) as $\hat{\mathcal{E}}_{j,O} = \hat{\mathcal{E}}_{eff}^{\parallel}$ using height h_j^O . Following this discretisation, the equivalent homogenised specimen portion with inclusion and therefore the equivalent homogenised specimen consists of a stack of serial layers, so that $\hat{\mathcal{E}}_{eff}$ is modelled using Eq. (5) resulting in overestimation $\hat{\mathcal{E}}_O = \hat{\mathcal{E}}_{eff}^{\perp}$ for $\hat{\mathcal{E}}_{j,O}$ or in underestimation $\hat{\mathcal{E}}_U = \hat{\mathcal{E}}_{eff}^{\perp}$ for $\hat{\mathcal{E}}_{j,U}$. The overall difference $\hat{\mathcal{E}}_O - \hat{\mathcal{E}}_U \geq 0$ for the equivalent homogenised specimen portion with inclusion of length l_{eq} (Fig. 10(c) and Fig. 10(d)) as well as for the equivalent homogenised specimen (Fig. 10(e) and Fig. 10(f)) decreases with discretisation step length l_j . Consequently, the sought model value $\hat{\mathcal{E}}_{eff}$ of the homogenised specimen is found for small enough discretisation step length l_j . The influence of the discretisation step length l_j is illustrated in Fig. 10 for molded specimen $E_{0.5}^{0.6}$ containing an inclined inclusion with $l_{eq} = 15.7$ mm (type E in Fig. 4(f)) and for molded specimen $F_{0.3}$ contain-

FIG. 9. Illustration of discretisation along the force direction (step length l_j) for inclusions (striped region) with arbitrary stacking: a) type E (specimen $E_{0.5}^{0.6}$), b) type F (specimen $F_{0.3}$). Rectangular inclusion portion approximations with height h_j^O (light gray shade) and h_j^U (dark gray shade) overestimating (O) and underestimating (U) the inclusion, respectively.

FIG. 10. Effect of discretisation step length l_j for molded ML specimens $E_{0.5}^{0.6}$ and $F_{0.3}$ on a,b) mean rectangle heights $\overline{h_j^O}(l_j)$ and $\overline{h_j^U}(l_j)$, c,d) modelled effective Young's modulus $\hat{\mathcal{E}}_O(l_j, h_j^O)$ and $\hat{\mathcal{E}}_U(l_j, h_j^U)$ and the difference $\hat{\mathcal{E}}_O - \hat{\mathcal{E}}_U$ for the homogenised specimen portion with inclusion of length l_{eq} , e,f) modelled effective Young's modulus $\hat{\mathcal{E}}_O(l_j, h_j^O)$ and $\hat{\mathcal{E}}_U(l_j, h_j^U)$ and the difference $\hat{\mathcal{E}}_O - \hat{\mathcal{E}}_U$ for the homogenised specimen. Values of $\hat{\mathcal{E}}_{O,U}$ for $l_j = 0.2$ mm are indicated (●).

ing a bent inclusion with $l_{eq} = 10.2$ mm (type F in Fig. 4(e)). For these specimens, $\hat{\mathcal{E}}_{eff}$ of both the equivalent homogeneous inclusion layer (with length l_{eq}) and of the specimen is approximated when the discretisation step length $l_j \leq 0.2$ mm as $\hat{\mathcal{E}}_O - \hat{\mathcal{E}}_U \leq 0.5$ kPa for both homogenised specimens. For $l_j = 0.2$ mm, the discretisation of l_{eq} corresponds to splitting l_{eq} into 78 ($E_{0.5}^{0.6}$) and 51 ($F_{0.3}$) equi-length layers. It is seen from Fig. 10(a) and Fig. 10(b) that also the mean of rectangle

heights for all discretised layers $\overline{h_j^O}$ and $\overline{h_j^U}$ converges as l_j decreases.

2. Geometrical approximation

Besides the discretisation approach outlined in section IV B 1, the quasi-analytical model approach outlined in section IV A can be applied when the inclusion of height h_{in} and length l_{in} with arbitrary stacking can be treated as an equivalent beam-shaped inclusion of length l_{eq} and height h_{eq} with serial, parallel or combined layer stacking. The length l_{eq} corresponds to the equivalent length of the inclusion in the force direction as illustrated for type E and type F specimens in Fig. 4 and in Fig. 9. The height h_{eq} of the equivalent inclusion is obtained when imposing area conservation and exploiting the model property that neither serial (Eq. (5)) or parallel (Eq. (6)) layer stacks depend on the stack order so that layers in both stacks can be split or permuted. The area conservation condition is defined as $h_{in}l_{in} = h_{eq}l_{eq}$ so that

$$h_{eq} = \frac{h_{in}l_{in}}{l_{eq}}. \quad (7)$$

The dimensions of the equivalent inclusions h_{eq} and l_{eq} can then be expressed in terms of geometrical parameters such as inclusion dimensions h_{in} and l_{in} .

For specimens of type E, such as $E_{0.5}^{0.6}$ containing an inclined inclusion as depicted in Fig. 4(e), l_{eq} and h_{eq} are expressed as:

$$\begin{aligned} l_{eq} &= \frac{h_{in} + h \cos(\alpha)}{\sin(\alpha)}, \\ h_{eq} &= \frac{l_{in}h_{in} \sin(\alpha)}{h_{in} + h \cos(\alpha)}. \end{aligned} \quad (8)$$

For specimens of type F, such as $F_{0.3}$ containing a bent inclusion as depicted in Fig. 4(f), l_{eq} and h_{eq} are given as:

$$\begin{aligned} l_{eq} &= \frac{l_{in} + h}{2} - \frac{\pi(h - h_{in})}{4}, \\ h_{eq} &= \frac{4h_{in}l_{in}}{2l_{in} + \pi h_{in} - (\pi - 2)h}. \end{aligned} \quad (9)$$

Analytical expressions of the geometry of equivalent inclusions, such as Eq. (8) and Eq. (9), are of interest when considering the influence of geometrical inclusion parameters on $\widehat{\mathcal{E}}_{eff}$. As an

FIG. 11. Modelled $\hat{\mathcal{E}}_{eff}$ as a function of inclination angle α ($0^\circ \leq \alpha \leq 90^\circ$) for specimens with an embedded inclusion of diagonal $\sqrt{l_{in}^2 + h_{in}^2} \leq h$ as schematically depicted (left). Curves $\hat{\mathcal{E}}_{eff}(\alpha)$ are plotted for $l_{in}/l_2 = 0.2$ and either $h_{in}/h = 0.3$ (dashed line) or $h_{in}/h = 0.8$ (full line).

additional example (not molded), expressions of the equivalent length l_{eq} and height h_{eq} of an inclusion with inclination angle α fully embedded in the superficial layer, *i.e.* with a diagonal shorter than the total specimen height h so that $\sqrt{l_{in}^2 + h_{in}^2} \leq h$ as schematically depicted in Fig. 11, are,

$$l_{eq} = l_{in} \cos(\alpha) + h_{in} \sin(\alpha),$$

$$h_{eq} = \frac{l_{in} h_{in}}{l_{in} \cos(\alpha) + h_{in} \sin(\alpha)}. \quad (10)$$

Modelled $\hat{\mathcal{E}}_{eff}$ for two fully embedded inclined inclusions from Table II, with length $l_{in} = 5.0$ mm (or $l_{in}/l_2 = 0.2$) and height $h_{in} \in \{2.6 \text{ mm}, 7.8 \text{ mm}\}$ (or $h_{in}/h \in \{0.3, 0.8\}$), as a function of inclination angle $0^\circ \leq \alpha \leq 90^\circ$ are plotted in Fig. 11. Although the overall tendency of $\hat{\mathcal{E}}_{eff}(\alpha)$ is similar, plotted curves show *e.g.* that angles associated with the minimum and maximum of the curves depend on the height ratio h_{in}/h .

3. $\hat{\mathcal{E}}_{eff}$ of molded specimens with arbitrary stacking

For the molded specimens with arbitrary stacking, both the discretisation approach (in section IV B 1 with step length $l_j = 0.2$ mm) and the geometrical approximation approach (in section IV B 2) result in the sought $\hat{\mathcal{E}}_{eff}$ as their difference is less than 0.9 kPa (or $\leq 4.8\%$) for

Low-strain inclusion

FIG. 12. Low-strain Young's moduli for molded ML specimens: a) measured $\mathcal{E}_{eff}$ (x) and modelled $\hat{\mathcal{E}}_{eff}$ (o). b) difference $\mathcal{E}_{eff} - \hat{\mathcal{E}}_{eff}$ (*) with the overall mean (dashed line) and standard deviation (shaded area).

specimen $E_{0.5}^{0.6}$ and less than 0.1 kPa (or $\leq 1.0\%$) for specimen $F_{0.3}$. Hereafter, $\hat{\mathcal{E}}_{eff}$ obtained with the geometrical approximation are reported for specimens with arbitrary stacked inclusions ($E_{0.5}^{0.6}$ and $F_{0.3}$) since in this case $\hat{\mathcal{E}}_{eff}$ does not depend on the applied discretisation step length l_j .

V. MODELLED $\hat{\mathcal{E}}_{eff}$ VERSUS MEASURED $\mathcal{E}_{eff}$

The model approach outlined in section IV is next used to predict the low-strain $\hat{\mathcal{E}}_{eff}$ of each of the 15 molded specimens. Modelled $\hat{\mathcal{E}}_{eff}$ and measured $\mathcal{E}_{eff}$ values are plotted in Fig. 12(a). The difference $\mathcal{E}_{eff} - \hat{\mathcal{E}}_{eff}$ between measured and modelled values is plotted in Fig. 12(b). The absolute value $|\mathcal{E}_{eff} - \hat{\mathcal{E}}_{eff}|$ for specimens with inclusions corresponds to a model error ranging from 1.0 kPa up to 7.1 kPa, which amounts to 7.4% up to 18.3% of the measured $\mathcal{E}_{eff}$. Overall $\mathcal{E}_{eff} - \hat{\mathcal{E}}_{eff}$ of molded specimens is characterised by a mean (and standard deviation) of 2.7 (2.7) kPa. Compared to the overall model accuracy of 0.0 (2.4) kPa obtained for two and three layer specimens reported in¹⁰, the overall model accuracy for specimens with inclusions is thus shifted due to the non-zero mean to the range from 0.0 kPa up to 5.2 kPa. The positive non-zero mean of 2.7 kPa indicates that the model tends to underestimate measured values. From Fig. 12 is seen that the underestimation (with 2.7 kPa up to 7.1 kPa) is associated with specimens

of type A and type B for which the influence of the inclusion on the Young's modulus is most prominent as their measured effective Young's modulus ($\mathcal{E}_{eff} \geq 30.6$ kPa) is at least tripled compared to $\mathcal{E}_{eff} = 10.2$ kPa, measured for the reference specimen without inclusion $A_{0.0}$. Despite this underestimation, the measured $\mathcal{E}_{eff}$ and modelled $\hat{\mathcal{E}}_{eff}$ values exhibit the same tendencies so that the experimental data validate the model approach for all specimen types (including the ones with arbitrary stacking) and also the model properties discussed in section IV such as:

- Comparing measured $\mathcal{E}_{eff}$ for specimens of type A (inclusion at the side) and type B (inclusion at the center) confirms that the transverse position of the inclusion within the superficial layer does not affect the effective Young's modulus of the specimen as the difference between $\mathcal{E}_{eff}$ measured for $A_{h_{in}/h}$ and $B_{h_{in}/h}$ is less than 1.6 kPa for all three assessed h_{in}/h ratios (0.1, 0.3, 0.6).
- Comparing measured $\mathcal{E}_{eff}$ for specimens of type A (inclusion with $l_{in}/l_2 = 1$) and type D (inclusion with $l_{in}/l_2 = 0.2$) confirms the influence of the inclusion size (length l_{in}/l_2 and height h_{in}/h ratios) on the modelled $\hat{\mathcal{E}}_{eff}$ shown in Fig. 8 so that in particular the length of the inclusion in the force direction (l_{in}/l_2) affects the effective Young's modulus $\mathcal{E}_{eff}$ for these specimens.
- Comparing measured $\mathcal{E}_{eff}$ for specimens $A_{0.8}$ (type A, inclusion in the superficial layer) and $C_{0.8}$ (type C, inclusion in both the superficial and the muscle layer) confirms that the influence of the inclusion on measured $\mathcal{E}_{eff}$ increases with the Young's modulus ratio $\mathcal{E}^{In}/\mathcal{E}$, which for the molded specimens (Table I) reduces from 74.5 in the superficial layer to 4.6 in the muscle layer.

The influence of the ratio $\mathcal{E}^{In}/\mathcal{E}$ is further explored using the model. The Young's modulus of the inclusion $\mathcal{E}^{In}$, and the Young's modulus ratio $\mathcal{E}^{In}/\mathcal{E}$ of the superficial and muscle layer given in Table I, are scaled as $\gamma\mathcal{E}^{In}$. The scalar γ is varied between 0.2 ($\mathcal{E}^{In}$ divided by 5) and 5 ($\mathcal{E}^{In}$ multiplied by 5) so that unscaled model values $\hat{\mathcal{E}}_{eff}$ for $\mathcal{E}^{In}$ are obtained for $\gamma = 1$. In particular, the scaled Young's modulus of the inclusion $\gamma\mathcal{E}^{In}$ ranges from 60 kPa up to 1.49 MPa so that the associated Young's modulus ratios $\mathcal{E}^{In}/\mathcal{E}$ for the superficial layer (ratio from 14.9 up to 372) and for the muscle layer (ratio from 2.6 up to 65) containing the inclusion remain greater than 1 (so an inclusion is embedded in a softer layer). Modelled $\hat{\mathcal{E}}_{eff}(\gamma)$ for all 14 composite types with inclusion are plotted in Fig. 13(a). Values $\hat{\mathcal{E}}_{eff}$ (circles) for $\gamma = 1$ obtained for the molded

TABLE III. Influence of scaling inclusions Young's modulus $\mathcal{E}^{In}$ with scalar $\gamma \in \{0.2, 1, 5\}$ on modelled $\hat{\mathcal{E}}_{eff}$ (in kPa) for all 14 composite types with inclusion and the relative maximum difference $\mathcal{D}$ (in %) with respect to $\hat{\mathcal{E}}_{eff}$ for $\gamma = 1$.

γ	$A_{0.1}$	$B_{0.1}$	$A_{0.3}$	$B_{0.3}$	$A_{0.6}$	$B_{0.6}$	$A_{0.8}$	$A_{1.0}$	$D_{0.3}^{0.2}$	$D_{0.8}^{0.2}$	$C_{0.8}$	$D_{0.5}^{0.6}$	$E_{0.5}^{0.6}$	$F_{0.3}$
0.2	18.4	18.0	22.3	22.6	26.4	26.3	27.8	29.2	11.6	11.7	16.8	16.6	17.3	10.4
1.0	27.3	27.2	30.0	29.8	31.3	31.0	31.7	32.8	11.9	11.8	18.1	18.0	19.3	10.7
5.0	31.7	31.8	32.6	32.2	32.6	32.2	32.7	33.6	12.0	11.8	18.4	18.4	19.8	10.7
$\mathcal{D}$	49	51	34	32	20	19	15	13	3	1	9	10	13	3

specimens are indicated as a reference. For each composite type, $\hat{\mathcal{E}}_{eff}(\gamma)$ increases with γ . Values of $\hat{\mathcal{E}}_{eff}(\gamma)$ for $\gamma \in \{0.2, 1.0, 5.0\}$ and the relative maximum difference (in %) of $\hat{\mathcal{E}}_{eff}(\gamma)$ with

FIG. 13. Effect of scaling the Young's modulus of the inclusion $\mathcal{E}^{In}$ with a scalar $0.2 \leq \gamma \leq 5$ on modelled $\hat{\mathcal{E}}_{eff}$ for all 14 ML composite types with inclusion: a) $\hat{\mathcal{E}}_{eff}(\gamma)$ for increasing $0.2 \leq \gamma \leq 5$ (gray scale) and $\hat{\mathcal{E}}_{eff}$ for $\gamma = 1$ (o), b) overall mean (full line) and standard deviation (std, shaded region) of $\hat{\mathcal{E}}_{eff} - \hat{\mathcal{E}}_{eff}(\gamma)$. As a reference, dotted lines indicate zero difference (horizontal) and $\gamma = 1$ (vertical).

respect to $\hat{\mathcal{E}}_{eff}$ for $\gamma = 1$

$$\mathcal{D} = \frac{\hat{\mathcal{E}}_{eff}(5) - \hat{\mathcal{E}}_{eff}(0.2)}{\hat{\mathcal{E}}_{eff}} \quad (11)$$

are summarised in Table III. The relative maximum difference $\mathcal{D}$ ranges from 1% up to 51%. As observed in Fig. 13(a), the influence of scaling the Young's modulus of the inclusion on $\hat{\mathcal{E}}_{eff}$ depends on the composite type. The overall (for all composite types) mean and standard deviation (std) of the difference $\hat{\mathcal{E}}_{eff} - \hat{\mathcal{E}}_{eff}(\gamma)$ as a function of γ is plotted in Fig. 13(b). Both the mean and std become zero at $\gamma = 1$ for which the difference is zero by definition (since $\hat{\mathcal{E}}_{eff} = \hat{\mathcal{E}}_{eff}(\gamma = 1)$). As $\hat{\mathcal{E}}_{eff}(\gamma)$ increases with γ , the overall mean of $\hat{\mathcal{E}}_{eff} - \hat{\mathcal{E}}_{eff}(\gamma)$ decreases monotonically as γ increases. Thus, the overall mean difference is positive for $\gamma < 1$ and negative for $\gamma > 1$ since $\hat{\mathcal{E}}_{eff} > \hat{\mathcal{E}}_{eff}(\gamma < 1)$ and $\hat{\mathcal{E}}_{eff} < \hat{\mathcal{E}}_{eff}(\gamma > 1)$. The rate at which the overall mean difference decreases slows down with γ . The model suggests that, for the assessed composite types, $\hat{\mathcal{E}}_{eff}$ becomes less sensitive to the exact value of $\mathcal{E}^{In}$ for large $\mathcal{E}^{In}/\mathcal{E}$ ratios. The overall standard deviation of $\hat{\mathcal{E}}_{eff} - \hat{\mathcal{E}}_{eff}(\gamma)$ increases with $|\gamma - 1|$, which expresses that the influence of scaling $\mathcal{E}^{In}$ on $\hat{\mathcal{E}}_{eff}$ differs between composite types.

VI. CONCLUSION

The effective low-strain elastic Young's modulus of silicone ML composites is measured on 15 molded bone-shaped specimens using uni-axial stress testing. A reference specimen is obtained from a three-layer vocal fold anatomical representation of the muscle, superficial and epithelium layers with Young's modulus between 4 kPa and 65 kPa. More complex ML composite types with at least 4 layers are obtained by embedding a stiffer (298 kPa) inclusion with variable size, position and stacking in the superficial or/and muscle layer of the reference specimen. Measured effective Young's moduli of all 15 molded ML composite specimens are compared to modelled values describing equivalent homogenised specimens based on the geometry of its layers, Young's moduli and stacking. For ML specimens consisting solely out of serial and/or parallel stacked layers an analytical model approach is applied, which exploits the hypothesis of homogeneous strain for parallel stacked layers and the hypothesis of homogeneous stress for serial stacked layers. A model approach is proposed for specimens for which the inclusion results in arbitrary stacking, first using spatial discretisation along the force direction and then using area conservation to propose a geometrical approximation for inclined or bent inclusions. Modelled effective Young's

moduli are validated against measured values (from 10 kPa up to 40 kPa) resulting in an overall model accuracy between 0.0 kPa and 5.2 kPa.

The validated quasi-analytical model allows one to explore the influence of its parameters on the predicted effective Young's modulus. Concretely, the influence of the dimensions of an inclusion in the superficial layer and of the influence of scaling the Young's modulus of the inclusion for different ML composite types are discussed. In the first case, the length of the inclusion in the force direction is shown to determine the modelled effective Young's modulus, whereas in the latter case it was found that the rate at which the effective Young's modulus increases with the inclusions slows down, so that eventually for stiff inclusions the exact value of its Young's modulus becomes less important. It is expected, that the proposed model and subsequent model parameter studies are of interest for the *a-priori* characterisation and design of silicone ML composite vocal fold replicas mimicking the complex ML anatomical vocal fold structure without or with inclusion, as in the case of a structural pathology or abnormality. Therefore, current results are important considering the understanding as well as the mitigation of structural pathologies entraining local vocal fold stiffening and their potential impact on human voice production.

ACKNOWLEDGEMENTS

This work was partly supported by Full3DTalkingHead project (ANR-20-CE23-0008-03) and a PhD grant from the French Ministry of Education and Research. Authors acknowledge Cristina Pérez Oms for experimental support. The fourth author would like to acknowledge the support from the project FEMVoQ (Ref: PID2020-120441GB-I00) from the Spanish Ministerio de Ciencia e Innovación.

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